

Cosmological constraints on a dynamical electron massJohn D Barrow¹ and João Magueijo²¹*DAMTP, Centre for Mathematical Sciences, Cambridge University, Wilberforce Road, Cambridge, England CB3 0WA*²*Theoretical Physics, The Blackett Laboratory, Imperial College, Prince Consort Road, London, England SW7 2BZ*

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Motivated by recent astrophysical observations of quasar absorption systems, we formulate a simple theory where the electron to proton mass ratio $\mu = m_e/m_p$ is allowed to vary in spacetime. In such a minimal theory only the electron mass varies, with α and m_p kept constant. We find that changes in μ will be driven by the electronic energy density after the electron mass threshold is crossed. Particle production in this scenario is negligible. The cosmological constraints imposed by recent astronomical observations are very weak, due to the low mass density in electrons. Unlike in similar theories for spacetime variation of the fine structure constant, the observational constraints on variations in μ imposed by the weak equivalence principle are much more stringent constraints than those from quasar spectra. Any time variation in the electron-proton mass ratio must be less than one part in 10^9 since redshifts $z \approx 1$. This is more than 1000 times smaller than current spectroscopic sensitivities can achieve. Astronomically observable variations in the electron-proton must therefore arise directly from effects induced by varying fine structure “constant” or by processes associated with internal proton structure. We also place a new upper bound of 2×10^{-8} on any large-scale spatial variation of μ that is compatible with the isotropy of the microwave background radiation. It is of course possible to bypass these constraints with the addition of a suitable potential to the dynamics.

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I. INTRODUCTION

This work is motivated by recent observational constraints [1–3] on variations of the electron-proton mass ratio $\mu = m_e/m_p$ which take advantage of new high-precision spectra to complement previous investigations of μ in combination with the fine structure constant α and the proton g factor g_p , [4–6]. It is important to relate these constraints to independent investigations on varying α [7–9]. The data reported in Ref. [3] from 21 cm and quasar UV spectra are drawn from absorption spectra in the redshift range $0.24 < z < 2.04$ and lead to an observational constraint on shifts in $x = g_p \alpha^2 \mu$ of $\Delta x/x = 0.35 \pm 1.09 \times 10^{-5}$. As discussed in detail in Ref. [3], using data on variations permitted in α by Murphy *et al.* [7] and Chand *et al.* [8] this results in constraints on variations in μ of $\Delta\mu/\mu = 1.5 \pm 1.1 \times 10^{-5}$ and $\Delta\mu/\mu = 0.5 \pm 1.1 \times 10^{-5}$, respectively, since we expect that the variations in the g factor $\Delta g_p/g_p \approx -0.1\Delta(m_q/\Lambda_{\text{QCD}})/m_q/\Lambda_{\text{QCD}}$ will be negligible [10]. The question we ask in this paper is how should theorists react should a varying μ be observed but not a varying alpha? Could a simple framework accommodate such a situation?

Theoretically there is a wide range of possible connections between α and μ . In previously considered models variations in α lead to variations in the electron mass (via the electron self-energy) and in the proton mass (via the electrostatic energy contained inside a proton). These are model dependent and in general quite complex to work out [11–13]. However it certainly seems the case that a varying alpha entails a varying μ .

However, superficially it also should be possible to have a variation in μ without a variation in α . The idea is simply

to induce such variations directly in the mass parameter for the electron. At least at tree level, such variations would have no effect upon the mass of the proton, or the value of α . This constitutes the simplest varying μ theory and in this paper we investigate some of its cosmological consequences together with the constraints that can be placed by considering the allowed level of violations of the weak equivalence theory. The analogous theory for variation of α developed in Ref. [14] produced observable effects on quasar spectra at high redshift ($z \approx 1$) with accompanying violations of the weak equivalence principle of order 10^{-13} , an order of magnitude below observational bounds. In contrast, if a varying μ exists at the astronomically observable level it results in violations of the weak equivalence principle that are unacceptably large by a factor of more than 10^3 .

