

Solving the supersymmetric CP problem with flavor breaking F terms

J. Lorenzo Diaz-Cruz*

*Cuerpo Académico de Partículas, Campos y Relatividad FCFM, BUAP, Ap. Postal 1364 Puebla Pue., 72000, México*Javier Ferrandis[†]*Theoretical Physics Group, Lawrence Berkeley National Laboratory, Cyclotron Road 1, Berkeley California 94720, USA*
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Supersymmetric flavor models for the radiative generation of fermion masses offer an alternative way to solve the SUSY- CP problem. We assume that the supersymmetric theory is flavor and CP conserving. CP violating phases are associated to the vacuum expectation values of flavor violating SUSY-breaking fields. As a consequence, phases appear at tree level only in the soft supersymmetry-breaking matrices. Using a $U(2)$ flavor model as an example we show that it is possible to generate radiatively the first and second generation of quark masses and mixings as well as the Cabibbo-Kobayashi-Maskawa (CKM) CP phase. The one-loop supersymmetric contributions to electric dipole moments are automatically zero since all the relevant parameters in the Lagrangian are flavor conserving and as a consequence real. The size of the flavor and CP mixing in the SUSY-breaking sector is mostly determined by the fermion mass ratios and CKM elements. We calculate the contributions to ϵ , ϵ' and to the CP asymmetries in the B decays to ψK_s , ϕK_s , $\eta' K_s$ and $X_s \gamma$. We analyze a case study with maximal predictivity in the fermion sector. For this worst case scenario the measurements of Δm_K , Δm_B and ϵ constrain the model requiring extremely heavy squark spectra.

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I. INTRODUCTION

It was suggested a few years ago that the huge number of possible string theory vacua in combination with eternal inflation may allow us to understand the smallness of the cosmological constant from an anthropic point of view [1,2]. Although there is no current general framework for examining these metastable vacua in string theory [3] some particular methods have been proposed [4]. It is still an open debate whether the landscape does or does not predict high scale supersymmetry (SUSY) [5,6]. Statistical analysis of the vacuum of certain string theories has derived formulas for the distribution of vacua, which support the idea of a very high scale of supersymmetry breaking [7]. It has been pointed out that these considerations may also be relevant to understand the gauge hierarchy problem [8]. This has motivated the recent interest in field-theoretic realizations of models with large numbers of vacua [9] as well as in the analysis of the collider, cosmological and other phenomenological implications of supersymmetric models with very heavy supersymmetric spectra [10].

In this paper we would like to revisit, under the light of these new considerations, certain supersymmetric flavor models that are not usually considered in the literature because it is naively expected that they require a very heavy supersymmetric spectra to be compatible with experimental constraints on flavor changing processes (FCNC). In particular we will show that supersymmetric flavor models for the radiative generation of fermion

masses generate very predictive Yukawa textures and at the same time offer a new insight in the SUSY CP and flavor problems.

After more than 30 years of its observation, the violation of the CP symmetry is poorly known in its origin in the present particle physics paradigm, despite its relevance in nature. The presence of CP violating phases in particle physics models can be tested, for instance, through precision measurements of the electric dipole moments (EDMs) in the leptonic sector (electron and muon) and in the quark sector (neutron and deuteron). At present there are very stringent upper limits [11], which are expected to improve by several orders of magnitude in the near future experiments. It is known that the standard model contribution to the neutron EDM, in the absence of a theta term, is approximately $10^{-30} e \text{ c.m.}$, which is still more than 4 orders of magnitude below the reach of the current experiments.

In the context of an unconstrained minimal supersymmetric standard model (MSSM) [12] the generic contribution to the electric dipole moments is several orders of magnitude larger than the SM contribution. This serious violation of the current experimental constraints is known as the SUSY CP problem. Several possible explanations have been considered in the literature to account for the suppression of the supersymmetric contributions to EDMs and other CP violating observables. Some of them are (1) CP suppression, (2) cancellations, (3) alignment, (4) sfermion decoupling, and (5) flavor off-diagonal CP violation.

- (i) The CP suppressed scenario assumes that all the CP phases are suppressed [13] because CP is an approximate symmetry of the full theory.

*Electronic address: lorenzo.diaz@fcfm.buap.mx[†]Electronic address: ferrandis@mac.com

- (ii) The cancellation scenario is based on the existence of certain regions of the SUSY parameter space where different contributions to EDMs cancel [14,15].

The first possibility is nowadays ruled out. The CP suppressed scenario would imply that all SM contributions to CP asymmetries are small, which we know today not to be the case in the B system [16]. The cancellation scenario is known to be ruled out in scenarios with universal soft masses if constraints from electron, neutron and mercury atom EDMs are imposed simultaneously [17]. On the other hand in the context of scenarios with nonuniversal soft masses it is possible to satisfy simultaneously all the EDM constraints through the cancellation mechanism [18]. Nevertheless we would like to point out that there seems to be no symmetry that can guarantee such cancellations.

- (iii) In the CP alignment case the phases associated to the relevant parameters are somehow related in such a way that the combinations contributing to EDMs cancel.
- (iv) Decoupling entails that the sfermion masses are heavy enough to strongly suppress the supersymmetric contributions even though the CP phases can be arbitrarily large [19].

The alignment scenario could arise naturally in the context of models that generate exact soft universality as gauge mediated supersymmetry-breaking models [20] or in models with approximate horizontal Abelian flavor symmetries [21]. The decoupling scenario is very plausible. It requires sfermion masses of the order of several TeV, which implies the existence of fine-tuning in the soft supersymmetry-breaking sector.

- (v) The scenario with flavor off-diagonal CP violation assumes that the origin of the CP violation is closely related to the origin of flavor structures in such a way that the flavor blind quantities as the μ term, soft bilinear terms gaugino masses are real and only flavor off-diagonal CP phases are nonzero.

We find that the scenarios with flavor off-diagonal CP phases are especially interesting. The models of this kind proposed in the literature to date [22–24] assumed that flavor violating Yukawa matrices and soft terms are both generated simultaneously at tree level at very high energies. Thus, they require that Yukawa matrices and soft-trilinear matrices are Hermitian, which forces flavor-diagonal phases to vanish (up to small renormalization group equation corrections). We would like to propose a new idea similar to the flavor off-diagonal scenario which has not been considered before.

- (vi) We propose that the underlying supersymmetric theory is exactly CP conserving while CP phases are only carried by flavor violating SUSY-breaking fields.

At first sight one may be tempted to think that this scenario cannot account for the observed CP violation in the SM,

especially the large mixing in the B - \bar{B} system, and therefore conclude that CP violation must be present in the superpotential. We will show in this paper that this is not the case and certain models for the radiative generation of first and second generation fermion masses and mixings recently proposed [25,26] allow us to generate radiatively the Cabibbo-Kobayashi-Maskawa (CKM) phase and offer an alternative solution for the SUSY CP problem.

In this model flavor and CP violation appear at tree level only in the soft supersymmetry-breaking parameters and are transmitted to the fermion sector at one loop through low-energy finite threshold corrections. CP violating phases could appear originally in the vacuum expectation values (VEVs) of certain flavor violating SUSY-breaking fields. These VEVs break spontaneously both flavor and the CP symmetry generating at tree-level flavor and CP violating soft mass matrices. This class of models makes use of the presence of soft supersymmetry-breaking terms for the radiative generation of quark and lepton masses through sfermion-gaugino loops, as originally suggested by Buchmuller and Wyler [27,28] and later analyzed in more detail in Refs. [29–35]. The gaugino mass would provide the violation of fermionic chirality required by a fermion mass while the soft breaking terms provide the violation of flavor and CP symmetries.

In this paper we have chosen to analyze a case study that as we will show is the worst case scenario from the point of view of FCNC constraints. We analyze a model of this kind because it achieves maximal predictivity in the quark sector. This case study, as we will see, tends to generate important contributions to some flavor changing processes, especially Δm_K , Δm_d and ϵ , which can only be avoided if both the squark and the gluino spectra are very heavy.

This paper is organized as follows. We begin in Sec. II by describing the model we propose for the radiative generation of first and second generation fermion masses and mixing angles. In Sec. III we analyze the radiative generation of Yukawa couplings in this model. In Sec. III A we study the predictions and constraints for quark mass ratios. In Sec. III B we study the radiative generation of the SM CKM phase and the predictions and constraints arising from measured CKM elements and CP phases. In Sec. IV we analyze in detail the calculation of the soft matrices in the SuperCKM basis. In Sec. V we argue that the contributions to EDMs are exactly zero in this model. In Sec. VI we study the contributions to direct and indirect CP violation in the K - \bar{K} system. In Sec. VII we study the contributions to CP asymmetries in the B - \bar{B} system.

II. THE MODEL

In this section we will consider a realistic three generation model based in a horizontal $U(2)_H$ symmetry [36]. This is a generalization of the model proposed in Ref. [25]. We will assume the usual MSSM particle content where third generation matter superfields,

$$\mathcal{Q}_3, \mathcal{D}_3, \mathcal{U}_3, \mathcal{L}_3, \mathcal{E}_3, \quad (1)$$

and up and down electroweak Higgs superfields, \mathcal{H}_u and \mathcal{H}_d , are singlets under $U(2)_H$. We will denote them abbreviately by $\bar{\phi}^L$ and ϕ^R . Let us assume that first and second generation left-handed superfields,

$$\bar{\Psi}_{\mathcal{Q}} = \begin{pmatrix} \mathcal{Q}_1 \\ \mathcal{Q}_2 \end{pmatrix}, \quad \bar{\Psi}_{\mathcal{L}} = \begin{pmatrix} \mathcal{L}_1 \\ \mathcal{L}_2 \end{pmatrix}, \quad (2)$$

as well as the first and second generation right-handed superfields,

$$\Psi_{\mathcal{U}} = \begin{pmatrix} \mathcal{U}_1 \\ \mathcal{U}_2 \end{pmatrix}, \quad \Psi_{\mathcal{D}} = \begin{pmatrix} \mathcal{D}_1 \\ \mathcal{D}_2 \end{pmatrix}, \quad \Psi_{\mathcal{E}} = \begin{pmatrix} \mathcal{E}_1 \\ \mathcal{E}_2 \end{pmatrix}, \quad (3)$$

transform as covariant vectors under $U(2)_H$. We will denote them abbreviately by $\bar{\Psi}_a^L$ and Ψ_a^R . We will introduce a set of supersymmetry-breaking chiral superfields,

$$S^{ab}, \quad \mathcal{A}^{ab}, \quad \mathcal{F}^a \quad (a, b = 1, 2), \quad (4)$$

that transform under $U(2)_H$ contravariantly as a symmetric tensor, an antisymmetric tensor and a vector, respectively. We will assume that only the auxiliary components of the flavor breaking superfields are nonzero. The most general form for the VEVs of the flavor breaking fields is

$$\langle S \rangle = \begin{pmatrix} v_s e^{i\phi_s} & 0 \\ 0 & V_S e^{i\phi_s} \end{pmatrix} \theta^2, \quad (5)$$

$$\langle \mathcal{A} \rangle = \begin{pmatrix} 0 & v_a e^{i\phi_a} \\ -v_a e^{i\phi_a} & 0 \end{pmatrix} \theta^2, \quad (6)$$

$$\langle \mathcal{F} \rangle = \begin{pmatrix} v_f e^{i\phi_f} \\ V_{\mathcal{F}} e^{i\phi_f} \end{pmatrix} \theta^2, \quad (7)$$

where the v_s, V_S, v_a, v_f and $V_{\mathcal{F}}$ are real parameters. We will assume the following particular hierarchy in the flavor breaking VEVs:

$$(v_f, v_a, V_S, V_{\mathcal{F}}) = (\lambda^2, \lambda^2, \lambda, 2\lambda) M_F \tilde{m}. \quad (8)$$

We will also assume that $v_s \ll V_S$ and for practical purposes set $v_s = 0$. Here λ is a flavor breaking perturbation parameter, M_F is the flavor breaking scale. We note that \tilde{m} is a new mass scale linked to the flavor violating SUSY-breaking fields. We do not have yet a predictive model for the $U(2)_H$ breaking, which is a relevant point under current investigation. The proposed VEVs in Eq. (8) are introduced *ad hoc*. These *ad hoc* assumptions will prove *a posteriori* to be very successful in reproducing fermion masses and mixings. Furthermore, in the case that $U(2)_H$ is a gauge symmetry broken spontaneously we expect the $U(2)_H$ -gauge fields to get masses of the order of the flavor breaking scale which can be very heavy in this scenario. Therefore any other phenomenological effects in the low-energy model beyond the flavor structure it gives rise to in

the soft supersymmetry-breaking sector would be very suppressed.

We will assume that the superpotential of the model is CP and $U(2)_H$ symmetric. Therefore the only couplings allowed in the renormalizable superpotential by the SM $SU(3)_C \times SU(2)_L \times U(1)_Y$ vertical symmetry, the $U(2)_H$ horizontal symmetry and CP conservation are the third generation ones and the so-called μ term,

$$\lambda_t \mathcal{Q}_3 \mathcal{U}_3 \mathcal{H}_u + \lambda_b \mathcal{Q}_3 \mathcal{D}_3 \mathcal{H}_d + \lambda_\tau \mathcal{L}_3 \mathcal{E}_3 \mathcal{H}_d + \mu \mathcal{H}_u \mathcal{H}_d. \quad (9)$$

We note that, in principle, two other couplings could be allowed in the superpotential: $\mathcal{L}_3 \mathcal{H}_u$ and $\mathcal{Q}_3 \mathcal{L}_3 \mathcal{D}_3$. There are different ways to remove these unwanted couplings. They could be forbidden imposing R -parity conservation defined as $R = (-)^{3B+L+2S}$, where B is the baryonic number, L the leptonic number and S the spin. A third possibility would be to extend the $U(2)_H$ symmetry to the maximal $U(3)_H$ horizontal symmetry. The breaking of the $U(3)_H$ symmetry in the direction of the third generation would leave us with our $U(2)_H$ symmetry; in such a case this bilinear interaction would not be allowed by the $U(3)_H$ symmetry. We also note that the couplings in the renormalizable superpotential cannot carry complex phases since CP is an exact symmetry at this level. Therefore, at tree level the Yukawa matrices are generically of the form,

$$\mathcal{Y} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & y \end{bmatrix}. \quad (10)$$

Additionally, trilinear soft supersymmetry-breaking terms are generated by operators generically of the form,

$$\sum_{Z=S, \mathcal{A}} \frac{1}{M_F} \int d^2\theta Z^{ab} \bar{\Psi}_a^L \Psi_b^R \mathcal{H}_\alpha, \quad (11)$$

$$\frac{1}{M_F} \int d^2\theta (\mathcal{F}^a \bar{\Psi}_a^L \phi_R + \bar{\phi}^L \mathcal{F}^a \Psi_a^R) \mathcal{H}_\alpha, \quad (12)$$

where M_F is the flavor breaking scale, $a = 1, 2$ are flavor indices, and \mathcal{H}_α ($\alpha = u, d$) represents any of the Higgs superfields. Soft supersymmetry-breaking mass matrices can receive diagonal flavor conserving contributions of the form,

$$\frac{1}{M_F^2} \int d^4\theta \left(\sum_{Z, Z'=S, \mathcal{A}} Z^\dagger Z' + \mathcal{F}^\dagger \mathcal{F} \right) (\Psi^\dagger \Psi + \phi^\dagger \phi). \quad (13)$$

