

Holographic dark energy reexamined

Yungui Gong*

*College of Electronic Engineering, Chongqing University of Posts and Telecommunications, Chongqing 400065, China*Bin Wang[†]*Department of Physics, Fudan University, Shanghai 200433, China*

Yuan-Zhong Zhang

CCAST (World Laboratory), P.O. Box 8730, Beijing 100080, China and Institute of Theoretical Physics, Chinese Academy of Sciences, P.O. Box 2735, Beijing 100080, China

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We have reexamined the holographic dark energy model by considering the spatial curvature. We have refined the model parameter and observed that the holographic dark energy model does not behave as phantom model. Comparing the holographic dark energy model to the supernova observation alone, we found that the closed Universe is favored. Combining with the Wilkinson microwave anisotropy probe (WMAP) data, we obtained the reasonable value of the spatial curvature of our Universe.

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I. INTRODUCTION

The total entropy of matter inside a black hole cannot be greater than the Bekenstein-Hawking entropy, which is one quarter of the area of the event horizon of the black hole measured in Planck unit. In view of the example of black hole entropy, Bekenstein proposed a universal entropy bound $S \leq 2\pi ER$ for a weak self-gravitating physical system with total energy E and size R in 1981 [1]. Later 't Hooft and Susskind proposed an influential holographic principle, relating the maximum number of degrees of freedom in a volume to its boundary surface area [2]. The extension of the holographic principle to the cosmological setting was first addressed by Fischler and Susskind (FS) [3]. Subsequently, various modifications of the FS version of the holographic principle was proposed [4]. The idea of the holographic principle is viewed as a real conceptual change in our thinking about gravity [5]. It has appeared in many examples of applying the holographic principle to study cosmology, such as understanding the possible value of the cosmological constant [6,7], selecting a physically acceptable model in inhomogeneous cosmology [8] and discussing upper limits on the number of e-foldings in inflation [9]. It is of great interest to generalize the application of holography to a much broader class of situations, especially to cosmology.

The type Ia supernova (SN Ia) observations suggest that the Universe is dominated by dark energy with negative pressure which provides the dynamical mechanism of the accelerating expansion of the Universe [10,11]. The simplest candidate of dark energy is the cosmological constant. However the unusual small value of the cosmological constant is a big challenge to theoretical physicists.

Whether holography can shed us some light in understanding the profound puzzle posed by the dark energy is a question we want to ask. Motivated by the assumption that for any state in the Hilbert space with energy E , the corresponding Schwarzschild radius $R_s \sim E$ is less than the infrared (IR) cutoff L [7], a relationship between the ultraviolet (UV) cutoff and the infrared cutoff is derived, i.e., $8\pi GL^3 \rho_D/3 \sim L$ [7]. We can express the holographic dark energy density as

$$\rho_D = \frac{3c^2 d^2}{8\pi GL^2}, \quad (1)$$

where c is the speed of light and d is a constant of the order of unity. This UV-IR relationship was also obtained by Padmanabhan by arguing that the cosmological constant is due to the vacuum fluctuation of energy density. Hsu found that the holographic dark energy model based on the Hubble scale as the IR cutoff will not give an accelerating universe [12]. In [13], Li showed that when choosing the particle horizon as the IR cutoff, an accelerating universe will not be produced either. However, by relating the IR cutoff to an event horizon, it was found that the holographic dark energy model can accommodate the accelerating universe [13,14]. The model in the flat universe was found inconsistent with current observations [15]. Here we would like to point out that the form $\rho_D \sim H^2$ also works for dark energy model building. For example, the model $\rho_D = \rho_\Lambda + 3c^2 d^2 H^2 / (8\pi G)$ with ρ_Λ a constant derived from the renormalization group models of the cosmological constant can explain the accelerating expansion of the Universe [16]. Ito also discovered a viable holographic dark energy model by using the Hubble scale as the IR cutoff with the use of nonminimal coupling to scalar field [17]. More recently, a dark energy model $\rho_D \sim H^2$ with an interaction between the dark energy and dark matter was proposed to explain the accelerating expansion [18]. The

*Electronic address: gongyg@cqupt.edu.cn[†]Electronic address: wangb@fudan.edu.cn

holographic dark energy model in the framework of Brans-Dicke theory was discussed in [19]. Some speculations about the deep reasons of the holographic dark energy were considered by several authors [20]. The holographic principle was also used to constrain dark energy models in [21]. In this paper, we reexamine the holographic dark energy model proposed in [13]. We give constraints on this model from both the theoretical argument and the observational data. Including the spatial curvature, we will find that the closed universe is marginally favored. This result agrees to the cosmic microwave background (CMB) anisotropy experiments [22–24] and recent supernova investigations [25].