II. THE FORMALISM

We shall use metric convention $+, -, -, -$ and set $\hbar = c = 1$ throughout. Consider the standard Dirac Lagrangian

$$\mathcal{L}_\Psi = i\bar{\Psi}\gamma^\mu\partial_\mu\Psi - m\bar{\Psi}\Psi, \quad (1)$$

and let the electron mass be controlled by a “dilaton” field ϕ defined by $m = m_0 \exp\phi$, where m_0 is the current electron mass (so that $\phi = 0$ today). The minimal dynamics for ϕ may be set by the kinetic Lagrangian

$$\mathcal{L}_\phi = \frac{\omega}{2}\partial_\mu\phi\partial^\mu\phi, \quad (2)$$

where ω is a coupling constant. This is the minimal theory; we can add a mass, a potential, or any other complication if required.

From this Lagrangian we obtain a Dirac equation with variable electron (and positron) mass

$$(\iota\gamma^\mu\partial_\mu - m)\Psi = 0. \quad (3)$$

The dynamical equation for the logarithm of the mass ($\phi = \ln(m/m_0)$) is the driven wave equation

$$\partial^2\phi = -\frac{m}{\omega}\bar{\Psi}\Psi. \quad (4)$$

Performing a simple calculation we can derive the following result concerning the driving term: the macroscopically averaged value of $\bar{\Psi}\Psi$ is negligible in the relativistic regime and is given approximately by the electron and positron number density ($n_e + n_p$) in the nonrelativistic regime. Thus, unlike the situation in the simplest theories for variations in α [14,15], variations in μ do occur in the radiation era and start as soon as the Universe cools down below the electron rest-mass threshold.

The calculation is straightforward [16]. We want to compute the macroscopically averaged value of $\bar{\Psi}\Psi$:

$$\langle\bar{\Psi}\Psi\rangle = \frac{1}{V\Delta t} \int d^3x dt \bar{\Psi}\Psi. \quad (5)$$

From the standard expansion

$$\Psi(x) = \int d^3p \sum_{s=1}^2 (a_s(\mathbf{p})u_s(\mathbf{p})e^{-ip\cdot x} + b_s^\dagger(\mathbf{p})v_s(\mathbf{p})e^{ip\cdot x}), \quad (6)$$

we arrive at

$$\begin{aligned} \int d^3x \bar{\Psi}\Psi &= \int d^3p \frac{m}{E} \sum_r (a_r^\dagger(\mathbf{p})a_r(\mathbf{p}) - b_r(\mathbf{p})b_r^\dagger(\mathbf{p})) \\ &+ \sum_{rs} (a_r^\dagger(\mathbf{p})b_s^\dagger(-\mathbf{p})\bar{u}_r(\mathbf{p})v_s(-\mathbf{p}))e^{2iEt} \\ &+ \sum_{rs} (a_r(\mathbf{p})b_s(-\mathbf{p})\bar{v}_r(\mathbf{p})u_s(-\mathbf{p}))e^{-2iEt}. \end{aligned} \quad (7)$$

The last two terms average to zero when integrated in time. The remaining terms, upon normal ordering, lead to

$$\langle\bar{\Psi}\Psi\rangle = \frac{1}{V} \int d^3p \frac{m}{E(p)} (N_e(\mathbf{p}) + N_p(\mathbf{p})). \quad (8)$$

Thus, in the relativistic regime $E \gg m$ and the driving term is negligible. In the nonrelativistic regime, on the other hand, (5) reduces to $n_e + n_p$.

This theory conserves lepton number. The term $e^\phi\bar{\Psi}\Psi$ leads to Feynman diagrams involving ϕ and the Ψ , but if electrons are created then an equal number of positrons also must be created. In general, a varying mass leads to particle production (see [17]; and compare to the case of pair production in an electric field [16]). In the nonrelativistic regime, however, this is negligible as shown below.

III. COSMOLOGICAL SOLUTIONS

We assume that the variations in ϕ that drive variations in μ are small in the sense that they do not produce significant contributions to the Friedmann-like equation governing the dynamics of the expansion scale factor of the Universe, $a(t)$. This assumption will be confirmed by the results obtained. In a cosmological setting the equation for ϕ is

$$\ddot{\phi} + 3\frac{\dot{a}}{a}\dot{\phi} \approx -\frac{m}{\omega}(n_e + n_p)\Theta(m - T), \quad (9)$$

where Θ is the theta function and $a(t)$ will be the scale factor for a spatially flat Friedmann universe containing radiation, matter, and quintessence. The limiting case of no mass variations corresponds to the $\omega \rightarrow \infty$ limit. We first try to constrain this parameter cosmologically. We write: $n_e \approx fn_b = f\eta s$, where n_L , n_b , and s are the lepton number, baryon number, and entropy densities; $f \approx 0.95$ under standard assumptions; and $\eta = n_b/s$ must be of order 10^{-10} for big bang nucleosynthesis to produce the observed light element abundances.

