CP asymmetries and branching ratios of $B \to K\pi$ in supersymmetric models

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We analyze the supersymmetric contributions to the direct and mixing *CP* asymmetries and also to the branching ratios of the $B \to K\pi$ decays in a model independent way. We consider both gluino and chargino exchanges and emphasize that a large gluino contribution is essential for saturating the direct and mixing *CP* asymmetries. We also find that combined contributions from the penguin diagrams with chargino and gluino in the loop could lead to a possible solution for the branching ratios puzzle and account for the results of R_c and R_n within $b \to s\gamma$ constraints. When all relevant constraints are satisfied, our result indicates that supersymmetry favors lower values of R_c. Finally, we study the correlations between the mixing *CP* asymmetry $S_{K^0\pi^0}$ and mixing *CP* asymmetries of the processes $B \to \phi K$ and $B \to \eta' K$. We show that it is quite possible for gluino exchanges to accommodate the results of the observables.

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I. INTRODUCTION

Recently, the *BABAR* and Belle Collaborations have measured the *CP* averaged branching ratios and the *CP* violating asymmetries of $B \to K\pi$ decays [1–3]. These results, in addition to those from $B \to \phi K$ and $B \to \eta' K$, offer an interesting avenue to understand the *CP* violation and flavor mixing of the quark sector in the standard model (SM).

In the SM, all *CP* violating observables should be explained by one complex phase $\delta_{\rm CKM}$ in the quark mixing matrix. The effect of this phase has been observed in the kaon system. In order to account for the observed *CP* violation in this sector, $\delta_{\rm CKM}$ has to be of order 1. With such a large value of $\delta_{\rm CKM}$, the experimental results of the *CP* asymmetry of $B \to J/\psi K_S$ are consistent with the SM. However, the experimental measurements of the *CP* asymmetries of $B \to \phi K$, $B \to \eta' K$, and $B \to K\pi$ decays exhibit a possible discrepancy from the SM predictions. Furthermore, it is well known that the strength of the SM *CP* violation cannot generate the observed size of the baryon asymmetry of the Universe, and a new source of *CP* violation beyond the $\delta_{\rm CKM}$ is needed.

In supersymmetric extensions of the SM, there are additional sources of *CP* violating phases and flavor mixings. It is also established that the supersymmetry (SUSY) flavor dependent (off-diagonal) phases could be free from the stringent electric dipole moment constraints [4]. These phases can easily provide an explanation for the above mentioned anomalies in the *CP* asymmetries of $B \to \phi K$ and $B \to \eta' K$ [5–7]. We aim in this article to prove that, in this class of SUSY models, it is also possible to accommodate the recent experimental results of $B \to K\pi$ *CP* asymmetries and branching ratios.

The latest experimental measurements for the four branching ratios and the four *CP* asymmetries of $B \to K\pi$ [1] are given in Table I. As can be seen from this table, the measured value of the direct *CP* violation in $\bar{B}^0 \to K^-\pi^+$ is $A^{CP}_{K^-\pi^+} = -0.113 \pm 0.019$, which corresponds to a 4.2σ deviation from zero, while the measured value of $A^{CP}_{K^+\pi^0}$, which may also exhibit a large asymmetry, is quite small. As we will see in the next section, it is very difficult in the SM to get such different values for the *CP* asymmetries.

Also from these results, one finds that the ratios R_c, R_n, and R of $B \to K\pi$ decays are given by

$$R_c = 2\left[\frac{\mathrm{BR}(B^+ \to K^+\pi^0) + \mathrm{BR}(B^- \to K^-\pi^0)}{\mathrm{BR}(B^+ \to K^0\pi^+) + \mathrm{BR}(B^- \to \bar{K}^0\pi^-)}\right] = 1.00 \pm 0.08, \quad (1)$$

$$R_n = \frac{1}{2}\left[\frac{\mathrm{BR}(B^0 \to K^+\pi^-) + \mathrm{BR}(\bar{B}^0 \to K^-\pi^+)}{\mathrm{BR}(B^0 \to K^0\pi^0) + \mathrm{BR}(\bar{B}^0 \to \bar{K}^0\pi^0)}\right] = 0.79 \pm 0.08, \quad (2)$$

$$R = \left[\frac{\mathrm{BR}(B^0 \to K^+\pi^-) + \mathrm{BR}(\bar{B}^0 \to K^-\pi^+)}{\mathrm{BR}(B^+ \to K^0\pi^+) + \mathrm{BR}(B^- \to \bar{K}^0\pi^-)}\right]\frac{\tau_B^+}{\tau_{B^0}} = 0.82 \pm 0.06. \quad (3)$$

In the SM, the R_c and R_n ratios are approximately equal; however, the experimental results in Eqs. (1) and (2) indicate a 2.4σ deviation from the SM prediction. On the

TABLE I. The current experimental results for the *CP* averaged branching ratios and *CP* asymmetries of $B \to K\pi$ decays.

Decay channel	BR × 10^6	A_{CP}	S_f
$\bar{K}^0\pi^-$	24.1 ± 1.3	−0.02 ± 0.034	⋯
$K^-\pi^0$	12.1 ± 0.8	0.04 ± 0.04	⋯
$K^-\pi^+$	18.2 ± 0.8	−0.113 ± 0.019	⋯
$\bar{K}^0\pi^0$	11.5 ± 1.0	−0.09 ± 0.14	0.34 ± 0.28

other hand, the quantity R is consistent with the SM value. Here $\tau_B^+/\tau_{B^0} = 1.089 \pm 0.017$. These inconsistencies between the $A_{K\pi}^{CP}$ and the $R_c - R_n$ measurements and the SM results are known as $K\pi$ puzzles.

These puzzles have created a lot of interest and much research work has been done to explain the experimental data [8,9]. It is tempting to conclude that any new physics contributions to $B \to K\pi$ should include a large electroweak penguin in order to explain these discrepancies. In SUSY models, the Z-penguin diagrams with chargino exchange in the loop contribute to the electroweak penguin significantly for a light right-handed stop mass. Also, the subdominant color suppressed electroweak penguin can be enhanced by the electromagnetic penguin with chargino in the loop. Therefore, the supersymmetric extension of the SM is an interesting candidate for explaining the $K\pi$ puzzles.

It is worth mentioning also that new precision determinations of the branching ratios and CP asymmetries of $B \to \pi\pi$ have been recently reported [2,3]. However, the SUSY contribution to $B \to \pi\pi$, at the quark level, is due to the loop correction for the process $b \to dq\bar{q}$, while the SUSY contribution to $B \to K\pi$ is due to the process $b \to sq\bar{q}$. Therefore, these two contributions are, in general, independent and SUSY could have a significant effect to $B \to K\pi$ and accommodates the new result, while its contribution to $B \to \pi\pi$ remains small. Thus, we will focus here only on SUSY contributions to $B \to K\pi$.

In this paper, we perform a detailed analysis of SUSY contributions to the CP asymmetries and the branching ratios of $B \to K\pi$ processes. We emphasize that chargino contribution has the potential to enhance the electroweak penguins and provides a natural solution to the above discrepancies. However, this contribution alone is not large enough to accommodate the experimental results and to solve the $K\pi$ puzzles. We argue that the gluino contribution plays an essential rule in explaining the recent measurements, especially the results of the CP asymmetries. Recall that other supersymmetric contributions such as the neutralino and charged Higgs are generally small and can be neglected. The charged Higgs contributions are relevant only at a very large $\tan\beta$ and small charged Higgs mass. Therefore, we are going to concentrate on the chargino and gluino contributions only.

The paper is organized as follows. In Sec. II we study the CP asymmetries and the branching ratios of $B \to K\pi$ in the SM. We show that within the SM the $K\pi$ puzzles cannot be resolved. In Sec. III we analyze the supersymmetric contributions, namely, the gluino and chargino contributions, to $B \to K\pi$. We show that a small value of the right-handed stop mass and a large mixing between the second and the third generation in the up-squark mass matrix are required to enhance the chargino Z penguin. Also, a large value of $\tan\beta$ is necessary to increase the effect of the chargino electromagnetic penguin.

Section IV is devoted to the constraints on SUSY flavor structure from the branching ratio of $b \to s\gamma$. New upper bounds on the relevant mass insertions are derived in the case of dominant gluino or chargino contribution. A correlation between the mass insertions $(\delta_{LR}^d)_{23}$ and $(\delta_{LL}^u)_{32}$ is obtained when both gluino and chargino exchanges are assumed to contribute significantly. In Sec. V the SUSY resolution for the $R_c - R_n$ puzzle is considered. We show that it is very difficult to explain this puzzle with a single mass insertion contribution. We emphasize that, with simultaneous contributions from gluino and chargino, one may be able to explain these discrepancies.

In Sec. VI we focus on the CP asymmetries in $B \to K\pi$ processes. We show that with a large gluino contribution it is quite natural to account for the recent experimental results of direct CP asymmetries. The SUSY contributions to the mixing CP asymmetry of $B^0 \to K^0\pi^0$ are also discussed. Finally, Sec. VII contains our main conclusions.

II. $B \to K\pi$ IN THE STANDARD MODEL

In this section, we analyze the SM predictions for the CP asymmetries and the branching ratios of $B \to K\pi$ decays. The effective Hamiltonian of $\Delta B = 1$ transition governing these processes can be expressed as

$$H_{\text{eff}}^{\Delta B=1} = \left\{\frac{G_F}{\sqrt{2}} \sum_{p=u,c} \lambda_p \left(C_1 Q_1^p + C_2 Q_2^p + \sum_{i=3}^{10} C_i Q_i + C_{7\gamma} Q_{7\gamma} + C_{8g} Q_{8g} \right)\right\} + \{Q_i \to \tilde{Q}_i, C_i \to \tilde{C}_i\}, \tag{4}$$

where $\lambda_p = V_{pb} V_{ps}^*$ and C_i are the Wilson coefficients and Q_i are the relevant local operators which can be found in Ref. [10]. The operators $\tilde{Q}_i \equiv \tilde{Q}_i(\mu_b)$ are obtained from Q_i by the chirality exchange $(\bar{q}_1 q_2)_{V\pm A} \to (\bar{q}_1 q_2)_{V\mp A}$. Notice that in the SM the coefficients $\tilde{C}_i$ identically vanish due to the V-A structure of charged weak currents, while in the minimal supersymmetric standard model, they can receive contributions from both chargino and gluino exchanges. The $b \to s$ transition can be generated in the SM through exchange of the W boson. The corresponding Wilson coefficients can be found in Ref. [10].

The calculation of the decay amplitudes of $B \to K\pi$ involves the evaluation of the hadronic matrix elements of the above operators in the effective Hamiltonian, which is the most uncertain part of this calculation. Adopting the QCD factorization [11], the matrix elements of the effective weak Hamiltonian can be written as

$$\langle \pi K | H_{\text{eff}} | \bar{B} \rangle = \frac{G_F}{\sqrt{2}} \sum_{p=u,c} \lambda_p \langle \pi K | (\mathcal{T}_p + \mathcal{T}_p^{\text{ann}}) | \bar{B} \rangle, \tag{5}$$

where

$$\langle \pi K | \mathcal{T}_p | \bar{B} \rangle = \sum_{i=1}^{10} a_i(\pi K) \langle \pi K | Q_i | \bar{B} \rangle_F \quad (6)$$

and

$$\langle \pi K | \mathcal{T}_p^{\rm ann} | \bar{B} \rangle = f_B f_K f_\pi \sum_{i=1}^{10} b_i(\pi K). \quad (7)$$

The term $\mathcal{T}_P$ arises from the vertex corrections, penguin corrections, and hard spectator scattering contributions which are involved in the parameters $a_i(\pi K)$. The $\langle \pi K | Q_i | \bar{B} \rangle_F$ are the factorizable matrix elements; i.e., if any operator $Q = j_1 \otimes j_2$, then $\langle \pi K | Q_i | \bar{B} \rangle_F = \langle \pi | j_1 | \bar{B} \rangle \times \langle K | j_2 | 0 \rangle$ or $\langle K | j_1 | \bar{B} \rangle \langle \pi | j_2 | 0 \rangle$. The other term $\mathcal{T}_p^{\rm ann}$ includes the weak annihilation contributions which are absorbed in the parameters $b_i(\pi K)$. Following the notation of Ref. [11], we write the decay amplitude of $B \to K\pi$ as:

$$A_{B^- \to \pi^- \overline{K}^0} = \sum_{p=u,c} \lambda_p A_{\pi \overline{K}} \Big[\delta_{pu} \beta_2 + \alpha_4^p - \frac{1}{2} \alpha_{4,\rm EW}^p + \beta_3^p + \beta_{3,\rm EW}^p \Big], \quad (8)$$

$$\sqrt{2} A_{B^- \to \pi^0 K^-} = \sum_{p=u,c} \lambda_p A_{\pi \overline{K}} [\delta_{pu}(\alpha_1 + \beta_2) + \alpha_4^p + \alpha_{4,\rm EW}^p + \beta_3^p + \beta_{3,\rm EW}^p] + \sum_{p=u,c} \lambda_p A_{\overline{K}\pi} \Big[\delta_{pu} \alpha_2 + \frac{3}{2} \alpha_{3,\rm EW}^p \Big], \quad (9)$$

$$A_{\overline{B}^0 \to \pi^+ K^-} = \sum_{p=u,c} \lambda_p A_{\pi \overline{K}} \Big[\delta_{pu} \alpha_1 + \alpha_4^p + \alpha_{4,\rm EW}^p + \beta_3^p - \frac{1}{2} \beta_{3,\rm EW}^p \Big], \quad (10)$$

$$\sqrt{2} A_{\overline{B}^0 \to \pi^0 \overline{K}^0} = \sum_{p=u,c} \lambda_p A_{\pi \overline{K}} \Big[-\alpha_4^p + \frac{1}{2} \alpha_{4,\rm EW}^p - \beta_3^p + \frac{1}{2} \beta_{3,\rm EW}^p \Big] + \sum_{p=u,c} \lambda_p A_{\overline{K}\pi} \Big[\delta_{pu} \alpha_2 + \frac{3}{2} \alpha_{3,\rm EW}^p \Big]. \quad (11)$$

Here the coefficients of the flavor operators $\alpha_i^p(\pi K)$ and $\beta_i^p(\pi K)$ are given in terms of the coefficients $a_i^p(\pi K)$ and $b_i^p(\pi K)$, respectively [11]. The parameter $A_{\pi \bar{K}}$ ($A_{\bar{K}\pi}$) is given by $i(G_F/\sqrt{2}) m_B^2 F_0^{B \to \pi(K)} f_{K(\pi)}$. Note that the parameters b_i of the weak annihilation and hard scattering contributions contain infrared divergence which are usually parametrized as

$$X_{A,H} \equiv (1 + \rho_{A,H} e^{i\phi_{A,H}}) \ln\Big(\frac{m_B}{\Lambda_h}\Big), \quad (12)$$

where $\rho_{A,H}$ are free parameters to be of order 1, $\phi_{A,H} \in [0, 2\pi]$, and $\Lambda_h = 0.5$. As discussed in Ref. [5], the experimental measurements of the branching ratios impose an upper bound on the parameter ρ_A. If one does not assume fine-tuning between the parameters ρ and ϕ, the typical upper bound on ρ_A is of the order of $\rho_A \lesssim 2$.