II. HOLOGRAPHIC DARK ENERGY MODEL WITH CURVATURE

We start from the homogeneous and isotropic Friedmann-Robertson-Walker (FRW) space-time metric

$$ds^2 = -c^2 dt^2 + a^2(t) \left[\frac{dr^2}{1 - kr^2} + r^2 d\Omega \right]. \quad (2)$$

If a light is emitted from a point r_1 at time t_1 , it will arrive at the origin at time t_0 . The light follows the null geodesics, so we have

$$\int_{t_1}^{t_0} \frac{cdt}{a(t)} = \int_0^{r_1} \frac{dr}{\sqrt{1 - kr^2}} \equiv f(r_1), \quad (3)$$

where

$$f(r_1) = \frac{1}{\sqrt{|k|}} \text{sinn}^{-1}(\sqrt{|k|r_1}) \\ = \begin{cases} \sin^{-1}(\sqrt{|k|r_1})/\sqrt{|k|}, & k = 1, \\ r_1, & k = 0, \\ \sinh^{-1}(\sqrt{|k|r_1})/\sqrt{|k|}, & k = -1. \end{cases}$$

With both an ordinary pressureless dust matter and the holographic dark energy as sources, the Friedmann equations are

$$H^2 + \frac{kc^2}{a^2} = \frac{8\pi G}{3}(\rho_m + \rho_r + \rho_D), \quad (4)$$

$$\dot{\rho}_D + 3H(\rho_D + p_D) = 0, \quad (5)$$

where the Hubble parameter $H = \dot{a}/a$, the matter density $\rho_m = \rho_{m0}(1/a)^3$, the radiation density $\rho_r = \rho_{r0}(1/a)^4$, the dot means derivative with respect to time and the subscript 0 means the value of the variable at present time and $a_0 = 1$ is set.

Now as done in [13] we choose the event horizon as the IR cutoff, where

$$R_{eh}(t) = a(t) \int_t^\infty \frac{cdt}{a(t)} = a(t) \int_{a(t)}^\infty \frac{cd\bar{a}}{\bar{a}^2 H} = \int_0^r \frac{d\bar{r}}{\sqrt{1 - k\bar{r}^2}}, \quad (6)$$

$$L = a(t)r = \frac{a(t) \text{sinn}[\sqrt{|k|R_{eh}(t)/a(t)}]}{\sqrt{|k|}}. \quad (7)$$

Apparently, we recover $L = R_{eh}$ for a spatially flat universe.

Let us rewrite Eq. (4) as

$$\Omega_m + \Omega_r + \Omega_D = 1 + \Omega_k, \quad (8)$$

where $\Omega_m = \rho_m/\rho_{cr} = \Omega_{m0}H_0^2/(H^2a^3)$, $\Omega_r = \rho_r/\rho_{cr} = \Omega_{r0}H_0^2/(H^2a^4)$, $\Omega_D = d^2c^2/(L^2H^2)$ and $\Omega_k = kc^2/(a^2H^2) = \Omega_{k0}H_0^2/(a^2H^2)$. Since

$$\frac{\Omega_k}{\Omega_m} = a \frac{\Omega_{k0}}{\Omega_{m0}} = a\gamma,$$

where $\gamma = \Omega_{k0}/\Omega_{m0}$, and

$$\frac{\Omega_r}{\Omega_m} = \frac{\Omega_{r0}}{a\Omega_{m0}} = \frac{\beta}{a},$$

where $\beta = \Omega_{r0}/\Omega_{m0} = 1/(1 + z_{eq})$ and the matter radiation equality redshift $z_{eq} = 3233$ [26], we have

$$\Omega_m = \frac{\Omega_{m0}H_0^2}{H^2a^3} = \frac{a(1 - \Omega_D)}{\beta + a - a^2\gamma}. \quad (9)$$

From the above equation, we get

$$\frac{1}{aH} = \frac{a}{H_0} \sqrt{\frac{1 - \Omega_D}{\Omega_{m0}(\beta + a - a^2\gamma)}}. \quad (10)$$

Combining Eqs. (7) and (10) and using the definition of Ω_D , we obtain

$$\sqrt{|k|} \frac{R_{eh}}{a} = \text{sinn}^{-1} \left[d\sqrt{|\gamma|} \sqrt{\frac{a^2(1 - \Omega_D)}{\Omega_D(\beta + a - a^2\gamma)}} \right] \\ = \text{sinn}^{-1}(d\sqrt{|\Omega_k|/\Omega_D}). \quad (11)$$

If $\Omega_k > 0$, then we require $d \leq \sqrt{\Omega_D/\Omega_k}$.