Additionally nondiagonal flavor violating contributions arise from operators generically of the form,

$$\frac{1}{M_F^2} \int d^4\theta \left(\sum_{Z, Z'=S, \mathcal{A}} \Psi^{\dagger a} Z_{ac}^\dagger Z'^{cb} \Psi_b + \Psi^{\dagger a} \mathcal{F}_a^\dagger \mathcal{F}^b \Psi_b \right), \quad (14)$$

where $a \neq b$. Flavor violating supersymmetry-breaking fields cannot generate masses for the gauginos. Therefore we need to introduce at least one flavor-singlet chiral superfield, \mathcal{G} , whose F -term component gets a nonzero VEV giving masses to gauginos from operators of the form,

$$\frac{1}{M} \int d^2\theta \mathcal{G} \tilde{\lambda} \tilde{\lambda}. \quad (15)$$

We note that $\langle \mathcal{G} \rangle$ breaks supersymmetry but not the flavor symmetry. We will identify M with the usual supersymmetry-breaking messenger scale. We note that the messenger scale is in general different from the flavor breaking scale even though the flavor breaking fields are supersymmetry-breaking fields themselves. The gaugino mass generated is given by $m_{\tilde{\lambda}} = \langle \mathcal{G} \rangle / M$. The flavor-singlet superfield responsible for generating gaugino masses, \mathcal{G} , will couple to matter fields too generating soft trilinear couplings,

$$\frac{\kappa}{M} \int d^2\theta \mathcal{G} \phi^L \phi^R \mathcal{H}_\alpha, \quad (16)$$

where κ is a real dimensionless coupling determined by some unknown underlying renormalizable theory. For practical purposes we will assume that κ can take arbitrary values. Soft masses would also be generated by operators generically of the form,

$$\frac{\eta}{M^2} \int d^4\theta \mathcal{G}^\dagger \mathcal{G} (\Psi^\dagger \Psi + \phi^\dagger \phi), \quad (17)$$

$$\frac{\eta'}{MM_F} \int d^4\theta \mathcal{G}^\dagger \phi^\dagger \mathcal{F} \Psi. \quad (18)$$

Here flavor indices have been omitted. η and η' are also real dimensionless couplings determined by the unknown

underlying renormalizable theory. Regarding the possible appearance of D terms in the scalar potential, D terms would appear when a local symmetry is spontaneously broken by scalar fields, which is not the case for the model under consideration. After the $U(2)_H$ flavor and the CP symmetry are broken spontaneously by the supersymmetry breaking fields defined in Eq. (7) the following boundary conditions for the soft-trilinear matrices are generated at the scale M_F :

$$\mathbf{A} = A \begin{bmatrix} 0 & \sigma \lambda^2 e^{i\phi_a} & \sigma \lambda^2 e^{i\phi_f} \\ -\sigma \lambda^2 e^{i\phi_a} & \sigma \lambda e^{i\phi_S} & 2\sigma \lambda e^{i\phi_F} \\ \sigma \lambda^2 e^{i\phi_f} & 2\sigma \lambda e^{i\phi_F} & 1 \end{bmatrix}, \quad (19)$$

where $A = \kappa m_{\tilde{\lambda}}$ and the dimensionless parameter σ is defined by $\sigma = \tilde{m}/A$. The mass parameter \tilde{m} , defined in Eq. (8), is a new mass scale introduced in the problem by the flavor violating SUSY-breaking fields. We note that only one combination of the complex phases in Eq. (19) will be transmitted to the Yukawa matrices. For convenience one can remove some of them from the soft-trilinear matrix through a redefinition of the phases of the matter fields, even though they will appear in the soft mass matrices. Without any loss of generality we can adopt a flavor basis where the soft-trilinear matrix takes the following form:

$$\mathbf{A} = A \begin{bmatrix} 0 & \sigma \lambda^2 & \sigma \lambda^2 e^{-i\gamma} \\ -\sigma \lambda^2 & \sigma \lambda & 2\sigma \lambda \\ \sigma \lambda^2 e^{-i\gamma} & 2\sigma \lambda & e^{-i\phi} \end{bmatrix}. \quad (20)$$

The phases γ and ϕ are related to the phases in Eq. (19) by $\gamma = -(\phi_f + \Phi_S - \phi_a - \Phi_F)$ and $\phi = (2\Phi_F - \Phi_S)$. After the $U(2)_H$ flavor breaking flavor violating soft mass matrices are also generated. In the flavor basis adopted in Eq. (20) they take the following form:

$$\tilde{\mathcal{M}}_{L,R}^2 = \tilde{m}_f^2 \begin{bmatrix} 1 + 5\rho \lambda^2 & \rho \lambda^3 (2e^{-i\gamma} - e^{-i\phi'}) & \rho' \lambda^2 e^{-i(\gamma-\phi)} \\ \rho \lambda^3 (2e^{i\gamma} - e^{i\phi'}) & 1 + 5\rho \lambda^2 & 2\rho' \lambda e^{i\phi} \\ \rho' \lambda^2 e^{i(\gamma-\phi)} & 2\rho' \lambda e^{-i\phi} & 1 + 5\rho \lambda^2 \end{bmatrix}, \quad (21)$$

where, $\tilde{m}_f^2 = \eta m_{\tilde{\lambda}}^2$ and $\phi' = (\phi + \phi_a - 2\Phi_F)$. ρ and ρ' are dimensionless parameters defined by $\rho = \tilde{m}^2 / \tilde{m}_f^2$ and $\rho' = (\eta' / \eta) (\tilde{m} / \tilde{m}_{\tilde{\lambda}})$. We note that in this scenario the amount of flavor violation in the soft mass matrices is determined not only by the powers of λ in the off-diagonal entries but also by the parameters σ , ρ and ρ' . We note that in the limit $\tilde{m} \rightarrow 0$ all the flavor violation will be suppressed. There are other interesting limits: if $\eta' \rightarrow 0$ the mixing between the third and first or the second generation in the soft mass matrices is suppressed; if $\tilde{m} \ll \tilde{m}_f$ the flavor mixing between the first and the second generation in the soft mass matrices is also suppressed and the sfermion masses will be nearly universal. In the case $A \simeq \tilde{m} \ll \tilde{m}_f$ only the contributions from soft masses to flavor vio-

lating processes would be suppressed while the flavor violation in the soft-trilinear matrices could be sizable.

III. RADIATIVE GENERATION OF YUKAWA COUPLINGS

In the presence of flavor violation in the soft sector, the left- and right-handed components of the sfermions mix. For instance, in the gauge basis the 6×6 down-type squarks mass matrix is given by

$$\mathcal{M}_D^2 = \begin{bmatrix} \tilde{\mathcal{M}}_{D_L}^2 + v^2 c_\beta^2 \mathbf{Y}_D^\dagger \mathbf{Y}_D & (\mathbf{A}_{D_C\beta}^\dagger - \mu \mathbf{Y}_{D_S\beta}) v \\ (\mathbf{A}_{D_C\beta} - \mu \mathbf{Y}_{D_S\beta}^\dagger) v & \tilde{\mathcal{M}}_{D_R}^2 + v^2 c_\beta^2 \mathbf{Y}_D \mathbf{Y}_D^\dagger \end{bmatrix}, \quad (22)$$

where $\tilde{\mathcal{M}}_{D_R}^2$ and $\tilde{\mathcal{M}}_{D_L}^2$ are the 3×3 right-handed and left-handed soft mass matrices (including D terms), \mathbf{A}_D is the 3×3 soft-trilinear matrix, \mathbf{Y}_D is the 3×3 tree-level Yukawa matrix. $\tan\beta$ is the ratio of Higgs expectation values in the MSSM, μ is the so-called μ term and $v = s_W m_W / \sqrt{2\pi\alpha_e} = 174.5$ GeV. $\tilde{\mathcal{M}}_D^2$ is diagonalized by a 6×6 unitary matrix, \mathbf{Z}^D . The presence of flavor violating entries at tree level in the soft supersymmetry breaking matrices will generate one-loop contributions to the Yukawa matrices. In general, the dominant finite one-loop contribution to the 3×3 down-type quark Yukawa matrix including CP phases [37] is given by the gluino-squark loop,

$$(\mathbf{Y}_D)_{ab}^{\text{rad}} = \frac{\alpha_s}{3\pi} m_{\tilde{g}}^* \sum_c \mathbf{Z}_{ac}^D \mathbf{Z}_{(b+3)c}^{D*} B_0(m_{\tilde{g}}, m_{\tilde{d}_c}), \quad (23)$$

where \tilde{d}_c ($c = 1, \dots, 6$) are mass eigenstates and $m_{\tilde{g}}$ is the gluino mass. B_0 is a known function that can be found elsewhere in the literature. The radiatively corrected 3×3 down-type quark mass matrix is given by

$$\mathbf{m}_D = v c_\beta (\mathbf{Y}_D + \mathbf{Y}_D^{\text{rad}}). \quad (24)$$

We note that the effective supersymmetric model proposed generates an approximately degenerate squark spectra. In the squark degenerate case one obtains a simple expression for $\mathbf{Y}_D^{\text{rad}}$,

$$\mathbf{Y}_D^{\text{rad}} = \frac{2\alpha_s}{3\pi} m_{\tilde{g}}^* (\mathbf{A}_D - \mu \mathbf{Y}_D \tan\beta) F(m_{\tilde{b}}, m_{\tilde{b}}, m_{\tilde{g}}), \quad (25)$$

where the function $F(x, y, z)$ is a form factor of the particles in the loop with units of $[\text{Mass}]^{-2}$. $F(x, y, z)$ is defined in Eq. (A2) of the Appendix. For the soft-trilinear texture in Eq. (19) predicted by our model one obtains a simple expression for the radiatively corrected down-type quark mass matrix,

$$\mathbf{m}_D = \hat{m}_b \begin{bmatrix} 0 & \omega \lambda^2 & \omega \lambda^2 e^{-i\gamma} \\ -\omega \lambda^2 & \omega \lambda & 2\omega \lambda \\ \omega \lambda^2 e^{-i\gamma} & 2\omega \lambda & 1 \end{bmatrix}, \quad (26)$$

where ω encodes the dependence on the supersymmetric spectra. For the case $m_{\tilde{b}} \geq m_{\tilde{g}}$, we obtain

$$\omega = c_\beta \left(\frac{v}{\hat{m}_b} \frac{2\alpha_s}{3\pi} \right) \left(\frac{m_{\tilde{g}}}{m_{\tilde{b}}} \right) \left(\frac{\tilde{m}}{m_{\tilde{b}}} \right). \quad (27)$$

We emphasize that \tilde{m} is not any squark mass scale but a new mass scale introduced in the problem by the VEVs of the flavor violating SUSY-breaking fields; see Eqs. (5)–(7). The parameter \hat{m}_b defined as

$$\hat{m}_b = v c_\beta \left(y_b + \omega_b \left(e^{-i\phi} - \frac{\mu}{A_b} y_b \tan\beta \right) \right) \quad (28)$$

is approximately the running bottom mass. $\omega_b = \omega \hat{m}_b / (v c_\beta)$. The phase $e^{-i\phi}$ is an overall phase absorbed

in the definition of \hat{m}_b in Eq. (28), which has no observable implications.

A. Quark masses

The implications for fermion masses arising from a matrix similar to the one in Eq. (26) were studied in Refs. [25,26,38]. In this subsection we briefly summarize results included in those references. Although not diagonal in the gauge basis, the matrix \mathbf{m}_D can be brought to diagonal form in the mass basis by a biunitary diagonalization, $(\mathcal{V}_L^d)^\dagger \mathbf{m}_D \mathcal{V}_R^d = (m_d, m_s, m_b)$. The down-type quark mass matrix given by Eq. (26) makes the following predictions for the quark mass ratios to leading order:

$$\frac{m_d}{m_s} = \lambda^2 + \mathcal{O}(\lambda^4), \quad \frac{m_s}{m_b} = \omega \lambda + \mathcal{O}(\lambda^4). \quad (29)$$

We can relate λ and ω with dimensionless fermion mass ratios. To first order,

$$\lambda = \left(\frac{m_d}{m_s} \right)^{1/2}, \quad \omega = \left(\frac{m_s^3}{m_b^2 m_d} \right)^{1/2}. \quad (30)$$

Using these relations and the running quark masses determined from experiment, see Ref. [38] for details, we can determine λ and ω given by $\lambda = 0.209 \pm 0.019$ and $\omega = 0.109 \pm 0.030$. We observe that constraints on the supersymmetric spectra can be derived from the parameter ω . To assess the viability of the model we must check if it is possible for ω to reach the values required by the observed quark masses. From Eq. (37) we obtain the following inequality for $m_{\tilde{g}} \leq m_{\tilde{b}}$:

$$\omega = c_\beta \left(\frac{v}{\hat{m}_b} \frac{2\alpha_s}{3\pi} \right) \left(\frac{\tilde{m}}{m_{\tilde{b}}} \right) \left(\frac{m_{\tilde{g}}}{m_{\tilde{b}}} \right). \quad (31)$$

Using the measured values $v = 174.5$ GeV, $m_b(m_b)_{\text{MS}} = 4.2 \pm 0.2$ and $\alpha_s = 0.117$ we obtain

$$\omega \leq 1.5 c_\beta \left(\frac{\tilde{m}}{m_{\tilde{b}}} \right). \quad (32)$$

Therefore the values of ω required by the measured quark masses can be easily reached without any *ad hoc* tuning in the supersymmetric parameter space. Let us examine with some detail the case $m_{\tilde{g}} \approx m_{\tilde{b}}$. For large $\tan\beta$, $\tan\beta = 50$, we obtain

$$\omega \approx 0.03 \left(\frac{\tilde{m}}{m_{\tilde{b}}} \right),$$

which would require $\tilde{m} \approx 3m_{\tilde{b}}$. On the other hand in the opposite gluino mass limit, $m_{\tilde{g}} > 2m_{\tilde{b}}$, one obtains

$$\omega = c_\beta \left(\frac{v}{\hat{m}_b} \frac{2\alpha_s}{3\pi} \right) \left(\frac{\tilde{m}}{m_{\tilde{b}}} \right) \ln \left(\frac{m_{\tilde{g}}}{m_{\tilde{b}}} \right). \quad (33)$$

For the large $\tan\beta$ case, $\tan\beta = 50$, we obtain similar results,

$$\omega \approx 0.03 \left(\frac{\tilde{m}}{m_{\tilde{g}}} \right) \ln \left(\frac{m_{\tilde{g}}}{m_{\tilde{b}}} \right).$$

If $\tilde{m} \simeq m_{\tilde{g}}$ we would need $m_{\tilde{g}} \simeq 20m_{\tilde{b}}$. This would imply $\tilde{m} \simeq 20m_{\tilde{b}}$. The mass matrix in Eq. (26) is very successful in reproducing the down-type quark mass ratios, but it cannot explain correctly the measured mass ratios in the up-type quark sector. We will propose a simple solution. Let us assume that down- and up-type quark fields transform differently under certain Z_2 symmetry. If this was the case the fields S , \mathcal{A} and \mathcal{F} would not generate any mixing in the up-type quark sector. Let us assume that there is an extra $U(2)_H$ symmetric tensor, S' , that gets a VEV of the form,

$$\langle S' \rangle = \begin{pmatrix} \lambda^6 & 0 \\ 0 & \lambda^2 \end{pmatrix} \theta^2. \quad (34)$$

If the couplings of S' with the up-type quark fields are allowed by the Z_2 symmetry they would induce a soft-trilinear matrix of the form,

$$\mathbf{A}_U = A_t \begin{bmatrix} \sigma \lambda^6 & 0 & 0 \\ 0 & \sigma \lambda^2 & 0 \\ 0 & 0 & 1 \end{bmatrix}. \quad (35)$$

