Baryonium with a phenomenological Skyrmion-type potential

Mu-Lin Yan and Si Li
Interdisciplinary Center for Theoretical Study, University of Science and Technology of China, Hefei, Anhui 230026, China

Bin Wu and Bo-Qiang Ma*
Department of Physics, Peking University, Beijing 1000871, China
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In this paper, we investigate the nucleon-antinucleon static energies in the Skyrme model with the product *Anzatz*. The calculation shows that, in the ungroomed $S\overline{S}$ (Skyrmion and anti-Skyrmion) channel which leads to rapid annihilation, there exists a quasistable bound state which may give a natural explanation for the near-threshold enhancement in the proton-antiproton ($p\overline{p}$) mass spectrum reported by the BES Collaboration and the Belle Collaboration. Similar to the phenomenological well potential of the deuteron, we construct a phenomenological Skyrmion-type potential to study this narrow $p\overline{p}$-resonance in $J/\psi \rightarrow \gamma p\overline{p}$. By this potential model, a $p\overline{p}$ baryonium with small binding energies is suggested and the decay width of this state is calculated by WKB approximation. In this picture the decay is attributed to quantum tunneling and $p\overline{p}$ annihilation. Prediction on the decay mode from the baryonium annihilation at rest is also pointed out.

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I. INTRODUCTION

Skyrme's old idea [1] that baryons are chiral solitons has been successful in describing the static nucleon properties [2] since Witten's illustration that the soliton picture of baryons is consistent with QCD in the large N_c approximation [3]. The Skyrme model has been widely used to discuss baryons and baryonic-system properties. The Skyrme model with the product *Ansatz* has also been applied to the nucleon-nucleon interaction [4] and to the static properties of the deuteron [5]. At the classical level, the deuteron mass has been successfully calculated by the *ab initio* approach [6]. Atiyah and Manton [7] revealed a mathematically elegant connection between the Skyrme soliton and the instantons in the SU(2) gauge field, in which the holonomy of the instanton along all lines parallel to the Euclidean time axis generates the Skyrmion with the same topological charge. Using the *Anzatz* generated in this approach, Leese *et al.* [8] reinvestigated the deuteron properties and gave an experimentally more agreeable result. However, since self-dual and anti-self-dual equations allow only trivial solutions of zero topological charge, it is difficult to apply the Skyrme model to the interaction between Skyrmion and anti-Skyrmion in such a manner. The nucleon-antinucleon static potential was studied in the Skyrme model using the product *Anzatz* by Lu and Amado [9], who showed that the Skyrme picture with the product *Ansatz* is the reasonable first step to obtain the real part of nucleon-antinucleon interaction.

Recently, the BES Collaboration observed a near-threshold enhancement in the proton-antiproton ($p\overline{p}$) mass spectrum from the radiative decay $J/\psi \rightarrow \gamma p\overline{p}$ [10]. This enhancement can be fitted with either an S- or P-wave Breit-Wigner resonance function. In the case of the S-wave fit, the peak mass is at $M = 1859^{+3}_{-10}(\mathrm{stat})^{+5}_{-25}(\mathrm{syst})$ with a total width $\Gamma < 30\ \mathrm{MeV}/c^2$ at 90% confidence level. The corresponding spin and parity are $J^{PC} = 0^{-+}$. Moreover, the Belle Collaboration also reported similar observations of the decays $B^+ \rightarrow K^+ p\overline{p}$ [11] and $\overline{B}^0 \rightarrow D^0 p\overline{p}$ [12], showing enhancements in the $p\overline{p}$ invariant mass distributions near $2m_p$. These observations could be interpreted as signals for baryonium $p\overline{p}$ bound states [13] or flavorless gluon states [14]. There are also suggestions that the near-threshold enhancement of $p\overline{p}$ is due to final state interactions [15] or as a result of the quark fragmentation process [14].

