Masses of heavy baryons in the relativistic quark model

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The masses of the ground state heavy baryons consisting of two light (u, d, s) and one heavy (c, b) quarks are calculated in the heavy-quark–light-diquark approximation within the constituent quark model. The light quarks, forming the diquark, and the light diquark in the baryon are treated completely relativistically. The expansion in v/c up to the second order is used only for the heavy (b and c) quarks. The diquark-gluon interaction is taken modified by the form factor describing the light diquark structure in terms of the diquark wave functions. An overall reasonable agreement of the obtained predictions with available experimental data and previous theoretical results is found.

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I. INTRODUCTION

The description of baryons within the constituent quark models is a very important problem in quantum chromodynamics (QCD). Since the baryon is a three-body system, its theory is much more complicated compared to the two-body meson system. The quark-diquark picture of a baryon [1,2] is the popular approximation widely used to describe the baryon properties [1–5]. The methods of heavy quark effective theory (HQET) proved to be very fruitful in predicting the properties of the heavy-light $q\bar{Q}$ mesons (B and D). This success suggests to apply these methods to heavy-light baryons. In our paper [6] we considered the simplest baryonic systems of this kind, the doubly-heavy baryons (qQQ). The two heavy quarks (b or c) compose in this case a bound diquark system in the antitriplet color state which serves as a localized color source. The light quark is orbiting around it and the resulting effective two-body system strongly resembles the heavy-light B and D mesons. The main distinction is that the quark-diquark interaction is not pointlike due to the heavy-diquark form factor [6]. The heavy-quark expansion in $1/m_Q$ can be used here, and the light quark is treated fully relativistically.

The experience acquired in the investigation of doubly-heavy baryons and the recent success in the relativistic description of light mesons [7] made it possible to study the baryons with one heavy quark (b or c), too. In this case we assume that the heavy-quark–light-diquark configuration dominates. Thus the three-body problem is again reduced to the two-body one. A crucial assumption of this quark-diquark picture consists in neglecting the influence of the third quark on the internal diquark dynamics. In the model with the harmonic-oscillator pair interactions, for instance, this effect increases the interaction strength by a factor $3/2$, and thus a significant increase by a factor $\sqrt{3/2}$ of the internal excitation energies of the diquark is achieved. It is hoped that for linear confinement and relativistic kinematics this effect would be modified and become smaller, but the answer could come only from a thorough treatment of the relativistic three-body problem and comparison with the quark-diquark approximation. In the considered version of the quark-diquark picture such influence could be partially reproduced by the account of the heavy-light diquark. We assume that such contributions are small and thus the heavy quark influence on the diquark dynamics is also small. Such assumption is necessary to preserve the presumed universal nature of the diquark [4]. Otherwise the diquark properties would be very different in such hadronic systems as baryons, tetraquarks, pentaquarks, etc. Unfortunately, the $1/m_Q$ expansion cannot be reliably applied for the c quark since its mass proves to be comparable with the mass of the light diquark. So we use instead the v/c expansion for the heavy quark and a completely relativistic description for the light quarks. Fortunately, our predictions can be compared with the rather large amount of experimental data for the ground-state baryons with one heavy quark (mainly the c).

II. RELATIVISTIC QUARK MODEL

In the quasipotential approach and quark-diquark picture of heavy baryons the interaction of two light quarks in a diquark and the heavy-quark interaction with a light diquark in a baryon are described by the diquark wave function (Ψ_d) of the bound quark-quark state and by the baryon wave function (Ψ_B) of the bound quark-diquark state, respectively, which satisfy the quasipotential equation [8] of the Schrödinger type [9]

$$\left(\frac{b^2(M)}{2\mu_R} - \frac{\mathbf{p}^2}{2\mu_R}\right)\Psi_{d,B}(\mathbf{p}) = \int \frac{d^3q}{(2\pi)^3} V(\mathbf{p}, \mathbf{q}; M)\Psi_{d,B}(\mathbf{q}), \quad (1)$$

where the relativistic reduced mass is

$$\mu_R = \frac{E_1E_2}{E_1 + E_2} = \frac{M^4 - (m_1^2 - m_2^2)^2}{4M^3}, \quad (2)$$

and E_1, E_2 are given by

$$E_1 = \frac{M^2 - m_2^2 + m_1^2}{2M}, \qquad E_2 = \frac{M^2 - m_1^2 + m_2^2}{2M}, \quad (3)$$