We note that we do not show any phases in the matrix \mathbf{A}_U . Possible phases in the diagonal entries of \mathbf{A}_U are not physical to leading order since they can be absorbed through a redefinition of the phases of the matter fields. Masses for the up and charm quarks are generated radiatively. One can perform an analysis similar to the analysis in the down-type quark sector and obtain a simple expression for the radiatively corrected up-type quark mass matrix,

$$\mathbf{m}_U = \hat{m}_t \begin{bmatrix} \omega \lambda^6 & 0 & 0 \\ 0 & \omega \lambda^2 & 0 \\ 0 & 0 & 1 \end{bmatrix}, \quad (36)$$

where ω for $m_{\tilde{t}} \geq m_{\tilde{g}}$ is given in this case by

$$\omega \leq s_\beta \left(\frac{v}{\hat{m}_t} \frac{2\alpha_s}{3\pi} \right) \left(\frac{m_{\tilde{g}}}{m_{\tilde{t}}} \right) \left(\frac{\tilde{m}}{m_{\tilde{t}}} \right), \quad (37)$$

and \hat{m}_t is the normalized top quark mass given by

$$\hat{m}_t = v s_\beta \left(y_t + \omega_t \left(1 - \frac{\mu}{A_t} y_t \cot \beta \right) \right), \quad (38)$$

with $\omega_t = \omega \hat{m}_t / (v s_\beta)$. For the soft-trilinear texture under consideration in Eq. (35) one obtains the following predictions for the up-type quark mass ratios:

$$\frac{m_u}{m_c} = \lambda^4 + \mathcal{O}(\lambda^6), \quad \frac{m_c}{m_t} = \omega \lambda^2 + \mathcal{O}(\lambda^5). \quad (39)$$

Again we can relate λ and ω with dimensionless quark mass ratios, to first order,

$$\lambda = \left(\frac{m_u}{m_c} \right)^{1/4}, \quad \omega = \left(\frac{m_c^3}{m_t^2 m_u} \right)^{1/2}. \quad (40)$$

Using the running quark masses determined from experiment, see Ref. [38] for details, we obtain $\lambda = 0.225 \pm 0.015$ and $\omega = 0.071 \pm 0.018$. We note the similarity in the values for λ and ω calculated in the up- and down-type quark sectors from Eqs. (30) and (40). Let us examine with some detail the case $m_{\tilde{g}} \approx m_{\tilde{t}}$. Using the measured top quark mass, $m_t = 178$ GeV, we obtain the condition,

$$\omega \approx 0.02 s_\beta \left(\frac{\tilde{m}}{m_{\tilde{t}}} \right).$$

For instance, for large $\tan \beta$, $\tan \beta = 50$ we obtain

$$\omega \approx 0.02 \left(\frac{\tilde{m}}{m_{\tilde{t}}} \right).$$

This constraint is compatible with the analogous constraint arising from the down-type quark sector, which was $\omega \approx 0.03(\tilde{m}/m_{\tilde{b}})$; see Eq. (32). Therefore no important splitting between the sbottom and stop quark masses is required for the viability of the model. Indeed both constraints could be satisfied simultaneously for $m_{\tilde{b}} \approx 1.5m_{\tilde{t}}$, which is a non-trivial consistency check both of the model and the *ad hoc* VEVs introduced in Eqs. (5)–(7).

B. Radiatively generated CP phase and CKM elements

Finally, one can calculate the CKM mixing matrix. This is defined by $\mathcal{V}_{\text{CKM}} = \mathcal{V}_L^{u\dagger} \mathcal{V}_L^d$. We have seen in the previous section that in the simple model here proposed the up-type quark mass matrix is diagonal. Therefore the CKM matrix is given by $\mathcal{V}_{\text{CKM}} = \mathcal{V}_L^d$. The diagonalization of the down-type quark mass matrix in Eq. (26) leads us to the following expression for \mathcal{V}_{CKM} to leading order in powers of λ :

$$\begin{bmatrix} 1 + 2is_\gamma \lambda \omega - \lambda^2/2 & -\lambda(1 + \omega \lambda(4 + 2is_\gamma)) & \omega \lambda^2 e^{-i\gamma} \\ -\lambda(1 + 4\omega \lambda) & \frac{1}{2}(\lambda^2 + 4\omega^2 \lambda^2) - 1 & 2\omega \lambda \\ \omega \lambda^2(2 - e^{i\gamma}) & 2\omega \lambda & 1 - 2\omega^2 \lambda^2 \end{bmatrix}. \quad (41)$$

It is easy to check that this CKM matrix is unitary to order λ^3 , i.e. $\mathcal{V}_{\text{CKM}}^\dagger \mathcal{V}_{\text{CKM}} = I + \mathcal{O}(\lambda^3)$. We note that the model predicts that to leading order $|V_{us}| = \lambda$. The measured value of $|V_{us}|$, $|V_{us}|_{\text{exp}} = 0.220 \pm 0.0026$, agrees

perfectly with the value of λ as calculated from quark mass ratios in Eqs. (30) and (40). The model also predicts that to leading order $\omega = |V_{cb}|/2|V_{us}|$. Using the measured value for $|V_{cb}|$, $|V_{cb}|_{\text{exp}} = 0.0413 \pm 0.0015$, we ob-

tain that $\omega = 0.093 \pm 0.005$. We note that this value of ω is surprisingly consistent with the value calculated from quark mass ratios from Eqs. (30) and (40). Finally using these values of λ and ω we can predict $|V_{ub}|$ to be $|V_{ub}| = \omega\lambda^2 = 0.0045 \pm 0.0003$, which is consistent with the measured value, $|V_{ub}|_{\text{exp}} = 0.00367 \pm 0.00047$. Using again the previous values of λ and ω calculated from $|V_{us}|$ and $|V_{cb}|$ we find the following predictions for $|V_{cs}|$ and $|V_{tb}|$, $|V_{cs}| = 0.9795 \pm 0.0007$ and $|V_{tb}| = 0.9991 \pm 0.0001$, which are also in agreement with experiment. It is a trivial check to prove that the angle γ introduced in the parametrization of the CKM matrix given in Eq. (41) coincides with the standard definition for $\gamma = \phi_2$,

$$\gamma = \arg\left[-\frac{V_{ud}V_{ub}^*}{V_{cd}V_{cb}^*}\right]. \quad (42)$$

The angle ϕ_1 is defined as usual by

$$\phi_1 = \arg\left[-\frac{V_{cd}V_{cb}^*}{V_{td}V_{tb}^*}\right]. \quad (43)$$

We note that we are using the notation ϕ_1 and not the usual β to avoid any confusion with the supersymmetric parameter $\tan\beta$, the ratio of the Higgs VEVs in the MSSM. The angle $\alpha = \phi_3$ can be obtained from $(\alpha + \phi_1 + \gamma) = \pi$. Using our parametrization for the CKM matrix we obtain to leading order in powers of λ a simple relation between the angles ϕ_1 and γ ,

$$\phi_1 = \arg[2 - e^{-i\gamma}]. \quad (44)$$

Using the measured value of the ϕ_1 phase, $\phi_1^{\text{exp}} = 23.3^\circ \pm 3.2^\circ$ (2σ) we predict to leading order the phase γ to be within the range $\gamma_{\text{theo}} = 103^\circ \pm 13^\circ$. This is far from the current 1σ global fit value for γ , $\gamma_{\text{fit}} = 61^\circ \pm 12^\circ$. To find better agreement with the experimental constraints on γ it is crucial to include in Eq. (44) the next to leading order (NLO) corrections to the unitarity of the CKM matrix. We find that the only relevant correction which affects the prediction for γ is the correction to the element V_{td} ,

$$V_{td}^{\text{NLO}} = (2(1 + 5\omega\lambda) - e^{i\gamma})\omega\lambda^2. \quad (45)$$

Including this correction we predict the phase γ to be within the range $\gamma_{\text{theo}} = 91^\circ \pm 18^\circ$, which intersects with the current 1σ global fit value for γ . For this range of γ , $|V_{td}|$ is predicted to be $|V_{td}| = 0.0043 \pm 0.0005$. Finally $|V_{ud}|$ is predicted to be $|V_{ud}| = 0.9765 \pm 0.0006$. To sum up, in this model not only the first and second generation fermion masses but also the CKM phase can be generated radiatively in perfect agreement with current measurements.

IV. SUPERCKM BASIS

Overcoming the present experimental constraints on supersymmetric contributions to flavor changing and CP

violating processes is a necessary requirement for the consistency of any supersymmetric model [39]. In our scenario, as a consequence of the approximate radiative alignment between Yukawa and soft-trilinear matrices there is an extra suppression of the supersymmetric contributions to some of these processes. Therefore for calculational purposes it is convenient to rotate the squarks to the so-called superCKM (SCKM) basis where this radiative alignment mechanism is manifest.

The superCKM basis is the basis where gaugino vertices are flavor diagonal [40–42]. In this basis, the entries in the soft-trilinear matrices are directly proportional to the corresponding contributions to flavor changing processes. For instance, the soft-trilinear matrix \mathbf{A}_D in the superCKM basis is given by

$$\mathbf{A}_D^{\text{SCKM}} = (\mathcal{V}_L^d)^\dagger \mathbf{A}_D \mathcal{V}_R^d, \quad (46)$$

where $\mathcal{V}_{L,R}^d$ are the down-type quark diagonalization matrices. The soft-trilinear matrix \mathbf{A}_D is given by Eq. (20). The Yukawa diagonalization matrices are given by $\mathcal{V}_L^d = \mathcal{V}_{\text{CKM}}^d$ in Eq. (41) while \mathcal{V}_R^d is completely determined to obtain real mass eigenvalues after the diagonalization of \mathbf{m}_D . We obtain, to leading order in λ ,

$$\text{Im}[\mathbf{A}_D^{\text{SCKM}}] = (-)A_b \begin{bmatrix} 0 & 2\omega s_\gamma \sigma \lambda^3 & s_\gamma \sigma \lambda^2 \\ 2s_\gamma \sigma \omega \lambda^3 & 0 & 2s_\phi \omega \lambda \\ s_\gamma \sigma \lambda^2 & 2s_\phi \omega \lambda & s_\phi \end{bmatrix}, \quad (47)$$

while the real part, $\text{Re}[\mathbf{A}_D^{\text{SCKM}}]$, is given by

$$A_b \begin{bmatrix} \sigma \lambda^3 & 4\sigma \omega \lambda^3 (c_\gamma - 1) & \sigma \lambda^2 (c_\gamma - 2) \\ 4\sigma \omega \lambda^3 (c_\gamma + 1) & \lambda \sigma & 2\lambda (c_\phi \omega - \sigma) \\ \sigma \lambda^2 (c_\gamma + 2) & 2\lambda (c_\phi \omega - \sigma) & c_\phi + 8\omega \sigma \lambda^2 \end{bmatrix}. \quad (48)$$

We note that the entries (2, 1) and (1, 2) contain an additional suppression factor $\omega\lambda$ compared with the soft-trilinear matrix in the flavor basis; see Eq. (20). This suppression is a consequence of the radiative alignment between Yukawa and soft-trilinear matrices. It is convenient when calculating supersymmetric contributions to flavor violating processes to use the parameters $(\delta_{ij}^d)_{LR}$ defined as

$$(\delta_{ij}^d)_{LR} = \frac{v c_\beta (\mathbf{A}_D^{\text{SCKM}})_{12}}{m_{\tilde{q}}^2}. \quad (49)$$

For consistency we also need to calculate the down-type squark soft mass matrices in the SCKM basis. For instance, for the left-handed soft mass matrix we obtain

$$(\tilde{\mathcal{M}}_{D_L}^2)^{\text{SCKM}} = (\mathcal{V}_L^d)^\dagger (\tilde{\mathcal{M}}_{D_L}^2) \mathcal{V}_L^d, \quad (50)$$

and analogously for the right-handed soft mass matrix. Assuming the soft-trilinear texture from Eq. (21) we obtain for $\text{Re}[(\tilde{\mathcal{M}}_{D_L}^2)^{\text{SCKM}}]$, to leading order in λ ,

$$m_{d_L}^2 \begin{bmatrix} 1 & y\lambda^3 & y'\lambda^2 \\ y\lambda^3 & 1 & -2c_\phi\rho'\lambda \\ y'\lambda^2 & -2c_\phi\rho'\lambda & (1 + (8c_\phi\rho'\omega - 5\sigma\lambda^2)\lambda^2) \end{bmatrix}. \quad (51)$$

Here $m_{d_L}^2$, y and y' are defined as

$$m_{d_L}^2 = m_f^2(1 + (1 + 5\rho)\lambda^2), \quad (52)$$

$$y = (4\rho'\omega(c_\gamma c_\phi - 2c_\phi) + \rho(c_{2\phi'} - 2c_\gamma)), \quad (53)$$

$$y' = (c_{(\phi-\gamma)} - 2c_\phi)\rho'. \quad (54)$$

We note that if the gluino mass is of the same order as the squark masses, $m_{\tilde{\chi}} \simeq \tilde{m}_f$, $\tilde{m}_f \simeq \tilde{m}$ and $\eta' \simeq \eta$, we expect that $\rho \simeq \rho'$. In that case the coefficient y simplifies to $y \simeq \rho(c_{2\phi'} - 2c_\gamma)$ since $\omega \simeq 2\lambda^2$. Furthermore if $\rho'/\rho = (\eta'/\eta)(\tilde{m}_f^2/\tilde{m}m_{\tilde{\chi}})$ the limit $\rho' \gg \rho$ would correspond to $\eta' \gg \eta$ or $\tilde{m} \ll \tilde{m}_f^2$ or $m_{\tilde{\chi}} \ll \tilde{m}_f^2$; if this is not the case we would expect the ρ term to dominate. The imaginary component, $\text{Im}[(\mathcal{M}_{D_L}^2)^{\text{SCKM}}]$, is given to leading order in λ by

$$m_{d_L}^2 \begin{bmatrix} 0 & -z\lambda^3 & -z'\lambda^2 \\ z\lambda^3 & 0 & -2s_\phi\rho'\lambda \\ z'\lambda^2 & 2s_\phi\rho'\lambda & 0 \end{bmatrix}, \quad (55)$$

where

$$z = (4\rho'\omega c_\gamma s_\phi + \rho(s_{2\phi'} - 2s_\gamma)), \quad (56)$$

$$z' = (2s_\phi - s_{(\phi-\gamma)})\rho'. \quad (57)$$

If $\rho \simeq \rho'$ the coefficient z reduces to $z \simeq \rho(s_{2\phi'} - 2s_\gamma)$ since $\omega \simeq 2\lambda^2$. Assuming the soft-trilinear texture from Eq. (21) we obtain for $\text{Re}[(\mathcal{M}_{D_R}^2)^{\text{SCKM}}]$, to leading order in λ ,

$$m_{d_R}^2 \begin{bmatrix} 1 & r\lambda^3 & r'\lambda^2 \\ r\lambda^3 & 1 & -2c_\phi\rho'\lambda \\ r'\lambda^2 & -2c_\phi\rho'\lambda & (1 + (8c_\phi\rho'\omega - 5\sigma\lambda^2)\lambda^2) \end{bmatrix}. \quad (58)$$

Here

$$m_{d_R}^2 = m_f^2(1 + (1 + 5\rho)\lambda^2), \quad (59)$$

$$r = (4\rho'\omega(c_{(\gamma-\phi)} + 2c_\phi) + \rho(c_{2\phi'} - 2c_\gamma)), \quad (60)$$

$$r' = (c_{(\gamma-\phi)} + 2c_\phi)\rho'. \quad (61)$$

Again if $\rho \simeq \rho'$ the coefficient r reduces to $r \simeq \rho(c_{2\phi'} - 2c_\gamma)$ since $\omega \simeq 2\lambda^2$. The imaginary component, $\text{Im}[(\mathcal{M}_{D_R}^2)^{\text{SCKM}}]$, is given to leading order in λ by

$$m_{d_R}^2 \begin{bmatrix} 0 & -t\lambda^3 & -t'\lambda^2 \\ t\lambda^3 & 0 & -2s_\phi\rho'\lambda \\ t'\lambda^2 & 2s_\phi\rho'\lambda & 0 \end{bmatrix}, \quad (62)$$

where

$$t = (s_{2\phi'} - 2s_\gamma)\rho, \quad (63)$$

$$t' = (s_{(\gamma-\phi)} - 2s_\phi)\rho'. \quad (64)$$

It is also convenient when calculating supersymmetric contributions to flavor violating processes to use the couplings $(\delta_{ij}^d)_{LL}$ defined by

$$(\delta_{ij}^d)_{LL} = \frac{(\tilde{\mathcal{M}}_{D_L}^2)^{\text{SCKM}}_{ij}}{m_{d_L}^2}. \quad (65)$$

One can define analogously the $(\delta_{ij}^d)_{RR}$ couplings.