By using Eqs. (1) and (5)–(7), and (8), we get the dark energy equation of state

$$w_D = -\frac{1}{3} \frac{d \ln \rho_D}{d \ln a} - 1 \\ = -\frac{1}{3} \left[1 + \frac{2}{d} \sqrt{\Omega_D} \cosn(\sqrt{|k|R_{eh}/a}) \right] \\ = -\frac{1}{3} \left[1 + \frac{2}{d} \sqrt{\Omega_D - d^2\Omega_k} \right], \quad (12)$$

where

$$\frac{1}{\sqrt{|k|}} \cosn(\sqrt{|k|x}) = \begin{cases} \cos(x), & k = 1, \\ 1, & k = 0, \\ \cosh(x), & k = -1. \end{cases}$$

It is obvious that $w_D \leq -1/3$, so we can have an accelerating universe.

Taking derivative with respect to a on both sides of Eq. (11) and using the redshift $z = 1/a - 1$ as the variable, we get the following differential equation by using Eqs. (6) and (10)

$$\begin{aligned} \frac{d\Omega_D}{dz} &= -\frac{2\Omega_D^{3/2}(1-\Omega_D)}{d(1+z)} \\ &\times \sqrt{1 - \frac{d^2\gamma(1-\Omega_D)}{\Omega_D[\beta(1+z)^2 + 1 + z - \gamma]}} \\ &- \frac{\Omega_D(1-\Omega_D)[1 + 2\beta(1+z)]}{\beta(1+z)^2 + 1 + z - \gamma}. \end{aligned} \quad (13)$$

With this expression, we can understand the evolution behavior of the dark energy.

Let us find the constraints on the parameter d in the holographic dark energy model. The entropy of the whole system is described by $S = \pi M_p^2 L^2$. To satisfy the second law of thermodynamics, we require that

$$\begin{aligned} \dot{L} &= LH - c \cosn[\sqrt{|k|}R_{eh}(t)/a(t)] \\ &= c \left(\frac{d}{\sqrt{\Omega_D}} - \sqrt{1 - \frac{d^2\gamma(1-\Omega_D)}{\Omega_D[\beta(1+z)^2 + 1 + z - \gamma]}} \right) \geq 0. \end{aligned} \quad (14)$$

Thus

$$d^2 \geq \frac{\Omega_D[\beta(1+z)^2 + 1 + z - \gamma]}{\beta(1+z)^2 + 1 + z - \gamma\Omega_D} = \frac{\Omega_D}{1 + \Omega_k}. \quad (15)$$

For the spatially flat universe, we recover $d^2 \geq \Omega_D$. When the dark energy dominates, $d^2 \geq 1$, which is the lower bound of d proposed in [14].

In addition to the lower bound on d , employing the argument that the total energy in a region of size L should not exceed the mass of a black hole of the same size, we have the upper bound $d \leq 1$. Alternatively $d \leq 1$ can be argued by using the condition $R_s \leq L$. For a dark energy dominated universe, we have

$$R_s = \frac{2GM}{c^2} = 2G\rho_D \left(\frac{4\pi}{3c^2} L^3 \right) \leq L, \quad (16)$$

so

$$\rho_D \leq \frac{3c^2}{8\pi GL^2}. \quad (17)$$

Comparing Eqs. (1) and (17), we get $d \leq 1$. Thus we find that d must lie in the range

$$\sqrt{\frac{\Omega_D}{1 + \Omega_k}} \leq d \leq 1. \quad (18)$$

As the dark energy gradually dominates the Universe, $\Omega_D \rightarrow 1$, the allowed range of d will become smaller. It is also interesting to note that the Bekenstein entropy bound

$$S \leq \frac{2\pi EL}{c} = \frac{8\pi^2 c \rho_D L^4}{3} \leq \frac{\pi c^3 L^2}{G} = S_{\text{BH}}. \quad (19)$$

Therefore, the maximum entropy is the Bekenstein-Hawking entropy S_{BH} .

Applying the constraint Eq. (18) to Eq. (12), we find that $w_D \geq -1$. Therefore, the holographic dark energy has no phantomlike behavior.