Fixing the experimental and the SM parameters to their center values, one can determine the explicit dependence of the decay amplitudes of the $B \to K\pi$ on the corresponding Wilson coefficients. For instance, with $\gamma = \pi/3$, and $\rho_{A,H}$ and $\phi_{A,H}$ are of order 1, the decay amplitude of $\bar{B}^0 \to K^-\pi^+$ is given by

$$A_{\bar{B}^0 \to \pi^+ K^-} \times 10^8 \simeq (1.05 - 0.02i) C_1 + (0.24 + 0.07i) C_2 + (3.1 + 14.5i) C_3 + (4.9 + 37.7i) C_4 - (2.9 - 13.1i) C_5 + (5.5 - 43.7i) C_6 + (1.7 + 10.4i) C_7 + (5.8 + 36.5i) C_8 + (2.8 + 12.7i) C_9 + (0.6 + 35.5i) C_{10} - (0.0006 + 0.04i) C_{7\gamma}^{\rm eff} - (0.04 + 2.5i) C_{8g}^{\rm eff}. \quad (13)$$

A similar expression can be obtained for $\bar{B}^0 \to K^0 \pi^0$:

$$A_{\bar{B}^0 \to \pi^0 K^0} \times 10^8 \simeq (-0.14 + 0.3i) C_1 + (0.4 + 0.2i) C_2 - (2.2 + 10.5i) C_3 - (3.5 + 26.7i) C_4 + (2.1 - 9.3i) C_5 + (3.9 - 30.9i) C_6 - (1.7 + 37.02i) C_7 - (1.9 + 1.7i) C_8 + (1.3 + 46.6i) C_9 + (2.2 + 28.8i) C_{10} - (0.0002 + 0.01i) C_{7\gamma}^{\rm eff} + (0.03 + 1.8i) C_{8g}^{\rm eff}. \quad (14)$$

The amplitude of $B^- \to K^- \pi^0$ can be written as

$$A_{B^- \to \pi^0 K^-} \times 10^8 \simeq (0.9 + 0.06i) C_1 + (0.6 + 0.3i) C_2 + (2.2 + 10.5i) C_3 + (3.5 + 26.7i) C_4 - (2.1 - 9.3i) C_5 - (3.9 - 30.9i) C_6 - (2.7 + 32.4i) C_7 - (5.4 - 17.8i) C_8 + (1.9 + 51.6i) C_9 + (2.4 + 42.8i) C_{10} - (0.0004 + 0.03i) C_{7\gamma}^{\rm eff} - (0.03 + 1.8i) C_{8g}^{\rm eff}. \quad (15)$$

Finally, the amplitude of $B^- \to K^0 \pi^-$ is given by

$$A_{B^-\to\pi^-K^0}\times 10^8 \simeq (0.4-0.4i)C_1-(0.00004-0.02i)C_2+(3.1+14.9i)C_3+(4.9+37.7i)C_4-(2.9-13.1i)C_5 + (5.5-43.7i)C_6-(3.2+3.8i)C_7-(10.8+13.8i)C_8-(1.9+5.7i)C_9-(0.3+19.7i)C_{10} + (0.0003+0.02i)C_{7\gamma}^{\rm eff}-(0.04+2.5i)C_{8g}^{\rm eff}, \quad (16)$$

where $C_{7\gamma}^{\rm eff}=C_{7\gamma}-\frac{1}{3}C_5-C_6$ and $C_{8g}^{\rm eff}=C_{8g}+C_5$. The SM contributions to the Wilson coefficients of $b\to s$ transition, which are the relevant ones for $B\to K\pi$, are given by

$$\mathbf{C}_1^{\rm SM}\simeq 1.077,\quad \mathbf{C}_2^{\rm SM}\simeq -0.175,\quad \mathbf{C}_3^{\rm SM}\simeq 0.012,\quad \mathbf{C}_4^{\rm SM}\simeq -0.33,\quad \mathbf{C}_5^{\rm SM}\simeq 0.0095,\quad \mathbf{C}_6^{\rm SM}\simeq -0.039,$$
$$\mathbf{C}_7^{\rm SM}\simeq 0.0001,\quad \mathbf{C}_8^{\rm SM}\simeq 0.0004,\quad \mathbf{C}_9^{\rm SM}\simeq -0.01,\quad \mathbf{C}_{10}^{\rm SM}\simeq 0.0019,\quad \mathbf{C}_{7\gamma}^{\rm SM}\simeq -0.315,\quad \mathbf{C}_{8g}^{\rm SM}\simeq -0.149. \quad (17)$$

From these values, it is clear that, within the SM, the dominant contribution to the $B\to K\pi$ decay amplitudes comes from the QCD penguin operator Q_4. However, the QCD penguin preserves the isospin. Therefore, this contribution is the same for all the decay modes. Isospin violating contributions to the decay amplitudes arise from the current-current operators Q_1^u and Q_2^u, which are called the "tree" contribution, and from the electroweak penguins, which are suppressed by a power α/α_s. As can be seen from the coefficients of C_{7-10} in Eqs. (13)–(16), the electroweak penguin contributions to the amplitudes of $B\to K\pi$ could be, in general, sizable and nonuniversal. However, due to the small values of the corresponding Wilson coefficients in the SM (17), these contributions are quite suppressed.

Note also that the Q_1 contribution to $A_{\bar{B}^0\to\pi^+K^-}$ and $A_{B^-\to\pi^0K^-}$ is 1 order of magnitude larger than its contribution to the other two decay amplitudes. Therefore, in the SM, the amplitudes $A_{\bar{B}^0\to\pi^+K^-}$ and $A_{B^-\to\pi^0K^-}$ can be approximated as functions of C_1 and C_4, while the amplitudes $A_{\bar{B}^0\to\pi^0K^0}$ and $A_{B^-\to\pi^-K^0}$ are approximately given in terms of C_4 only. It is worth noting that the difference between the coefficients of C_4 in the amplitudes $A_{\bar{B}^0\to\pi^+K^-}$ and $A_{B^-\to\pi^0K^-}$ is just due to the factor of $\sqrt{2}$ in Eq. (9), which is the same difference between the corresponding coefficients in $A_{\bar{B}^-\to\pi^-K^0}$ and $-A_{B^0\to\pi^0K^0}$.

We are now in a position to determine the SM results for the CP asymmetries and the CP average branching ratios of $B\to K\pi$ decays within the framework of the QCD factorization approximation. The direct CP violation may arise in the decay $B\to K\pi$ from the interference between the tree and the penguin diagrams. The direct CP asymmetry of $B^0\to K^-\pi^+$ decay $A^{CP}_{K^-\pi^+}$ is defined as

$$A^{CP}_{K^-\pi^+}=\frac{|A(\bar{B}^0\to K^-\pi^+)|^2-|A(B^0\to K^+\pi^-)|^2}{|A(\bar{B}^0\to K^-\pi^+)|^2+|A(B^0\to K^+\pi^-)|^2}, \quad (18)$$

and similar expressions for the asymmetries $A^{CP}_{\bar{K}^0\pi^-}$, $A^{CP}_{K^-\pi^0}$, and $A^{CP}_{\bar{K}^0\pi^0}$. Also, the branching ratio can be written in terms of the corresponding decay amplitude as

$$\mathrm{BR}\,(B\to K\pi)=\frac{1}{8\pi}\frac{|P|}{M_B^2}|A(B\to K\pi)|^2\frac{1}{\Gamma_{\rm tot}}, \quad (19)$$

where

$$|P|=\frac{[(M_B^2-(m_K+m_\pi)^2)(M_B^2-(m_K-m_\pi)^2)]^2}{2M_B}. \quad (20)$$

The SM results are summarized in Tables II and III. In Table II, we present the predictions for the branching ratios of the four decay modes of $B\to\pi K$. We assume that $\gamma=\pi/3$ and consider some representative values of $\rho_{A,H}$ and $\phi_{A,H}$ to check the corresponding uncertainty. Namely, $\rho_{A,H}=0,1,3$ and $\phi_{A,H}=\mathcal{O}(1)$ are considered. From these results, one can see that for $\rho_{A,H}\in[0,1]$ the SM predicted values for the branching ratios of $B\to K\pi$ are less sensitive to the hadronic parameters. Larger values of $\rho_{A,H}$ enhance the branching ratios, and eventually they

TABLE II. The SM predictions for the branching ratios of the four decay modes of $B\to K\pi$ with $\gamma=\pi/3$.

Branching ratio	$\rho_{A,H}=0$	$\rho_{A,H}=1$ and $\phi_{A,H}\sim 1$	$\rho_{A,H}=3$ and $\phi_{A,H}\sim 1$
$\mathrm{BR}_{\bar{K}^0\pi^-}\times 10^6$	31.06	33.35	43.92
$\mathrm{BR}_{K^-\pi^0}\times 10^6$	17.31	18.45	23.36
$\mathrm{BR}_{K^-\pi^+}\times 10^6$	25.87	27.98	39.55
$\mathrm{BR}_{\bar{K}^0\pi^0}\times 10^6$	11.41	12.47	18.66
R_n	1.13	1.12	1.059
R_c	1.11	1.106	1.063
R	0.83	0.838	0.9

TABLE III. The SM predictions for the direct *CP* asymmetries of the four decay modes of $B \to \pi K$ with $\gamma = \pi/3$.

CP asymmetry	$\rho_{A,H} = 0$	$\rho_{A,H} = 1$ and $\phi_{A,H} \sim 1(-1)$	$\rho_{A,H} = 3$ and $\phi_{A,H} \sim 1(-1)$
$A^{CP}_{\bar{K}^0\pi^-}$	0.007	0.0086 (0.005)	0.0078 (0.001)
$A^{CP}_{K^-\pi^0}$	0.029	0.063 (−0.006)	0.185 (−0.15)
$A^{CP}_{K^-\pi^+}$	0.0044	0.057 (−0.049)	0.194 (−0.19)
$A^{CP}_{\bar{K}^0\pi^0}$	−0.02	−0.013 (−0.025)	−0.019 (−0.002)

exceed the experimental limits presented in Table I for $\rho > 2$. It is also remarkable that the SM results for $\mathrm{BR}(B^- \to \bar{K}^0\pi^-)$ and $\mathrm{BR}(\bar{B}^0 \to \bar{K}^0\pi^0)$ are larger than the experiment measurements, while the results for $\mathrm{BR}(B^- \to K^-\pi^0)$ and $\mathrm{BR}(\bar{B}^0 \to K^-\pi^+)$ are consistent with their experimental values. This discrepancy does not seem to be resolved in the SM, even if we consider large hadronic uncertainties. The parameters R_c and R_n, defined in Eqs. (1) and (2) as the ratio of the *CP* average branching ratios of $B \to K\pi$, exhibit this deviation from the SM prediction in a clear way. The results in Table II show that in the SM $R_c \simeq R_n > 1$. However, the recent experimental measurements reported in Table I imply that $R_c \sim 1$ and $R_n < 1$. It is very difficult to have this situation within the SM. As emphasized above, in the SM the amplitudes of $B \to K\pi$ can be approximately written as

$$A_{\bar{B}^0\to\pi^+K^-} \simeq (a_1 + b_1 i)C_1 + (a_2 + b_2 i)C_4, \tag{21}$$

$$A_{\bar{B}^0\to\pi^0K^0} \simeq -\frac{1}{\sqrt{2}}(a_2 + b_2 i)C_4, \tag{22}$$

$$A_{B^-\to\pi^0K^-} \simeq \frac{1}{\sqrt{2}}(a_1 + b_1 i)C_1 + \frac{1}{\sqrt{2}}(a_2 + b_2 i)C_4, \tag{23}$$

$$A_{B^-\to\pi^-K^0} \simeq (a_2 + b_2 i)C_4. \tag{24}$$

Thus, the parameters R_c and R_n are given by

$$R_c = R_n = \frac{|(a_1 + b_1 i)C_1 + (a_2 + b_2 i)C_4|^2}{|(a_2 + b_2 i)C_4|^2} \gtrsim 1, \tag{25}$$

which is consistent with the result given in Table II, using the full set of Wilson coefficients.

Now we turn to the SM predictions for the *CP* asymmetries of $B \to K\pi$. Let us start by considering the approximation that the decay amplitudes for $B^- \to \bar{K}^0\pi^-$ and $\bar{B}^0 \to \bar{K}^0\pi^0$ are dominated by the pure gluon penguin operator Q_4, while the amplitudes for $B^- \to K^-\pi^0$ and $\bar{B}^0 \to K^-\pi^+$ are given by Q_4 and also by the tree contribution of the current-current operator Q_1. In this case, the following results are expected: The direct *CP* asymmetries $A^{CP}_{\bar{K}^0\pi^-}$ and $A^{CP}_{\bar{K}^0\pi^0}$ should be very tiny (equal zero in the exact limit of this approximation). The direct *CP* asymmetries $A^{CP}_{K^-\pi^0}$ and $A^{CP}_{K^-\pi^+}$ should be of the same order and larger than the other two asymmetries.

The SM results of the *CP* asymmetries for the different decay modes, including the effect of all local operators Q_i, are given in Table III. As in the case of the branching ratios, we assume that $\gamma = \pi/3$ and $\rho_{A,H} = 0, 1, 3$. Respecting the strong phases $\phi_{A,H}$, we take it to be of order 1 as before. Because of the sensitivity of the *CP* asymmetry on their sign, we consider both cases of $\phi_{A,H} = \mathcal{O}(\pm 1)$. A few comments on the results of the direct *CP* asymmetries given in Table II are in order:

(1) The *CP* asymmetries $A^{CP}_{K^-\pi^0}$ and $A^{CP}_{K^-\pi^+}$ are sensitive to the sign ϕ_A (note that ϕ_H is irrelevant for these processes). On the contrary, the *CP* asymmetries $A^{CP}_{\bar{K}^0\pi^-}$ and $A^{CP}_{\bar{K}^0\pi^0}$ are insensitive to this sign.

(2) As expected, the results of the *CP* asymmetries $A^{CP}_{\bar{K}^0\pi^-}$ and $A^{CP}_{\bar{K}^0\pi^0}$ are very small even with large values of ρ_A.