V. SUPPRESSED CONTRIBUTIONS TO EDMS

In an unconstrained MSSM the generic contribution to the neutron EDM [13,27,43] is around 8 orders of magnitude larger than the SM contribution, i.e. about 4 orders of magnitude above the current experimental constraint [44]. This is the so-called SUSY CP problem or to be more specific the flavor conserving SUSY CP problem. The disparity between the current experimental constraint and the generic supersymmetric contribution in the MSSM is due to the, in principle, allowed presence of CP phases in the superpotential and in the soft supersymmetry-breaking sector. Numerous papers have examined this topic in the context of supersymmetric models [45] and a few solutions have been proposed, which were summarized in the Introduction. We will explain with some detail how generic supersymmetric models for radiative mass generation can ameliorate this problem. We will analyze separately the one-loop, two-loop and higher order contributions to EDMs.

Interestingly, the one-loop supersymmetric contributions to EDMs always appear as combinations of six possible physical phases of the generic form [14,46,47],

$$\arg(A^* m_{\tilde{\chi}}), \quad \arg(B^* \mu m_{\tilde{\chi}}), \quad (66)$$

where A are first generation flavor-diagonal trilinear soft supersymmetry-breaking parameters, m are gaugino masses, B is the bilinear soft supersymmetry-breaking term and μ is the superpotential bilinear term. In the special case of universal soft supersymmetry-breaking terms these reduce to only two physical phases. First let us focus our attention on the term $\arg(B^* \mu m_{\tilde{\chi}})$. The tree-level gaugino masses are flavor conserving parameters generated by the supersymmetry-breaking flavor-singlet field \mathcal{G} as we explained before and as a consequence cannot carry complex phases, which are linked to flavor breaking VEVs. The μ term is allowed in the CP conserving superpotential at tree level. This parameter is linked to

the flavor blind operator $\mathcal{H}_u \mathcal{H}_d$, which obviously cannot carry CP phases at tree level. For the same reason the bilinear soft supersymmetry-breaking term, B , is also a real parameter since the term $h_u h_d$ in the scalar potential is also a flavor singlet.

Regarding the contributions of the form $\arg(A^* m_{\tilde{\chi}})$, in the case of the neutron and mercury EDMs the relevant terms arise from the up, down and strange quark EDMs which are generically of the form $\arg(A_u^* m_{\tilde{\chi}})$, $\arg(A_d^* m_{\tilde{\chi}})$ and $\arg(A_s^* m_{\tilde{\chi}})$. We have seen that in our model the 3×3 matrix \mathbf{A}_U is diagonal; see Eq. (35). The diagonal entries carry no CP phases. Even if they existed they could have been absorbed in a redefinition of the phases of the up-type matter fields. Furthermore, the entries (11) and (22) of the 3×3 soft-trilinear matrix in the down-type squark sector corresponding to A_d and A_s in Eq. (47) are in general real. Therefore all the one-loop contributions to the EDMs in this model are exactly zero.

Regarding the two-loop contributions to EDMs, it has been pointed out that the two-loop supersymmetric contributions can also constrain the supersymmetric parameter space, even though not so severely as the one-loop contributions [48–51].

For instance, there is a two-loop contribution to the quark EDMs of the purely gluonic dimension six operator [52], which is of the form $f_{\alpha\beta\gamma} G_{\alpha\mu\rho} G_{\beta\nu}^{\rho} G_{\lambda\gamma\sigma} \epsilon^{\mu\nu\lambda\sigma}$ where $f_{\alpha\beta\gamma}$ are the Gell-Mann matrices, $\epsilon^{\mu\nu\lambda\sigma}$ is the totally antisymmetric tensor and $G_{\alpha\mu\rho}$ is the gluon field strength. Supersymmetry-breaking terms generate a contribution to this operator [14,51]. In the case under consideration, since the matrix \mathbf{A}_U is diagonal and can be brought to real form through redefinition of the quark fields, only the complex phase in the term A_b can give a contribution. For instance, using dimensional analysis the dimension six gluonic contribution to the neutron EDM will be given by

$$\left(\frac{d_n}{e}\right)_{2\text{-loop}}^G = 3 \left(\frac{g_s}{4\pi}\right)^5 \frac{m_b^2 |A_b| M_C}{m_{\tilde{g}}^5} H(z_{\tilde{t}_1}, z_{\tilde{t}_2}, z_t) \sin\phi \quad (67)$$

where $M_C = 1.19$ GeV is the chiral symmetry breaking scale, $z_i = m_i^2/m_{\tilde{g}}$ and $H(x, y, z)$ is a dimensionless form factor, whose full expression can be found in Ref. [51]. Sizable radiative contributions to fermion masses require $m_{\tilde{g}} \approx m_{\tilde{q}}$. If $A_b \ll m_{\tilde{q}}$ this contribution will be much below the current constraint even for $\sin\phi = 1$. Nevertheless, in the worst case scenario, when $\sin\phi = 1$ and $A_b \approx m_{\tilde{q}}$ (maximal sbottom mixing), we would obtain that this contribution is below the current experimental constraint, $|d_n| < 10^{-25} e \text{ c.m.}$, for squark masses above ≈ 1 TeV.

Another well-known two-loop contribution to d_d is the Barr-Zee-type diagram with exchange of stops and the CP -odd Higgs. This contribution is of the form [48],

$$\left(\frac{d_d}{e}\right)_{2\text{-loop}}^i = \frac{-\alpha_e}{77\pi^3} \frac{m_d m_t}{v^2} \frac{\mu \sin(2\theta_t)}{s_{2\beta} m_A^2} \sin\delta_t C(\tilde{t}_1, \tilde{t}_2, A) \quad (68)$$

where $\delta_t = \arg[A_t + \cot\beta\mu^*]$, θ_t is the stop mixing, $v = 175$ GeV, A is the CP -odd Higgs and C stands for a dimensionless two-loop form factor which can be found in Ref. [48]. For the large $\tan\beta$ case under consideration $\delta_t \approx \arg[A_t] = 0$ since A_t and μ are real parameters. On the other hand there is an analogous contribution from bottom squarks that is proportional to $\text{Im}[A_b e^{i\delta_b}]$ with $\delta_b = \arg[A_b + \tan\beta\mu^*]$. For the large $\tan\beta$ case under consideration the second term in δ_b would dominate in general and we obtain that the contribution is proportional to $-A_b \sin\phi$, as can be seen from Eq. (47). If the sbottom mixing is small, $\sin(2\theta_b) \ll 0.1$, this contribution would be below the experimental constraint for CP -odd Higgs masses above 250 GeV. Nonetheless in the worst case scenario, if $\sin\phi = 1$ and the sbottom mixing is maximal $A_b \approx m_{\tilde{q}}$, the experimental constraint requires the CP -odd Higgs mass to be ≥ 1 TeV.

Some of the two-loop gluino contributions to the neutron EDM have been recently studied, neglecting all other possible sources of flavor violation except those related to the CKM matrix [53]. For the model under consideration, where the only source of a CP phase is the sbottom soft-trilinear term, these contributions are always smaller than the two-loop contributions analyzed previously.

One may wonder if the previous arguments for the cancellation of the one-loop contribution to EDMs could be extended in a variant of this model to not only suppress but cancel the two-loop contributions. We note that if the flavor model generates Hermitian soft-trilinear matrices the two-loop contributions, which are proportional to the phases of A_b and A_t , would be zero. It may be possible in principle that small phases are generated in the “flavor conserving” parameters in the Lagrangian, as the μ term, the B parameter or the gaugino masses. Nonetheless, we note that many of the higher order operators which could contribute to the radiative corrections to the μ term have to be flavor conserving operators of the form $\propto Z^{\dagger ab} Z_{ab}$ ($Z = S, \mathcal{A}$) or $\propto \mathcal{F}^{\dagger a} \mathcal{F}_a$, etc., ... After the breaking of the flavor symmetry these operators cannot generate complex phases since the presence of a complex phase would be an indication of flavor violation.

To sum up, because of the intrinsic flavor off-diagonal nature of the CP violating phases in this model, the one-loop contributions to EDMs are zero. The constraints imposed by the two-loop contributions depend on the parameter space of the model. In the worst case scenario, with maximal sbottom mixing, the dimension six gluonic operator imposes the strongest constraint requiring $m_A \geq 1$ TeV. Nevertheless, if the sbottom mixing is small the two-loop contributions can be below the current experimental constraint without requiring a heavy squark spectra.

VI. CONTRIBUTIONS TO DIRECT AND INDIRECT CP VIOLATION IN THE KAON SYSTEM

A. ϵ

The measure of indirect CP violation in the kaon system is given by the parameter ϵ defined by

$$\epsilon = \frac{A(K_L \rightarrow \pi\pi)}{A(K_S \rightarrow \pi\pi)} \approx \frac{e^{i\pi/4}}{\sqrt{2}\Delta m_K} \text{Im}[\mathcal{M}_{12}] \quad (69)$$

where Δm_K is the $K_L K_S$ mass difference, $\mathcal{M}_{12} = \mathcal{M}(K^0) = \langle K^0 | \mathcal{H}_{\text{eff}}^{\Delta S=2} | \bar{K}^0 \rangle$ is the $K^0 \bar{K}^0$ mixing amplitude and $\mathcal{H}_{\text{eff}}^{\Delta S=2}$ is the effective $\Delta S = 2$ Hamiltonian. The parameters Δm_K and $|\epsilon|$ have received considerable attention in supersymmetric models since their measured values have been known with good precision for a long time [40,54,55]. We will separate the SM and supersymmetric contributions to the mixing amplitude in the form $\mathcal{M}_{12} = \mathcal{M}_{12}^{\text{SUSY}} + \mathcal{M}_{12}^{\text{SM}}$. We define ϕ_ϵ and ϕ'_ϵ as the phases of the SM and the supersymmetric contributions, respectively, $\mathcal{M}_{12}^{\text{SUSY}} = |\mathcal{M}_{12}^{\text{SUSY}}| e^{i\phi'_\epsilon}$ and $\mathcal{M}_{12}^{\text{SM}} = |\mathcal{M}_{12}^{\text{SM}}| e^{i\phi_\epsilon}$. It is also convenient to introduce the ratio $R_K = |\mathcal{M}_{12}^{\text{SUSY}}|/|\mathcal{M}_{12}^{\text{SM}}|$. This ratio and the complex phase ϕ'_ϵ are constrained by the experimental measurements of Δm_K and $|\epsilon|$. We can expand in powers of R_K to obtain the following expression for the new physics contributions to Δm_K and $|\epsilon|$:

$$\frac{\Delta m_K - \Delta m_K^{\text{SM}}}{\Delta m_K^{\text{SM}}} \approx R_K \frac{\cos(2\phi'_\epsilon)}{\cos(2\phi_\epsilon)}, \quad (70)$$

$$\frac{|\epsilon| - |\epsilon^{\text{SM}}|}{|\epsilon^{\text{SM}}|} \approx R_K \frac{\sin(2\phi'_\epsilon)}{\sin(2\phi_\epsilon)}. \quad (71)$$

We note that Δm_K has been measured with an uncertainty of approximately 0.2%, $\Delta m_K^{\text{exp}} = (3.490 \pm 0.006) \times 10^{-12}$ MeV. In the SM roughly 70% of the measured Δm_K is described by the real parts of the box diagrams with charm quark and top quark exchanges. Some non-negligible contribution comes from the box diagrams with simultaneous charm and top exchanges while approximately the remaining 20% of the measured Δm_K is attributed to long distance contributions. On the other hand these are potentially sizable and up to date incalculable. While a precise prediction is not possible, the observation is roughly compatible with the SM expectation. Assuming that the supersymmetric contribution saturates one-half of the experimental measurement and using Eq. (70) we can obtain an approximate upper constraint on R_K ,

$$R_K \cos(2\phi'_\epsilon) \lesssim \frac{1}{2} \cos(2\phi_\epsilon). \quad (72)$$

$|\epsilon|$ has been measured with an uncertainty of approximately 0.6%. A fit to the $K \rightarrow \pi\pi$ data yields $|\epsilon| = (2.284 \pm 0.014) \times 10^{-3}$. The calculation of $|\epsilon|$ is also affected by large distance corrections. It is usual when

calculating $|\epsilon|$ to input the experimental measurement for Δm_K in the denominator of Eq. (69). We will allow again for the supersymmetric contribution to saturate one-half of the experimental measurement. The resulting constraint on R_K and ϕ'_ϵ can be expressed in the form,

$$R_K \sin(2\phi'_\epsilon) \lesssim \frac{1}{2} \sin(2\phi'_\epsilon). \quad (73)$$

