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Five-dimensional gauged supergravity black holes with independent rotation parameters

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We construct new nonextremal rotating black hole solutions in $SO(6)$ gauged five-dimensional supergravity. Our solutions are the first such examples in which the two rotation parameters are independently specifiable, rather than being set equal. The black holes carry charges for all three of the gauge fields in the $U(1)^3$ subgroup of $SO(6)$, albeit with only one independent charge parameter. We discuss the BPS limits, showing that these include the first examples of regular supersymmetric black holes with independent angular momenta in gauged supergravity. We also find nonsingular BPS solitons. Finally, we obtain another independent class of new rotating nonextremal black hole solutions with just one nonvanishing rotation parameter, and one nonvanishing charge.

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Constructing nonextremal charged rotating black hole solutions in gauged supergravity is quite a complicated problem. This is because, unlike the case of ungauged supergravity, there are no known solution-generating techniques that could be used to add charges to the the already-known neutral rotating black hole solutions found in four dimensions in [1], five dimensions in [2], and $D \geq 6$ dimensions in [3,4]. Aside from the four-dimensional Kerr-Newman-AdS black holes, which were found in [5], the known nonextremal charged rotating black hole solutions comprise recently-discovered examples in five-dimensional gauged supergravities in [6,7]; in four-dimensional gauged supergravity in [8]; and in seven-dimensional gauged supergravity in [9]. In the five and seven-dimensional cases, the problem was simplified greatly by taking the *a priori* independent rotation parameters of the orthogonal 2-planes in the transverse space to be equal. This reduces the problem to studying cohomogeneity-1 metrics, with nontrivial coordinate dependence on only the radial variable, rather then metrics of cohomogeneity 2 or cohomogeneity 3.

In this paper, we shall present some new results on nonextremal rotating black holes in five-dimensional gauged supergravity, in which the two rotation parameters a and b can be independently specified. Our black holes can be viewed as solutions in $N = 8$ gauged $SO(6)$ supergravity, with three charge parameters associated with the gauge fields of the $U(1)^3$ abelian subgroup. They can also be viewed as solutions in $N = 2$ gauged supergravity coupled to two vector multiplets.

After presenting the solutions, we then calculate the charges, angular momenta, angular velocities, electrostatic potentials, temperature and entropy. From these, we follow the procedure that was used in [10], and more recently in [11], for calculating the energy by integration of the first law of thermodynamics. Then, we study the conditions under which supersymmetric limits will arise, by looking for zero eigenvalues of the Bogomol'nyi matrix arising from anticommutators of the supercharges. We obtain by this means families of supersymmetric configurations, characterized by a mass parameter and the two independent rotation parameters. In general these BPS solutions have naked closed timelike curves (CTC's) lying outside an horizon. (More precisely, the CTC's lie outside a "pseudohorizon," which we define later in the paper.) However, for a particular choice of the mass, we obtain completely regular black holes with no singularities or closed timelike curves on or outside the horizon. These are similar to the regular black holes of five-dimensional gauged supergravity that were found in [12,13], except that in our new solutions the two angular momenta can be independently specified. Indeed, the rotating BPS black holes that we find in this paper are the first such examples with independent rotation parameters. We also find other special cases, describing completely regular solitons.

We also obtain a further class of new nonextremal rotating black hole solutions of five-dimensional gauged supergravity, in which only one rotation parameter is nonvanishing, and only one of the three $U(1)$ charges is turned on. These solutions are therefore independent of any found previously in this paper or elsewhere. We again study the thermodynamics and obtain expressions for the conserved energy, angular momentum and charge. From the BPS limit we again obtain supersymmetric solutions. In this case, unlike the one discussed above, there are no regular BPS black holes or solitons, but only solutions with naked CTC's.