(3) The value of $A^{CP}_{K^-\pi^0}$ and $A^{CP}_{K^-\pi^+}$ can be enhanced by considering a large value of ρ_A and one gets values for $A^{CP}_{K^-\pi^+}$ of the order of the experimental result given in Table I. However, it is very important to note that, in this case, the *CP* asymmetry $A^{CP}_{K^-\pi^0}$ is also enhanced in the same way and it becomes 1 order of magnitude larger than its experimental value.

While a confirmation with more accurate experimental data is necessary, the above results of the branching ratio and the direct *CP* asymmetries of $B \to \pi K$ show that within the SM the current experimental measurements listed in Table I do not seem to be accommodated even if one considers large hadronic uncertainties. It is worth stressing that the QCD correction would not play an essential role in solving this $K\pi$ puzzle. Furthermore, since we are interested here in the ratio of the amplitudes, many of the theoretical uncertainties cancel. So it cannot be the source of these discrepancies.

Another useful way of parametrizing the decay amplitudes can be obtained by factorizing the dominant penguin amplitude P, where P is defined as [12]

$$Pe^{i\delta_P} = \alpha_4^c - \tfrac{1}{2}\alpha_{4,\mathrm{EW}}^c + \beta_3^c + \beta_{3,\mathrm{EW}}^c. \tag{26}$$

In this case, one can write the above expressions for the decay amplitude as follows:

$$\begin{aligned} A_{B^-\to\pi^-\overline{K}^0} &= \lambda_c A_{\pi\overline{K}} P[1 + r_A e^{i\delta_A} e^{-i\gamma}], \\ \sqrt{2}A_{B^-\to\pi^0 K^-} &= \lambda_c A_{\pi\overline{K}} P[1 + (r_A e^{i\delta_A} + r_C e^{i\delta}) e^{-i\gamma} + r_{\rm EW} e^{i\delta_{\rm EW}}], \\ A_{\overline{B}^0\to\pi^+ K^-} &= \lambda_c A_{\pi\overline{K}} P[1 + (r_A e^{i\delta_A} + r_T e^{i\delta_T}) e^{-i\gamma} + r^C_{\rm EW} e^{i\delta^C_{\rm EW}}], \\ -\sqrt{2}A_{\overline{B}^0\to\pi^0\overline{K}^0} &= \lambda_c A_{\pi\overline{K}} P[1 + (r_A e^{i\delta_A} + r_T e^{i\delta_T} - r_C e^{i\delta_C}) e^{-i\gamma} + r^C_{\rm EW} e^{i\delta^C_{\rm EW}} - r_{\rm EW} e^{i\delta_{\rm EW}}], \end{aligned} \quad (27)$$

where

$$r_A e^{i\delta_A} = \epsilon_{KM}[\beta_2 + \alpha_4^u - \tfrac{1}{2}\alpha^u_{4,\rm EW} + \beta_3^u + \beta^u_{3,\rm EW}]/P, \quad (28)$$

$$r_T e^{i\delta_T} = \epsilon_{KM}[\alpha_1 + \tfrac{3}{2}\alpha^u_{4,\rm EW} - \tfrac{3}{2}\beta^u_{3,\rm EW} - \beta_2)]/P, \quad (29)$$

$$r_C e^{i\delta_P} = \epsilon_{KM}[\alpha_1 + R_{K\pi}\alpha_2 + \tfrac{3}{2}(R_{K\pi}\alpha^u_{3,\rm EW} + \alpha^u_{4,\rm EW})]/P, \quad (30)$$

$$r_{\rm EW} e^{i\delta_{\rm EW}} = [\tfrac{3}{2}(R_{K\pi}\alpha^c_{3,\rm EW} + \alpha^c_{4,\rm EW})]/P, \quad (31)$$

$$r^C_{\rm EW} e^{i\delta^C_{\rm EW}} = [\tfrac{3}{2}(\alpha^c_{4,\rm EW} - \beta^c_{3,\rm EW})]/P. \quad (32)$$

Here we define $\lambda_u/\lambda_c \equiv \epsilon_{KM} e^{-i\gamma}$, $R_{K\pi} = A_{\pi\overline{K}}/A_{\overline{K}\pi}$, and δ_A, δ_T, δ_C, $\delta_{\rm EW}$, and $\delta^C_{\rm EW}$ as strong interaction phases. The SM contributions within the QCD factorization lead to the following results:

$$\begin{aligned} (Pe^{i\delta_P})_{\rm SM} &= -0.11e^{0.051i}, \qquad (r_A e^{i\delta_A})_{\rm SM} = 0.019e^{0.26i}, \\ (r_C e^{i\delta_C})_{\rm SM} &= 0.186e^{2.9i}, \qquad (r_T e^{i\delta_T})_{\rm SM} = 0.191e^{2.9i}, \\ (r_{\rm EW} e^{i\delta_{EW}})_{\rm SM} &= 0.13e^{-0.2i}, \\ (r^C_{\rm EW} e^{i\delta^C_{\rm EW}})_{\rm SM} &= 0.012e^{-2.5i}. \end{aligned} \quad (33)$$

As can be seen from this result, within the SM r_A and $r^C_{\rm EW}$ are much smaller than r_C, r_T, and $r_{\rm EW}$, so that they can be easily neglected. In this case, the parameters R_c and R_n can be expressed by the following approximated expressions:

$$R_c \simeq 1 + 2r_C \cos\delta_C \cos\gamma + 2r_{\rm EW}\cos\delta_{\rm EW}, \quad (34)$$

$$R_n \simeq \frac{1 + 2r_T\cos\delta_T\cos\gamma}{1 + 2r_T\cos\delta_T\cos\gamma - 2r_C\cos\delta_C\cos\gamma - 2r_{\rm EW}\cos\delta_{\rm EW}}, \quad (35)$$

which confirms our previous conclusion that in the SM $R_n \sim R_c \gtrsim 1$. Explicitly, using the results of Eq. (33), one finds that

$$R_c = 1.08(1.45), \qquad R_n = 1.13(1.6), \qquad R = 0.757(0.673) \quad (36)$$

for $\gamma = \pi/3(2\pi/3)$, which is quite close to the full result that we obtained in Table II, with $\rho_A \sim 1$.

Now we would like to comment on the mixing CP asymmetry of $B \to K\pi$. CP violation in the interference between mixing and decay can be observed as time dependent oscillation of the CP asymmetry. The amplitude of the oscillation in charmonium decay modes provides a theoretical clean determination of the parameter $\sin 2\beta$ of the unitary triangle. The SM predicts the B-decay modes, dominated by a single penguin amplitude such that $B \to \phi K$, $B \to \eta' K$, and $B \to K^0\pi^0$ have the same time dependent CP asymmetry equal to $\sin 2\beta$. Again, this result contradicts the experimental measurement given in Table I. Note that the latest experimental results on the mixing CP asymmetry of $B \to \phi K_S$ process are given by [2,3]

$$\begin{aligned} S_{\phi K_S} &= 0.50 \pm 0.25^{+0.07}_{-0.04}(BABAR) \\ &= 0.06 \pm 0.33 \pm 0.09(\text{Belle}), \end{aligned} \quad (37)$$

where the first errors are statistical and the second systematic. Thus, the average of this CP asymmetry is $S_{\phi K_S} = 0.34 \pm 0.20$. On the other hand, the most recent measured CP asymmetry in the $B^0 \to \eta' K_S$ decay is found by *BABAR* [2] and Belle [3] Collaborations as

$$\begin{aligned} S_{\eta' K_S} &= 0.27 \pm 0.14 \pm 0.03(BABAR) \\ &= 0.65 \pm 0.18 \pm 0.04(\text{Belle}), \end{aligned} \quad (38)$$

with an average $S_{\eta' K_S} = 0.41 \pm 0.11$, which shows a 2.5σ discrepancy from the SM expectation. This difference among $S_{\phi K}$, $S_{\eta' K}$, $S_{K^0\pi^0}$, and $\sin 2\beta$ is also considered as a hint for new physics beyond the SM, in particular for supersymmetry.

III. $B \to K\pi$ IN SUSY MODELS

As mentioned in the previous section, due to the asymptotic freedom of QCD, the calculation of the hadronic decay amplitude of $B \to K\pi$ can be factorized by the product of long and short distance contributions. The short distance contributions, including the SUSY effects, are contained in the Wilson coefficients C_i.

The SUSY contributions to the $b \to s$ transition could be dominated by the gluino or the chargino intermediated penguin diagrams [5]. It turns out that the dominant effect in both contributions is given by a chromomagnetic penguin (Q_{8g}). However, in the case of $B \to K\pi$, it was observed that this process is more sensitive to the isospin violating interactions [8,9], namely, the contributions from the electromagnetic penguin ($Q_{7\gamma}$) and photon- and Z-penguin contributions to Q_7 and Q_9. Therefore, in our discussion we will focus only on these contributions, although in our numerical analysis we keep all the contributions of the gluino and the chargino.

For the gluino exchange, it turns out that the Z-penguin contributions to $C_{7,9}$ are quite small and can be neglected with respect to the photon-penguin contributions. At the first order in the mass insertion approximation, the gluino contributions to the Wilson coefficients $C_{7\gamma,8g}$, C_7, and C_9 at SUSY scale M_S are given by

$$C_7^{\tilde{g}}(M_S) = C_9(M_S) = \frac{2\alpha_s \alpha}{9\sqrt{2} G_F m_{\tilde{q}}^2} \frac{1}{3} (\delta_{LL}^d)_{23} P_{042}(x, x), \tag{39}$$

$$C_{7\gamma}^{\tilde{g}}(M_S) = \frac{8\alpha_s \pi}{9\sqrt{2} G_F m_{\tilde{q}}^2} \bigg[(\delta_{LL}^d)_{23} M_3(x) + (\delta_{LR}^d)_{23} \times \frac{m_{\tilde{g}}}{m_b} M_1(x) \bigg], \tag{40}$$

$$C_{8g}^{\tilde{g}}(M_S) = \frac{\alpha_s \pi}{\sqrt{2} G_F m_{\tilde{q}}^2} \bigg[(\delta_{LL}^d)_{23} \bigg(\frac{1}{3} M_3(x) + 3M_4(x) \bigg) + (\delta_{LR}^d)_{23} \frac{m_{\tilde{g}}}{m_b} \bigg(\frac{1}{3} M_3(x) + 3M_2(x) \bigg) \bigg], \tag{41}$$

where $x = m_{\tilde{g}}^2 / m_{\tilde{q}}^2$ and the functions $M_1(x)$, $M_2(x)$, and $P_{ijk}(x, x)$ can be found in Refs. [13,14]. The coefficients $\tilde{C}_{7\gamma,8g}$ and $\tilde{C}_{7,9}$ are obtained from $C_{7\gamma,8g}$ and $C_{7,9}$, respectively, by the chirality exchange $L \leftrightarrow R$. As can be seen from Eqs. (40) and (41), the term proportional to $(\delta_{LR}^d)_{23}$ in the coefficients $C_{7\gamma,8g}$ has a large enhancement factor $m_{\tilde{g}}/m_b$. This enhancement factor is responsible for the dominant gluino effects in B decays, although this mass insertion is strongly constrained from $b \to s\gamma$. Note also that, since the photon penguin gives the same contributions to C_7 and C_9, and we neglect the Z-penguin contributions, we have $C_7 = C_9$. Finally, it is clear that the coefficient $C_{7,9}$ is suppressed with respect to $C_{7\gamma,8g}$ by a factor $\alpha/4\pi$ at least.

It is worth mentioning that the mass insertion $(\delta_{LR}^d)_{23}$ can be generated by the mass insertion $(\delta_{LL}^d)_{23}$ as follows :

$$(\delta_{LR}^d)_{23} = (\delta_{LL}^d)_{23} (\delta_{LR}^d)_{33},$$

where

$$(\delta_{LR}^d)_{33} \sim \frac{m_b(A_b - \mu \tan\beta)}{m_{\tilde{d}}^2} \sim \frac{m_b}{m_{\tilde{d}}^2} \tan\beta \sim 10^{-2} \tan\beta.$$

Therefore,

$$(\delta_{LR}^d)_{23} \simeq 10^{-2} \tan\beta (\delta_{LL}^d)_{23}.$$

Hence, for a moderate value of $\tan\beta$ and $(\delta_{LL}^d)_{23} \sim \mathcal{O}(0.1)$, one obtains $(\delta_{LR}^d)_{23}$ of order 10^{-2}, which can easily imply significant contributions for the $S_{\phi K}$ and also account for the different results between $S_{\phi K}$ and $S_{\eta' K}$. Thus, in our analysis we define

$$(\delta_{LR}^d)_{23_{\rm eff}} = (\delta_{LR}^d)_{23} + (\delta_{LL}^d)_{23} (\delta_{LR}^d)_{33}. \tag{42}$$

It is important to stress that, in the case of $(\delta_{LR}^d)_{23_{\rm eff}}$ dominated by double mass insertions, we still call this scenario as LR contribution. This is due to the fact that the main SUSY contribution is still through the C_{8g}, which is enhanced by the chirality flipped factor $m_{\tilde{g}}/m_b$. In the literature [16], this contribution has been considered in analyzing the CP asymmetry of $B \to \phi K$ and it was called as LL contribution, as an indication for the large mixing in the squark mass matrix and dominant effect of $(\delta_{LL}^d)_{23}$. However, we prefer to work with the notation $LR_{\rm eff}$ to be able to trace the effective operators that may lead to dominant contributions for different B decay channels.