In this paper, we provide a possible explanation that the enhancement might be explained as a baryonium $p\overline{p}$ bound state in a phenomenological potential inspired by the investigation of the static energy in the ungroomed $S\overline{S}$ channel. In Sec. II, we scrutinize the nucleon-antinucleon static energy in the ungroomed $S\overline{S}$ channel from the $SU(2)$ Skyrme model. In Sec. III, inspired by this energy, we construct a phenomenological Skyrmion-type potential, investigate the bound state in this potential, and calculate the width by WKB approximation through the quantum tunneling effect. Then, in Sec. IV, we give our conclusion and discussion, with emphasis on the significant implication on the decay mode of the baryonium annihilation at rest.

*Author to whom correspondence should be addressed.
Electronic address: mabq@phy.pku.edu.cn

II. THE STATIC ENERGY IN THE UNGROOMED $S\bar{S}$ CHANNEL

The Lagrangian for the SU(2) Skyrme model is

$$\mathcal{L} = \frac{1}{16}F_\pi^2 \mathrm{Tr}(\partial_\mu U^\dagger \partial^\mu U) + \frac{1}{32e^2}\mathrm{Tr}([(\partial_\mu U)U^\dagger, (\partial_\nu U)U^\dagger]^2) + \frac{1}{8}m_\pi^2 F_\pi^2 \mathrm{Tr}(U-1), \tag{1}$$

where $U(t, \mathbf{x})$ is the SU(2) chiral field, expressed in terms of the pion fields:

$$U(t, \mathbf{x}) = \sigma(t, \mathbf{x}) + i\pi(t, \mathbf{x}) \cdot \tau. \tag{2}$$

We fix the parameters F_π and e as in [2], and our units are related to conventional units via

$$\frac{F_\pi}{4e} = 5.58 \text{ MeV}, \qquad \frac{2}{eF_\pi} = 0.755 \text{ fm}. \tag{3}$$

Skyrme avoided the obstacle of minimizing the energy by invoking the hedgehog *Ansatz*,

$$U_H(\mathbf{r}) = \exp[i\tau \cdot \hat{r} f(r)]. \tag{4}$$

The product *Ansatz* describing the behavior of the Skyrmion and the anti-Skyrmion system with relatively arbitrary rotation in the isospace is of the form

$$U_s = CU_H\left(\mathbf{r} + \frac{\rho}{2}\hat{z}\right)C^\dagger U_H^\dagger\left(\mathbf{r} - \frac{\rho}{2}\hat{z}\right), \tag{5}$$

where C is an element in the isospin SU(2) group. The Skyrmion and the anti-Skyrmion are separated along the $\hat{z}$-axis by a distance ρ. The interactions between various groomed Skyrmion and anti-Skyrmion have been studied in Ref. [9], in which the configuration lies in a manifold of relatively higher dimension. We find that the static potential in the ungroomed $S\bar{S}$ channel is physically more interesting with $C = I$, which satisfies

$$U \to 1 \quad \text{when } \rho \to 0. \tag{6}$$

Fixing our *Ansatz* as above, we can get the static energy from the Skyrmion Lagrangian:

$$M(\rho) = \int d^3\mathbf{r}\left[-\frac{1}{2}\mathrm{Tr}(R_i R_i) - \frac{1}{16}\mathrm{Tr}([R_i, R_j]^2) - m_\pi^2 \mathrm{Tr}(U-1)\right], \tag{7}$$

where $i, j = 1, 2, 3$. The right-currents R_μ are defined via

$$R_\mu = (\partial_\mu U)U^\dagger, \tag{8}$$

and we express the energy in the units defined above. In this picture, the binding energies for $S\bar{S}$ (which correspond to the classical binding energies of $p\bar{p}$) are

$$\Delta E_B = 2m_p^c - M(\rho), \tag{9}$$

where $m_p^c = 867$ MeV is the mass of a classical nucleon (or classical Skyrmion). A stable or quasistable $p\bar{p}$-binding state corresponds to the Skyrmion configuration $U_s(r, \rho_B) = U_H(\mathbf{r} + \frac{\rho_B}{2}\hat{z})U_H^\dagger(\mathbf{r} - \frac{\rho_B}{2}\hat{z})$ with $\Delta E_B(\rho_B) < 0$ and $\frac{d}{d\rho}(\Delta E_B(\rho))|_{\rho=\rho_B} = 0$.