The rotating black hole metrics that we shall construct arise as solutions of $SO(6)$ gauged five-dimensional supergravity. The first class we construct are charged under all three $U(1)$ factors in the Cartan subgroup of $SO(6)$, with specific relations between the three charges. The two rotation parameters can be specified independently. The relevant part of the supergravity Lagrangian that describes these solutions is given by

$$e^{-1}\mathcal{L} = R - \frac{1}{2}\partial\vec{\varphi}^2 - \frac{1}{4}\sum_{i=1}^{3} X_i^{-2}(F^i)^2 + 4g^2\sum_{i=1}^{3} X_i^{-1} + \frac{1}{4}\epsilon^{\mu\nu\rho\sigma\lambda}F^1_{\mu\nu}F^2_{\rho\sigma}A^3_\lambda, \tag{1}$$

where $\vec{\varphi} = (\varphi_1, \varphi_2)$, and

$$X_1 = e^{-1/\sqrt{6}\varphi_1 - 1/\sqrt{2}\varphi_2}, \qquad X_2 = e^{-1/\sqrt{6}\varphi_1 + 1/\sqrt{2}\varphi_2}, \qquad X_3 = e^{2/\sqrt{6}\varphi_1}. \tag{2}$$

The three $U(1)$ gauge fields A^i_μ are labeled by the upper triplet index.

In what follows, we shall first construct the nonextremal rotating metrics, and then calculate their associated conserved quantities, namely, their mass E, their angular momenta J_a and J_b, and the three electric charges Q_i. Next, by using the BPS conditions derived from the AdS superalgebra, we determine the restrictions on the parameters of the solutions that lead to supersymmetry. We investigate the global structure of these BPS limits, showing, in particular, that there exist regular supersymmetric black holes with no naked singularities or closed timelike curves.

We find that the following provides a solution of the five-dimensional gauged supergravity equations:

$$\begin{aligned}
ds^2 &= H^{-4/3}\Big[-\frac{X}{\rho^2}\Big(dt - a\sin^2\theta\frac{d\phi}{\Xi_a} - b\cos^2\theta\frac{d\psi}{\Xi_b}\Big)^2 + \frac{C}{\rho^2}\Big(\frac{ab}{f_3}dt - \frac{b}{f_2}\sin^2\theta\frac{d\phi}{\Xi_a} - \frac{a}{f_1}\cos^2\theta\frac{d\psi}{\Xi_b}\Big)^2 \\
&\quad + \frac{Z\sin^2\theta}{\rho^2}\Big(\frac{adt}{f_3} - \frac{d\phi}{f_2\Xi_a}\Big)^2 + \frac{W\cos^2\theta}{\rho^2}\Big(\frac{bdt}{f_3} - \frac{d\psi}{f_1\Xi_b}\Big)^2\Big] + H^{2/3}\Big(\frac{\rho^2}{X}dr^2 + \frac{\rho^2}{\Delta_\theta}d\theta^2\Big), \\
H &= \tilde{\rho}^2/\rho^2, \qquad \rho^2 = r^2 + a^2\cos^2\theta + b^2\sin^2\theta, \qquad \tilde{\rho}^2 = \rho^2 + 2ms^2, \\
f_1 &= a^2 + r^2, \qquad f_2 = b^2 + r^2, \qquad f_3 = (a^2 + r^2)(b^2 + r^2) + 2mr^2s^2; \qquad \Delta_\theta = 1 - a^2g^2\cos^2\theta - b^2g^2\sin^2\theta, \\
X &= \frac{1}{r^2}(a^2 + r^2)(b^2 + r^2) - 2m + g^2(a^2 + r^2 + 2ms^2)(b^2 + r^2 + 2ms^2), \qquad C = f_1f_2(X + 2m - 4m^2s^4/\rho^2), \\
Z &= -g^2f_2f_3(a^2 - b^2)(a^2 + r^2 + 2ms^2)\cos^2\theta - b^2C + \frac{f_2f_3^2}{r^2}, \\
W &= g^2f_1f_3(a^2 - b^2)(b^2 + r^2 + 2ms^2)\sin^2\theta - a^2C + \frac{f_1f_3^2}{r^2}, \qquad \Xi_a = 1 - a^2g^2, \qquad \Xi_b = 1 - b^2g^2,
\end{aligned} \tag{3}$$

where $s \equiv \sinh\delta$ and $c \equiv \cosh\delta$.