The dominant chargino contributions are found to be also due to the chromomagnetic-penguin, magnetic-penguin, and Z-penguin diagrams. As emphasized in Ref. [5], these contributions depend on the up sector mass insertion $(\delta_{LL}^u)_{32}$ and $(\delta_{RL}^u)_{32}$, while the LR and RR contributions are suppressed by λ^2 or λ^3, where λ is the Cabibbo mixing. At the first order in the mass insertion approximation, the chargino contributions to the Wilson coefficients are given by [5]

$$C_7^{\chi}(M_S) = \frac{\alpha}{6\pi} (4C_\chi + D_\chi), \tag{43}$$

$$C_9^{\chi}(M_S) = \frac{\alpha}{6\pi} \bigg(4 \bigg(1 - \frac{1}{\sin^2\theta_W} \bigg) C_\chi + D_\chi \bigg), \tag{44}$$

$$C_{7\gamma}^{\chi} = M_\gamma, \tag{45}$$

$$C_{8g}^{\chi} = M_g, \tag{46}$$

where the functions $F \equiv C_\chi$ (Z penguin), D_χ (photon penguin), M^γ (magnetic penguin), and M^g (chromomagnetic penguin) are given by [5]

$$F_\chi = [(\delta_{LL}^u)_{32} + \lambda (\delta_{LL}^u)_{31}] R_F^{LL} + [(\delta_{RL}^u)_{32} + \lambda (\delta_{RL}^u)_{31}] Y_t R_F^{RL}. \tag{47}$$

The functions R_F^{LL} and R_F^{RL}, F depend on the SUSY parameters through the chargino masses (m_{χ_i}), squark masses ($\tilde{m}$), and the entries of the chargino mass matrix. For the Z and magnetic (chromomagnetic) dipole penguins $R_C^{LL,RL}$ and $R_{M^{\gamma(g)}}^{LL,RL}$, respectively, we have [5]

$$\begin{aligned}
R_C^{LL} &= \sum_{i=1,2} |V_{i1}|^2 P_C^{(0)}(\bar{x}_i) + \sum_{i,j=1,2} \bigg[U_{i1} V_{i1} U_{j1}^\star V_{j1}^\star P_C^{(2)}(x_i, x_j) + |V_{i1}|^2 |V_{j1}|^2 \bigg(\frac{1}{8} - P_C^{(1)}(x_i, x_j) \bigg) \bigg], \\
R_C^{RL} &= -\frac{1}{2} \sum_{i=1,2} V_{i2}^\star V_{i1} P_C^{(0)}(\bar{x}_i, \bar{x}_{it}) - \sum_{i,j=1,2} V_{j2}^\star V_{i1} (U_{i1} U_{j1}^\star P_C^{(2)}(x_i, x_{it}, x_j, x_{jt}) + V_{i1}^\star V_{j1} P_C^{(1)}(x_i, x_j)), \\
R_{M^{\gamma,g}}^{LL} &= \sum_i |V_{i1}|^2 x_{Wi} P_{M^{\gamma,g}}^{LL}(x_i) - Y_b \sum_i V_{i1} U_{i2} x_{Wi} \frac{m_{\chi_i}}{m_b} P_{M_{\gamma,g}}^{LR}(x_i), \\
R_{M^{\gamma,g}}^{RL} &= -\sum_i V_{i1} V_{i2}^\star x_{Wi} P_{M_{\gamma,g}}^{LL}(x_i, x_{it}),
\end{aligned} \tag{48}$$

where Y_b is the Yukawa coupling of bottom quark, $x_{Wi} = m_W^2/m_{\chi_i}^2$, $x_i = m_{\chi_i}^2/\tilde{m}^2$, $\bar{x}_i = \tilde{m}^2/m_{\chi_i}^2$, and $x_{it} = m_{\chi_i}^2/m_{\tilde{t}_R}^2$. The loop functions $P_{M_{\gamma,g}}^{LL(LR)}$ can be found in Ref. [5]. Finally, U and V are the matrices that diagonalize the chargino mass matrix.

Notice that the terms in $R_{M_\gamma}^{LL}$ and $R_{M_g}^{LL}$ which are enhanced by m_{χ_i}/m_b in Eq. (48) lead to the large effects of chargino contributions to $C_{7\gamma}$ and C_{8g}, respectively. Also, the dependence of these terms on Yukawa bottom Y_b enhance the LL contributions in $C_{7\gamma,8g}$ at large $\tan\beta$. In the case of light stop-right, the function R_C^{RL} of the Z-penguin contribution is largely enhanced. In order to understand the impact of the chargino contributions in the $B \to K\pi$ process, it is very useful to present the explicit dependence of the Wilson coefficients $C_{7,9,7\gamma,8g}$ in terms of the relevant mass insertions. For gaugino mass $M_2 = 200$ GeV, squark masses $\tilde{m} = 500$ GeV, light stop $\tilde{m}_{\tilde{t}_R} = 150$ GeV, $\mu = 400$ GeV, and $\tan\beta = 10$, we obtain

$$\begin{aligned} C_7^\chi \simeq{}& 0.000002(\delta_{LL}^u)_{32} - 0.000011(\delta_{RL}^u)_{31} \\ &- 0.000046(\delta_{RL}^u)_{32}, \end{aligned} \tag{49}$$

$$\begin{aligned} C_9^\chi \simeq{}& 0.00000039(\delta_{LL}^u)_{32} + 0.000037(\delta_{RL}^u)_{31} \\ &+ 0.000165(\delta_{RL}^u)_{32}, \end{aligned} \tag{50}$$

$$\begin{aligned} C_{7\gamma}^\chi \simeq{}& -0.011(\delta_{LL}^u)_{31} - 0.05(\delta_{LL}^u)_{32} - 0.00043(\delta_{RL}^u)_{31} \\ &- 0.002(\delta_{RL}^u)_{32}, \end{aligned} \tag{51}$$

$$\begin{aligned} C_{8g}^\chi \simeq{}& -0.0032(\delta_{LL}^u)_{31} - 0.0014(\delta_{LL}^u)_{32} - 0.0003(\delta_{RL}^u)_{31} \\ &- 0.0012(\delta_{RL}^u)_{32}. \end{aligned} \tag{52}$$

From these results, it is clear that the Wilson coefficient $C_{7\gamma}^\chi$ seems to give the dominant contribution, especially through the LL mass insertion. However, one should be careful with this contribution since it is also the main contribution to the $b \to s\gamma$, and stringent constraints on $(\delta_{LL}^u)_{32}$ are usually obtained, especially with large $\tan\beta$. Finally, as expected from Eq. (48), only LL contributions to $C_{7\gamma}^\chi$ and C_{8g}^χ have strong dependence on the value of $\tan\beta$. For instance, with $\tan\beta = 40$, these contributions are enhanced with a factor 4, while the result of $C_{7,9}^\chi$ and LR part of $C_{7\gamma}^\chi$ and C_{8g}^χ change from the previous ones by less than 2%.

IV. ON THE CONSTRAINTS FROM BR($B \to X_s\gamma$)

In this section, we revise the constraints on SUSY flavor structure which arise from the experimental measurements of the branching ratio of the $B \to X_s\gamma$ [17]:

$$2 \times 10^{-4} < \mathrm{BR}(b \to s\gamma) < 4.5 \times 10^{-4} \quad \text{(at 95\% C.L.)}. \tag{53}$$

In supersymmetric models, there are additional contributions to $b \to s\gamma$ decay besides the SM diagrams with a W-gauge boson and an up quark in the loop. The SUSY particles running in the loop are charged Higgs bosons ($H^\pm$) or chargino with up quarks and gluino or neutralino with down squarks. The total amplitude for this decay is the sum of all these contributions. As advocated in the introduction, the neutralino contributions are quite small and can be safely neglected. Also, the charged Higgs contributions are relevant only at very large $\tan\beta$ and small charged Higgs mass. Therefore, we consider chargino and gluino contributions only to analyze the possible constraints on the mass insertions $(\delta_{AB}^u)_{32}$ and $(\delta_{AB}^d)_{23}$, where $A \equiv L, R$.

Although the gluino contribution to $b \to s\gamma$ is typically very small in models with minimal flavor structure, it is significantly enhanced in models with nonminimal favor structure [18]. In this class of models, both chargino and gluino exchanges give a large contribution to the amplitude of $b \to s\gamma$ decay, and, hence, they have to be simultaneously considered in analyzing the constraints of the branching ratio BR($b \to s\gamma$).

The relevant operators for this process are Q_2, $Q_{7\gamma}$, and Q_{8g}. The contributions of the other operators in Eq. (4) can be neglected. The branching ratio BR($b \to s\gamma$), conventionally normalized to the semileptonic branching ratio $\mathrm{BR}^{\mathrm{exp}}(B \to X_c e\nu) = (10.4 \pm 0.4)\%$ [19], is given by [20]

$$\mathrm{BR}^{\mathrm{NLO}}(B \to X_s\gamma) = \mathrm{BR}^{\mathrm{exp}}(B \to X_c e\nu)\frac{|V_{ts}^* V_{tb}|^2}{|V_{cb}|^2} \times \frac{6\alpha_{em}}{\pi g(z)k(z)}\left(1 - \frac{8}{3}\frac{\alpha_s(m_b)}{\pi}\right) \times (|D|^2 + A)(1 + \delta_{np}), \quad (54)$$

with

$$D = C_7^{(0)}(\mu) + \frac{\alpha_s(\mu)}{4\pi}\left(C_7^{(1)}(\mu) + \sum_{i=1}^{8} C_i^{(0)}(\mu)\left[r_i(z) + \gamma_{i7}^{(0)}\log\frac{m_b}{\mu}\right]\right),$$

$$A = (e^{-\alpha_s(\mu)\log\delta(7+2\log\delta)/3\pi} - 1)|C_7^{(0)}(\mu)|^2 + \frac{\alpha_s(\mu)}{\pi} \times \sum_{i\leq j=1}^{8} C_i^{(0)}(\mu)C_j^{(0)}(\mu)f_{ij}(\delta),$$

where $z = m_c^2/m_b^2$, μ is the renormalization scale which is chosen of order m_b, and ρ is photon energy resolution. The expressions for $C_i^{(0)}$, $C_i^{(1)}$, and the anomalous dimension matrix γ, together with the functions $g(z)$, $k(z)$, $r_i(z)$, and $f_{ij}(\delta)$, can be found in Ref. [20]. The term δ_{np} (of order a few percent) includes the nonperturbative $1/m_b$ [21] and $1/m_c$ [22] corrections. From the formula above, we obtain the theoretical result for $\mathrm{BR}(B \to X_s\gamma)$ in the SM, which is given by

$$\mathrm{B\,R}^{\mathrm{NLO}}(B \to X_s\gamma) = (3.29 \pm 0.33) \times 10^{-4}, \quad (55)$$

where the main theoretical uncertainty comes from uncertainties in the SM input parameters, namely, m_t, $\alpha_s(M_Z)$, α_{em}, m_c/m_b, m_b, V_{ij}, and the small residual scale dependence. The central value in Eq. (55) corresponds to the following central values for the SM parameters: $m_t^{\mathrm{pole}} \simeq m_t^{\overline{\mathrm{MS}}}(m_Z) \simeq 174$ GeV, $m_b^{\mathrm{pole}} = 4.8$ GeV, $m_c^{\mathrm{pole}} = 1.3$ GeV, $\mu = m_b$, $\alpha_s(m_Z) = 0.118$, $\alpha_e^{-1}(m_Z) = 128$, $\sin^2\theta_W = 0.23$, and a photon energy resolution corresponding to $\rho = 0.9$ is assumed.

The SUSY contributions to the Wilson coefficients $C_{7\gamma}$ and $C_8 g$ at leading order are given in the previous section. In general, the SUSY effects in $b \to s\gamma$ decay can be parametrized by introducing $R_{7,8}$ and $\tilde{R}_{7,8}$ parameters defined at the electroweak scale as

$$R_{7,8} = \frac{(C_{7\gamma,8g} - C_{7\gamma,8g}^{\mathrm{SM}})}{C_{7\gamma,8g}^{\mathrm{SM}}}, \qquad \tilde{R}_{7,8} = \frac{\tilde{C}_{7\gamma,8g}}{C_{7\gamma,8g}^{\mathrm{SM}}}, \quad (56)$$

where $C_{7\gamma,8g}$ include the total contribution, while $C_{7\gamma,8g}^{\mathrm{SM}}$ contains only the SM ones. Note that in $\tilde{C}_{7\gamma,8g}$, which are the corresponding Wilson coefficients for $\tilde{Q}_{7\gamma,8g}$, respectively, we have set to zero the SM contribution. Inserting these definitions into the $\mathrm{BR}(B \to X_s\gamma)$ formula in Eq. (54) yields a general parametrization of the branching ratio [18,23]

$$\mathrm{BR}(B \to X_s\gamma) = \mathrm{BR}^{\mathrm{SM}}(B \to X_s\gamma)(1 + 0.681\,\mathrm{Re}(R_7) + 0.116[|R_7|^2 + |\tilde{R}_7|^2] + 0.083\,\mathrm{Re}(R_8) + 0.025[\mathrm{Re}(R_7R_8^*) + \mathrm{Re}(\tilde{R}_7\tilde{R}_8^*)] + 0.0045[|R_8|^2 + |\tilde{R}_8|^2]). \quad (57)$$

From this parametrization, it is clear that $C_{7\gamma}$ would give the dominant new contribution (beyond the SM one) to $\mathrm{BR}(B \to X_s\gamma)$. Using the allowed experimental range given in Eq. (53), one can impose stringent constraints on $C_{7\gamma}$ and, hence, on the corresponding mass insertions. It is also remarkable that R_7 and $\tilde{R}_7$ have different contributions to the $\mathrm{BR}(B \to X_s\gamma)$; therefore, the possible constraints on $C_{7\gamma}$ and, hence, on the *LL* and *LR* mass insertions would be different from the constraints on $\tilde{C}_{7\gamma}$ and, hence, on the *RR* and *RL* mass insertions, unlike what has been assumed in the literature. Furthermore, since the leading contribution to the branching ratio is due to $\mathrm{Re}(R_7)$, the *CP* violating phase of $C_{7\gamma}$ will play a crucial role in the possible constraints imposed by $\mathrm{BR}(B \to X_s\gamma)$.

Note that the constraints obtained in Ref. [13], namely, $(\delta_{LR}^d)_{23} \leq 1.6 \times 10^{-2}$ and $(\delta_{LL}^d)_{23}$ is unconstrained, are based on the assumption that the gluino amplitude is the dominant contribution to $b \to s\gamma$, even dominant with respect to the SM amplitude. Although this a very acceptable assumption in order to derive conservative constraints on the relevant mass insertions, it is unrealistic and usually leads to unuseful constraints. The aim of this section is to provide a complete analysis of the $b \to s\gamma$ constraints by including the SM, chargino, and gluino contributions.

Let us start first with the gluino contribution as the dominant SUSY effect to $b \to s\gamma$ decay. We assume that the average squark mass of order 500 GeV, and we consider three representative values for $x = (m_{\tilde{g}}/m_{\tilde{q}})^2 = 0.3$, 1, and 4. We also assume that the SM value for $\mathrm{BR}(B \to X_s\gamma)$ is given by 3.29×10^{-4}, which is the central value of the results in Eq. (55). In these cases, we find that both the mass insertions $|(\delta_{LL}^d)_{23}|$ and $|(\delta_{RR}^d)_{23}|$ are unconstrained by the branching ratio of $b \to s\gamma$ for any values of their phases. The upper bounds on $|(\delta_{LR}^d)_{23}|$ and $|(\delta_{RL}^d)_{23}|$ from $b \to s\gamma$ decay are give in Table IV. As can be seen from these results, the limits on $|(\delta_{LR}^d)_{23}|$ are quite sensitive to the phase of this mass insertion, unlike the bounds on $|(\delta_{RL}^d)_{23}|$. Also, as suggested by Eq. (57), the bounds on *LR* coincide with the ones on *RL* only if $\arg(\delta_{LR(RL)}^d)_{23} = \pi/2$. Note that in this case $\mathrm{Re}(R_7)$ vanishes and the expression of the branching ratio is a symmetric under exchange R_7 and $\tilde{R}_7$.