The numerical result of the static energy as a function of ρ is showed in Fig. 1. From it, we find that there is a quasistable $p\bar{p}$-binding state:

$$\rho_B \approx 2.5 \text{ fm}, \tag{10}$$

$$\Delta E_B(\rho_B) \approx 10 \text{ MeV}. \tag{11}$$

In the above investigation, we actually conjecture the binding energy between S and $\bar{S}$ under the *Anzatz* of $U_s(r, \rho_B) = U_H(\mathbf{r} + \frac{\rho_B}{2}\hat{z})U_H^\dagger(\mathbf{r} - \frac{\rho_B}{2}\hat{z})$ being approximately the binding energy of $p\bar{p}$. This conjecture is based on the considerations in the studies of deuteron as a soliton in the Skyrme model done by Braaten and Carson [5]. In their works, the two-Skyrmion configuration with $B = 2$ has been treated as a single soliton $U_2(\mathbf{r})$ by using the product *Ansatz*, and the spin- and isospin-quantum numbers of the physical states arise from the semiclassical quantization of the collective coordinates of the soliton. In this way, the deuteron state with $(I = 0, J = 1)$ has been identified, and its mass (or its corresponding binding energy) has been obtained. In this formulation, the deuteron's mass (or energies) is divided into two parts: (i) the classical energies of statical soliton configuration $U_2(\mathbf{r})$; (ii) the semiclassical corrections. The latter one is suppressed in large N_c-expansion limit. Namely, the classical part of the energies of the Skyrmion, i.e., the toroidal configuration, is qualitatively dominant. Moreover, the detailed analysis by Forest *et al.* does indeed exhibit the toroidal configuration in their quantum mechanical deuteron wave function, which, though making up only a very small component, could have a revealing relation to certain aspects of QCD [16]. We argue that, when one uses similar product *Ansatz*

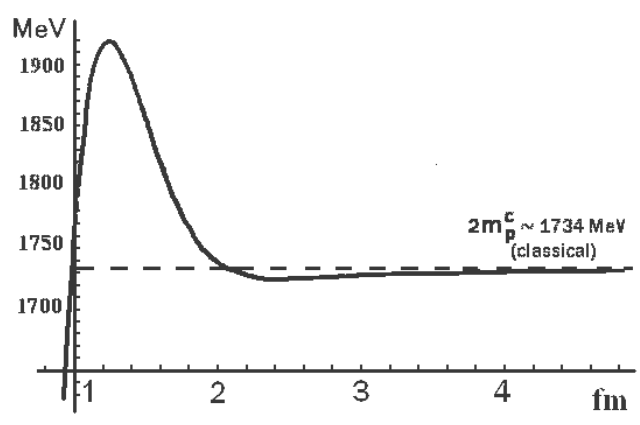

FIG. 1. The static energy of the Skyrmion–anti-Skyrmion system, where m_p^c is the classical single Skyrmion mass without quantum correction.

to deal with the Skyrmion–anti-Skyrmion system, the situation should be similar to the case of Skyrmion-Skyrmion. Namely, the $(S\overline{S})$-binding energy equation (11) should roughly be the $(p\overline{p})$-binding energy and the spin- and isospin-quantum numbers of the $(p\overline{p})$-bound state arise from the semiclassical quantization of the collective coordinates of $U_s(r, \rho_B)$. Since $U_s(r, \rho_B)$ serves as a classical soliton *Ansatz* of $(p\overline{p})$ [rather than (NN) for the deuteron case], it could be expected that the state with spin-0 and isospin-0 would be the lowest semiclassical state. However, the real situation might be complicated and we leave this to be one of the topics for future studies.