The gauge potentials and scalar fields are given by

$$\begin{aligned}
A^1 &= A^2 = \frac{2msc}{\tilde{\rho}^2}\Big(dt - a\sin^2\theta\frac{d\phi}{\Xi_a} - b\cos^2\theta\frac{d\psi}{\Xi_b}\Big), \\
A^3 &= \frac{2ms^2}{\rho^2}\Big(b\sin^2\theta\frac{d\phi}{\Xi_a} + a\cos^2\theta\frac{d\psi}{\Xi_b}\Big), \\
X_1 &= X_2 = H^{-1/3}, \qquad X_3 = H^{2/3}.
\end{aligned} \tag{4}$$

The solution reduces to the uncharged Kerr-AdS metric of [2] if the charge parameters are set to zero, and it reduces to a special case of the solutions in [7] if instead the two rotation parameters are set equal. Various special cases in the BPS limit are discussed later.

It should be noted that the solution above is presented in a coordinate frame that is rotating at infinity. One can pass to coordinates that are asymptotically static by making the redefinitions $\phi = \tilde{\phi} + ag^2t$ and $\psi = \tilde{\psi} + bg^2t$. It is helpful to make this transformation in order to simplify the calculation of the thermodynamic quantities. One might think from the expressions for the gauge potentials in (4) that there are just two nonvanishing (and equal) charges, since A^3 has no electric component. However, this is a somewhat misleading artefact of the original rotating coordinate system. After transforming to the asymptotically nonrotating frame, one finds that A^3 also has an electric component, and indeed, as we shall see below, the third electric charge is nonzero too.

It is straightforward to calculate the temperature, entropy, angular velocities on the horizon, and the electrostatic potentials on the horizon, referred to the asymptotically static frame. We find

$$\begin{aligned}
T &= \frac{2g^2r_+^6 + [1 + g^2(a^2 + b^2 + 4ms^2)]r_+^4 - a^2b^2}{2\pi r_+[r_+^4 + (a^2 + b^2 + 2ms^2)r_+^2 + a^2b^2]}, \\
S &= \frac{\pi^2[r_+^4 + (a^2 + b^2 + 2ms^2)r_+^2 + a^2b^2]}{2\Xi_a\Xi_b r_+}, \\
\Omega_a &= \frac{a[g^2r_+^4 + (b^2 + 2ms^2)g^2r_+^2 + b^2]}{r_+^4 + (a^2 + b^2 + 2ms^2)r_+^2 + a^2b^2}, \\
\Omega_b &= \frac{b[g^2r_+^4 + (a^2 + 2ms^2)g^2r_+^2 + a^2]}{r_+^4 + (a^2 + b^2 + 2ms^2)r_+^2 + a^2b^2}, \\
\Phi_1 &= \Phi_2 = \frac{2mr_+^2sc}{r_+^4 + (a^2 + b^2 + 2ms^2)r_+^2 + a^2b^2}, \\
\Phi_3 &= \frac{2mabs^2}{r_+^4 + (a^2 + b^2 + 2ms^2)r_+^2 + a^2b^2},
\end{aligned} \tag{5}$$

where r_+, the largest root of the metric function $X(r)$, is the location of the outer horizon.