Now we consider the chargino contribution as the dominant SUSY effect to $b \to s\gamma$ in order to analyze the bounds on the relevant mass insertions in the up-squark sector. From the expression of $C_{7\gamma}^{\chi}$ in Eq. (51), which provides the leading contribution to the branching ratio of $b \to s\gamma$, it is

TABLE IV. Upper bounds of $|(\delta^d_{LR(RL)})_{23}|$ from $b \to s\gamma$ decay for $m_{\tilde{q}} = 500$ GeV and $\arg(\delta^d_{LR(RL)})_{23} = 0$ (a), $\pi/2$ (b), π (c), respectively.

x	$\lvert(\delta^d_{LR})_{23}\rvert$	$\lvert(\delta^d_{RL})_{23}\rvert$
	(a) 0.0116	
0.3	(b) 0.0038	0.0038
	(c) 0.0012	
	(a) 0.02	
1	(b) 0.006	0.006
	(c) 0.002	
	(a) 0.006	
4	(b) 0.015	0.016
	(c) 0.0045	

clear that one can derive strong constraints on $(\delta^u_{LL})_{32}$ and $(\delta^u_{LL})_{31}$ and much weaker constraints (essentially no constraint) on $(\delta^u_{RL})_{32}$ and $(\delta^u_{RL})_{31}$. The resulting bounds on $(\delta^u_{LL})_{32}$ and $(\delta^u_{LL})_{31}$ as functions of the gaugino mass M_2 and the average squark mass $\tilde{m}$ are presented in Table V, for $\tan\beta = 10$ and $\mu = 400$ GeV.

The results in Table V correspond to a positive sign of μ. If one assumed a negative sign of μ, the constraints on $|(\delta^u_{LL})_{32}|$ and $|(\delta^u_{LL})_{31}|$ with $\arg(\delta^u_{LR})_{32(31)}$ will be exchanged with the corresponding ones with $\arg(\delta^u_{LL})_{32(31)} + \pi$. Thus, in Table V, the results of case (a) will be replaced with the results of (c) and vice versa. For larger values of $\tan\beta$, the above constraints will be reduced by the factor $(\tan\beta/10)$. Note also that, because of the $SU(2)$ gauge invariance, the soft scalar mass M^2_Q is common for the up and down sectors. Therefore, one gets the following relations between the up and down mass insertions:

$$(\delta^d_{LL})_{ij} = [V^+_{\rm CKM}(\delta^u_{LL})V_{\rm CKM}]_{ij}. \tag{58}$$

Hence,

$$(\delta^d_{LL})_{32} = (\delta^u_{LL})_{32} + \mathcal{O}(\lambda^2). \tag{59}$$

As a result, the constraints obtained from the chargino contribution to $b \to s\gamma$ transition on $|(\delta^u_{LL})_{32}|$ can be conveyed to a constraint on $|(\delta^d_{LL})_{32}|$ which equals to $|(\delta^d_{LL})_{23}|$, due to the Hermiticity of $(M^2_D)_{LL}$. This is the strongest constraint one may obtain on $|(\delta^d_{LL})_{23}|$, and, therefore, it should be taken into account in analyzing the LL part of the gluino contribution to the $b \to s$.

Finally, we consider the scenario in which both gluino and chargino exchanges are assumed to contribute to $b \to s\gamma$ simultaneously with relevant mass insertions, namely, $(\delta^d_{LR})_{23}$ and $(\delta^u_{LL})_{32}$. It is known that these two contributions could give rise to a substantial destructive or constructive interference with the SM amplitude, depending on the relative sign of these amplitudes. Recall that, in the minimal supersymmetric standard model with the universality assumptions, the gluino amplitude is negligible, since $(\delta^d_{LR})_{23} \lesssim \mathcal{O}(10^{-6})$, and the chargino contribution at large $\tan\beta$ is the only relevant SUSY contribution. In this class of model, depending on the sign of μ, the chargino contribution gives destructive interference with the SM result.

In a generic SUSY model, the situation is different, and the experimental results of the branching ratio of $b \to s\gamma$ can be easily accommodated by any one of these contributions. Also, since the gluino and the chargino contributions are given in terms of the parameters of the up- and down-squark sectors, they are, in principle, independent and could have destructive interference between themselves or with the SM contribution. We stress that we are not interested in any fine-tuning region of the parameter space that may lead to a large cancellation. We are rather considering the general scenario with large down and up mass insertions favored by the CP asymmetries of different B processes. In this case, both gluino and chargino contributions to $b \to s\gamma$ are large and cancellation of the order 20%–50% can take place.

Now it is clear that the previous constraint obtained on $(\delta^d_{LR})_{23}$ and $(\delta^u_{LL})_{32}$ in Tables IV and V will be relaxed. We plot the corresponding results for the correlations between $(\delta^d_{LR})_{23}$ and $(\delta^u_{LL})_{32}$ in Fig. 1. Here we consider the relation $(\delta^d_{LL})_{23} = (\delta^u_{LL})_{32}$ into account and also set $(\delta^u_{RL})_{32}$ to

TABLE V. Upper bounds of $|(\delta^u_{LL})_{32}|$ (left) and $|(\delta^u_{LL})_{31}|$ (right) from $b \to s\gamma$ decay for $\tan\beta = 10$ and $\mu = 400$ GeV and $\arg(\delta^u_{LL})_{32(31)} = 0$(a), $\pi/2$(b), π(c), respectively.

$M_2\backslash m$	300	500	700	900
	(a) 0.04	0.065	0.095	0.14
150	(b) 0.14	0.24	0.37	0.54
	(c) 0.51	0.85	⋯	⋯
	(a) 0.053	0.075	0.1	0.15
250	(b) 0.20	0.28	0.4	0.55
	(c) 0.70	⋯	⋯	⋯
	(a) 0.07	0.09	0.12	0.16
350	(b) 0.26	0.33	0.45	0.6
	(c) 0.92	⋯	⋯	⋯
	(a) 0.085	0.105	0.14	0.16
450	(b) 0.33	0.4	0.5	0.6
	(c) ⋯	⋯	⋯	⋯
$M_2\backslash m$	300	500	700	900
	(a) 0.17	0.28	0.45	0.65
150	(b) 0.65	⋯	⋯	⋯
	(c) ⋯	⋯	⋯	⋯
	(a) 0.24	0.34	0.48	0.67
250	(b) 0.86	⋯	⋯	⋯
	(c) ⋯	⋯	⋯	⋯
	(a) 0.32	0.4	0.52	0.73
350	(b) ⋯	⋯	⋯	⋯
	(c) ⋯	⋯	⋯	⋯
	(a) 0.45	0.48	0.62	0.8
450	(b) ⋯	⋯	⋯	⋯
	(c) ⋯	⋯	⋯	⋯

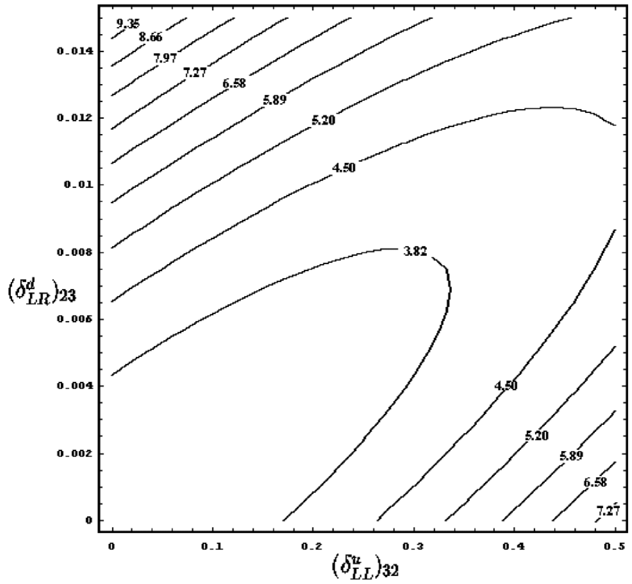

FIG. 1. Contour plot for $\mathrm{BR}(b \to s\gamma) \times 10^4$ as a function of $(\delta^d_{LR})_{23}$ and $(\delta^u_{LL})_{32}$.

zero. The phases of $(\delta^d_{LR})_{23}$ and $(\delta^u_{LL})_{32}$ are assumed to be of order $\pi/2$ as favored by the *CP* asymmetry of $B \to \phi K$. From this plot, we can see that constraints on these mass insertions, particularly $(\delta^u_{LL})_{32}$, are relaxed.

V. SUSY SOLUTION TO THE $R_c - R_n$ PUZZLE

Now we analyze the supersymmetric contributions to the $B \to K\pi$ branching ratio. We will show that the simultaneous contributions from penguin diagrams with chargino and gluino in the loop could lead to a possible solution to the $R_c - R_n$ puzzle. As mentioned in Sec. III, these penguin contributions have three possible sources of a large SUSY contribution to $B \to K\pi$ processes:

(1) Gluino mass enhanced $O_{7\gamma}$ and O_{8g} which depend on $(\delta^d_{LR})_{23}$ and $(\delta^d_{RL})_{23}$;

(2) Chargino mass enhanced $O_{7\gamma}$ and O_{8g} which depend on $\tan\beta(\delta^u_{LL})_{23}$;

(3) Right-handed stop mass enhanced Z penguin which is given in terms of $(\delta^u_{RL})_{32}$.

For the same inputs of SUSY parameters that we used above, $m_{\tilde{g}} = 500$ GeV, $m_{\tilde{q}} = 500$ GeV, $m_{\tilde{t}_R} = 150$ GeV, $M_2 = 200$ GeV, $\mu = 400$ GeV, and $\tan\beta = 10$, one finds the following SUSY contributions to the amplitudes of $B \to K\pi$:

$$A_{\bar{B}^0\to\pi^0\bar{K}^0} \times 10^7 \simeq -9.82i[(\delta^d_{LR})_{23} + (\delta^d_{RL})_{23}] + 0.036i(\delta^u_{LL})_{32} - 0.02i(\delta^u_{RL})_{32},$$

$$A_{\bar{B}^0\to\pi^+K^-} \times 10^7 \simeq 14.04i[(\delta^d_{LR})_{23} + (\delta^d_{RL})_{23}] + 0.06i(\delta^u_{LL})_{32} - 0.001i(\delta^u_{RL})_{32},$$

$$A_{B^-\to\pi^0K^-} \times 10^7 \simeq 9.9i[(\delta^d_{LR})_{23} + (\delta^d_{RL})_{23}] - 0.04i(\delta^u_{LL})_{32} + 0.024i(\delta^u_{RL})_{32},$$

$$A_{B^-\to\pi^-K^0} \times 10^7 \simeq 13.89i[(\delta^d_{LR})_{23} + (\delta^d_{RL})_{23}] + 0.05i(\delta^u_{LL})_{32} - 0.006i(\delta^u_{RL})_{32}.$$

It is remarkable that for the amplitudes $A_{\bar{B}^0\to\pi^0\bar{K}^0}$ and $A_{B^-\to\pi^0K^-}$, which suffer from a large discrepancy between their SM values and their experimental measurements, the SUSY contributions have the following features: (i) the effect of $(\delta^u_{RL})_{32}$ is not negligible as in the other amplitudes; (ii) there can be a distractive interference between the $(\delta^d_{LR})_{23}$ and $(\delta^u_{LL})_{32}$ contributions. As we will see below, these two points are important in saturating the experimental results by supersymmetry. Also note that the effect of the gluino contribution through $O_{7\gamma}$ is very small and the contribution of $(\delta^d_{LR})_{23}$ is mainly due to O_{8g}. However, the chargino effect of $O_{7\gamma}$ can be enhanced by $\tan\beta$.

We present our numerical results for the correlation between the total contributions (SM + SUSY) to the R_n and R_c in Fig. 2. We have scanned over the relevant mass insertions: $(\delta^u_{LL})_{32}$, $(\delta^d_{LR})_{23}$, and $(\delta^u_{RL})_{32}$, since we have assumed $(\delta^u_{LL})_{32} \simeq (\delta^d_{LL})_{23}$ and $(\delta^d_{LR})_{23} \simeq (\delta^d_{RL})_{23}$. We considered $|(\delta^u_{LL})_{32}| \in [0.1, 1]$, $|(\delta^d_{LR})_{23}| \in [0.001, 0.01]$, $\arg[(\delta^u_{LL})_{32}] \in [-\pi, \pi]$, $\arg[(\delta^d_{LR})_{23}] \simeq \pi/3$ (which is preferred by $S_{\phi K_S}$), and $(\delta^u_{RL})_{32}] = 1$ (in order to maximize the difference between R_n and R_c). As can be seen from the results in Fig. 2, the experimental results of R_n and R_c at 2σ can be naturally accommodated by the SUSY contributions. However, the results at 1σ can be obtained only by a smaller region of parameter space. In fact, the value of R_c is predicted to be less than one for most of the parameter space. Therefore, it will be a nice accordance with SUSY results if the experimental result of R_c goes down.

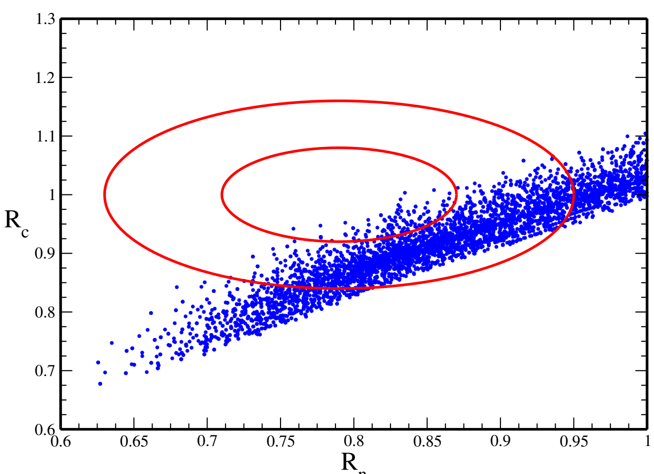

FIG. 2 (color online). $R_c - R_n$ correlation in SUSY models with $|(\delta^u_{LR})_{32}| \simeq 1$, $|(\delta^d_{LR})_{23}| \in [0.001, 0.01]$ and $|(\delta^u_{LL})_{32}| \in [0.1, 1]$; see the text for the other parameters. The small and large ellipses correspond to 1σ and 2σ experimental results, respectively.