Lepton-number (and charge) conservation in this theory implies that $n_L \propto 1/a^3$. This imposes the requirement that

$$\dot{n}_e + 3\frac{\dot{a}}{a}n_e = P(t) = \dot{n}_p + 3\frac{\dot{a}}{a}n_p, \quad (10)$$

where $P(t)$ is the particle production rate density. Energy conservation in turn requires that

$$\dot{\rho}_\phi + 3\frac{\dot{a}}{a}(\rho_\phi + p_\phi) = -mn_L\dot{\phi} = -\dot{\rho}_L - 3\frac{\dot{a}}{a}\rho_L, \quad (11)$$

where $p_\phi = \rho_\phi = \omega\dot{\phi}^2/2$, and (in the nonrelativistic regime) $\rho_L = m(n_e + n_p)$, $p_L \approx 0$. Combining energy and lepton-number conservation we therefore conclude that $P(t) = 0$ and we have $n_e = B/a^3$ and $n_p = A/a^3$; A, B constants. Although the electron number is conserved the varying mass induces a corresponding variation in the electron's energy density. This is supplied (or absorbed) by the ϕ field so that the overall energy is conserved.

We now consider a number of approximate solutions with cosmological implications. In general, we can write (9) as

$$(\dot{\phi}a^3) = -M \exp[\phi], \quad (12)$$

with

$$M = \frac{a^3 n_L m_0}{\omega} \approx \frac{\rho_{e0} a_0^3}{\omega}. \quad (13)$$

If the mass variations are small, as observations demand, we may set $e^\phi \approx 1$ in the right-hand side of (12) and integrate to obtain

$$\dot{\phi}a^3 = -Mt + \tilde{A}, \quad (14)$$

where \tilde{A} is an integration constant. In the dust era, we take

$a \propto t^{2/3}$ and assume that the driven solution dominates [18]; hence $\tilde{A} \approx 0$, and so $\phi = -Mt_0^2 \ln(t/t_0)$, that is

$$\phi = \frac{3}{2}Mt_0^2 \ln(1+z), \quad (15)$$

where t_0 is the present time and z is the redshift corresponding to comoving proper time t . But

$$Mt_0^2 = \frac{\Omega_e}{\omega} \rho_c t_0^2 = \frac{\Omega_b}{\omega} \frac{m_e}{m_p} \left(1 - \frac{f_{\text{He}}}{2}\right) \frac{1}{6\pi G}, \quad (16)$$

where f_{He} is the helium-4 number fraction, and so the shift in μ between redshift z and the present is given by

$$\frac{\Delta\mu}{\mu} \approx \phi = \frac{1}{4\pi G\omega} \Omega_b \mu \left(1 - \frac{f_{\text{He}}}{2}\right) \ln(1+z). \quad (17)$$

The observational constraints from redshifts of order 1 [1–3] therefore convert into a bound on ω the one free parameter defining the theory and

$$\frac{|\Delta\mu|}{\mu} < 10^{-5} \rightarrow G|\omega| > 0.2. \quad (18)$$

This is a very poor constraint, a fact to be expected because the electron density is so low. It means that ω can be extremely “small”—slightly smaller even than the minimal scale of $E_{\text{Planck}}^2 = G^{-1}$. Recall that the no- m_e -variation model has $\omega = \infty$ and current astronomical observations at high redshift barely constrain this maximal variation case. Similar bounds on the combination $\alpha^2\mu$ arise by introducing an additional scalar field to carry variations in α as in [14] and the bounds on the time variation of this combination are dominated by the constraints on changes in μ if the two scalar fields are coupled only by gravity.