Using the usual Wolfenstein parametrization of the CKM matrix to second order in powers of λ we obtain that the dominant SM contribution to \mathcal{M}_{12} is proportional to $(V_{cs}V_{cd}^*)^2$. This is given by $V_{cs}V_{cd}^* = -\lambda(1 - \frac{\lambda^2}{2})(1 + A^2\lambda^4(\rho - i\eta))$. This can be written as $V_{cs}V_{cd}^* \approx -\lambda(1 - \frac{\lambda^2}{2})e^{-i\phi_\epsilon}$ where ϕ_ϵ is defined as $\phi_\epsilon = \tan^{-1}(\eta A^2\lambda^4)$. This is a very small number. Using updated extractions of A , η and λ we obtain $\phi_\epsilon \approx 0.03^\circ$. Therefore the constraints from Eqs. (72) and (73) can be written in the form,

$$R_K \cos(2\phi'_\epsilon) \lesssim 0.5, \quad (74)$$

$$R_K \sin(2\phi'_\epsilon) \lesssim 6 \times 10^{-4}. \quad (75)$$

Next we need to calculate the expressions for R_K and ϕ'_ϵ in our model. We will see that ϕ'_ϵ in the model under consideration is completely determined given the measured value of γ and the quark mass ratios. Therefore the previous constraints in Eqs. (74) and (75) will translate into lower bounds on the squark mass spectra. The supersymmetric contribution to \mathcal{M}_{12} contains contributions proportional to the different δ couplings in the soft supersymmetry-breaking sector. There are qualitatively four different contributions to $\mathcal{M}_{12}^{\text{SUSY}}$ that in our model, to leading order in powers of λ , take the following values:

$$(\delta_{LR}^d)_{12}^2 + (\delta_{RL}^d)_{12}^2 = 2\delta_{LR}^{12} \left(\frac{5}{4} c_{2\gamma} + \frac{11}{4} - i s_{2\gamma} \right), \quad (76)$$

$$(\delta_{LR}^d)_{12}(\delta_{RL}^d)_{12} = \delta_{LR}^{12} \left(\frac{5}{4} c_{2\gamma} - \frac{5}{4} - i s_{2\gamma} \right), \quad (77)$$

where

$$\delta_{LR}^{12} = 8\omega^2 \lambda^6 c_\beta^2 \frac{v^2}{m_b^2} \frac{\tilde{m}^2}{m_b^2}, \quad (78)$$

and

$$(\delta_{LL}^d)_{12}^2 + (\delta_{RR}^d)_{12}^2 = 2\delta_{LL}^{12} (e^{-2i\phi'} - 2e^{-i\gamma})^2, \quad (79)$$

$$(\delta_{LL}^d)_{12}(\delta_{RR}^d)_{12} = \delta_{LL}^{12} (e^{-2i\phi'} - 2e^{-i\gamma})^2, \quad (80)$$

where

$$\delta_{LL}^{12} = \rho^2 \lambda^6. \quad (81)$$

For instance, we noted in Sec. III A that for large $\tan\beta$ and $m_{\tilde{g}} \approx m_{\tilde{b}}$ the parameter \tilde{m} (new mass scale introduced by the flavor violating SUSY-breaking fields) is required to be approximately $\tilde{m} \approx 3m_{\tilde{b}}$. Therefore the LR couplings in

our model have an important additional suppression factor, $\delta_{LR}^2/\delta_{LL}^2 \propto c_\beta^2 \omega^2 v^2/m_{\tilde{b}}^2$. As a consequence in this model the δ_{LL} and δ_{RR} couplings dominate the contribution to $\mathcal{M}_{12}^{\text{SUSY}}$. All the δ_{LL} and δ_{RR} contributions to the Wilson coefficients can be added up in a simple expression. We obtain

$$\mathcal{M}_{12}^{\text{SUSY}} = \frac{\alpha_s^2 m_K f_K^2 X_K^2}{60 m_{\tilde{b}}^2} [((\delta_{LL}^d)_{12}^2 + (\delta_{RR}^d)_{12}^2) D(x) - (\delta_{LL}^d)_{12} (\delta_{RR}^d)_{12} C(x)]. \quad (82)$$

Here X_K^2 is a dimensionless factor defined as $X_K^2 = m_K^2/(m_s + m_d)^2$, numerically $X_K \approx 4.07$, f_K is the K -meson decay constant and x is the gluino-squark mass ratio squared, $x = m_{\tilde{g}}^2/m_{\tilde{b}}^2$. $C(x)$ and $D(x)$ are given by $C(x) = C_f f(x) + C_g g(x)$ and $D(x) = D h(x)$ where $f(x)$, $g(x)$ and $h(x)$ are dimensionless form factors defined in Eqs. (A3)–(A5), of the Appendix. The functions $f(x)$, $g(x)$ and $h(x)$ have been conveniently normalized so that in the limit $m_{\tilde{g}} \approx m_{\tilde{b}}$ they tend to 1. We note that in our approach the constant coefficients C_f , C_g and D absorb the dependency on the method of calculation of the hadronic matrix elements as well as the renormalization effects on the Wilson coefficients. Following Ref. [55], where lattice QCD methods were used to calculate the relevant hadronic matrix elements and including NLO renormalization effects to the Wilson coefficients, we obtain the following numerical values, $C_f \approx 9.13$, $C_g \approx 0.75$ and $D \approx 0.002$. We note that the naive vacuum insertion approximation at tree level gives the values $C_f \approx 1.8$, $C_g \approx 0.067$ and $D \approx 0.0055$, which are significantly different. We also note that these constant parameters, C_f , C_g and D , do not depend on the flavor mixing structure in the soft supersymmetric breaking sector. From the numerical values of these coefficients we note that the contribution of the form $\delta_{LL} \delta_{RR}$ in Eq. (80) dominates the supersymmetric contribution. Finally, using the well-known expression for the SM contribution to the amplitude, see for instance Eq. (3.39) in Ref. [56], we can write the ratio of the supersymmetric contribution over the SM amplitude in the form,

$$R_K \approx 2 \frac{\eta_K^2}{m_{\tilde{b}}^2} f(x) \rho^2 \lambda^6 \left(\frac{5}{4} - \cos(\gamma - 2\phi') \right)^{1/2}, \quad (83)$$

where

$$\eta_K \approx \frac{\alpha_s \pi X_K C_f^{1/2}}{\sqrt{5} G_F V_{cs} V_{cd} B_K^{1/2} \eta_1^{1/2} m_c} \approx 666 \text{ TeV}. \quad (84)$$

Here $\hat{B}_K = 0.85 \pm 0.15$ is a renormalization group invariant form of the B parameter arising from the hadronic matrix element, m_c is the charm quark mass and η_1 is a short distance QCD correction factor, at NLO η_1 is given by $\eta_1 = 1.38 \pm 0.20$ [56]. Using for λ the value determined from CKM elements, $\lambda \approx 0.22$, and for γ the 1σ

global fit $\gamma_{\text{fit}} = 61^\circ \pm 11^\circ$ we obtain

$$R_K \approx \left(\frac{\tilde{m}}{m_{\tilde{b}}} \right)^4 \left(\frac{10 \text{ TeV}}{m_{\tilde{b}}} \right)^2 h(x). \quad (85)$$

The phase of the supersymmetric contribution, ϕ'_ϵ , can be calculated from Eq. (80). This phase depends strongly on the phase ϕ' , which is not constrained by the CKM matrix. For instance, for $\phi' = 0$ we obtain

$$\tan(\phi'_\epsilon) \approx \frac{s_\gamma(1 - 2c_\gamma)}{\left(\frac{1}{4} - c_\gamma - c_{2\gamma}\right)}. \quad (86)$$

For $\gamma = 60^\circ$ we would obtain $\phi'_\epsilon = 0$. If this was the case the measurement of $|\epsilon|$ would not constrain the supersymmetric spectra; see Eq. (75). The only phase independent constraint comes from the Δm_K measurement; see Eq. (74). Let us analyze with more detail the large $\tan\beta$ case. For $\tan\beta = 50$ and $m_{\tilde{g}} \approx m_{\tilde{b}}$ we noted in Sec. III A that for the hierarchy of flavor breaking VEVs postulated in Eqs. (5)–(7) we need to have $\tilde{m} \approx 3m_{\tilde{b}}$, i.e. $\rho \approx 9$. Therefore using Eqs. (74), (75), and (83) we obtain for $\tan\beta = 50$ the constraints,

$$m_{\tilde{b}} \geq 130 \text{ TeV}, \quad \Delta m_K, \quad (87)$$

$$m_{\tilde{b}} \geq 3670 \text{ TeV}, \quad |\epsilon| \quad (\phi' \neq 0). \quad (88)$$

To obtain the second constraint we assume that the phase ϕ' is arbitrary. If this phase was zero as we mentioned this second constraint would not be effective. We would like to emphasize that these constraints are nongeneric. They only apply for the case large $\tan\beta$ case study and the particular texture considered in this paper. This case study must be considered a worst case scenario for these kinds of models. Several variants of this model may allow us to lower considerably these bounds. For instance we could lower the bounds at the price of less predictivity in the fermion sector by increasing the number of parameters in the flavor breaking sector.

B. ϵ'/ϵ

The measure of direct CP violation in the kaon system is given by the parameter ϵ'/ϵ . Direct CP violation originates from direct transitions of the CP -odd state into the CP -even $\pi\pi$ final state. The direct CP violation in the neutral $K \rightarrow \pi\pi$ decays can be described through the ratio,

$$\frac{\epsilon'}{\epsilon} = e^{i\Phi} \frac{w}{\sqrt{2}|\epsilon|} \left[\frac{\text{Im}A_2}{\text{Re}A_2} - \frac{\text{Im}A_0}{\text{Re}A_0} \right] \quad (89)$$

where $A_{0,2}$ are the isospin amplitudes for the $\Delta I = 1/2, 3/2$ transitions. $\text{Im}A_{0,2}$ are calculated from the general low-energy effective Hamiltonian for $\Delta S = 1$ transitions [57]. $w = \text{Re}A_2/\text{Re}A_0$ and Φ is a strong phase shift difference between the two amplitudes. In 1999 the NA48 experiment [58] at CERN and the KTeV [59] experiment at FNAL demonstrated that this observable is actually differ-

ent from zero as expected in the SM. The present world average is [60]

$$\text{Re}\left[\frac{\epsilon'}{\epsilon}\right]_{\text{exp}} = (16.6 \pm 1.6) \times 10^{-4}. \quad (90)$$

There is no simple approximate expression for the SM contribution to ϵ'/ϵ . This calculation is affected by large hadronic uncertainties. It has been recently pointed out [61] that to lowest order (in $1/N_c$ and in the chiral expansion) $\text{Re}[\epsilon'/\epsilon]$ is governed by the competition between two different decay topologies and suffers from a strong cancellation between them. Nevertheless to higher orders chiral loops generate an enhancement of the isoscalar amplitude and a reduction of A_2 . Taking this into account, and following Ref. [61], the latest SM prediction is

$$\text{Re}\left[\frac{\epsilon'}{\epsilon}\right]_{\text{SM}} = (19_{-18}^{+17}) \times 10^{-4}. \quad (91)$$

Therefore even though the SM prediction is consistent with the measurement, it does not allow us at present to perform stringent tests of the CKM mechanism of CP violation. For a 2003 review of several calculations see Ref. [62].

Since the 1999 measurements, the parameter ϵ'/ϵ has received considerable attention in the context of supersymmetric theories [63]. The dominant supersymmetric contributions to ϵ'/ϵ come from the chromomagnetic operators like O_g , $O_g = g_s/(16\pi^2)d_L\sigma^{\mu\nu}t^A s_R G_{\mu\nu}^A$. The Wilson coefficient C_g corresponding to this operator is given by

$$C_g = (-) \frac{\alpha_s \pi}{2m_{\tilde{d}}} \left[\frac{4m_s}{m_{\tilde{d}}} N(x)(\delta_{LL}^d)_{12} + M(x)(\delta_{LR}^d)_{12} \right], \quad (92)$$

where $N(x)$ and $M(x)$ are dimensionless form factors defined in Eqs. (A7) and (A8) of the Appendix. Taking into account that the relevant hadronic matrix element is given by

$$\langle (\pi\pi)_{I=0} | O_g | K^0 \rangle = \sqrt{\frac{3}{2}} \frac{11}{16\pi^2} \frac{\langle q\bar{q} \rangle}{F_\pi^3} m_\pi^2 B_G, \quad (93)$$

with $F_\pi = 131$ MeV and where the B_G factor is not well known, $B_G = 1-4$ we obtain that the total LR supersymmetric contribution to $\text{Re}[\epsilon'/\epsilon]$, can be conveniently written as

$$\text{Re}\left[\frac{\epsilon'}{\epsilon}\right]_{LR+RL} = \left(\frac{\eta_{\epsilon'}}{m_{\tilde{b}}}\right) |\text{Im}[(\delta_{LR}^d)_{21} - (\delta_{LR}^d)_{12}^*]| N(x), \quad (94)$$

where

$$\eta_{\epsilon'} = \frac{11\sqrt{3}}{64\pi} \frac{w}{|\epsilon|\text{Re}A_0} \frac{m_K^2 m_\pi^2}{F_\pi(m_s + m_d)} \alpha_s(m_{\tilde{b}}) \eta B_G. \quad (95)$$

We used for w and $\text{Re}A_0$ experimental values $w \approx 1/22$ and $\text{Re}A_0 = 3.326 \times 10^{-4}$ MeV. For the rest of the pa-

rameters we used $m_K = 490$ MeV, $m_\pi = 140$ MeV. η is a well-known dimensionless strong coupling renormalization factor defined in Ref. [64]. Taking into account the important uncertainties in the current determination of the B_G factor and the lighter quark masses we obtain the estimate $\eta_{\epsilon'} = 100-600$ TeV. We note that for the model under consideration

$$\text{Im}[(\delta_{LR}^d)_{12}] = \text{Im}[(\delta_{LR}^d)_{21}] = (-) 2\omega s_\gamma c_\beta \lambda^3 \frac{v}{m_{\tilde{b}}} \frac{\tilde{m}}{m_{\tilde{b}}}.$$

For the worst case scenario, $\eta_{\epsilon'} = 600$ TeV, and assuming that the supersymmetric contribution saturates the experimental measurement we obtain the constraint,

$$s_\gamma c_\beta \left(\frac{\tilde{m}}{m_{\tilde{b}}}\right) \left(\frac{220 \text{ GeV}}{m_{\tilde{b}}}\right)^2 N(x) \lesssim \text{Re}\left[\frac{\epsilon'}{\epsilon}\right]_{\text{exp}}. \quad (96)$$

For the large $\tan\beta$ case, $\tan\beta = 50$ with $m_{\tilde{g}} \approx m_{\tilde{d}}$, examined in Sec. III A, the hierarchy $\tilde{m} \approx 3m_{\tilde{b}}$ was required. In this case using for λ and ω the values determined from the CKM elements and for γ , $\gamma = 60^\circ$ we obtain

$$m_{\tilde{b}} \gtrsim 1.3 \text{ TeV} \quad (\epsilon'/\epsilon). \quad (97)$$

Let us analyze separately the size of the LL and RR contributions. We see that because of an extra $m_s/m_{\tilde{b}}$ suppression factor the $LL + RR$ contribution to ϵ' is much smaller than the LR contribution. If the gluino mass is of the same order as the squark masses (which is required to maximize the loop generated quark masses), and $\rho \approx \rho'$ we obtain a simple approximate expression for the LL and RR couplings,

$$\text{Im}[(\delta_{LL}^d)_{12}] = \text{Im}[(\delta_{RR}^d)_{21}] = (2s_\gamma - s_{2\phi'}) \lambda^3 \frac{\tilde{m}^2}{m_{\tilde{d}}^2}.$$

The total $LL + RR$ supersymmetric contribution to $\text{Re}[\epsilon'/\epsilon]$ can be conveniently written as

$$\text{Re}\left[\frac{\epsilon'}{\epsilon}\right]_{LL+RR} = \left(\frac{4m_s \eta_{\epsilon'}}{m_{\tilde{b}}^2}\right) \text{Im}[(\delta_{RR}^d)_{21} - (\delta_{LL}^d)_{12}^*] M(x). \quad (98)$$

Using for λ the value determined from the CKM elements and assuming that the supersymmetric contribution saturates the experimental measurement, we obtain the constraint,

$$(2s_\gamma - s_{2\phi'}) \left(\frac{\tilde{m}}{m_{\tilde{b}}}\right)^2 \left(\frac{1.7 \text{ GeV}}{m_{\tilde{b}}}\right)^2 \lesssim \text{Re}\left[\frac{\epsilon'}{\epsilon}\right]_{\text{exp}}. \quad (99)$$

For the large $\tan\beta$ case, $\tan\beta = 50$, with $m_{\tilde{g}} \approx m_{\tilde{b}}$ and $\tilde{m} \approx 3m_{\tilde{b}}$ we obtain $m_{\tilde{b}} \gtrsim 220$ GeV.