III. PHENOMENOLOGICAL CONSEQUENCES

Now we use the 157 gold sample SN Ia data compiled in [27] to fit the model. The parameters d , Ω_{m0} , and Ω_{k0} in the model are determined by minimizing

$$\chi^2 = \sum_i \frac{[\mu_{\text{obs}}(z_i) - \mu(z_i)]^2}{\sigma_i^2}, \quad (20)$$

where the extinction-corrected distance modulus $\mu(z) = 5\log_{10}(d_L(z)/\text{Mpc}) + 25$, the luminosity distance is

$$\begin{aligned} d_L &= (1+z)r(z) \\ &= \frac{c(1+z)}{H_0\sqrt{|\Omega_{k0}|}} \text{sinn}(\sqrt{|k|}[(1+z)R_{eh}(z) - R_{eh}(0)]) \\ &= \frac{c(1+z)}{H_0\sqrt{|\Omega_{k0}|}} \text{sinn} \left[-\text{sinn}^{-1} \left(\sqrt{\frac{d^2|\Omega_{k0}|}{\Omega_{D0}}} \right) \right. \\ &\quad \left. + \text{sinn}^{-1} \left(\sqrt{\frac{d^2|\gamma|(1-\Omega_D)}{\Omega_D[\beta(1+z)^2 + 1 + z - \gamma]}} \right) \right], \end{aligned} \quad (21)$$

σ_i is the total uncertainty in the observation. The nuisance parameter H_0 is marginalized over with a flat prior assumption. Since H_0 appears linearly as the form of $5\log_{10}H_0$ in χ^2 , the marginalization by integrating $\mathcal{L} = \exp(-\chi^2/2)$ over all possible values of H_0 is equivalent to finding the value of H_0 which minimizes χ^2 if we also include the suitable integration constant. Therefore we marginalize H_0 by minimizing $\chi'^2 = \chi^2(y) -$

$2 \ln(10)y/5 - 2 \ln[\ln(10)\sqrt{(2\pi/\sum_i 1/\sigma_i^2)}/5]$ over y , where $y = 5\log_{10}H_0$. We also assume a prior $\Omega_{m0} = 0.3 \pm 0.1$. The parameter space for Ω_{m0} is $[0, 1]$, the parameter space for Ω_{k0} is $[-1, 1]$, and the parameter space for d is coming from the constraint Eq. (15). The best fit parameters are $\Omega_{m0} = 0.35_{-0.10}^{+0.11}$, $\Omega_{k0} = 0.35_{-0.38}^{+0.17}$, and $d = 1.0_{-0.17}$ with $\chi^2 = 173.35$. Note that d has reached the upper bound 1, so there is no positive error for d . The error is referred to 1σ error throughout this paper. For the flat universe, the best fit parameters are $\Omega_{m0} = 0.30_{-0.08}^{+0.04}$ and $d = 0.84_{-0.03}^{+0.16}$ with $\chi^2 = 176.33$. For comparison, the best fit to the flat Λ CDM model gives $\chi^2 = 176.51$. Therefore using the holographic dark energy model from the supernova data fitting, the closed universe is marginally favored compared to the flat case.

To further constrain the model, we combine the SN Ia data with the WMAP data. The main effect of changing the

values of Ω_{m0} and Ω_{k0} on the CMB anisotropy can be found from the shift parameter \mathcal{R} with which the l -space positions of the acoustic peaks in the angular power spectrum shift [28],

$$\begin{aligned} \mathcal{R} &= \sqrt{\Omega_{m0}} H_0 r(z_{ls}) / c \\ &= \frac{1}{\sqrt{|\gamma|}} \operatorname{sinn} \left[-\operatorname{sinn}^{-1} \left(\sqrt{\frac{d^2 |\Omega_{k0}|}{\Omega_{D0}}} \right) \right. \\ &\quad \left. + \operatorname{sinn}^{-1} \left(\sqrt{\frac{d^2 |\gamma| (1 - \Omega_D)}{\Omega_D [\beta (1 + z_{ls})^2 + 1 + z_{ls} - \gamma]}} \right) \right] \\ &= 1.710 \pm 0.137, \end{aligned} \quad (22)$$

where $z_{ls} = 1089 \pm 1$ [26]. Therefore we use the above shift parameter along with the SN Ia data to fit the model. The best fit parameters are $\Omega_{m0} = 0.29^{+0.06}_{-0.08}$, $\Omega_{k0} = 0.02 \pm 0.10$, and $d = 0.84^{+0.16}_{-0.03}$ with $\chi^2 = 176.12$. It is interesting to note that this best fitting result presents us the same curvature of the Universe as that from the WMAP observation. This result suggests that the WMAP data prefers an almost spatially flat universe while the SN Ia data gives a closed universe. By using the best fit parameters, we plot the evolutions of Ω_D , Ω_m , and Ω_k in Fig. 1.