Of course, we should not take this potential too seriously for two reasons. On one hand, the product *Anzatz* is an exact solution to the equation of motion of the system only when $\rho_B \rightarrow \infty$. On the other hand, even in the case of deuteron [5,8], the toroidal configuration gives a binding energy of the order interaction scale, which is ~pion decay constant (~ 100 MeV), not of the nuclear scale, ~a few MeV, and the size and quadrupole moment are simply too small. However, the potential in Fig. 1 seems suggestive to give the observed near-threshold $p\overline{p}$ enhancement in experiments [10–12] a possible phenomenological explanation, as simple as possible, just like that in Ref. [13], based on the inspiration from it. Moreover, by such a phenomenological potential reflecting the character of Fig. 1, we can also obtain a picture of how the $p\overline{p}$ bound state decays: there is a $p\overline{p}$ bound state in a well, which will mostly annihilate through a barrier penetration by the tunnel effect, and this will be discussed in detailed in the next section.

III. A PHENOMENOLOGICAL MODEL WITH A SKYRMION-TYPE POTENTIAL

In this section, we employ the phenomenological model induced from the Skyrmion picture of $(p\overline{p})$-interactions for the nucleons. We favor such a potential as shown in Fig. 1 because it seems that the potential can be physically substantiated. For over 50 years there has been a general understanding of the nucleon-nucleon interaction as one in which there is, in potential model terms, a strong repulsive short distance core together with a longer range weaker attraction. The attractive potential at the middle range binds the neutron and the proton to form a deuteron. In comparison with the Skyrmion result on the deuteron [5,7,8], we notice several remarkable features of the static energy $M(\rho)$ and the corresponding $(p\overline{p})$-potential $V(\rho)$. First, the potential is attractive at $\rho > 2.0$ fm, similar to the deuteron case. This is due to the fact that the interaction via pseudoscalar π-meson exchange is attractive for both quark-quark qq and quark-antiquark $q\overline{q}$ pairs. Physically, the attractive force between p and $\overline{p}$ should be stronger than that of pn. Therefore the fact that our result of the $p\overline{p}$-binding energy [see Eq. (11)] $\Delta E_B(\rho_B)(\approx 10$ MeV) is larger than that of deuteron (2.225 MeV) is quite reasonable physically. Second, there is a static Skyrmion energy peak at $\rho \sim 1$ fm in Fig. 1. This means that the corresponding potential between p and $\overline{p}$ is repulsive at that range. This is an unusual and also an essential feature. The possible explanation for it is that the Skyrmions are extended objects, and there would emerge a repulsive force to counteract the deformations of their configuration shapes when they are close to each other. Similar repulsive potential has also been found in previous numerical calculation [9]. Third, a well potential at middle ρ-range is formed due to the competition between the repulsive and attractive potentials mentioned above, similar to the deuteron case. The depth should be deeper than that in the deuteron case, as argued in a QCD based discussion [13]. The p and $\overline{p}$ will be bound to form a baryonium in this well potential. Finally, the potential turns to decrease quickly from ~2000 MeV to zero when $\rho \rightarrow 0$. This means that there is a strong attractive force at $\rho \sim 0$. Physically, $p\overline{p}$ are annihilated.

The qualitative features of the proton-neutron potential for the deuteron can be well described by a simple phenomenological model of a square well potential [17–19] with a depth which is sufficient to bind the pn $3S_1$-state with a binding energy of -2.225 MeV. Numerically, the potential width a_{pn} is about 2.0 fm, and the depth is about $V_{pn} = 36.5$ MeV. Similarly, from the above illustration on the features of the potential between p and $\overline{p}$ based on the Skyrmion picture, we now construct a phenomenological potential model for the $p\overline{p}$ system, as shown in Fig. 2, and it will hereafter be called the Skyrmion-type potential.