VII. CONTRIBUTIONS TO CP ASYMMETRIES IN THE B SYSTEM

The CP violation measured in neutral K meson decays, taking into account current experimental and theoretical

uncertainties, can be simply explained with the CKM phase. B factories have verified, especially through measurements of the CP asymmetry in the $B_d \rightarrow \psi K_S$ decay [65,66], that the CP symmetry is significantly violated in the B sector, in agreement with standard model predictions, providing a confirmation of the so-called CKM paradigm [67,68]. This fact does not rule out the possibility that the effects of CP phases of a different origin, as for instance the phases in the soft supersymmetry-breaking sector [69–71], could manifest in the near future [72] through other CP violating observables, especially penguin dominated amplitudes such as $B \rightarrow \phi K^0, \eta' K_S$. In this section we will study the constraints that the currents measurements of CP asymmetries in several B decays impose on the model under consideration.

A. CP asymmetry in $B \rightarrow \psi K_S$

The $B^0 \bar{B}^0$ mixing amplitude is defined by the matrix element of the effective $\Delta B = 2$ Hamiltonian as $\mathcal{M}_b = M_{12}(B_d^0) = \langle \bar{B}_d | \mathcal{H}_{\text{eff}}^{\Delta B=2} | B_d \rangle$. The phase of the mixing amplitude is related with the mixing CP asymmetry in the decay $B \rightarrow \psi K_S$ by

$$S_{\psi K_S} = \sin[\arg[\mathcal{M}_b]]. \quad (100)$$

According to the most recent averaged experimental results of $BABAR$ and $Belle$ $S_{\psi K_S} = 0.736 \pm 0.049$. This can be simply accounted to date with the CKM phase. If future measurements reduce considerably the experimental uncertainty in $S_{\psi K_S}$, there is hope that deviations from the SM prediction could be elucidated. It is convenient to separate the SM and supersymmetric contributions to the mixing amplitude in the form $\mathcal{M}_b = \mathcal{M}_b^{\text{SM}} + \mathcal{M}_b^{\text{SUSY}}$. We also find it convenient to define ϕ_1 and ϕ'_1 as the phases of the SM and supersymmetric component of the amplitude, respectively, i.e. $\mathcal{M}_b^{\text{SM}} = e^{i2\phi_1} |\mathcal{M}_b^{\text{SM}}|$ and $\mathcal{M}_b^{\text{SUSY}} = e^{i2\phi'_1} |\mathcal{M}_b^{\text{SUSY}}|$. Since the SM prediction can account perfectly for the experimental result we expect that the supersymmetric contribution is a small correction and expand the expression for the CP asymmetry in powers of the ratio $R_\psi = |\mathcal{M}_b^{\text{SUSY}}|/|\mathcal{M}_b^{\text{SM}}|$. To leading order,

$$S_{\psi K_S} = \sin 2\phi_1 [1 - \sin(2(\phi_1 - \phi'_1))R_\psi] + \sin(2\phi'_1)R_\psi. \quad (101)$$

It is known that in the absence of new physics contributions the SM CP phase can account for the present experimental results for $S_{\psi K_S}$. From Eq. (101) we can obtain constraints on R_ψ and ϕ'_1 . Assuming that the new physics contribution saturates 50% of the experimental uncertainty we obtain to leading order in R_ψ ,

$$R_\psi \sin(2(\phi'_1 - \phi_1)) \lesssim 0.5. \quad (102)$$

We note that even in the limit where the complex phase of the SUSY amplitude goes to zero the mixing CP asymmetry, $S_{\psi K_S}$, is affected by the SUSY contributions through

their effects on the absolute value of the amplitude, $S_{\psi K_S} = \sin 2\phi_1^{\text{SM}} [1 - \sin(2\phi_1^{\text{SM}})R_\psi]$. The mass difference in the $B_d \bar{B}_d$ system, $\Delta m_d = 2 \text{Abs}[\mathcal{M}_b]$, also puts a stringent constraint on R_ψ . Δm_d is an observable well known experimentally, to the level of 1.5%. The experimental measurement yields $\Delta m_d = (3.22 \pm 0.05) \times 10^{-10}$ MeV. The SM prediction for Δm_d is about $\Delta m_d^{\text{SM}} = (2.9 \pm 2.2) \times 10^{-10}$ MeV. We note that even though the theoretical uncertainty is about 75% the central value is only 10% from the central experimental value. Assuming that the supersymmetric contribution saturates 50% of the experimental measurement and expanding \mathcal{M}_b in powers of R_ψ we obtain the constraint,

$$R_\psi \cos(2(\phi'_1 - \phi_1)) \lesssim 0.5. \quad (103)$$

We note that we have assumed that $(\phi_1 - \phi'_1) \neq \pm \pi/4$. If this was not the case, then the second order term in the expansion would be dominant and we would obtain a milder constraint, $R_\psi^2 \lesssim 0.5$.

Next we need to calculate the expressions for R_ψ and ϕ'_1 in our model. The previous constraints in Eqs. (102) and (103) will translate into lower bounds on the squark mass spectra. Again, we identify four qualitatively different contributions to $\mathcal{M}_b^{\text{SUSY}}$, that in our model, working to leading order in powers of λ , take the following values:

$$(\delta_{LR}^d)_{13}^2 + (\delta_{RL}^d)_{13}^2 = 2\delta_{LR}^{13}(3 + 2c_\gamma^2 - 2is_{2\gamma}), \quad (104)$$

$$(\delta_{LR}^d)_{12}(\delta_{RL}^d)_{12} = \delta_{LR}^{13}(2c_\gamma^2 - 5 - is_{2\gamma}) \quad (105)$$

where

$$\delta_{LR}^{13} = \lambda^4 c_\beta^2 \frac{v^2}{m_b^2} \frac{\tilde{m}^2}{m_b^2}, \quad (106)$$

and

$$(\delta_{LL}^d)_{13}^2 + (\delta_{RR}^d)_{13}^2 = 2\delta_{LL}^{13} e^{2i(\phi-\gamma)}(1 + 4e^{2i\gamma}), \quad (107)$$

$$(\delta_{LL}^d)_{13}(\delta_{RR}^d)_{13} = \delta_{LL}^{13} e^{2i(\phi-\gamma)}(1 - 4e^{2i\gamma}), \quad (108)$$

where

$$\delta_{LL}^{13} = \rho^2 \lambda^4. \quad (109)$$

We see that in this model the LL and RR delta couplings in general are expected to dominate the contribution to $\text{Im}[\mathcal{M}_b^{\text{SUSY}}]$ since the LR couplings contain an additional suppression factor, $c_\beta^2 v^2/m_b^2$. It is possible to add all the dominant δ_{LL} and δ_{RR} contributions to the Wilson coefficients to give a simple approximate expression for the supersymmetric contribution to \mathcal{M}_b . We obtain

$$\begin{aligned} \mathcal{M}_b^{\text{SUSY}} &= \frac{\alpha_s^2 m_B f_B^2 X_B^2}{60 m_b^2} [((\delta_{LL}^d)_{13}^2 + (\delta_{RR}^d)_{13}^2) D(x) \\ &\quad + (\delta_{LL}^d)_{13} (\delta_{RR}^d)_{13} C(x)]. \end{aligned} \quad (110)$$

Here X_B^2 is a dimensionless factor defined as $X_B^2 = m_B^2/(m_b(m_b) + m_d(m_b))^2$, numerically $X_B \approx 1.08$. x is defined as $x = m_g^2/m_b^2$. $C(x)$ and $D(x)$ are dimensionless form factors which were already introduced previously in Sec. VIA as $C(x) = C_f f(x) + C_g g(x)$ and $D(x) = Dh(x)$. $f(x)$, $g(x)$ and $h(x)$ are dimensionless form factors defined in Eqs. (A2)–(A4) of the Appendix. The constant coefficients C_f , C_g and D , like in the $\Delta S = 2$ case, absorb the dependency on the method of calculation of the hadronic matrix elements as well as the renormalization effects on the Wilson coefficients. We have evaluated C_f , C_g and D following Ref. [73] where lattice QCD methods were used to calculate the relevant hadronic matrix elements. We obtain that $D \ll C_g \ll C_f$ and $C_f \approx 7.33$. Had we used the naive vacuum insertion approximation at tree level we would obtain $C_f = (48X_B^2 + 9)/(27X_B^2) \approx 2.06$ which would imply an underestimation of the dominant term by a factor of order 1/10. The coefficients C_f , C_g and D also depend on the scale of the supersymmetric spectra. We have calculated our numerical values at the scale $M_S = 1$ TeV using the renormalization factors given in Ref. [73]. The contribution of the form $\delta_{LL}\delta_{RR}$ in Eq. (109) clearly dominates the supersymmetric contribution. We use the well-known expression for the SM contribution to the amplitude \mathcal{M}_b , see for instance Eq. (3.60) in Ref. [56],

$$\mathcal{M}_b^{\text{SM}} = \frac{G_F^2}{12\pi^2} \eta_B \hat{B}_{B_d} f_B^2 m_{B_d} m_W^2 |V_{td} V_{tb}^*|^2 S(x_t), \quad (111)$$

where $\eta_B = 0.55(1)$ is a QCD correction factor, \hat{B}_{B_d} is a renormalization group invariant parameter available in the literature [74], $\hat{B}_{B_d} = 1.30 \pm 0.12$, and $S(x_t)$ is a dimensionless form factor given by $S(x_t) = 2.46(m_t/170 \text{ GeV})^{1.52}$. We can write the ratio of the supersymmetric contribution over the SM amplitude in the form,

$$R_\psi|_{LL+RR} \approx \frac{\eta_\psi^2}{m_{\tilde{b}}^2} \rho^2 \lambda^4 f(x) (17 - 8c_{2\gamma})^{1/2}, \quad (112)$$

where

$$\eta_\psi \approx \frac{\alpha_s \pi X_B C_f^{1/2}}{\sqrt{5} G_F |V_{td} V_{tb}| B_B^{1/2} \eta_B^{1/2} m_W S^{1/2}(x_t)} \approx 78 \text{ TeV}. \quad (113)$$

We will substitute in the expression (112), the value of λ determined from the quark data and for γ we will use the central value of the 1σ global fit. $\gamma = 60^\circ$. For the large $\tan\beta$ case, $\tan\beta = 50$, examined in Sec. III A, assuming that $\rho' \simeq \rho \approx 9$ and $m_{\tilde{g}} \approx m_{\tilde{b}}$. The constraints in Eqs. (102) and (103) reduce to

$$\left(\frac{700 \text{ TeV}}{m_{\tilde{b}}}\right)^2 c_{2(\phi'_1 - \phi_1)} \lesssim 0.5, \quad (114)$$

$$\left(\frac{700 \text{ TeV}}{m_{\tilde{b}}}\right)^2 s_{2(\phi'_1 - \phi_1)} \lesssim 0.13. \quad (115)$$

The phase ϕ'_1 can be calculated from Eq. (108). We obtain

$$\tan(2\phi'_1) = \frac{4s_{2\phi} + s_{2(\gamma - \phi)}}{4c_{2\phi} - c_{2(\gamma - \phi)}}. \quad (116)$$

The phase ϕ is not constrained by the quark masses and mixings. The value of ϕ'_1 depends strongly on the phase ϕ . For $\phi = 0$ and $\gamma = 60^\circ$ we obtain $\phi'_1 = 10.9^\circ$ while for $\phi = 30^\circ$ and $\gamma = 60^\circ$ we obtain $\phi'_1 = 70.9^\circ$. When $\cos(2(\phi'_1 - \phi_1)) \approx 1$ the strongest phase independent constraint on the squark spectra comes from Eq. (114),

$$m_{\tilde{b}} \gtrsim 1000 \text{ TeV} \quad (\Delta m_d \quad \phi'_1 \approx \phi_1), \quad (117)$$

$$m_{\tilde{b}} \gtrsim 1000 \text{ TeV} \quad (S_{\psi K_s} \quad \phi'_1 \approx \phi_1 \pm \pi/2). \quad (118)$$

On the other hand if $\rho' \ll 1$ these constraints on the squark spectra would be milder. This will happen for instance when $\eta' \ll \eta$.

B. CP asymmetry in $B \rightarrow \phi K_S$

The latest results from the Belle Collaboration for the time dependent CP asymmetry coefficient $S_{\phi K}$ derived from the combined ϕK^0 data set are [75] $S_{\phi K}^{\text{Belle}} = +0.06 \pm 0.42$ while the latest results from the BABAR Collaboration for the same coefficient are [76] $S_{\phi K}^{\text{BABAR}} = +0.50 \pm 0.32$. Combining the results from both experiments one obtains the world average, $S_{\phi K}^{\text{BABAR+Belle}} = +0.34 \pm 0.21$ [68]. Taking into account that the SM prediction for the time dependent CP asymmetry is $S_{\phi K} = \sin(2\phi_1) = 0.726 \pm 0.037$ (here we used the world averaged CP asymmetry determined from charmonium final states) the current world average seems to differ from the SM expectation by about a 2σ level. Therefore this process is one of the best candidates for the manifestation of new physics in the quark sector. $S_{\phi K}$ in the context of supersymmetric theories has received considerable attention recently [69,70,77–90] since any clear deviation from the SM prediction would imply the existence of new CP phases other than the CKM phase.

The total decay amplitude, $A_{\phi K_S} = \langle \phi K_S | \mathcal{H}_{\Delta B=1}^{\text{eff}\dagger} | B^0 \rangle$ can be written in the form, $A_{\phi K_S} = A_{\phi K_S}^{\text{SM}} + A_{\phi K_S}^{\text{SUSY}}$. Additionally one can parametrize the SM and supersymmetric contributions to the amplitude in the form, $A_{\phi K_S}^{\text{SUSY}} = |A_{\phi K_S}^{\text{SUSY}}| e^{i\phi_{\text{NP}}} e^{i\delta_{\text{SUSY}}}$ and $A_{\phi K_S}^{\text{SM}} = |A_{\phi K_S}^{\text{SM}}| e^{i\delta_{\text{SM}}}$, where ϕ_{NP} is the CKM-like complex phase of the supersymmetric contribution and δ_{SM} and δ_{SUSY} are the SM and supersymmetric CP conserving strong phases, respectively. Assuming that the SUSY contribution to the amplitude is smaller than the SM one, and expanding in powers of the ratio $R_\phi = |A_{\phi K_S}^{\text{SUSY}}|/|A_{\phi K_S}^{\text{SM}}|$ it is possible to obtain approximate expressions for the direct and mixing CP asymme-

tries [78,91]. To leading order in R_ϕ ,

$$S_{\phi K_S} = s_{2\phi_1} + 2s_{\phi_{NP}} c_\delta c_{2\phi_1} R_\phi, \quad (119)$$

where δ is the difference of supersymmetric and SM CP conserving strong phases, $\delta = (\delta_{SM} - \delta_{SUSY})$. We will constrain the supersymmetric contribution to $S_{\phi K_S}$ assuming that this contribution accounts for the difference between the experimental measurement and the SM prediction, i.e.,

$$2s_{\phi_{NP}} c_\delta c_{2\phi_1} R_\phi \lesssim 0.40 \pm 0.26, \quad (120)$$

where for $s_{2\phi_1}$ we used the experimental value of $S_{\psi K_S}$, $S_{\psi K_S} = 0.736 \pm 0.049$. There are two basic contributions to the supersymmetric amplitude: the contributions coming from the δ_{LR} couplings and the contributions coming from the δ_{LL} and δ_{RR} couplings. We will first obtain a simple expression for the δ_{LR} contributions. We note that at the SUSY scale there is only one Wilson coefficient which contains the coupling $(\delta_{LR}^d)_{23}$, the chromomagnetic operators like O_g , $O_g = g_s/(16\pi^2) d_L \sigma^{\mu\nu} t^A b_R G_{\mu\nu}^A$. The Wilson coefficient C_g corresponding to this operator at the SUSY scale is given by

$$C_g(m_{\bar{b}}) = (-) \frac{\alpha_s \pi}{2m_{\bar{b}}} \left[\frac{4m_b}{m_{\bar{b}}} N(x) (\delta_{LL}^d)_{23} + M(x) (\delta_{LR}^d)_{23} \right]. \quad (121)$$