From Fig. 1, we see that $\Omega_D \rightarrow 1$, $\Omega_m \rightarrow 0$, and $\Omega_k \rightarrow -1 + \Omega_D = 0$. Combining Eqs. (13) and (12), we get the evolution of w_D . The result is plotted in Fig. 2.

From Fig. 2, we see that as expected the holographic dark energy does not have phantomlike behavior.

Using Eqs. (4) and (5), we get the acceleration equation

$$\begin{aligned} \frac{\ddot{a}}{a} &= -\frac{4\pi G}{3} (\rho_m + \rho_D + 3p_D) \\ &= -\frac{H^2}{2} [\Omega_m + (1 + 3w_D)\Omega_D]. \end{aligned} \quad (23)$$

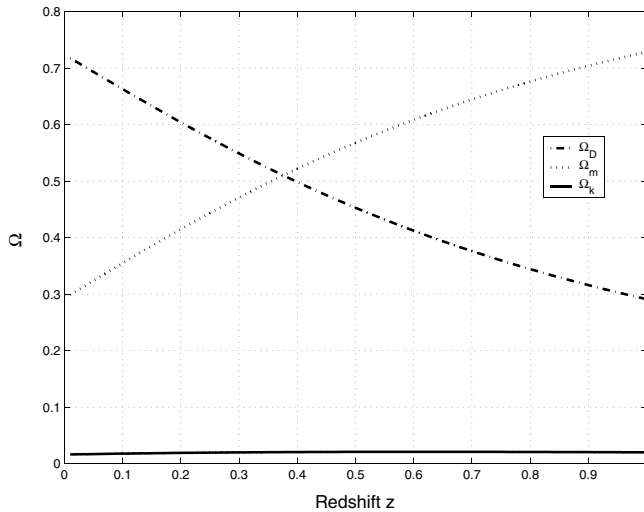


FIG. 1. The evolution of Ω_D , Ω_m , and Ω_k by using the best fit parameters $\Omega_{m0} = 0.29$, $\Omega_{k0} = 0.02$, and $d = 0.84$.

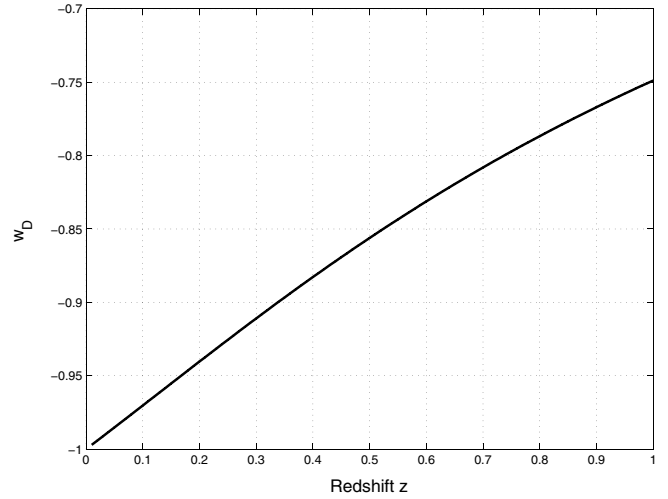


FIG. 2. The evolution of w_D by using the best fit parameters $\Omega_{m0} = 0.29$, $\Omega_{k0} = 0.02$, and $d = 0.84$.

It is clear that the sign of $\Omega_m + (1 + 3w_D)\Omega_D$ determines the sign of \ddot{a} . Combining the behaviors of Ω_D , Ω_m , and w_D , we plot the evolution of $\Omega_m + (1 + 3w_D)\Omega_D = -2\ddot{a}/(aH^2)$ which shows the behavior of acceleration in Fig. 3.

From Fig. 3, we see that the Universe experienced the transition from deceleration to acceleration around $z_t = 0.6$. By fixing Ω_{k0} at its best fit value $\Omega_{k0} = 0.02$, we give the contour plot for Ω_{m0} and d in Fig. 4. For the spatially flat holographic model, the best fit parameters are $\Omega_{m0} = 0.28 \pm 0.05$ and $d = 0.85^{+0.15}_{-0.03}$ with $\chi^2 = 176.18$. Again, for comparison, the best fit parameter of the flat Λ CDM model is $\Omega_{m0} = 0.31^{+0.04}_{-0.03}$ with $\chi^2 = 176.61$. Thus combining with the WMAP data, the closed universe still cannot be ruled out.