We take the width of the square well potential, denoted as $a_{p\overline{p}}$, as close to that of the deuteron, i.e., $a_{p\overline{p}} \sim a_{pn} \simeq 2.0$ fm. According to QCD inspired considerations [13,20,21], the well potential between q and $\overline{q}$ should be twice as attractive as the qq case, i.e., the depth of the $p\overline{p}$ square well potential is $V_{p\overline{p}} \simeq 2V_{pn} = 73$ MeV. The width for the repulsive force revealed by the Skyrme model can be fitted by the decay width of the baryonium, and we take it to be $\lambda = 1/(2m_p) \sim 0.1$ fm, the Compton wavelength of the bound state of $p\overline{p}$. The square barrier potential

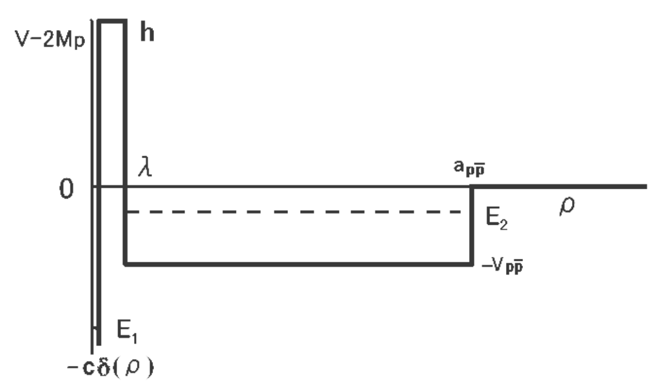

FIG. 2. The Skyrmion-type potential of the $p\overline{p}$-system.

begins from $\rho \sim \lambda$, and the height of the potential barrier, which should be constrained by both the decay width and the binding energy of the baryonium, is taken as $2m_p + h$, where $h \sim m_p/4$. At $\rho \sim 0$, $V^{(p\bar{p})}(\rho) \sim -c\delta(\rho)$ with a constant $c > 0$.

Analytically, the potential $V(\rho)$ is expressed as follows:

$$V(\rho) = 2m_p - c\delta(\rho) + V_c(\rho), \tag{12}$$

where

$$V_c(\rho) = \begin{cases} h = m_p/4, & 0 < \rho < \lambda, \\ -V_{p\bar{p}} = -73 \text{ MeV}, & \lambda < \rho < a_{p\bar{p}}, \\ 0, & \rho > a_{p\bar{p}}. \end{cases} \tag{13}$$

With this potential, the Schrödinger equation for S-wave bound states is

$$\frac{-1}{2(m_p/2)} \frac{\partial^2}{\partial \rho^2} u(\rho) + [V(\rho) - E]u(\rho) = 0, \tag{14}$$

where $u(\rho) = \rho\psi(\rho)$ is the radial wave function and $m_p/2$ is the reduced mass. This equation can be solved analytically, and there are two bound states $u_1(\rho)$ and $u_2(\rho)$: $u_1(\rho)$ with binding energy $E_1 < -V_{p\bar{p}} = -73$ MeV is due to the $-c\delta(\rho)$-function potential mainly, and $u_2(\rho)$ with binding energy $E_2 > -73$ MeV is due to the attractive square well potential at the middle range mainly. $u_1(\rho)$ is the vacuum state, and, clearly, $u_2(\rho)$ should correspond to a deuteronlike molecule state, and it may be interpreted as the new $p\bar{p}$ resonance reported by BES [10]. It is also expected that corresponding binding energies $-E_2$ in the potential model provided in the above $\Delta E_B(\rho_B)$ [see Eq. (11)] are all in agreement with the data within errors of BES [10]. By fitting experimental data, we have

$$E_1 = -(2m_p - m_{\eta_0}) \simeq -976 \text{ MeV}, \tag{15}$$

$$E_2 = -17.2 \text{ MeV}. \tag{16}$$

Considering its decay width which will be derived soon [see Eq. (20)], we conclude that the near-threshold narrow enhancement in the $p\bar{p}$ invariant mass spectrum from $J/\psi \to \gamma p\bar{p}$ might be interpreted as a state of protonium in this potential model.