The two angular momenta can be evaluated from the (convergent) Komar integrals

$$J = \frac{1}{16\pi}\int_{S^3} *dK, \tag{6}$$

where K is the 1-form obtained by lowering the index on the angular Killing vector $\partial/\partial\phi$ or $\partial/\partial\psi$. The charges are given by Gaussian integrals

$$Q_i = \frac{1}{16\pi}\int_{S^3}(X_i^{-2} * F^i - A^j \wedge F^k), \tag{7}$$

where $(j, k) = (2, 3), (3, 1), (1, 2)$ for $i = 1, 2, 3$ respectively. With these angular momenta and charges, we can now integrate the first law of thermodynamics

$$dE = TdS + \Omega_a dJ_a + \Omega_b dJ_b + \sum_i \Phi_i dQ_i \tag{8}$$

in order to obtain the energy E of the black hole solution. Our results for the conserved quantities are

$$\begin{aligned} E &= \frac{\pi m}{4\Xi_a^2\Xi_b^2}[2\Xi_a + 2\Xi_b - \Xi_a\Xi_b + (2\Xi_a^2 + 2\Xi_b^2 \\ &\quad + 2\Xi_a\Xi_b - \Xi_a^2\Xi_b - \Xi_b^2\Xi_a)s^2], \\ J_a &= \frac{\pi m a(1 + s^2\Xi_b)}{2\Xi_b\Xi_a^2}, \qquad J_b = \frac{\pi m b(1 + s^2\Xi_a)}{2\Xi_a\Xi_b^2}, \\ Q_1 &= Q_2 = \frac{\pi m s c}{2\Xi_a\Xi_b}, \qquad Q_3 = -\frac{\pi a b m s^2 g^2}{2\Xi_a\Xi_b}. \end{aligned} \tag{9}$$

As discussed recently in [11], the BPS limit of the nonextremal solution can conveniently be discussed by studying the eigenvalues of the Bogomol'nyi matrix that arises from the anticommutator of the supercharges of the AdS superalgebra. Thus the BPS limit of nonextremal five-dimensional gauged supergravity solutions is attained when $E + gJ_a + gJ_b - \sum_i Q_i = 0$ (modulo unimportant sign choices). Substituting our expressions for the conserved mass, angular momenta and charges given in (9), we find that the BPS condition is satisfied if the parameter δ is chosen so that

$$e^{2\delta} = 1 + \frac{2}{(a + b)g}. \tag{10}$$

It is interesting to note that with a and b as independently specificable parameters, we can make contact with previous results in two inequivalent special cases. Firstly, if we take $a = b$, the solutions we have obtained in this paper reduce to particular cases of the 3-charge rotating black holes with equal angular momenta that were found in [7]. In particular, the BPS condition (10) reduces to one that was found for the $a = b$ solutions in [11]. An inequivalent special case arises if instead we take $a = -b$. Now, the BPS condition (10) reduces to the condition that $e^{2\delta} \rightarrow \infty$, which also arose, as a disjoint case, in the analysis in [11]; it again can be viewed as a situation with "equal angular momenta," after making an orientation reversal. Because in the present work we have the possibility to specify a and b independently, we can actually describe a continuous interpolation between two BPS limits that were seen as disjoint possibilities in the earlier work.

To analyze the global structure of the metric (3), we first rewrite it in the form

$$\begin{aligned} ds^2 = H^{2/3}\Big[&-\frac{r^2 X\Delta_\theta \sin^2 2\theta}{4\Xi_a\Xi_b H^2 B_\phi B_\psi}dt^2 + \rho^2\Big(\frac{dr^2}{X} + \frac{d\theta^2}{\Delta_\theta}\Big) \\ &+ B_\psi(d\psi + v_1 d\phi + v_2 dt)^2 + B_\phi(d\phi + v_3 dt)^2\Big]. \end{aligned} \tag{11}$$

The functions B_ψ, B_ϕ, v_1, v_2 and v_3 can be read off straightforwardly by comparing (11) with (3), and we shall not present them explicitly here because their detailed forms play no essential role in the following discussion. From (11), it is evident that there is an outer Killing horizon located at $r = r_+$, the largest root of $X(r)$. There is a velocity of light surface (VLS), located at the boundary $r = r_L$ of the region where $B_\psi B_\phi$ changes sign from positive (at large r) to negative. Inside the VLS, the metric develops closed timelike curves (CTC's). If $r_+ > r_L$, then the Killing horizon lies outside the VLS, and so the Killing horizon is an event horizon. In these circumstances, the solution describes a regular black hole, in which there are neither curvature singularities nor CTC's outside the horizon. If the largest root r_+ is inside the VLS, the solution instead describes a naked time machine.