here $M = E_1 + E_2$ is the bound state mass (diquark or baryon), $m_{1,2}$ are the masses of light quarks (q_1 and q_2) which form the diquark or of the light diquark (d) and heavy quark (Q) which form the heavy baryon (B), and $\mathbf{p}$ is their relative momentum. In the center of mass system the relative momentum squared on mass shell reads

$$b^2(M) = \frac{[M^2 - (m_1 + m_2)^2][M^2 - (m_1 - m_2)^2]}{4M^2}. \quad (4)$$

The kernel $V(\mathbf{p}, \mathbf{q}; M)$ in Eq. (1) is the quasipotential operator of the quark-quark or quark-diquark interaction. It is constructed with the help of the off-mass-shell scattering amplitude, projected onto the positive energy states. In the following analysis we closely follow the similar construction of the quark-antiquark interaction in mesons which were extensively studied in our relativistic quark model [10]. For the quark-quark interaction in a diquark we use the relation $V_{qq} = V_{q\bar{q}}/2$ arising under the assumption about the octet structure of the interaction from the difference of the qq and $q\bar{q}$ color states. An important role in this construction is played by the Lorentz-structure of the confining interaction. In our analysis of mesons while constructing the quasipotential of the quark-antiquark interaction, we adopted that the effective interaction is the sum of the usual one-gluon exchange term with the mixture of long-range vector and scalar linear confining potentials, where the vector confining potential contains the Pauli terms. We use the same conventions for the construction of the quark-quark and quark-diquark interactions in the baryon. The quasipotential is then defined by [6,10]

(a) for the quark-quark (qq) interaction

$$V(\mathbf{p}, \mathbf{q}; M) = \bar{u}_1(p)\bar{u}_2(-p)\mathcal{V}(\mathbf{p}, \mathbf{q}; M)u_1(q)u_2(-q), \quad (5)$$

with

$$\mathcal{V}(\mathbf{p}, \mathbf{q}; M) = \frac{1}{2}\Big[\frac{4}{3}\alpha_s D_{\mu\nu}(\mathbf{k})\gamma_1^\mu\gamma_2^\nu + V^V_{\rm conf}(\mathbf{k})\Gamma_1^\mu(\mathbf{k})\Gamma_{2;\mu}(-\mathbf{k}) + V^S_{\rm conf}(\mathbf{k})\Big],$$

(b) for quark-diquark (Qd) interaction

$$\begin{aligned} V(\mathbf{p}, \mathbf{q}; M) = {} & \frac{\langle d(P)|J_\mu|d(Q)\rangle}{2\sqrt{E_d(p)E_d(q)}}\bar{u}_Q(p)\frac{4}{3}\alpha_S D_{\mu\nu}(\mathbf{k})\gamma^\nu u_Q(q) \\ & + \psi_d^*(P)\bar{u}_Q(p)J_{d;\mu}\Gamma_Q^\mu(\mathbf{k})V^V_{\rm conf}(\mathbf{k})u_Q(q)\psi_d(Q) \\ & + \psi_d^*(P)\bar{u}_Q(p)V^S_{\rm conf}(\mathbf{k})u_Q(q)\psi_d(Q), \end{aligned} \quad (6)$$

where α_s is the QCD coupling constant, $\langle d(P)|J_\mu|d(Q)\rangle$ is the vertex of the diquark-gluon interaction which is discussed in detail below [$P = (E_d, -\mathbf{p})$ and $Q = (E_d, -\mathbf{q})$, $E_d = (M^2 - m_Q^2 + M_d^2)/(2M)$]. $D_{\mu\nu}$ is the gluon propagator in the Coulomb gauge

$$D^{00}(\mathbf{k}) = -\frac{4\pi}{\mathbf{k}^2}, \qquad D^{ij}(\mathbf{k}) = -\frac{4\pi}{k^2}\Big(\delta^{ij} - \frac{k^ik^j}{\mathbf{k}^2}\Big), \qquad D^{0i} = D^{i0} = 0, \quad (7)$$

and $\mathbf{k} = \mathbf{p} - \mathbf{q}$; γ_μ and $u(p)$ are the Dirac matrices and spinors

$$u^\lambda(p) = \sqrt{\frac{\epsilon(p) + m}{2\epsilon(p)}}\begin{pmatrix} 1 \\ \frac{\boldsymbol{\sigma}\mathbf{p}}{\epsilon(p)+m}\end{pmatrix}\chi^\lambda, \quad (8)$$

with $\epsilon(p) = \sqrt{\mathbf{p}^2 + m^2}$.