In the above derivation we have neglected the acceleration of the Universe. This will weaken the bound further, as acceleration stabilizes μ . If we solve (12) in a flat background universe containing dust and a cosmological constant, Λ , and scale factor $a^3 \propto \sinh^2[t\sqrt{3\Lambda}/2]$ then the driven solution becomes

$$\phi = \phi_* + \frac{M\Omega_{b0}}{\Omega_{\Lambda 0}\lambda} t \coth \lambda t - \frac{M\Omega_{b0}}{\Omega_{\Lambda 0}\lambda^2} \ln \sinh \lambda t, \quad (19)$$

where $\lambda = \sqrt{3\Lambda}/2$ and as $t \rightarrow \infty$ we have $\phi \rightarrow$ constant which we neglect as small. Similar solutions have been found in [19,20]. To a reasonable approximation ϕ evolves as (15) until the epoch $z_\Lambda = \sqrt{[3]7/3} - 1 \approx 0.3$ when the cosmological constant begins to dominate the expansion and is approximately constant thereafter. So the bound (18) will be weakened by a factor of order 0.3. The variation of ϕ is turned off in a similar way whenever the dynamics are dominated by a fluid with equation of state satisfying $\rho + 3p < 0$ (such as quintessence) and this includes the case of negative curvature ($a = t$) also. We see that our distance from the future asymptotic value, $\phi(t_0) - \phi_*$, is also small.

In the radiation epoch, where $a = t^{1/2}$, we find a different solution of (12): $\dot{\phi}a^3 = -M(t - t_m)$, where $t_m \approx 100s$ is the time when the electron mass threshold is crossed at $T \sim m_e$. This integrates to give

$$\phi = C - 2Mt_0^2 \frac{a}{a_0} \left(1 + \frac{t_m}{t}\right), \quad (20)$$

where the constant C should be chosen so that at equality $\phi = \phi_{\text{eq}}$ matches the result obtained from the dust solution (17). As $z_m \gg z_{\text{eq}}$ we have

$$\phi = \phi_{\text{eq}} + 2Mt_0^2 \left(\frac{1}{z_{\text{eq}}} - \frac{1}{z}\right) \quad (21)$$

(there is a small correction if $z \approx z_m \approx 10^9$). We see that most of the variation in μ in the radiation epoch happens near the epoch of matter-radiation equality, so that the redshift of the electron mass threshold, z_m , is numerically irrelevant.

We can now consider the total variation in μ as we go up in redshift deep into the radiation epoch, $z \rightarrow \infty$. This is given by

$$\frac{\Delta\mu}{\mu} = \phi_\infty = Mt_0^2 \left(\frac{3}{2} \ln z_{\text{eq}} + \frac{2}{z_{\text{eq}}}\right). \quad (22)$$

The first term contains the variation that happened in the matter epoch; the second adds on the variation that happened in the radiation epoch. The first term dominates, and so most of the variation in the electron mass in this theory occur in the matter epoch. Thus we do not expect detailed consideration of primordial nucleosynthesis at $z \sim 10^9 - 10^{10}$ to constrain it further. Note that our approximation was based on neglecting the contribution of the scalar-field energy density to the Friedmann equation. This requires $\omega\dot{\phi}^2/t^{-2}$ to be bounded at early times and this holds for our solution (22).

This leaves us with a worrying possibility. In the above discussion we have assumed that before $t_m \sim m_e^{-1}$ there are no variations in m_e since the driving term is absent. But this need not necessarily be the case. We could add to the driven solution any solution to the free equation, $\dot{\phi}a^3 = -A$, where A is an arbitrary constant. In the radiation-dominated case [21] this would add to ϕ solution $\phi_{\text{free}} = -2A/t^{1/2}$. As long as A is chosen so that ϕ variations are suitably small at nucleosynthesis, this component cannot be ruled out. Its effect, however, would be dramatic. It would imply that the electron becomes massless (or infinitely massive) at the big bang. However, if this solution mode is included our approximation of neglecting the back-reaction of the ϕ motion on the Friedmannian dynamics of the Universe begins to fail. The kinetic term for this mode is of order $\omega\dot{\phi}^2 \approx \omega A^2 a^{-6}$ and will dominate the radiation plasma ($\rho \propto a^{-4}$) in the Friedmann equation as $t \rightarrow 0$ —leading to scalar-dominated background expansion with $a = t^{1/3}$. The resulting exact general solution of

(12) is

$$m = \exp[\phi] = -\frac{2C^2}{Mt} \left(\frac{t}{T}\right)^{\pm C} \frac{1}{[1 - (t/T)^{\pm C}]^2}$$

with C constant. With $C = 2n + 1$, $n \in \mathbb{Z}^+$, $T = -\beta < 0$, we have $m \propto t^{2n} \rightarrow 0$ as $t \rightarrow 0$.