In the supersymmetric limit, there is a Killing vector

$$\ell = \frac{\partial}{\partial t} - g\frac{\partial}{\partial\tilde\phi} - g\frac{\partial}{\partial\tilde\psi} \tag{12}$$

that has a spinorial square root, in the sense that $\ell \sim \bar\eta\gamma^\mu\eta\partial_\mu$, where η is the Killing spinor. (See [11] for a recent detailed discussion of this.) This Killing vector is necessarily nonspacelike, and in fact we find that the explicit expression for its norm is a manifestly nonpositive quantity. From this, we find the identity

$$\begin{aligned} &-\frac{X\Delta_\theta\sin^2 2\theta}{4\Xi_a\Xi_b H^2 B_\psi B_\phi} + B_\psi(v_2 + g - bg^2 \\ &+ v_1(g - ag^2))^2 + B_\phi(v_3 + g - ag^2)^2 \\ &= -\Big[\frac{(1 + ag + bg)(1 + ag\cos^2\theta + bg\sin^2\theta)}{(1 + ag)(1 + bg)H} \\ &-\frac{ag\cos^2\theta}{1 + bg} - \frac{bg\sin^2\theta}{1 + ag}\Big]^2, \end{aligned} \tag{13}$$

where the right-hand side comes from the evaluation of ℓ^2 in the metric (3), and the left-hand side is obtained from evaluating ℓ^2 in the form (11) for the metric.

It follows from (13) that at least one of B_ϕ and B_ψ will be negative at the largest root $r = r_+$ where $X(r)$ vanishes. Furthermore, since the determinant of the metric is given by $\sqrt{-g} = H^{2/3} r\rho^2 \sin 2\theta/(2\sqrt{\Xi_a \Xi_b})$, which does not pass through zero as $X(r)$ approaches zero, it follows that the spacetime signature is unchanged, and hence only one of B_ϕ or B_ψ changes sign. Which of the two changes sign depends on the choice of the parameters. Suppose, for example, that it is B_ψ that changes sign. Inside the VLS, ∂_ψ is timelike, while the first term in (11) is positive. The metric on the quotient of (11) by the $SO(2)$ action of ∂_ψ is therefore positive definite inside the VLS, and spacetime comes to an end at $r = r_+$ where $X(r)$ vanishes. This is therefore not an horizon, but more like an origin of polar coordinates in a Euclidean-signature space. We shall, for convenience, refer to it as a "pseudohorizon." As discussed in [11], in general one must identify t with an appropriate (real) periodicity in order to avoid a conical singularity on the pseudohorizon. Similar remarks apply, *mutatis mutandis*, if it is B_ϕ, rather than B_ψ, that changes sign inside the VLS.

As in the cases discussed in [11], there are two ways of avoiding such naked CTC's. The first is if the parameters are chosen so that right-hand side of (13) vanishes at $r = r_+ \equiv r_0$. This occurs if

$$m = \frac{(a+b)^2(1+ag)(1+bg)(2+ag+bg)}{2(1+ag+bg)}. \qquad (14)$$

When this condition is satisfied, the function X becomes

$$X = \frac{(r-r_0)^2((ab-r_0)^2 r^2 + a^2b^2(a+b)^2)}{(a+b)^2 r_0^4 r^2}, \qquad (15)$$