The diquark state in the confining part of the quark-diquark quasipotential (6) is described by the wave functions

$$\psi_d(p) = \begin{cases} 1 & \text{for scalar diquark} \\ \varepsilon_d(p) & \text{for axial vector diquark} \end{cases}, \quad (9)$$

where the four vector

$$\varepsilon_d(p) = \Big(\frac{(\boldsymbol{\varepsilon}_d\mathbf{p})}{M_d}, \boldsymbol{\varepsilon}_d + \frac{(\boldsymbol{\varepsilon}_d\mathbf{p})\mathbf{p}}{M_d(E_d(p) + M_d)}\Big), \qquad \varepsilon_d^\mu(p)p_\mu = 0, \quad (10)$$

is the polarization vector of the axial vector diquark with momentum $\mathbf{p}$, $E_d(p) = \sqrt{\mathbf{p}^2 + M_d^2}$ and $\varepsilon_d(0) = (0, \boldsymbol{\varepsilon}_d)$ is the polarization vector in the diquark rest frame. The effective long-range vector vertex of the diquark can be presented in the form

$$J_{d;\mu} = \begin{cases} \frac{(P+Q)_\mu}{2\sqrt{E_d(p)E_d(q)}} & \text{for scalar diquark} \\ \frac{(P+Q)_\mu}{2\sqrt{E_d(p)E_d(q)}} - \frac{i\mu_d}{2M_d}\Sigma_\mu^\nu\tilde{k}_\nu & \text{for axial vector diquark} \end{cases}, \quad (11)$$

where $\tilde{k} = (0, \mathbf{k})$. Here the antisymmetric tensor

$$(\Sigma_{\rho\sigma})_\mu^\nu = -i(g_{\mu\rho}\delta_\sigma^\nu - g_{\mu\sigma}\delta_\rho^\nu) \quad (12)$$

and the axial vector diquark spin $\mathbf{S}_d$ is given by $(S_{d;k})_{il} = -i\varepsilon_{kil}$. We choose the total chromomagnetic moment of the axial vector diquark $\mu_d = 2$ [6].

The effective long-range vector vertex of the quark is defined by [10,11]

$$\Gamma_\mu(\mathbf{k}) = \gamma_\mu + \frac{i\kappa}{2m}\sigma_{\mu\nu}\tilde{k}^\nu, \qquad \tilde{k} = (0, \mathbf{k}), \quad (13)$$

where κ is the Pauli interaction constant characterizing the anomalous chromomagnetic moment of quarks. In the configuration space the vector and scalar confining potentials in the nonrelativistic limit reduce to

$$V^V_{\rm conf}(r) = (1 - \varepsilon)V_{\rm conf}(r), \qquad V^S_{\rm conf}(r) = \varepsilon V_{\rm conf}(r), \quad (14)$$

with

$$V_{\rm conf}(r) = V^S_{\rm conf}(r) + V^V_{\rm conf}(r) = Ar + B, \quad (15)$$

where ε is the mixing coefficient.

The constituent quark masses $m_b = 4.88$ GeV, $m_c = 1.55$ GeV, $m_u = m_d = 0.33$ GeV, $m_s = 0.5$ GeV and the parameters of the linear potential $A = 0.18$ GeV^2 and $B = -0.3$ GeV have the usual values of quark models. The value of the mixing coefficient of vector and scalar confining potentials $\varepsilon = -1$ has been determined from the consideration of charmonium radiative decays [12] and the heavy-quark expansion [13]. Finally, the universal Pauli interaction constant $\kappa = -1$ has been fixed from the analysis of the fine splitting of heavy quarkonia ${}^3P_J$ states [12]. In the literature it is widely discussed the 't Hooft-like interaction between quarks induced by instantons [14]. This interaction can be partly described by introducing the quark anomalous chromomagnetic moment having an approximate value $\kappa = -0.744$ (Diakonov [14]). This value is of the same sign and order of magnitude as the Pauli constant $\kappa = -1$ in our model. Thus the Pauli term incorporates at least some part of the instanton contribution to the $q\bar{q}$ interaction. Note that the long-range chromomagnetic contribution to the potential in our model is proportional to $(1 + \kappa)$ and thus vanishes for the chosen value of $\kappa = -1$.