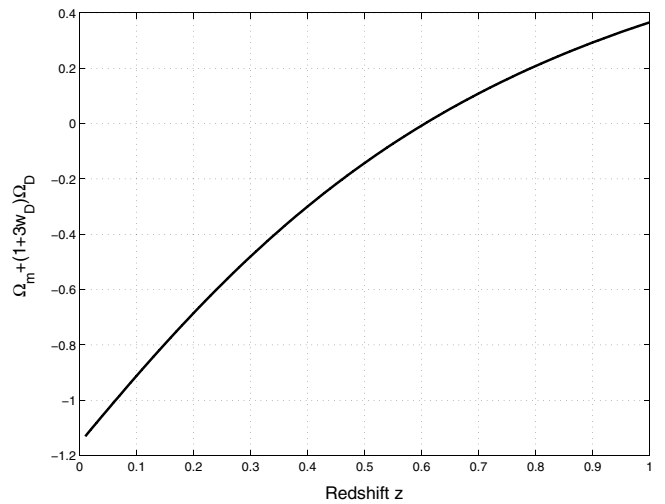


FIG. 3. The evolution of $-2\ddot{a}/(aH^2)$ by using the best fit parameters $\Omega_{m0} = 0.29$, $\Omega_{k0} = 0.02$, and $d = 0.84$.

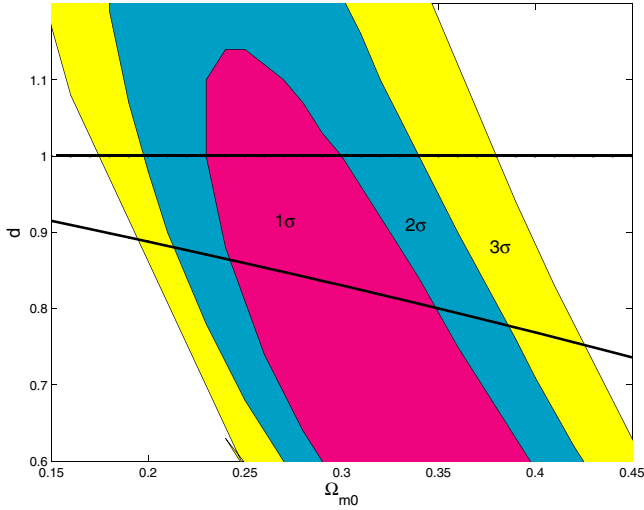


FIG. 4 (color online). The 1σ , 2σ , and 3σ contour plots for Ω_{m0} and d by using $\Omega_{k0} = 0.02$. The contours are those regions intersecting with the two black lines due to the constraint Eq. (18).

IV. CONCLUSIONS

In conclusion, we have reexamined the holographic dark energy model and given a constraint on its parameter. By comparing to observations, we found that the holographic model is an effective model in describing dark energy. A spatially closed universe is favored by using the SN Ia data alone. Combining with the WMAP data, the best fitting result gives us a reasonable value of the curvature of our universe and the closed universe cannot be ruled out. Statistically the closed universe plays the same role as the flat universe in comparing with observations. By investigating the evolution of the dark energy, we observed that the transition of our universe from the deceleration to

the acceleration happens at $z_t = 0.6$. In Ref. [15], one of us discussed the spatially flat holographic dark energy model and found that $\Omega_{m0} = 0.46$ and $d = 0.20$, the model behaved like phantom. In this paper, we used the arguments of the second law of thermodynamics and the holographic principle to get the lower and upper bounds on the parameter d . Because of the constraint Eq. (18), the holographic model discussed in this paper has no phantomlike behavior. Furthermore, we get a lower value of Ω_{m0} which is more consistent with other observations on the value of the nonrelativistic matter energy density.

Comparing with Ref. [15], we have included the curvature of the Universe in our discussion. The SN Ia data alone favors the closed universe with a bit bigger Ω_k , while combining with the WMAP data, Ω_k decreases to a value around 0.02. This discussion is not trivial. Although our result is consistent with the viewpoint that our universe is approximately flat, the small curvature of the Universe is still interesting since it may contribute to the small l suppress of the CMB spectrum [24].

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