In order to understand the results in Fig. 2 and the impact of the SUSY on the correlation between R_n and R_c, we extend the parametrization introduced in Sec. II for the relevant amplitudes by including the SUSY contribution [8]. In this case, Eqs. (27) can be written as

$$A_{B^-\to\pi^-\overline{K}^0} = \lambda_c A_{\pi\bar{K}} P[e^{i\theta_P} + r_A e^{i\delta_A} e^{-i\gamma}], \tag{60}$$

$$\sqrt{2}A_{B^-\to\pi^0K^-} = \lambda_c A_{\pi\bar{K}} P[e^{i\theta_P} + (r_A e^{i\delta_A} + r_C e^{i\delta_C})e^{-i\gamma} + r_{\rm EW} e^{i\theta_{\rm EW}} e^{i\delta_{\rm EW}}], \tag{61}$$

$$A_{\overline{B}^0\to\pi^+K^-} = \lambda_c A_{\pi\overline{K}} P[e^{i\theta_P} + (r_A e^{i\delta_A} + r_T e^{i\delta_T})e^{-i\gamma} + r^C_{\rm EW} e^{i\theta_{\rm EW}} e^{i\delta^C_{\rm EW}}], \tag{62}$$

$$-\sqrt{2}A_{\overline{B}^0\to\pi^0\overline{K}^0} = \lambda_c A_{\pi\overline{K}} P[e^{i\theta_P} + (r_A e^{i\delta_A} + r_T e^{i\delta_T} - r_C e^{i\theta_C} e^{i\delta_C})e^{-i\gamma} + r^C_{\rm EW} e^{i\theta^C_{\rm EW}} e^{i\delta^C_{\rm EW}} - r_{\rm EW} e^{i\theta_{\rm EW}} e^{i\delta_{\rm EW}}]. \tag{63}$$

The parameters δ_A, δ_C, δ_T, $\delta_{\rm EW}$, $\delta^C_{\rm EW}$ and θ_P, $\theta_{\rm EW}$, $\theta^C_{\rm EW}$ are the CP conserving (strong) and the CP violating phase, respectively. Note that the parameters P, $r_{\rm EW}$, $r^C_{\rm EW}$ are now defined as

$$\begin{aligned} Pe^{i\theta_P}e^{i\delta_P} &= \alpha_4^c - \tfrac{1}{2}\alpha^c_{4,\rm EW} + \beta_3^c + \beta^c_{3,\rm EW}, \\ r_{\rm EW}e^{i\theta_{\rm EW}}e^{i\delta_{\rm EW}} &= [\tfrac{3}{2}(R_{K\pi}\alpha^c_{3,\rm EW} + \alpha^c_{4,\rm EW})]/P, \\ r^C_{\rm EW}e^{i\theta^C_{\rm EW}}e^{i\delta^C_{\rm EW}} &= [\tfrac{3}{2}(\alpha^c_{4,\rm EW} - \beta^c_{3,\rm EW})]/P. \end{aligned} \tag{64}$$

First, let us include some assumptions to simplify our formulas. As is mentioned before, α_4^p, $\alpha^p_{3,\rm EW}$, $\alpha^p_{4,\rm EW}$, β_3^p, $\beta^p_{3,\rm EW}$, $\beta^p_{4,\rm EW}$ receive SUSY contributions through the Wilson coefficients. The upper index p takes both u and c; however, the contribution with the u index is always suppressed by the factor $\epsilon_{KM} \simeq 0.018$ so that its SUSY contributions can be safely neglected comparing to the one with the index c. As a result, $(r_A e^{i\delta_A})$, $(r_C e^{i\delta_C})$, and $(r_T e^{i\delta_T})$ receive a correction of a factor $1/|1 + (P^{\rm SUSY}/P^{\rm SM})|$.

Second, we assume that the strong phase for SM and SUSY are the same. We found that this is a reasonable assumption in QCD approximation in which the main source of the strong phase comes from hard spectator and weak annihilation diagrams. This leads us to the following parametrization:

$$Pe^{i\delta_P}e^{i\theta_P} = P^{\rm SM}e^{i\delta_P}(1 + ke^{i\theta'_P}), \tag{65}$$

$$r_{\rm EW}e^{i\delta_{\rm EW}}e^{i\theta_{\rm EW}} = (r_{\rm EW})^{\rm SM}e^{i\delta_{\rm EW}}(1 + le^{i\theta'_{\rm EW}}), \tag{66}$$

$$r^C_{\rm EW}e^{i\delta^C_{\rm EW}}e^{i\theta^C_{\rm EW}} = (r^C_{\rm EW})^{\rm SM}e^{i\delta^C_{\rm EW}}(1 + me^{i\theta^{C'}{\rm EW}}), \tag{67}$$

where

$$ke^{i\theta'_P} \equiv \frac{(\alpha_4^c - \frac{1}{2}\alpha^c_{4,\rm EW} + \beta_3^c + \beta^c_{3,\rm EW})_{\rm SUSY}}{(\alpha_4^c - \frac{1}{2}\alpha^c_{4,\rm EW} + \beta_3^c + \beta^c_{3,\rm EW})_{\rm SM}}, \tag{68}$$

$$le^{i\theta'_q} \equiv \frac{(R_{K\pi}\alpha^c_{3,\rm EW} + \alpha^c_{4,\rm EW})_{\rm SUSY}}{(R_{K\pi}\alpha^c_{3,\rm EW} + \alpha^c_{4,\rm EW})_{\rm SM}}, \tag{69}$$

$$me^{i\theta'_{qC}} \equiv \frac{(\alpha^c_{4,\rm EW} - \beta^c_{3,\rm EW})_{\rm SUSY}}{(\alpha^c_{4,\rm EW} - \beta^c_{3,\rm EW})_{\rm SM}}. \tag{70}$$

The index SM (SUSY) means to keep only SM (SUSY) Wilson coefficients in $\alpha^p_{i(,\rm EW)}$ and $\beta^p_{i(,\rm EW)}$. Using these parameters, we also have

$$r_A e^{i\delta_A} = \frac{(r_A e^{i\delta_A})_{\rm SM}}{|1 + ke^{i\theta'_P}|}, \qquad r_C e^{i\delta_C} = \frac{(r_C e^{i\delta_C})_{\rm SM}}{|1 + ke^{i\theta'_P}|}, \qquad r_T e^{i\delta_T} = \frac{(r_T e^{i\delta_T})_{\rm SM}}{|1 + ke^{i\theta'_P}|}. \tag{71}$$

Now let us investigate the $R_c - R_n$ puzzle. We shall follow the standard procedure to simplify and expand the formulas. Considering the numbers obtained above, we shall simplify our formulas by assuming

(1) the strong phases are negligible; i.e., δ_P, δ_A, δ_C, $\delta_{\rm EW}$, $\delta^C_{\rm EW}$ are all zero;
(2) the annihilation tree contribution is negligible; i.e., $r_A \simeq 0$;
(3) the color suppressed tree contribution is negligible; i.e., $r_C e^{i\delta_C} \sim r_T e^{i\delta_T}$.

Using these assumptions, we expand R_c, R_n, and $R_c - R_n$. We expand in terms of r_T and $r_{\rm EW}$ and $r^C_{\rm EW}$ up to the second order. As a result, we obtain

$$R_c \simeq 1 + r_T^2 - 2r_T\cos(\gamma + \theta_P) + 2r_{\rm EW}\cos(\theta_P - \theta_{\rm EW}) - 2r_T r_{\rm EW}\cos(\gamma + \theta_{\rm EW}), \tag{72}$$

$$R_c - R_n \simeq 2r_T r_{\rm EW}\cos(\gamma + 2\theta_P - \theta_{\rm EW}) - 2r_T r^C_{\rm EW}\cos(\gamma + 2\theta_{\rm EW} - \theta^C_{\rm EW}). \tag{73}$$

Now let us find the configuration which leads to $R_c - R_n > 0.2$. Looking at Eq. (73), we can find that, in general, the larger the values of r_T, $r_{\rm EW}$, and $r^C_{\rm EW}$ are, the larger the splitting between R_c and R_n we would acquire. The phase combinations $\theta_P - \theta_{\rm EW}$ and $\theta_P + \gamma$ also play an important role. The possible solution of the $R_c - R_n$ puzzle by enhancing $r_{\rm EW}$, which we parametrize as l, has been intensively studied in the literature [9]. As we will see in the following, r_T can also be enhanced due to the factor $ke^{i\theta'_P}$, which contributes destructively against the SM and diminishes P. However, since P is the dominant contribution to the $B \to K\pi$ process, the branching ratio is very sensitive to $ke^{i\theta'_P}$. Therefore, we are allowed to vary $ke^{i\theta'_P}$ only in a range of the theoretical uncertainty of QCD factorization, which gives about right sizes of the $B \to K\pi$ branching ratios. As shown in Ref. [8], we would be able to reduce P at most by 30%, which can be easily compensated by the error in the transition form factor $F^{B\to\pi,K}$.

Considering the tiny effect from the second term in Eq. (73), in order to achieve $R_c - R_n \gtrsim 0.2$, we need $r_T r_{\rm EW}$ larger than about 0.1 or, equivalently, $r_{\rm EW}$ larger than about 0.5 with $r_T^{\rm SM}$. In Ref. [8], it was emphasized that, with $k = 0$, one needs $l \gtrsim 2$ to reproduce the experimental values, while an inclusion of a small amount of k lowers this bound significantly. For the SUSY parameters that we have considered above, the following results for our SUSY parameters k, l, and m are obtained:

$$ke^{i\theta_P} = -0.0019\tan\beta(\delta^u_{LL})_{32} - 35.0(\delta^d_{LR})_{23} + 0.061(\delta^u_{LR})_{32}, \tag{74}$$

$$le^{i\theta_q} = 0.0528\tan\beta(\delta^u_{LL})_{32} - 2.78(\delta^d_{LR})_{23} + 1.11(\delta^u_{LR})_{32}, \tag{75}$$

$$me^{i\theta_{qC}} = 0.134\tan\beta(\delta^u_{LL})_{32} + 26.4(\delta^d_{LR})_{23} + 1.62(\delta^u_{LR})_{32}. \tag{76}$$

Note that we do not consider $(\delta^d_{23})_{RL}$ here but it is the same as $(\delta^d_{LR})_{23}$ with an opposite sign (see also [7]). Let us first discuss the contributions from a single mass insertion $(\delta^u_{LL})_{32}$, $(\delta^d_{LR})_{23}$, or $(\delta^u_{LR})_{32}$ to $\{k, l, m\}$; keeping only one mass insertion and switching off the other two. In this case, one finds that the maximum value of $\{k, l, m\}$ with $|(\delta^u_{LR})_{32}| = 1$ is $\{k, l, m\} = \{0.061, 1.11, 1.62\}$. Thus, in this case where k is almost negligible, we would need $l \simeq 2$ to explain the experimental data. We have a chance to enlarge the coefficients for $(\delta^u_{LR})_{32}$ by, for instance, increasing the averaged squark mass $\tilde{m}_{\tilde{q}}$. However, even if we choose $\tilde{m}_{\tilde{q}} = 5$ TeV, we find that l is increased only by 20% to 30%. The maximum contributions from $(\delta^d_{LR})_{23}$ and $(\delta^u_{LL})_{32}$ are found to be $\{k, l, m\} = \{0.18, 0.014, 0.13\}$ and $\{0.0019, 0.053, 0.13\}$, which are far too small to explain the experimental data. The coefficients for $(\delta^d_{LR})_{23}$ depend on the overall factor $1/\tilde{m}_{\tilde{q}}$ and also on the variable of the loop function $x = m_{\tilde{g}}/\tilde{m}_{\tilde{q}}$, and we found that $m_{\tilde{g}} = \tilde{m}_{\tilde{q}} = 250$ GeV can lead to a 100% increase. However, the value of l is still too small to deviate $R_c - R_n$ significantly. As a whole, it is extremely difficult to have $R_c - R_n \gtrsim 0.2$ from a single mass insertion contribution.

Let us try to combine two main contributions, $(\delta^d_{LR})_{23}$ and $(\delta^u_{LR})_{32}$ terms. Using the previous input parameters and including the $b \to s\gamma$ constraint $|(\delta^d_{LR})_{23}|$, the maximum value is found to be $\{k, l, m\} = \{0.24, 1.12, 1.48\}$. In this case, it is easy to check that the experimental data are not reproduced very well [8]. As discussed above, for a large value of the averaged squark masses, l increases while k decreases. On the contrary, k also depends on the ratio of gluino and squark masses. Hence, we need to optimize these masses so as to increase k and l simultaneously. For instance, with $m_{\tilde{g}} = 250$ GeV and $\tilde{m}_{\tilde{q}} = 1$ TeV, we obtain $\{k, l, m\} = \{0.30, 1.36, 1.90\}$, which leads to a result within the experimental bounds of R_c and R_n. Finally, we consider the case with the three nonzero mass insertions. The main feature of this scenario is that we expect a relaxation of the constraints on $|\tan\beta \times (\delta^u_{LL})_{32}|$ and $|(\delta^d_{LR})_{23}|$ from the cancellation between $(\delta^d_{LR})_{23}$ and $(\delta^u_{LL})_{32}$ contributions to $b \to s\gamma$. Under this circumstance, we observe much larger $R_c - R_n$ for various combinations of the phases in this scenario.

VI. SUSY CONTRIBUTIONS TO THE CP ASYMMETRY OF $B \to K\pi$

We start this section by summarizing our convention for CP asymmetry in $B \to K\pi$ processes. The time dependent CP asymmetry for $B \to K\pi$ can be described by

$$A_{K\pi}(t) = A_{K\pi}\cos(\Delta M_{B_d} t) + S_{K\pi}\sin(\Delta M_{B_d})t, \tag{77}$$

where $A_{K\pi}$ and $S_{K\pi}$ represent the direct and the mixing CP asymmetry, respectively, and they are given by

$$A_{K\pi} = \frac{|\bar{\rho}(K\pi)|^2 - 1}{|\bar{\rho}(K\pi)|^2 + 1}, \qquad S_{K\pi} = \frac{2\,\mathrm{Im}(\bar{\rho}(K\pi))}{|\bar{\rho}(K\pi)|^2 + 1}, \tag{78}$$

where $\bar{\rho}(K\pi) = e^{-i\phi_B}\frac{\bar{A}(K\pi)}{A(K\pi)}$. The phase ϕ_B is the phase of M_{12}, the $B^0 - \bar{B}^0$ mixing amplitude. The $A(K\pi)$ and $\bar{A}(K\pi)$ are the decay amplitudes for B^0 and $\bar{B}^0$ to $K\pi$, respectively.

The SM predicts that the mixing and direct asymmetry of $B \to K\pi$ decay are given by

$$S_{K\pi} = \sin 2\beta, \qquad A_{K\pi} = 0. \tag{79}$$

The recent measurements of the CP asymmetries in $B \to K\pi$, reported in Table I, show significant discrepancies with the SM predictions. As mentioned above, SUSY can affect the results of the CP asymmetries in B decay, due to the new source of CP violating phases in the corresponding amplitude. Therefore, deviation on CP asymmetries from the SM expectations can be sizable, depending on the relative magnitude of the SM and the SUSY amplitudes. In this respect, SUSY models with nonminimal flavor structure and new CP violating phases in the squark mass matrices can generate large deviations in the $B \to K\pi$ asymmetry. In this section, we present and discuss our results for SUSY contributions to the direct and the mixing CP asymmetries in $B \to K\pi$.