III. PROPERTIES OF LIGHT DIQUARKS

At a first step, we calculate the masses and form factors of the light diquark. As it is well known, the light quarks are highly relativistic, which makes the v/c expansion inapplicable and thus, a completely relativistic treatment is required. To achieve this goal in describing light diquarks, we closely follow our recent consideration of the spectra of light mesons [7] and adopt the same procedure to make the relativistic quark potential local by replacing $\epsilon_{1,2}(p) \equiv \sqrt{m^2_{1,2} + \mathbf{p}^2} \to E_{1,2}$ (see discussion in Ref. [7]). As a result, the light quark-quark interaction (5) in the diquark state, which is 1/2 of the $q\bar{q}$ interaction in light mesons, consists of the sum of the spin-independent and spin-dependent parts [7]

$$V(r) = V_{\rm SI}(r) + V_{\rm SD}(r), \quad (16)$$

where the spin-independent potential for the S-wave states ($\mathbf{L}^2 = 0$) has the form

$$V_{\rm SI}(r) = \frac{1}{2}\Bigg[V_{\rm Coul}(r) + V_{\rm conf}(r) + \frac{(E_1^2 - m_1^2 + E_2^2 - m_2^2)^2}{4(E_1 + m_1)(E_2 + m_2)}\Bigg\{\frac{1}{E_1E_2}V_{\rm Coul}(r) + \frac{1}{m_1m_2}\Bigg(1 + (1+\kappa)\Bigg[(1+\kappa) \times \frac{(E_1+m_1)(E_2+m_2)}{E_1E_2} - \Bigg(\frac{E_1+m_1}{E_1} + \frac{E_1+m_2}{E_2}\Bigg)\Bigg]\Bigg)V^V_{\rm conf}(r) + \frac{1}{m_1m_2}V^S_{\rm conf}(r)\Bigg\} + \frac{1}{4}\Bigg(\frac{1}{E_1(E_1+m_1)}\Delta\tilde{V}^{(1)}_{\rm Coul}(r) + \frac{1}{E_2(E_2+m_2)}\Delta\tilde{V}^{(2)}_{\rm Coul}(r)\Bigg) - \frac{1}{4}\Bigg[\frac{1}{m_1(E_1+m_1)} + \frac{1}{m_2(E_2+m_2)} - (1+\kappa)\Bigg(\frac{1}{E_1m_1} + \frac{1}{E_2m_2}\Bigg)\Bigg]\Delta V^V_{\rm conf}(r) + \frac{(E_1^2 - m_1^2 + E_2^2 - m_2^2)}{8m_1m_2(E_1+m_1)(E_2+m_2)}\Delta V^S_{\rm conf}(r)\Bigg], \quad (17)$$

and the spin-dependent potential is given by

$$V_{\rm SD}(r) = \frac{1}{3E_1E_2}\Bigg[\Delta\bar{V}_{\rm Coul}(r) + \Bigg(\frac{E_1-m_1}{2m_1} - (1+\kappa)\frac{E_1+m_1}{2m_1}\Bigg)\Bigg(\frac{E_2-m_2}{2m_2} - (1+\kappa)\frac{E_2+m_2}{2m_2}\Bigg)\Delta V^V_{\rm conf}(r)\Bigg]\mathbf{S}_1\mathbf{S}_2, \quad (18)$$

with [7,10]

$$V_{\rm Coul}(r) = -\frac{4}{3}\frac{\alpha_s}{r}, \qquad \tilde{V}^{(i)}_{\rm Coul}(r) = V_{\rm Coul}(r)\frac{1}{(1 + \eta_i\frac{4}{3}\frac{\alpha_s}{E_i}\frac{1}{r})(1 + \eta_i\frac{4}{3}\frac{\alpha_s}{E_i+m_i}\frac{1}{r})}, \quad (i = 1, 2),$$
$$\bar{V}_{\rm Coul}(r) = V_{\rm Coul}(r)\frac{1}{(1 + \eta_1\frac{4}{3}\frac{\alpha_s}{E_1}\frac{1}{r})(1 + \eta_2\frac{4}{3}\frac{\alpha_s}{E_2}\frac{1}{r})}, \qquad \eta_{1,2} = \frac{m_{2,1}}{m_1+m_2}. \quad (19)$$