Here x is defined as $x = m_{\bar{g}}^2/m_{\bar{d}}^2$. $M(x)$ and $N(x)$ are invariant dimensionless form factors that we have conveniently normalized so that $M(x), N(x) \approx 1$ when $x \rightarrow 1$, see Eqs. (A7) and (A8) of the Appendix. When calculating the supersymmetric contributions to the asymmetry one has to take into account the renormalization of the Wilson coefficients from the SUSY scale down to the bottom mass scale. Following Ref. [92] we have included the NLO corrections using the generalized factorization approach assuming that $m_{\bar{b}} \approx 1$ TeV. We note that all the δ_{LR} contributions to the low-energy effective Wilson coefficients arise originally from the Wilson coefficient $C_g(m_{\bar{b}})$ in Eq. (121). Therefore all the contributions to each effective Wilson coefficient coming from the flavor violating soft SUSY-breaking trilinear couplings (i.e. δ_{LR} couplings) can be added up since they are proportional to the same gluino-squark form factor $M(x)$. The resulting contribution can always be written in the form,

$$A_{\phi K_S}^{\text{SUSY}}|_{LR+RL} = \frac{\alpha_s^2 f_{\phi K} M(x)}{18m_b m_{\bar{b}}} ((\delta_{LR}^d)_{23} + (\delta_{LR}^d)_{32}^*). \quad (122)$$

In our notation the coefficient $f_{\phi K}$ absorbs the dependency on the method of calculation of the hadronic matrix elements. We have calculated $f_{\phi K}$ using the generalized factorization approach, following Ref. [92]. $f_{\phi K}$ is parametrized in the form $f_{\phi K} = f_q X_\phi$, f_q is a factor associated with the momentum carried by the gluon in the

corresponding penguin diagram, $f_q = m_b/\sqrt{\langle q^2 \rangle}$ (in the rest of the paper we will assume that $f_q = \sqrt{2}$). X_ϕ arises from the hadronic matrix elements. It is given by $X_\phi = 2F_1^{B \rightarrow K}(m_\phi^2) f_\phi m_\phi (p_K \cdot \epsilon_\phi)$. The numerical value of the parameter X_ϕ is irrelevant for our purposes because it cancels with the same factor coming from the SM contribution. The coefficient $f_{\phi K}$ could be calculated using other more recent and precise approaches which are available in the literature: as the perturbative QCD approach [93] or the QCD factorization approach [94]. Nevertheless it has been pointed out that in that case one would obtain slightly different values for the relevant coefficients [80,85,86,89]. Therefore for our purpose, which is to obtain a good estimate of the constraint on the new physics contributions, the generalized factorization approach is precise enough.

The contributions to the supersymmetric amplitude coming from the δ_{LL} and δ_{RR} couplings can also be written in a similar fashion,

$$A_{\phi K_S}^{\text{SUSY}}|_{LL+RR} = \frac{\alpha_s^2 X_\phi L(x)}{47m_{\bar{g}}^2} ((\delta_{LL}^d)_{23} + (\delta_{RR}^d)_{23}). \quad (123)$$

$L(x)$ is a dimensionless polynomial conveniently normalized such that $L(x) \rightarrow 0$ when $x \rightarrow 0$ and $L(x) \approx 1$ when $x \rightarrow 1$. We note that in this case, since there are δ_{LL} and δ_{RR} contributions coming from different Wilson coefficients, the coefficients of the polynomial $L(x)$ depend on the method used to evaluate the hadronic matrix elements and to a lesser extent on the scale of the supersymmetric spectra but they do not depend on the flavor mixing structure in the SUSY-breaking sector. We have evaluated the coefficients of $L(x)$ numerically using the generalized factorization approach following Ref. [92]. We obtain approximately,

$$L(x)_{\text{GF}} = c_0 + c_1(x-1), \quad c_0 \approx 1, \quad c_1 \approx -\sqrt{3}, \quad (124)$$

in the limit $x \rightarrow 1$. Additionally, if one is interested in the limit $x \approx 0$, i.e. $m_{\bar{g}} \ll m_{\bar{q}}$, $L(x)$ is approximated by

$$L(x)_{\text{GF}} = (-)4x(d_0 + d_1 \ln x), \quad d_0 \approx 18, \quad d_1 \approx 7, \quad (125)$$

where the coefficients $c_{0,1}$ and $d_{0,1}$ shown here are good approximations to the actual values calculated numerically. The expressions in Eqs. (122) and (123) are practical expressions of general interest irrespective of the form of the matrices δ_{LR} , δ_{LL} and δ_{RR} . We see from Eqs. (122) and (123) that one naively expects that the δ_{LR} contributions dominate since the δ_{LL} and δ_{RR} contributions receive in general an additional suppression factor of the order $m_b/(5m_{\bar{b}})$. Nevertheless for the model under consideration we obtain

$$(\delta_{LR}^d)_{23} + (\delta_{LR}^d)_{32}^* = 4\lambda \left(c_\phi \omega - \frac{\tilde{m}}{A_b} \right) \frac{v A_b}{m_{\tilde{b}}^2} c_\beta, \quad (126)$$

$$(\delta_{LL}^d)_{23} + (\delta_{RR}^d)_{23} = -4e^{i\phi} \rho' \lambda. \quad (127)$$

We note that the total δ_{LR} contribution to $S_{\phi K_s}$ is zero since $\text{Im}[(\delta_{LR}^d)_{23}^* + (\delta_{LR}^d)_{32}] = 0$. Therefore in our model we find that only $(LL + RR)$ couplings contribute to $S_{\phi K_s}$. We find the following simple expression for the ratio of the dominant supersymmetric contribution to the amplitude over the standard model contribution,

$$R_\phi|_{LL+RR} = \left(\frac{\eta_\phi^2}{m_{\tilde{q}}^2} \right) L(x) |(\delta_{LL}^d)_{23} + (\delta_{RR}^d)_{23}| \quad (128)$$

where η_ϕ is a coefficient independent of the supersymmetric parameter space given by

$$\eta_\phi^2 \approx \frac{\sqrt{2} \alpha_s^2}{45 G_F |V_{ib}^* V_{ts}| h_\phi} \approx (189 \text{ GeV})^2. \quad (129)$$

Here h_ϕ parametrizes the dependence of the SM contribution on the Wilson coefficients and hadronic matrix elements. We used the value for h_ϕ calculated numerically in Ref. [92] using the generalized factorization approach (GF). For instance if $m_{\tilde{q}} = 500 \text{ GeV}$ we obtain $|R_\phi|_{LL+RR} \approx 0.14 L(x) |(\delta_{LL}^d)_{23} + (\delta_{RR}^d)_{23}|$, which agrees with previous numerical calculations [78,90]. We note that Eq. (128) provides some analytical insight in the dependency of the supersymmetric contributions on the supersymmetric spectra, especially on the gluino-squark mass ratio through the form factor $L(x)$.

Finally we will use the expression for $((\delta_{LL}^d)_{23} + (\delta_{RR}^d)_{23})$ in our model given in Eq. (127) and the general expression for the amplitude $A_{\phi K_s}$ given in Eq. (128) to rewrite the constraint from Eq. (120). Using for λ , $\lambda \approx 0.22$, we obtain

$$\rho' s_{\phi_{\text{NP}}} c_\delta \left(\frac{212 \text{ GeV}}{m_{\tilde{b}}} \right)^2 L(x) \leq 0.40 \pm 0.26. \quad (130)$$

We note that the phase ϕ_{NP} as well as the strong phases difference δ is not constrained by the data on quark masses and mixings. If $\phi_{\text{NP}} = 0$ this contribution to the asymmetry $S_{\phi K}$ would cancel. Let us assume in the worst case scenario that $\phi_{\text{NP}} = \pi/2$, $\delta = \pi/2$ and $\rho' = \rho = 9$ (which is the value for the large $\tan\beta$ scenario analyzed in Sec. III A). We would obtain only a mild constraint on the squark mass scale, of the order $m_{\tilde{b}} \gtrsim 1 \text{ TeV}$.

Finally we would like to mention that, as it has been pointed out, in the case when the δ_{LR} contribution is much smaller than the δ_{LL} or δ_{RR} contributions, the chargino contributions to the amplitude may be relevant since they could be of the same order as the δ_{LL} and δ_{RR} gluino contributions [81–84,89,90]. A more precise calculation would require the inclusion of these contributions.

C. CP asymmetry in $B \rightarrow \eta' K_s$

Recent results on the measurements of the CP asymmetries on the $b \rightarrow s$ processes have reported possible anomalies not only in $B \rightarrow \phi K_s$ but also in other processes, including $B \rightarrow \eta' K_s$. The latest results from Belle [75] and BABAR [76] Collaborations for the time dependent CP asymmetry coefficient $S_{\eta' K_s}$, $(S_{\eta' K}^{\text{Belle}})^{2004} = +0.06 \pm 0.42$ and $(S_{\eta' K}^{\text{BABAR}})^{2004} = +0.50 \pm 0.32$, seems to differ from the SM expectation. Combining the results from both experiments one obtains the world average, $(S_{\eta' K}^{\text{BABAR+Belle}})^{2004} = +0.41 \pm 0.11$ [68]. This has motivated the recent interest in the supersymmetric contributions to the CP asymmetry in the decay $B \rightarrow \eta' K_s$ versus $B \rightarrow \phi K_s$ [90,95] as well as in correlations with other supersymmetric processes [89,96]. It is known that because vector mesons (ϕ, ρ, \dots) and pseudo-scalar mesons (π, K, η', \dots) have opposite parity the B decays to these two final states will be sensitive to different combinations of the relevant Wilson coefficients [97]. For instance, in supersymmetric theories the gluino loop effects coming from δ_{LR} couplings will contribute by a factor proportional to $((\delta_{LR})_{23} + (\delta_{LR})_{32}^*)$ in the vector case and to $((\delta_{LR})_{23} - (\delta_{LR})_{32}^*)$ in the pseudo-scalar case, respectively. For the model under consideration, the contributions from δ_{LR} couplings, which exactly cancel for the $S_{\phi K_s}$ asymmetry, not only do not cancel but dominate the CP asymmetry in the decay $B \rightarrow \eta' K_s$. We can obtain an expression for the δ_{LR} contribution to $A_{\eta' K_s}$ similar to $A_{\phi K_s}$ in Eq. (122) with the change $((\delta_{LR})_{23} + (\delta_{LR})_{32}^*) \rightarrow ((\delta_{LR})_{23} - (\delta_{LR})_{32}^*)$. Using the resulting expression we obtain the following simple formula for the supersymmetric contribution to $S_{\eta' K_s}$,

$$S_{\eta' K_s} \approx 4 c_\beta \omega \lambda s_\phi M(x) \left(\frac{3.4 \text{ TeV}}{m_{\tilde{b}}} \right)^2 \left(\frac{A_b}{m_{\tilde{b}}} \right). \quad (131)$$

Using the values of λ and ω determined from quark masses and mixings, and assuming that $A_b \approx m_{\tilde{b}}$ we obtain the following constraint on $S_{\eta' K}$:

$$|s_\phi c_\beta c_{2\phi_1} s_\delta| \left(\frac{1 \text{ TeV}}{m_{\tilde{b}}} \right)^2 M(x) \leq 0.33 \pm 0.16. \quad (132)$$

We note that the phase ϕ as well as δ , the difference between strong phases, is not constrained by the data. In $\phi \approx 0$ this contribution to the asymmetry would cancel. In the worst case scenario, assuming that $x \approx 1$, $\phi = \pi/2$ and $\delta = \pi/2$ the constraint depends strongly on the value of $\tan\beta$. For large $\tan\beta$, $\tan\beta = 50$, we would obtain a mild lower constraint on the squark mass scale, $m_{\tilde{b}} \gtrsim 250 \text{ GeV}$.

D. CP asymmetry in $B \rightarrow X_s \gamma$

The CLEO Collaboration has set a range on the direct CP asymmetry in the $b \rightarrow s \gamma$ decay, $A_{CP}^{b \rightarrow s \gamma}$, at 90% C.L. as $A_{CP}^{b \rightarrow s \gamma} = (-3.5 \pm 13.5)\%$ [98] while the

Belle Collaboration also set a range as $A_{CP}^{b \rightarrow s\gamma} = (-0.8 \pm 10.7)\%$ [99]. According to the SM theoretical prediction $A_{CP}^{b \rightarrow s\gamma}$ is smaller than 1% [100]. Therefore $A_{CP}^{b \rightarrow s\gamma}$ is an observable potentially sensitive to the presence of new physics. Furthermore it is expected that the experimental uncertainty will be reduced to less than 1% at a super B factory. $A_{CP}^{b \rightarrow s\gamma}$ in supersymmetric theories has received considerable interest recently [79,89,96,101]. It is known that a CP violating phase in the entries $(\delta_{LR,RL}^d)_{23}$ or $(\delta_{LL,RR}^d)_{23}$ will generate CP violation in the decay $B \rightarrow X_s \gamma$ [97,102]. The direct CP asymmetry in $b \rightarrow s\gamma$ decay can be written in terms of the effective Wilson coefficients of the low-energy effective weak Hamiltonian [97],

$$A_{CP}^{b \rightarrow s\gamma} = \frac{1}{|C_7^L|^2 + |C_7^R|^2} [a_{27} \text{Im}[C_2(C_7^{L*} + C_7^{R*})] + a_{g7} \text{Im}[C_g^L C_7^{L*} + C_g^R C_7^{R*}] + a_{2g} \text{Im}[C_2(C_g^{L*} + C_g^{R*})]], \quad (133)$$

where $C_7^L = C_7^{L\text{eff}}(m_b)$, $C_g^L = C_g^{L\text{eff}}(m_b)$ and C_2 multiply the chromomagnetic dipole operators, $O_7 = \frac{e}{16\pi^2} \bar{s}_L \sigma_{\mu\nu} F^{\mu\nu} b_R$, $O_g = (g_s/16\pi^2) \bar{s}_L \sigma_{\mu\nu} G^{\mu\nu} b_R$, and the current-current operator, $O_2 = \bar{s}_L \gamma_\mu q_L \bar{q}_L \gamma^\mu b_L$, respectively. C_7^R and C_g^R are the corresponding coefficients of the nonstandard dipole operators involving right-handed light-quark fields, which appear in supersymmetric theories. We will use the numerical values of the coefficients a_{ij} as computed using the parton model in Ref. [97]: $a_{27} \approx 0.0123$, $a_{g7} \approx -0.0952$ and $a_{2g} \approx 0.0010$. In order to explore the implications of supersymmetric flavor models it is useful to express the effective coefficients in terms of the new physics contributions to the Wilson coefficients at the scale m_W . To this end numerical expressions were given in Ref. [97] including NLO renormalization effects from m_W down to the m_b mass scale,

$$C_7 = C_7^0 + \eta_{77} C_7(m_W) + \eta_{7g} C_g(m_W), \quad (134)$$

$$C_g = C_g^0 + \eta_g C_g(m_W). \quad (135)$$

Here $\eta_{77} = 0.67$, $\eta_{7g} = 0.09$ and $\eta_g = 0.70$. The supersymmetric contributions to C_7^L were given in Eq. (121). $C_7^L(m_{\bar{q}})$ is given by

$$C_7^L(m_{\bar{q}}) = \frac{\alpha_s \pi}{2m_{\bar{b}}} \left[\frac{m_b}{3m_{\bar{b}}} M_3(x) (\delta_{LL}^d)_{23} + \frac{1}{10} M_1(x) (\delta_{LR}^d)_{23} \right], \quad (136)$$

where $M_1(x)$ and $M_3(x)$ are dimensionless form factors defined in Eqs. (A11) and (A12) of the Appendix. $M_1(x)$ and $M_3(x)$ have been normalized to 1 when $x \rightarrow 1$. We note that for simplicity we have defined C_7 as the whole coefficient accompanying the operator O_7 . Therefore in our notation the SM contribution to the Wilson coefficient C_7^L at m_W is given by $C_7^{\text{SM}}(m_W) = -\sqrt{2} m_b G_F V_{ts}^* V_{tb} K(x_t)$,

where $x_t = m_t/m_W$ and $K(x)$ is a dimensionless form factor given in Eq. (A13) of the Appendix. The supersymmetric contributions to C_2 are negligible. We will use the SM value, $C_2(m_b) \approx 1.11 \times G_F V_{ts}^* V_{tb} / \sqrt{2}$. It is straightforward to obtain C_g^R and C_7^R by the exchange $L \leftrightarrow R$ in the expressions for C_7^L and C_g^L . Barring cancellations between δ_{LR} and δ_{LL} terms we will obtain an approximate bound from the LR contribution. We can see from Eq. (133) that the total δ_{LR} contribution is proportional to a coupling of the form $((\delta_{LR}^d)_{23}^* + (\delta_{LR}^d)_{32}^*)$. We obtain the following approximate expression for the asymmetry:

$$A_{CP}^{b \rightarrow s\gamma}|_{LR+RL} \approx (-) \frac{\alpha_s \pi}{2m_{\bar{b}}} \frac{C_2 \text{Im}[(\delta_{LR}^d)_{23}^* + (\delta_{LR}^d)_{32}^*] A(x)}{|C_7^{\text{SM}}|^2}. \quad (137)$$