IV. THE WEAK EQUIVALENCE PRINCIPLE

Stronger bounds on this theory are possible if we consider the observational constraints on any local violations of the weak equivalence principle (WEP). All varying constant theories imply the existence of a ‘‘fifth force’’ but this need not violate the equivalence principle. Varying e theories, like BSBM, which match the level of variations in α consistent with quasar data of Refs. [7] predict WEP violations within 1 order of magnitude of current constraints [14], and there is a real prospect of testing this prediction with the first generation of space-based tests of the WEP. In the varying- μ theory introduced here the situation turns out to be quite different.

Defining $\zeta_t = \rho_e/\rho$, test particles may be represented by

$$\mathcal{L}(y) = -\int d\tau m((1 - \zeta_t) + \zeta_t e^\phi) [-g_{\mu\nu} \dot{x}^\mu \dot{x}^\nu]^{1/2} \times \frac{\delta(x - y)}{\sqrt{-g}}, \quad (23)$$

where overdots are derivatives with respect to the proper time τ . This leads to equations of motion:

$$\ddot{x}^\mu + \Gamma_{\alpha\beta}^\mu \dot{x}^\alpha \dot{x}^\beta - \frac{\zeta_t e^\phi}{(1 - \zeta_t) + \zeta_t e^\phi} \partial^\mu \phi = 0, \quad (24)$$

which in the nonrelativistic limit (with $\zeta_t \ll 1$) reduce to

$$\frac{d^2 x^i}{dt^2} = -\nabla_i \Phi + \zeta_t \nabla_i \phi, \quad (25)$$

where Φ is the gravitational potential. Linearizing (4) around a spherically symmetric body with mass M_s and $\zeta = \zeta_s$ we have

$$\phi = -\frac{\zeta_s}{4\pi\omega} \frac{GM_s}{r}, \quad (26)$$

so that we predict an anomalous acceleration equal to

$$a = \frac{GM_s}{r^2} \left(1 + \frac{\zeta_s \zeta_t}{G\omega\pi}\right). \quad (27)$$

Violations of the WEP occur because ζ_t is substance dependent. For two test bodies with ζ_1 and ζ_2 the Eötvös parameter is

$$\eta \equiv \frac{2|a_1 - a_2|}{a_1 + a_2} = \frac{\zeta_s |\zeta_1 - \zeta_2|}{G\omega\pi}. \quad (28)$$

Since $\zeta_s \approx |\zeta_1 - \zeta_2| = \mathcal{O}(10^{-4})$ then in order to produce

$\eta < \mathcal{O}(10^{-12})$ we need $G\omega > 10^3$. Similar bounds are obtained by considering the motion of the Earth-Moon system due to their compositional differences, [22,23] Under this constraint on ω the largest tolerated variations in μ at redshifts of order 1 from Eq. (17) are of the order

$$|\Delta\mu|/\mu < 10^{-9}. \quad (29)$$

It is easy to see why the varying- μ theory is so much more strongly constrained by tests of the WEP than the BSBM varying- α theory [14,15]. The latter is a varying- e theory, driven by $E^2 - B^2$. It is possible that the dark matter is dominated by E^2 or B^2 field: this enhances the cosmological variations permitting variations in alpha at $z \approx 1$ of order 10^{-5} without conflict with the WEP, as explained in [14]. A similar structure for the theory proposed in this paper would require the dark matter to be dominated by electrons, that is $\zeta_D \approx 1$. This is not a possibility. Still one should bear in mind this possible loophole, as well as the one raised in [24].

V. COSMOLOGICAL INHOMOGENEITIES

Another astronomical consideration is that of the non-uniform matter distribution in the Universe. In line with usual practice we have considered the evolution of μ in a spatially homogeneous universe. Since the real Universe exhibits significant inhomogeneity in the matter distribution on sub-Gpc scales we expect there will be inhomogeneity in μ also. From Eqs. (21) and (22) we see that the fact that the dominant contributions to variations in μ come from changes at low redshifts in the dust-dominated era. The largest effects arising from the effects of growing density inhomogeneities will arise at these times. The leading-order effect in (22) will be inhomogeneity in M which arises directly from inhomogeneity in ρ_e . Once protoclusters separate out from the expansion of the Universe and virialize the time evolution of μ will cease inside them while μ continues to decrease logarithmically in the background universe until the expansion starts to accelerate [25]. This will create small spatial variation in the value of μ between clusters and the background universe and we can solve (14) to determine the evolution of ϕ and μ inside the over-dense region. If we model a growing spherical density inhomogeneity in the dust era by a closed Friedmann universe [25,26] with greater than average density we obtain $\phi = -Mt_0^2 [\ln(t/t_0) + \Gamma(\vec{x})(t/t_0)^{2/3}]$ where $\Gamma(\vec{x})$ is a positive function of the space coordinates that fixes the amplitude of the growing perturbation mode in the linear approximation. Hence, substituting for redshift