and there is a double root at $r = r_0$, where r_0 is given by

$$r_0^2 = \frac{ab}{1+ag+bg}. \qquad (16)$$

It is straightforward to verify that the VLS lies inside the Killing horizon at $r = r_0$, and so it is an event horizon. Thus the solution describes a supersymmetric black hole that is regular on and outside the event horizon. The fact that $X(r)$ has a double root at $r = r_0$ implies that the Hawking temperature is zero. This is the first example of a supersymmetric black hole with two independent angular momenta in gauged supergravity. If a and b are set equal, the solution reduces to a special case of the regular black holes found in [13]. (Note that a black hole is not possible if $a + b = 0$. As we discussed earlier, it corresponds to a BPS limit that was also studied in [11], which was associated with BPS solutions found in [14,15]. The solutions in [14,15] all describe configurations with naked CTC's, or, as shown in [11], a nonsingular soliton; there is no solution in that family that describes a regular black hole.) The rotation parameters a and b must in general be restricted to an appropriate range, in order to ensure that B_ϕ and B_ψ remain positive and hence that there are no CTC's outside the horizon.

The other way to avoid the naked CTC's of the generic supersymmetric solutions is by restricting the parameters so that $B_\psi B_\phi$ goes to zero at the same radius as X goes to zero, i.e. so that $r_L = r_+$. This occurs when the parameter m is given by

$$m = \frac{(a+b)(1+ag)(1+bg)(2+ag+bg)}{2g(1+ag+bg)^2} \times (1+2ag+bg)(1+ag+2bg). \qquad (17)$$

The solution then describes a smooth finite-energy configuration of the type that was called a *topological soliton* in [11]. It is a completely nonsingular globally stationary spacetime, with no horizon, defined on the product of time with a spatial manifold having the nontrivial topology of an IR^2 bundle over S^2. Defining $R = r^2 + a^2b^2g^2/(1+ag+bg)^2$, the coordinate R runs from 0 to ∞. Requiring no conical singularity at $R = 0$, where $B_\phi = 0$, implies the quantisation condition

$$1 = \frac{1 + 3(a+b)g + (3a^2 + 5ab + 3b^2)g^2}{bg(1-ag)(1+ag+bg)(1+2ag+bg)} + \frac{(a+b)(a^2+b^2)g^3 - ab(a^2+4ab+b^2)^4}{bg(1-ag)(1+ag+bg)(1+2ag+bg)}. \qquad (18)$$

In the special cases $a = b$ or $a = -b$, these topological solitons are encompassed within the soliton solutions obtained in [11]. The rotation parameters a and b must in general be restricted to an appropriate range, in order to ensure that B_ϕ and B_ψ remain positive and hence that there are no CTC's for all $R \geq 0$.

Aside from the above two possibilities, the supersymmetric solutions have naked CTC's in general. As in the examples in [11], a conical singularity at the pseudohorizon can be avoided by periodically identifying the asymptotic time coordinate t with an appropriate period. If the pseudohorizon is associated with a double root of $X(r)$, then such an identification is unnecessary.

To close, we obtain another new solution describing a nonextremal rotating black hole in gauged five-dimensional supergravity. In this case, just one of the two rotation parameters is nonzero, and only one of the three gauge fields in the $U(1)^3$ subgroup of $SO(6)$ is turned on. This solution is therefore not a special case of the solution obtained above, nor indeed of any other previously-obtained solutions. Having presented the solution, we then evaluate the conserved mass, angular momentum and charge, and from this we study the BPS limit.

Again, we shall omit the details of the lengthy process of conjecture and verification whereby we arrived at the solution, and just present our final result here. We find that the metric for the five-dimensional black hole with one nonvanishing rotation parameter and one charge is given by