A. SUSY contributions to the direct CP asymmetry in $B \to K\pi$

Using the general parametrization of the decay amplitudes of $B \to K\pi$ given in Eqs. (60)–(63), one can write the direct CP asymmetries $A^{CP}_{K\pi}$ as follows:

$$A^{CP}_{K^-\pi^+} \simeq 2r_T \sin\delta_T \sin(\theta_P + \gamma) + 2r^C_{\rm EW} \sin\delta^C_{\rm EW} \sin(\theta_P - \theta^c_{\rm EW}) - r_T^2 \sin 2\delta_T \sin 2(\theta_P + \gamma) + 2r_T r^C_{\rm EW} \sin(\delta^C_{\rm EW} - \delta_T) \sin(\theta^C_{\rm EW} + \gamma) - 4r_T r^C_{\rm EW} \sin\delta^C_{\rm EW} \sin(\theta_P - \theta^C_{\rm EW}) \cos\delta_T \cos(\theta_P + \gamma) - 4r_T r^C_{\rm EW} \sin\delta_T \sin(\theta_P + \gamma) \cos\delta^c_{\rm EW} \cos(\theta_P - \theta^C_{\rm EW}), \tag{80}$$

$$A^{CP}_{K^0\pi^-} \simeq 2r_A \sin\delta_A \sin(\theta_P + \gamma), \tag{81}$$

$$A^{CP}_{K^0\pi^0} \simeq 2r^C_{\rm EW} \sin\delta^C_{\rm EW} \sin(\theta_P - \theta^C_{\rm EW}) - 2r_{\rm EW} \sin\delta_{\rm EW} \sin(\theta_P - \theta_{\rm EW}), \tag{82}$$

$$A^{CP}_{K^-\pi^0} \simeq 2r_T \sin\delta_T \sin(\theta_P + \gamma) - 2r_{\rm EW} \sin\delta_{\rm EW} \sin(\theta_P - \theta_{\rm EW}) - r_T^2 \sin 2\delta_T \sin 2(\theta_P + \gamma) - 2r_T r_{\rm EW} \sin(\delta_{\rm EW} - \delta_T) \sin(\theta_{\rm EW} + \gamma) - 4r_T r_{\rm EW} \sin\delta_{\rm EW} \sin(\theta_P - \theta_{\rm EW}) \cos\delta_T \cos(\theta_P + \gamma) - 4r_T r_{\rm EW} \sin\delta_T \sin(\theta_P + \gamma) \cos\delta_{\rm EW} \cos(\theta_P - \theta_{\rm EW}). \tag{83}$$

From these expressions, it is clear that if we ignore the strong phases, then the direct *CP* asymmetries would vanish. However, Belle and *BABAR* Collaborations observed nonzero values for the $A^{CP}_{K\pi}$; thus, we should consider nonvanishing strong phases in this analysis. It is also remarkable that the leading contributions to the direct *CP* asymmetries are given by the linear terms of $r_i \equiv r_T, r_A, r_{\rm EW}, r^C_{\rm EW}$, unlike the difference $R_c - R_n$ which receives corrections of order $r_i r_j$. As in the previous section, we have assumed that the color suppressed contributions are negligible, i.e., $r_C e^{i\delta_C} = r_T e^{i\delta_T}$, and we have neglected terms of order r_i^2 except for r_T which is typically larger than $r_{\rm EW}$, $r^C_{\rm EW}$, and r_A.

The rescattering effects parametrized by r_A are quite small $[r_A^{\rm SM} \simeq \mathcal{O}(0.01)]$; therefore, the *CP* asymmetry in the decays $B^\pm \to K^0\pi^\pm$ is expected to be very small as can be easily seen from Eq. (81). This result is consistent with the experimental measurements reported in Table I. The sign of this asymmetry will depend on the relative sign of $\sin\delta_A$ and $\sin(\theta_P + \gamma)$. Note that the value of the angle γ is fixed by the *CP* asymmetry in $B \to \pi\pi$ to be of order $\pi/3$. The angle θ_P can also be determined from the *CP* asymmetry $S_{\phi(\eta')K}$.

In the SM, the parameters r_A, $r^C_{\rm EW}$ are much smaller than r_T, $r_{\rm EW}$ and $\theta_P = 0$; therefore, the following relation among the direct *CP* asymmetries $A^{CP}_{K\pi}$ is obtained:

$$A^{CP}_{K^-\pi 0} \gtrsim A^{CP}_{K^-\pi^+} \gtrsim A^{CP}_{K^0\pi^0} > A^{CP}_{K^0\pi^-}.$$

This relation is in agreement with the numerical results listed in Table III for the direct *CP* asymmetries in the SM with $\rho_{A,H}$, $\phi_{A,H} \simeq 1$. To change this relation among the *CP* asymmetries and to get consistent correlations with experimental measurements, one should enhance the electroweak penguin contributions to $\bar{B}^0 \to K^-\pi^+$ decay amplitude, parametrized by $r^C_{\rm EW}$. Furthermore, a nonvanishing value of θ_P, which is also required to account for the recent measurements of $S_{\phi K_S}$ and $S_{\eta' K_S}$, is favored in order to obtain $A^{CP}_{K^-\pi^+} > A^{CP}_{K^-\pi^0}$. It is worth mentioning that, in the SM and due to the fact that $\theta_P = 0$, the second terms in Eqs. (80) and (83) gives destructive and constructive interferences, respectively, with the first terms. Thus, one finds $A^{CP}_{K^-\pi^0}$ is larger than $A^{CP}_{K^-\pi^+}$. In SUSY models, the gluino contribution leads to a large value of θ_P, and, depending on the sign of this angle, the parameter r_T could be enhanced or reduced; see Eq. (71). As will be seen below, in this case we can explain the *CP* asymmetry results with moderate values of the electroweak penguin parameter $r^C_{\rm EW}$. Note that, in other models studied in the literature, the value of this parameter is required to be larger than 1 in order to account for the *CP* asymmetry results.

Now let us discuss the SUSY contribution to the *CP* asymmetries $A^{CP}_{K\pi}$. As can be seen from Table I, the experimental measurements of $A^{CP}_{K^0\pi^0}$ suffer from a large uncertainty. It turns out that it is very easy to have the SUSY results for this asymmetry within the range of 2σ measurements. Thus, this decay mode is not useful in constraining the SUSY parameter space and can be ignored in our discussion for the correlation among the *CP* asymmetries of $B \to K\pi$ in generic SUSY models.

We will consider, as in the previous section, three scenarios with a single mass insertion, two mass insertions, and three mass insertions. In the first case, if we consider the contribution due to the mass insertion $(\delta^u_{LR})_{32}$, the maximum values of $\{k, l, m\}$ are given by $\{0.061, 1.11, 1.62\}$, while from $(\delta^d_{LR})_{23}$ and $(\delta^u_{LL})_{32}$, one finds that the maximum values of $\{k, l, m\}$ are $\{0.18, 0.014, 0.13\}$ and $\{0.0019, 0.053, 0.13\}$, respectively. Note that k is almost negligible in the case of dominant chargino contribution which depends on $(\delta^u_{LL})_{32}$ and $(\delta^u_{LR})_{32}$ and can be significantly enhanced by the gluino contribution that depends on $(\delta^d_{LR})_{23}$ as emphasized in Ref. [5]. Also from Eqs. (66), (67), and (71), one finds

$$r_{\rm EW} = r^{\rm SM}_{\rm EW}(1 + l^2 + 2l\cos\theta'_{\rm EW})^{1/2}, \tag{84}$$

$$r^c_{\rm EW} = (r^C_{\rm EW})^{\rm SM}(1 + m^2 + 2m\cos\theta^{C'}_{\rm EW})^{1/2}, \tag{85}$$

$$r_T = \frac{r_T}{|1 + ke^{i\theta'_P}|}. \tag{86}$$

Since $(r_{\rm EW}^C)^{\rm SM} \simeq 0.01$, the enhancement of $r_{\rm EW}^C$ remains quite limited in SUSY models and it is impossible to enhance it to be of order 1. Hence, the contribution of $r_{\rm EW}^C$ to $A_{K^-\pi^+}^{CP}$ is negligible with respect to the contribution of $r_{\rm EW}$ to $A_{K^-\pi^0}^{CP}$. To overcome this problem and get the desired relation between $A_{K^-\pi^+}^{CP}$ and $A_{K^-\pi^0}^{CP}$, a kind of cancellation between r_T and $r_{\rm EW}$ contributions to $A_{K^-\pi^0}^{CP}$ is required. Such a cancellation can be obtained naturally without fine-tuning the parameters if $r_T \sim r_{\rm EW}$, i.e., the total value of $r_T < r_T^{\rm SM}$. This could happen if k is not very small. Therefore, one would expect that the scenarios with dominant chargino contribution, where $k = 0.061$ or $k = 0.0019$, will not be able to saturate the experimental results of $A_{K^-\pi^+}^{CP}$ and $A_{K^-\pi^0}^{CP}$ simultaneously. This observation is confirmed in Fig. 3 (top left), where the results of $A_{K^-\pi^+}^{CP}$ are plotted versus the results of $A_{K^-\pi^0}^{CP}$ for $\{k, l, m\} = \{0.061, 1.11, 2.62\}$ and the other parameters vary as follows: $\delta_i \equiv -\pi, \pi/2$, and π. The angles $\theta_{\rm EW}$ and $\theta_{\rm EW}^C \in [-\pi, \pi]$. Also, θ_P is assumed to be in the region $[\pi/4, \pi/2]$. Note that in this plot we have taken the $A_{K^0\pi^-}^{CP}$ as a constraint. Thus, all the points in the plot correspond to consistent values of $A_{K^0\pi^-}^{CP}$ with the experimental results.

Now we consider the second scenario with dominant gluino contribution, i.e., $(\delta_{LR}^d)_{23} \simeq 0.005 e^{i\pi/3}$, $(\delta_{LL}^u)_{32} = (\delta_{RL}^u)_{32} = 0$. In this case, one finds that the maximum values of $\{k, l, m\}$ are give by $\{k, l, m\} = \{0.18, 0.014, 0.13\}$; hence, r_T is reduced from $r_T^{\rm SM} \simeq 0.2$ to $r_T \simeq 0.12$, while $r_{\rm EW}$ and $r_{\rm EW}^C$ approximately remain the same as in the SM. In Fig. 3 (top right), we plot the *CP* asymmetries $A_{K^-\pi^+}^{CP}$ and $A_{K^-\pi^0}^{CP}$ in this scenario, varying the relevant parameter as before. It is remarkable that a large number of points of the parameter space can simultaneously accommodate the experimental results of these *CP* asymmetries. It is slightly surprising to get the values of the *CP* asymmetries $A_{K\pi}^{CP}$ within the experimental range, i.e., $A_{K^-\pi^+}^{CP} \in [-0.075, -0.151]$ and $A_{K^-\pi^0}^{CP} \in [-0.04, 0.12]$, by just one mass insertion in dominant

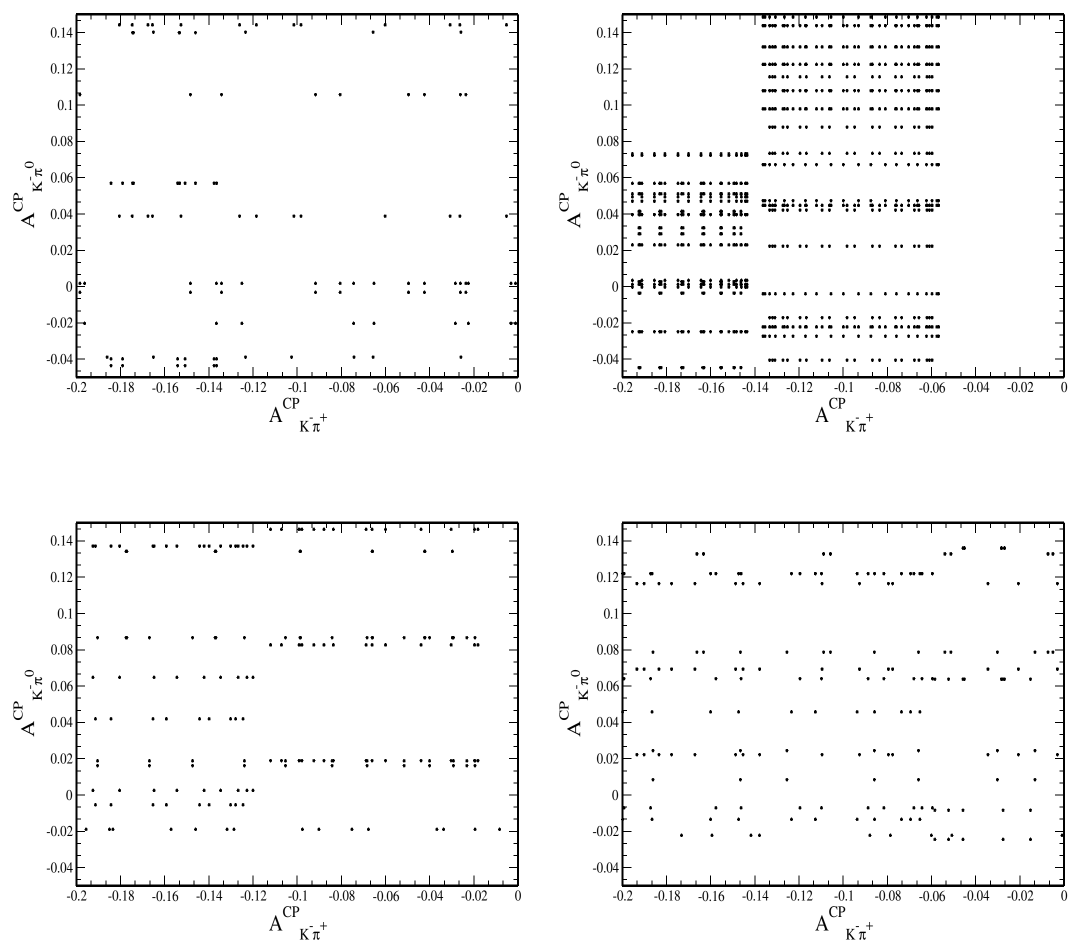

FIG. 3. *CP* asymmetry of $B \to K^-\pi^+$ versus *CP* asymmetry of $B \to K^-\pi^0$ for $\{k, l, m\} = \{0.061, 1.11, 2.62\}, \{0.18, 0.014, 0.13\}, \{0.24, 1.12, 1.48\}, \{0.32, 0.95, 2.26\}$, respectively, from left to right and top to bottom. Strong phases $\delta_i \equiv -\pi, \pi/2, \pi$ and *CP* violating phases $\theta_{\rm EW}$ and $\theta_{\rm EW}^C$ reside between $-\pi$ and π. Finally, θ_P is assumed to be in the region $[\pi/4, \pi/2]$.

gluino models. This is contrary to the $R_c - R_n$ results, which need gluino and chargino combination in order to be within the experimental range. This result can be explained by the cancellation that occurs in $A^{CP}_{K^-\pi^0}$ between the r_T and $r_{\rm EW}$ contributions and the negligible effect of $r^C_{\rm EW}$ to $A^{CP}_{K^-\pi^+}$.