$$\phi = \frac{3}{2} Mt_0^2 \ln(1+z) - \frac{\Gamma Mt_0^2}{1+z}.$$

To leading order, the spatial inhomogeneity in μ is

$$\frac{\delta\mu}{\mu} = \frac{\mu - \bar{\mu}}{\bar{\mu}} \approx -Mt_0^2 \Gamma(\vec{x}) \left(\frac{t}{t_0}\right)^{2/3} = \frac{-Mt_0^2 \Gamma(\vec{x})}{1+z},$$

where $\bar{\mu}$ is the spatially uniform value of μ in the background (where $\Gamma = 0$). Therefore we expect inhomogeneities in μ to grow in time at the same rate as inhomogeneities in the matter density which drive them. There is therefore observational motivation to search for spatial variations in μ and in $\dot{\mu}$. Since the dominant evolution of the electron mass occurs in the dust-dominated era there appears to be little effect on primordial nucleosynthesis. There will be effects on the Thomson scattering of the microwave photons (as $\sigma_T \sim m_e^{-2}$) but they are also expected to be unobservably small.

It is possible to place a strong bound on the magnitude of any spatial variations in μ by noting that inhomogeneity in μ is driven by inhomogeneity in the density ρ_e and the latter will lead to metric potential perturbations on the scale of the inhomogeneity which will produce temperature anisotropies in the microwave background on large angular scales [27]. If we assume that the inhomogeneity in the electron density is approximately proportional to that in the matter distribution, so

$$\frac{\delta\rho_e}{\rho_e} = \beta \frac{\delta\rho_m}{\rho_m},$$

then the spatial inhomogeneity in the electron-proton mass ratio μ is

$$\frac{\delta\mu}{\mu} \sim \frac{0.3}{G\omega} \frac{\delta\rho_e}{\rho_e} \left(\frac{L}{t}\right)^2 \sim \frac{0.9\beta}{G\omega} \frac{\Delta T}{T} \sim \frac{2 \times 10^{-5} \beta}{G\omega} < 2 \times 10^{-8}.$$

Here, we have used the WEP lower bound on $G\omega$, (29), to improve on the cosmological bound given in [27] on spatial inhomogeneity in the electron-proton mass ratio on large cosmological scales that have not undergone gravitational binding and nonlinear evolution.

VI. CONCLUSIONS

In summary, we have formulated the first theory which describes the cosmological evolution of the electron-proton mass ratio, μ . Although “limits” exist on the

time variation this quantity in the literature and the consequences of altering its constant value are well known [28–32]. There has been no self-consistent theoretical description of the time-evolution of μ that is consistent with the conservation of energy and momentum and cosmology. In this paper we have presented the first (and simplest) such theory. It can be related to dilaton couplings beyond tree-level found in string theory [33], to the covariant varying- c model [34], or to the standard model with varying parameters (as long as the variations envisaged here are due to a dynamic Yukawa coupling rather than a shift in the Higgs VEV [35]).

The theory has appealing features which result in it being relatively insensitive to uncertainties in defining parameters and we have found that it is remarkably weakly constrained by the high-precision astronomical data provided by observations of quasar spectra. However, unlike the case of theories of varying α [14], the theory is significantly more strongly constrained by the data supporting the weak equivalence principle. The deviations from composition-independent free fall under gravity place constraints upon varying μ which are about 1000 times stronger than the current quasar bounds and indicate that direct effects from varying μ on atomic energy levels (in particular to the observable combination $\alpha^2\mu$) at redshifts $O(1)$ will be unobservably small [at most $O(10^{-9})$] if the variations in μ are driven by scalar-field variation of the electron mass as assumed here. Thus if observational evidence arises for cosmological time variations in μ we expect complementary variations in α to exist from which the μ variations arise from radiative corrections or electrostatic effects internal to the proton.

It is of course possible to bypass these constraints by adding to the dynamics of our field ϕ a suitable potential; however such theory would no longer be “minimal” in the sense defined above and would have to be very finely tuned.

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