$$ds^2 = -H^{-2/3}\frac{wY}{F(r,\theta)}\Big(cdt - a\sin^2\theta\frac{d\phi}{w\Xi}\Big)^2 + H^{1/3}\Big(\frac{w\Delta_\theta\sin^2\theta}{F(r,\theta)}\Big(f_1 dt - cf_2\frac{d\phi}{w\Xi}\Big)^2 + \frac{\rho^2}{Y}dr^2 + \frac{\rho^2}{\Delta_\theta}d\theta^2 + r^2\cos^2\theta d\psi^2\Big),$$
$$F(r,\theta) = c^2 f_2(r) - a\sin^2\theta f_1(r), \qquad H = \tilde\rho^2/\rho^2, \qquad \tilde\rho^2 = a^2 + r^2 + 2ms^2, \qquad \rho^2 = a^2 + r^2,$$
$$f_1 = \frac{a}{w}(1 - ws^2g^2(a^2+r^2)), \qquad f_2 = w(a^2+r^2), \qquad Y = (a^2+r^2)(1+g^2r^2) - 2mf_1/a,$$
$$\Delta_\theta = 1 - a^2g^2\cos^2\theta, \qquad \Xi = 1 - a^2g^2. \tag{19}$$

where again $c = \cosh\delta$, $s = \sinh\delta$, and the constant w, which satisfies $c^2w^2 - s^2w\Xi = 1$, is given by

$$w = \frac{\Xi s^2 + \sqrt{4(1+s^2) + \Xi^2 s^4}}{2(1+s^2)}. \tag{20}$$

The gauge potentials and scalar fields are given by

$$A^1 = \frac{2ms\sqrt{w}}{\tilde\rho^2}\Big(cdt - a\sin^2\theta\frac{d\phi}{w\Xi}\Big),$$
$$A^2 = A^3 = 0, X_1 = H^{-2/3}, \qquad X_2 = X_3 = H^{\frac{1}{3}}. \tag{21}$$

We find that the conserved energy, angular momentum and charge for this black hole solution are given by

$$E = \frac{\pi m[\Xi - w(2+\Xi) + w^2\Xi(1+\Xi)]}{4\Xi^2 w(\Xi - w)},$$
$$J = \frac{\pi ma\sqrt{1-w\Xi}}{2\Xi^2\sqrt{w(w-\Xi)}}, \qquad Q = \frac{\pi m\sqrt{(1-w^2)(1-w\Xi)}}{2\Xi\sqrt{w}(\Xi - w)}. \tag{22}$$

The supersymmetry condition (i.e. a vanishing eigenvalue of the Bogomol'nyi matrix) is now given (modulo equivalent sign choices) by $E - gJ - Q = 0$. Using our expressions (22) for the conserved energy, angular momentum and charge, this BPS condition implies

$$a^2g^2 = \frac{1-w}{w^2}. \tag{23}$$

Alternatively, it can be expressed as $ag = \frac{c}{s^2}$.

To analyze the global properties of the solution, it is helpful to rewrite the metric as

$$ds^2 = H^{1/3}\Big(-\frac{Y\Delta_\theta\sin^2\theta dt^2}{(1-a^2g^2)^2HB_\phi} + \frac{\rho^2dr^2}{Y} + \frac{\rho^2d\theta^2}{\Delta_\theta} + B_\phi(d\phi + vdt)^2 + r^2\cos^2\theta d\psi^2\Big), \tag{24}$$

where again, we shall not need the detailed expressions for the metric functions. This expression is valid both in the BPS limit and in the nonextremal case. In the supersymmetric limit there exists a Killing vector $\ell = \partial/\partial t - g\partial/\partial\tilde\phi - g\partial/\partial\psi$ with a spinorial square root, where $\tilde\phi = \phi + ag^2wct$, and $(\tilde\phi, \psi)$ are asymptotically nonrotating coordinates. From the norm of ℓ, which is manifestly nonpositive, we can read off the identity

$$-H^{-1} = -\frac{Y\Delta_\theta\sin^2\theta}{(1-a^2g^2)^2HB_\phi} + B_\phi(v - ag^2wc + g)^2 + g^2r^2\cos^2\theta. \tag{25}$$

Thus in general, when $Y = 0$ either B_ϕ or r^2 is negative, implying the existence of naked CTC's. It is straightforward to verify that, unlike the solutions we presented earlier, here naked CTC's are unavoidable when there is only one charge and one nonvanishing rotation. The BPS solutions therefore all describe naked time machines.

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