In the Skyrmion-type potential of $p\bar{p}$, there are two attractive potential wells: one is at $\rho \sim 0$ and the other is at middle scale, together with a potential barrier between them. At $\rho \sim 0$, the baryon and antibaryon pair annihilates. The baryon-antibaryon annihilation has been studied in the Skyrme model in the literature (see, e.g., Refs. [22,23]). Naturally, we postulate that the bound states decay dominantly by annihilation and, therefore, we can derive the width of protonium state $u_2(\rho)$ by calculating the quantum tunneling effect for $u_2(\rho)$ passing through the potential barrier. By WKB approximation, the tunneling coefficient (i.e., barrier penetrability) reads [19]

$$T_0 = \exp\left[-2\int_0^\lambda dr\sqrt{m_p(h - E_2)}\right] = \exp[-2\lambda\sqrt{m_p(h - E_2)}]. \tag{17}$$

In the square well potential from λ to $a_{p\bar{p}}$, the time period θ of a round-trip for the particle is

$$\theta = \frac{2[a_{p\bar{p}} - \lambda]}{v} = [a_{p\bar{p}} - \lambda]\sqrt{\frac{m_p}{V_{p\bar{p}} + E_2}}. \tag{18}$$

Thus, the state $u_2(r)$'s lifespan is $\tau = \theta T_0^{-1}$, and hence the width of that state reads

$$\Gamma \equiv \frac{1}{\tau} = \frac{1}{a_{p\bar{p}} - \lambda}\sqrt{\frac{V_{p\bar{p}} + E_2}{m_p}}\exp[-2\lambda\sqrt{m_p(h - E_2)}]. \tag{19}$$

Numerically, substituting $E_2 = -17.2$ MeV, $a_{p\bar{p}} = 2.0$ fm into (19), we obtain the prediction of Γ:

$$\Gamma \simeq 15.5 \text{ MeV}, \tag{20}$$

which is compatible with the experimental data [10].

IV. CONCLUSION

In conclusion, we investigated the $S\bar{S}$ static potential in the Skyrme model and, similar to the phenomenological potential model of the deuteron, we constructed a Skyrmion-type potential model to study the recent discovery of a narrow $N\bar{N}$-resonance in the decay $J/\psi \to \gamma p\bar{p}$ by BES, and also in the decays $B^+ \to K^+ p\bar{p}$ and $\bar{B}^0 \to D^0 p\bar{p}$ by Belle. The parameters in the model are guided by the parameters in the deuteron model and by QCD inspired considerations. We found that this Skyrmion-type potential model has one baryonium solution, which might be explained as the $p\bar{p}$ bound state. By fitting the mass, we found that the width can be compatible with the experimental data.

We have learned from the studies in this paper that, based on the Skyrmion considerations, the baryonium decays are mainly due to the annihilations of nucleon-antinucleon. This is a significant feature for the particle decay of the baryonium. It has been well known that the nucleon-antinucleon annihilation at rest mostly favors processes with 4 to 7 pions in the final states over those with two or three pions [23]. Considering that the binding energy of the nucleon (or antinuleon) is rather small (compared with the mass of nucleon), the annihilation of the baryonium occurs nearly at rest. Therefore, we would predict that the baryonium hadronic decay should also mostly favor processes with 4 to 7 pseudoscalar mesons in the final states over those with two or three mesons, even though the phase spaces for the latter are larger than the former. This prediction is nontrivial and needs to be tested by experiments.

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