For the $\delta_{LL,RR}$ couplings we will obtain a similar expression proportional to the coupling $((\delta_{LL}^d)_{23}^* + (\delta_{RR}^d)_{32}^*)$. Here $A(x)$ is a dimensionless form factor defined by

$$A(x) = \left(a_{27} \eta_{77} \frac{1}{3} M_1(x) + a_{2g} \left(\eta_{7g} \frac{1}{3} M_1(x) + \eta_g M(x) \right) \right). \quad (138)$$

For the model under consideration,

$$(\delta_{LR}^d)_{23}^* + (\delta_{LR}^d)_{32}^* = (-) 4\lambda s_\phi \omega \frac{v A_b}{m_{\bar{b}}^2} c_\beta. \quad (139)$$

The SM contribution to the effective coefficient C_7^0 is related to the Wilson coefficient at the m_W scale by a renormalization factor, $C_7^0 = \eta_{bW} C_7^{\text{SM}}$, which can be extracted from Ref. [97]. Assuming that $x \approx 1$, i.e. $m_{\bar{g}} \approx m_{\bar{b}}$, and using the values of λ and ω as determined from quark masses and mixings we obtain the constraint,

$$A_{CP}^{b \rightarrow s\gamma}|_{LR+RL} \approx s_\phi c_\beta \left(\frac{40 \text{ GeV}}{m_{\bar{b}}} \right)^2 \left(\frac{A_b}{m_{\bar{d}}} \right) \lesssim 0.1. \quad (140)$$

In the worst case scenario, assuming that $s_\phi \approx 1$, $A_b \approx m_{\bar{d}}$, $\tan\beta \approx 1$ the current experimental bound requires $m_{\bar{b}} \gtrsim 230 \text{ GeV}$. On the other hand, for large $\tan\beta$ one would obtain a milder constraint. We would like to point out that the phase ϕ is not constrained by the CKM phase. If $s_\phi \ll 1$ the squark masses would not be constrained by $A_{CP}^{b \rightarrow s\gamma}$. One would naively expect that the δ_{LR} gives the dominant contribution to $A_{CP}^{b \rightarrow s\gamma}$ because of the m_b/v suppression factor of the δ_{LL} contributions to C_7 and C_g . Nevertheless for the model under consideration the contributions coming from $\delta_{LL,RR}$ couplings are of the same order of magnitude. For the model under consideration,

$$(\delta_{LL}^d)_{23}^* + (\delta_{RR}^d)_{32}^* = -4\lambda \rho' e^{-i\phi}. \quad (141)$$

We obtain a similar expression,

$$A_{CP}^{b \rightarrow s\gamma}|_{LL+RR} \approx s_\phi \rho' \left(\frac{30 \text{ GeV}}{m_{\bar{b}}} \right)^2 \lesssim 0.1. \quad (142)$$

The constraint on the squark spectra depends on the value of ρ' . For the large $\tan\beta$ case we noted in Sec. III A that $\rho = \tilde{m}^2/m_{\tilde{b}}^2 \approx 9$. If $\rho' = \rho$ we would obtain the constraint $m_{\tilde{b}} \gtrsim 300$ GeV. This constraint could be avoided if $\phi \approx 0$ or $\eta' \ll 1$.

E. $\Gamma(b \rightarrow s\gamma)$

The supersymmetric contributions to the $b \rightarrow s\gamma$ decay are indirectly correlated with the CP asymmetries in $B \rightarrow \phi K_s$ and $b \rightarrow s\gamma$ decays since the same flavor mixing couplings contribute to the relevant Wilson coefficients. The $b \rightarrow s\gamma$ decay rate is also proportional to the C_7 Wilson coefficients,

$$\Gamma(b \rightarrow s\gamma) \propto (|C_7^L|^2 + |C_7^R|^2). \quad (143)$$

The current world average of the CLEO [103] and Belle [104] results is given by $B(b \rightarrow s\gamma)_{\text{exp}} = (3.3 \pm 0.4) \times 10^{-4}$, which can perfectly be accounted for the SM theoretical prediction, $B(b \rightarrow s\gamma)_{\text{theo}} = (3.29 \pm 0.33) \times 10^{-4}$ [105], which leaves a small window open for new physics. There is no SM contribution to C_7^R . A full expression for the main supersymmetric contributions, i.e. gluino exchange, to this branching ratio were first given in Ref. [106]. Further improvements in the calculation as chargino diagrams and QCD corrections were subsequently included [107]. Therefore if the supersymmetric contribution is just a correction to the SM one we can expand in powers of $R_{s\gamma} = |C_7^{L\text{SUSY}}/C_7^{\text{SM}}|$ and obtain to leading order,

$$\frac{\Gamma(b \rightarrow s\gamma)}{\Gamma(b \rightarrow s\gamma)^{\text{SM}}} - 1 \approx 2R_{s\gamma}. \quad (144)$$

Allowing the supersymmetric contribution to saturate the 2σ experimental uncertainty we obtain to leading order in $R_{s\gamma}$ the constraint,

$$R_{s\gamma} \lesssim \frac{\Delta(\text{Br}(b \rightarrow s\gamma))_{\text{exp}}}{\text{Br}(b \rightarrow s\gamma)_{\text{exp}}} \lesssim 0.12. \quad (145)$$

Using the expression for the supersymmetric contribution to C_7^L from Eq. (136) we obtain for the δ_{LR} contribution to $R_{s\gamma}$ the expression,

$$R_{s\gamma}|_{LR} = \frac{\eta_{s\gamma}}{m_{\tilde{q}}} |(\delta_{LR}^d)_{23}| M_3(x). \quad (146)$$

Here $\eta_{s\gamma}$ is a coefficient independent of the supersymmetric parameter space with mass units. Using the SM expression for C_7^L and the expression for $(\delta_{LR}^d)_{23}$ that our model predicts for $x \approx 1$,

$$|(\delta_{LR}^d)_{23}| \approx 2\lambda c_\beta \frac{v}{m_{\tilde{b}}} \frac{\tilde{m}}{m_{\tilde{b}}},$$

we obtain the constraint,

$$c_\beta \left(\frac{130 \text{ GeV}}{m_{\tilde{b}}} \right)^2 \left(\frac{\tilde{m}}{m_{\tilde{b}}} \right) \lesssim 0.12. \quad (147)$$

For the large $\tan\beta$ case with $m_{\tilde{g}} \approx m_{\tilde{b}}$ examined in Sec. III A, $\tan\beta = 50$, \tilde{m} was required to be $\tilde{m} \approx 3m_{\tilde{b}}$. In this case we obtain the following constraint on the squark mass scale, $m_{\tilde{b}} \gtrsim 370$ GeV. An analysis with similar results can be implemented for the $\delta_{(LL+RR)}$ contribution to $\Gamma(b \rightarrow s\gamma)$. In this case,

$$|(\delta_{LL}^d)_{23}| \approx 2\lambda\rho'.$$

We obtain the constraint,

$$\rho' \left(\frac{16 \text{ GeV}}{m_{\tilde{b}}} \right)^2 \lesssim 0.12. \quad (148)$$

For the large $\tan\beta$ case with $m_{\tilde{g}} \approx m_{\tilde{b}}$ examined in Sec. III A, $\tan\beta = 50$, ρ was required to be $\rho \approx 3$. If $\rho' = \rho$ we would obtain the following lower bound on the squark mass scale, $m_{\tilde{b}} \gtrsim 140$ GeV.

VIII. CONCLUSIONS

We have shown that generic supersymmetric flavor models exist for the radiative generation of fermion masses, mixings and CP phases. We have studied in detail the phenomenological implications of a particular supersymmetric flavor model for the radiative generation of first and second generation quark masses, focusing our attention especially in the CP violating phenomenology. The basic idea underlying this kind of flavor models is that the flavor breaking fields are also supersymmetry-breaking fields.

We have shown that these models generically solve the SUSY CP problem in a very simple fashion. The one-loop contributions to EDMs are exactly zero due to the real character of the relevant parameters in the calculation while in the worst case scenario the two-loop contributions are suppressed below current experimental constraints for a supersymmetric spectra above $\gtrsim 1$ TeV.

Our main goal was to present a flavor model as predictive as possible. To this end we have proposed a particular implementation of this scenario using a $U(2)$ flavor symmetry where the required hierarchy of flavor breaking VEVs is expressed in powers of a single parameter, λ . As a consequence the model generates quark Yukawa matrices that contain only three parameters, λ, θ, γ and can fit the data with precision. Therefore the quark masses and mixings determine the amount of flavor violation in the soft sector requiring a very heavy SUSY spectra especially to overcome the constraints on $\Delta m_K, \epsilon$ and Δm_d .

We would like to emphasize that this case study can be considered the worst case scenario from the point of view of FCNC constraints. Between the extreme case study here and the usual models with scalar flavor breaking VEVs there is a continua of possibilities which would ameliorate the FCNC constraints. For instance, we could increase the

number of parameters in the flavor breaking sector, use a different flavor symmetry or generate radiatively only the first generation of fermion masses. If that was the case one could lower considerably the constraints on the sfermion spectra, probably at the price of decreasing the predictivity of the flavor model.

We believe these models are a scenario worthy of a more dedicated exploration. They generically allow us to simplify the “flavor vacuum,” or in other words the hierarchies of the flavor breaking VEVs, through the introduction of a natural hierarchy, the loop factor, and they offer a new insight in the SUSY CP and flavor problems.

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APPENDIX

For completeness we include expressions for the dimensionless form factors that were used in the main text. The form factor $F(x, y, z)$ is defined as

$$F(x, y, z) = \frac{[(x^2 y^2 \ln \frac{y^2}{x^2} + y^2 z^2 \ln \frac{z^2}{y^2} + z^2 x^2 \ln \frac{x^2}{z^2})]}{(x^2 - y^2)(y^2 - z^2)(z^2 - x^2)}. \quad (\text{A1})$$

$f(x)$, $g(x)$ and $h(x)$ are given by

$$f(x) = \frac{10x}{3} \left[\frac{(x^2 - 8x - 17)}{(x-1)^4} + \frac{6(1+3x)\ln(x)}{(x-1)^5} \right], \quad (\text{A2})$$

$$g(x) = 10 \left[\frac{(x^2 + 10x + 1)}{(x-1)^4} - \frac{6x(1+x)\ln(x)}{(x-1)^5} \right], \quad (\text{A3})$$

$$h(x) = \frac{11g(x) - 6f(x)}{5}. \quad (\text{A4})$$

These functions appear in the calculation of the supersymmetric contributions to the Wilson coefficients. The functions $f(x)$, $g(x)$ and $h(x)$ have been conveniently normalized so that in the limit $x \rightarrow 1$ they tend to 1. Approximate expressions in the limits $x \rightarrow 0, 1$ are given by

$$f(x) = \begin{cases} 1 - \frac{1}{3}\zeta + \mathcal{O}(\zeta^2), & x \rightarrow 1, (\zeta = x - 1), \\ -\frac{10x}{3}(17 + 6\ln(x)) + \mathcal{O}(x^2), & x \rightarrow 0, \end{cases} \quad (\text{A5})$$

$$g(x) = \begin{cases} 1 - \zeta + \mathcal{O}(\zeta^2), & x \rightarrow 1, (\zeta = x - 1), \\ 10(1 + 2x(3\ln(x) + 7)) + \mathcal{O}(x^2), & x \rightarrow 0. \end{cases} \quad (\text{A6})$$

$N(x)$ and $M(x)$ are dimensionless form factors given by

$$N(x) = \left[\frac{(x^2 + 172x + 19)}{36(x-1)^4} - x \ln(x) \frac{(x^2 - 15x - 18)}{6(x-1)^5} \right], \quad (\text{A7})$$

$$M(x) = \sqrt{x} \left[\frac{(54x^4 + 216x^3 - 287x^2 - 8x + 1)}{9(x-1)^4} - 2x^2 \ln(x) \frac{(36x^2 - 19x - 21)}{3(x-1)^5} \right]. \quad (\text{A8})$$

In the limit $x \simeq 1$ $M(x)$ is given by

$$M(x) = a_0 + a_1(x-1) + \mathcal{O}((x-1)^2),$$

$$a_0 = \frac{31}{30}, \quad a_1 = \frac{233}{180}. \quad (\text{A9})$$

If one is interested in the limit $x \simeq 0$, i.e. $m_{\tilde{g}} \ll m_{\tilde{q}}$, it is also possible to obtain an approximate expression for $M(x)$,

$$M(x) = \frac{1}{9} \sqrt{x} (b_0 + b_1 x + \mathcal{O}(x^2)),$$

$$b_0 = 1, \quad b_1 = -4. \quad (\text{A10})$$

The functions $M_1(x)$ and $M_3(x)$ are defined by

$$M_1(x) = \frac{12x^2 \ln(x)}{(1-x)^4} + \frac{6(1+5x)}{(1-x)^3}, \quad (\text{A11})$$

$$M_3(x) = \frac{10\sqrt{x}}{3} \left[\frac{(1-8x-17x^2)}{(x-1)^4} + \ln(x) \frac{(18x^2+6x^3)}{(x-1)^5} \right]. \quad (\text{A12})$$

Here $M_1(x)$ and $M_3(x)$ have also been “normalized” so that in the limit $x \rightarrow 1$ they tend to 1. Finally the dimensionless form factor $K(x)$, which appears in the SM contribution to the Wilson coefficient C_7 , is given by,

$$K(x) = \frac{x}{2(x-1)} \left(\frac{(8x^2+5x-7)}{12} - \left(\frac{2x^2}{3} - x \right) \frac{\ln(x)}{(x-1)} \right). \quad (\text{A13})$$

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