To be more quantitative, let us consider the following example where $(\delta^d_{LR})_{23} \simeq 0.005e^{i\pi/3}$ and $(\delta^u_{LL})_{32} = (\delta^u_{RL})_{32} = 0$. In this case, one get $r_T = 0.12$, $r_{\rm EW} = 0.13$, and $r^C_{\rm EW} = 0.01$. Therefore, the main contribution to $A^{CP}_{K^-\pi^+}$ is due to the linear term in r_T, which is $r_T \sin\delta_T \sin(\theta_P + \gamma)$. With $\theta_P \sim \pi/3$ and $\delta_T \sim -\pi/4$, this contribution leads to $A^{CP}_{K^-\pi^+} \simeq -0.113$. Since r_T gives the same contributions to $A^{CP}_{K^-\pi^0}$, a significant positive contribution from $r_{\rm EW}$ is required to change the $A^{CP}_{K^-\pi^0}$ and make it positive. With $r_{\rm EW} = 0.13$, the $A^{CP}_{K^-\pi^0}$ is approximately given by $A^{CP}_{K^-\pi^0} \simeq -0.113 + 0.26\sin\delta_{\rm EW}\sin(\theta_P - \theta_{\rm EW})$. It is worth mentioning that, although θ'_P and $\theta'_{\rm EW}$ are equal in the case of a single mass insertion, the values of θ_P and $\theta_{\rm EW}$ are different due to the different values of k and l. In this example, it turns out that $\theta_P - \theta_{\rm EW} \sim \pi/9$. Hence, one gets $A^{CP}_{K^-\pi^0} \simeq -0.113 + 0.22\sin\delta_{\rm EW}$. So that for $\delta_{\rm EW} \sim \pi/4$, one finds $A^{CP}_{K^-\pi^0} \simeq 0.04$, which is the central value of the experimental measurements reported in Table I.

We turn to the contributions from two mass insertions: $(\delta^d_{LR})_{23}$ and $(\delta^u_{RL})_{32}$, which reflect simultaneous contributions from the penguin diagrams with chargino and gluino in the loop. Applying the $b \to s\gamma$ constraints on these mass insertions, the maximum values of $\{k, l, m\}$ are found to be $\{0.24, 1.12, 1.148\}$. In this case, we obtain $r_T = 0.11$, $r_{\rm EW} = 0.54$, and $r^C_{\rm EW} = 0.06$. Therefore, the CP asymmetry $A^{CP}_{K^-\pi^0}$ is dominated by $r_{\rm EW}$ contribution, and, in order to get $A^{CP}_{K^-\pi^0}$ of order $\mathcal{O}(0.04)$, a small value of the strong phase $\delta_{\rm EW}$ should be used. This makes the possibility of saturating the results of $A^{CP}_{K^-\pi^+}$ and $A^{CP}_{K^-\pi^0}$ less possible than the previous case. In Fig. 3 (bottom left), we present the results of this scenario for the same set of input parameters used before. This figure confirms our expectation and it can be easily seen that it has less points of the parameter space that account for the experimental results of the CP asymmetries than Fig. 3 (top right). Note also that with two mass insertions, the phases θ'_P and $\theta'_{\rm EW}$ can be considered independent; hence, the angles θ_P and $\theta_{\rm EW}$ are also independent.

Finally, we consider the case of three nonvanishing mass insertions: $(\delta^d_{LR})_{23}$, $(\delta^u_{RL})_{32}$, and $(\delta^u_{LL})_{32}$. Including the $b \to s\gamma$ constraints, we find that the maximum value of $\{k, l, m\}$ is $\{0.32, 0.95, 2.26\}$. The corresponding values of r_i are $r_T = 0.10$, $r_{\rm EW} = 0.48$, and $r^C_{\rm EW} = 0.09$. It is clear that r_T and $r_{\rm EW}$ are slightly changed than in the previous scenario, while $r^C_{\rm EW}$ is enhanced a bit. In this case, it will be easier to accommodate for $A^{CP}_{K^-\pi^0}$. The numerical results for this scenario are given in Fig. 3 (bottom right) for the same set of parameter space used in previous cases. As can be seen from this figure, the probability of accommodating the experimental results of different CP asymmetries in this class of models is higher than in models with two mass insertions. However, it remains that the model with dominated gluino contributions provides the largest possibility of saturating the experimental results of CP asymmetries of $B \to K\pi$.

B. SUSY contributions to the mixing CP asymmetry in $B \to K^0\pi^0$

We turn our attention now to the mixing CP asymmetry of $B \to K^0\pi^0$. As mentioned before, this decay is dominated by $b \to s$ penguin. Thus, within the SM, the CP asymmetry $S_{K^0\pi^0}$ should be very close to the value of $\sin 2\beta \simeq 0.73$. However, the current experimental measurements summarized in Table I show that $S_{K^0\pi^0}$ is lower than the expected value of $\sin 2\beta$, namely,

$$S_{K^0\pi^0} \simeq 0.34 \pm 0.28. \tag{87}$$

In this section we aim to interpret this discrepancy in terms of supersymmetry contributions. It is useful to parametrize the SUSY effects by introducing the ratio of SM and SUSY amplitudes as follows:

$$\left(\frac{A^{\rm SUSY}}{A^{\rm SM}}\right)_{K\pi} \equiv R_\pi e^{i\theta_\pi} e^{i\delta_\pi}, \tag{88}$$

where R_π stands for the absolute value of $|[A^{\rm SUSY}(B \to K^0\pi^0)]/[A^{\rm SM}(B \to K^0\pi^0)]|$ and the angle θ_π is the SUSY CP violating phase. The strong (CP conserving) phase δ_π is defined by $\delta_\pi = \delta^{\rm SM}_\pi - \delta^{\rm SUSY}_\pi$. This parametrization is analogous for those of $S_{K\phi}$ and $S_{K\eta'}$ [5,7]. Using this parametrization, one finds that the mixing CP asymmetry $S_{K^0\pi^0}$ in Eq. (78) takes the following form:

$$S_{K^0\pi^0} = \frac{\sin 2\beta + 2R_\pi \cos\delta_\pi \sin(\theta_\pi + 2\beta) + R^2_\pi \sin(2\theta_\pi + 2\beta)}{1 + 2R_\pi \cos\delta_\pi \cos\theta_\pi + R^2_\pi}. \tag{89}$$

Assuming that the SUSY contribution to the amplitude is smaller than the SM one, i.e., $R_\pi \ll 1$, one can simplify the above expressions as:

$$S_{K^0\pi^0} = \sin 2\beta + 2\cos 2\beta \sin\theta_\pi \cos\delta_\pi R_\pi + \mathcal{O}(R^2_\pi). \tag{90}$$

In order to reduce $S_{K^0\pi^0}$ smaller than $\sin 2\beta$, the relative sign of $\sin\theta_\pi$ and $\cos\delta_\pi$ has to be negative. If one assumes that

$\sin\theta_\pi \cos\delta_\pi \simeq -1$, then $R_\pi \gtrsim 0.2$ is required in order to get $S_{K^0\pi^0}$ within 1σ of the experimental range.

In the QCD factorization approach, the decay amplitude of $B \to K^0\pi^0$ is given by Eq. (11). As in the case of $B \to \phi(\eta')K$ [5], we will provide the numerical parametrization of this amplitude in terms of the Wilson coefficients $\mathbf{C_i}$ and $\tilde{\mathbf{C}}_\mathbf{i}$ defined according to the parametrization of the effective Hamiltonian in Eq. (4)

$$H_{\rm eff}^{\Delta B=1} = \frac{G_F}{\sqrt{2}} \sum_i \{\mathbf{C}_i Q_i + \tilde{\mathbf{C}}_i \tilde{Q}_i\}, \qquad (91)$$

where the operators basis Q_i and $\tilde{Q}_i$ are the same ones of Eq. (4). By fixing the hadronic parameters with their center values as in Table 1 of Ref. [11], we obtain

$$A(B \to K^0\pi^0) = -i\frac{G_F}{\sqrt{2}} m_B^2 F_+^{B\to K} f_\pi \times \sum_{i=1\ldots10,7\gamma,8g} H_i(\pi)(\mathbf{C}_i - \tilde{\mathbf{C}}_i), \qquad (92)$$

where

$$\begin{aligned} &H_1(\pi) \simeq -0.7 + 0.0003i, \qquad H_2(\pi) \simeq -0.21 + 0.037i - 0.006X_H, \\ &H_3(\pi) \simeq 0.22 - 0.076i + 0.0045X_A + 0.0003X_A^2 + 0.0065X_H, \qquad H_4(\pi) \simeq 0.68 - 0.078i, \\ &H_5(\pi) \simeq 0.2 - 0.001X_A + 0.004X_A^2, \qquad H_6(\pi) \simeq 0.68 - 0.078i - 0.007X_A + 0.014X_A^2, \\ &H_7(\pi) \simeq 0.95 + 0.0004X_A - 0.0014X_A^2, \qquad H_8(\pi) \simeq -0.068 + 0.08i + 0.002X_A - 0.0047X_A^2 - 0.009X_H, \\ &H_9(\pi) \simeq -1.16 + 0.026i - 0.0015X_A - 0.0001X_A^2 - 0.003X_H, \qquad H_{10}(\pi) \simeq -0.67 + 0.08i - 0.0096X_H, \\ &H_{7\gamma}(\pi) \simeq 0.0004, \qquad H_{8g}(\pi) \simeq -0.045. \end{aligned} \qquad (93)$$

The different sign between $\mathbf{C}_i$ and $\tilde{\mathbf{C}}_i$ appearing in Eq. (92) is due to the fact that $\langle K^0\pi^0|Q_i|B\rangle = -\langle K^0\pi^0|\tilde{Q}_i|B\rangle$, since the initial and the final states have different parity. Comparing the coefficients $H_i(\pi)$ with $H_i(\phi)$ and $H_i(\eta')$ in Ref. [5], one finds that the Wilson coefficients in these decay amplitudes are different. Thus, it is natural to have different CP asymmetries $S_{K^0\pi^0}$, $S_{K\phi}$, and $S_{K\eta'}$, unlike the SM prediction.

In order to understand the dominant SUSY contribution to the CP asymmetry $S_{K^0\pi^0}$, it is useful to present a numerical parametrization of the ratio of the amplitude R_π in terms of the relevant mass insertions. For the usual SUSY configurations that we have used in the previous sections, we obtain

$$\begin{aligned} R_\pi \simeq &\{0.02 \times e^{-i0.4}(\delta_{LL}^d)_{23} - 40.4 \times e^{-i0.01}(\delta_{LR}^d)_{23}\} \\ &- \{L \leftrightarrow R\} + 0.15 \times e^{-i0.002}(\delta_{LL}^u)_{32} \\ &- 0.08 \times e^{-i0.013}(\delta_{RL}^u)_{32}. \end{aligned} \qquad (94)$$

From this result, it is clear that the largest SUSY effect is provided by the gluino contribution to the chromomagnetic operator which is proportional to $(\delta_{LR}^d)_{23}$ and $(\delta_{RL}^d)_{23}$. For $(\delta_{LR}^d)_{23} \simeq 0.006 \times e^{i\pi/3}$ and all the other mass insertions set to zero, one finds $S_{K^0\pi^0} \simeq 0.34$, which coincides with the central value of the experimental results reported in Table I. It is important to note that with such a value of $(\delta_{LR}^d)_{23}$ the gluino contribution can account for the CP asymmetries $S_{K\phi}$ and $S_{K\eta'}$ as well [5]. Furthermore, if we consider the scenario where both chargino and gluino exchanges are contributed simultaneously, the result of R_π is enhanced and we can get smaller values of $S_{K^0\pi^0}$.

VII. CONCLUSIONS

In this paper, we have analyzed the supersymmetric contributions to the direct and mixing CP asymmetries and also to the branching ratios of the $B \to K\pi$ decays in a model independent way.

We have shown that, in the SM, the $R_c - R_n$ puzzle which reflects the discrepancy between the experimental measurements of the branching ratios and their expected results cannot be resolved. Also, the direct CP asymmetries $A_{\bar{K}^0\pi^-}^{CP}$ and $A_{\bar{K}^0\pi^0}^{CP}$ are very small, while $A_{K^-\pi^0}^{CP}$ and $A_{K^-\pi^+}^{CP}$ are of the same order and can be larger. These correlations among the CP asymmetries are inconsistent with the recent measurements. Moreover, the mixing CP asymmetry $S_{K^0\pi^0}$, which is expected to be $\sin 2\beta$, differs from the corresponding experimental data. The confirmation of these discrepancies will be a clear signal for new physics beyond the SM.

We have emphasized that the Z-penguin diagram with chargino in the loop and the chargino electromagnetic penguin can enhance the contribution of the electroweak penguin to $B \to K\pi$, which is supposed to play a crucial role in explaining the above mentioned discrepancies. We found, however, that these contributions alone are not enough to solve the $R_c - R_n$ puzzle. It turns out that a combination of gluino and chargino contributions is necessary to account for the results of R_c and R_n within the $b \to s\gamma$ constraints. Nevertheless, our numerical results confirmed that the general trend of SUSY models favors that the experimental result of R_c goes down.

We have also provided a systematic study of the SUSY contributions to the direct CP asymmetries for $B \to K\pi$ decays. We found that a large gluino contribution is essential to explain the recent experimental data. It is worth mentioning that a large gluino contribution is also important to accommodate other controversial results measured in the B factories, namely, the mixing CP asymmetries $S_{\phi K}$ and $S_{\eta' K}$. Unlike the $R_c - R_n$ puzzle, we found that the CP asymmetries $A^{CP}_{K\pi}$ can be saturated by a single mass insertion $(\delta^d_{LR})_{23}$ contribution. It has been noticed that a large electroweak penguin is less favored by the CP asymmetries $A^{CP}_{K\pi}$. Therefore, one needs to optimize the gluino and the chargino contributions in order to satisfy simultaneously the branching ratios and the CP asymmetries of $B \to K\pi$.

Finally, we have considered the mixing CP asymmetry $S_{K^0\pi^0}$. We found, as in $S_{\phi K}$ and $S_{\eta' K}$, that the gluino contribution through the LR or RL mass insertion gives the largest contribution to $S_{K^0\pi^0}$. On the other hand, it is quite possible for the gluino exchanges to account for $S_{K^0\pi^0}$, $S_{\phi K}$, and $S_{\eta' K}$ at the same time.

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