Here we put $\alpha_s \equiv \alpha_s(\mu^2_{12})$ with $\mu_{12} = 2m_1m_2/(m_1+m_2)$ and use for $\alpha_s(\mu^2)$ the simplest model with freezing [15]

$$\alpha_s(\mu^2) = \frac{4\pi}{\beta_0\ln\frac{\mu^2+M_0^2}{\Lambda^2}}, \qquad \beta_0 = 11 - \frac{2}{3}n_f, \quad (20)$$

where the background mass is $M_0 = 2.24\sqrt{A} = 0.95$ GeV [15] and $\Lambda = 413$ MeV was fixed from fitting the ρ mass. We put the number of flavors $n_f = 2$ for ud, us diquarks and $n_f = 3$ for ss diquark, cf. [7] . As a result we obtain $\alpha_s(\mu^2_{ud}) = 0.730$, $\alpha_s(\mu^2_{us}) = 0.711$ and $\alpha_s(\mu^2_{ss}) = 0.731$.

The quasipotential Eq. (1) is solved numerically for the complete relativistic potential (16) which depends on the diquark mass in a complicated highly nonlinear way. The obtained ground-state masses of scalar and axial vector light diquarks are presented in Table I. These masses are in good agreement with values found in Ref. [16] within the Nambu–Jona-Lasinio model. It follows from Table I that

TABLE I. Masses of light ground state diquarks (in MeV). S and A denote scalar and axial vector diquarks antisymmetric $[q, q']$ and symmetric $\{q, q'\}$ in flavor, respectively.

Quark content	Diquark type	Mass this work	Mass Ref. [16][a]
$[u, d]$	S	710	705
$\{u, d\}$	A	909	875
$[u, s]$	S	948	895
$\{u, s\}$	A	1069	1050
$\{s, s\}$	A	1203	1215

[a]For $G_1/G_{\rm meson} = 1.1$

the mass difference between the scalar and vector diquark decreases from ~200 to ~120 MeV, when one of the u, d quarks is replaced by the s quark in accord with the statement of Ref. [4].

In order to determine the diquark interaction with the gluon field, which takes into account the diquark structure, it is necessary to calculate the corresponding matrix element of the quark current between diquark states. This diagonal matrix element can be parametrized by the following set of elastic form factors.

(a) scalar diquark (S)

$$\langle S(P)|J_\mu|S(Q)\rangle = h_+(k^2)(P+Q)_\mu, \qquad (21)$$

(b) axial vector diquark (A)

$$\begin{aligned}\langle A(P)|J_\mu|A(Q)\rangle = &-[\varepsilon_d^*(P)\cdot\varepsilon_d(Q)]h_1(k^2)(P+Q)_\mu \\ &+ h_2(k^2)\{[\varepsilon_d^*(P)\cdot Q]\varepsilon_{d;\mu}(Q) \\ &+ [\varepsilon_d(Q)\cdot P]\varepsilon^*_{d;\mu}(P)\} + h_3(k^2) \\ &\times \frac{1}{M_A^2}[\varepsilon_d^*(P)\cdot Q] \\ &\times [\varepsilon_d(Q)\cdot P](P+Q)_\mu, \end{aligned} \qquad (22)$$

where $k = P - Q$ and $\varepsilon_d(P)$ is the polarization vector of the axial vector diquark (10).

In the quasipotential approach, the matrix element of the quark current $J_\mu = \bar{q}\gamma^\mu q$ between the diquark states (d) has the form [17]

$$\langle d(P)|J_\mu(0)|d(Q)\rangle = \int \frac{d^3p\, d^3q}{(2\pi)^6}\bar{\Psi}_P^d(\mathbf{p})\Gamma_\mu(\mathbf{p},\mathbf{q})\Psi_Q^d(\mathbf{q}), \qquad (23)$$

where $\Gamma_\mu(\mathbf{p},\mathbf{q})$ is the two-particle vertex function and Ψ_P^d are the diquark wave functions projected onto the positive energy states of quarks and boosted to the moving reference frame with momentum P. In the impulse approximation the vertex function Γ is shown in Fig. 1. The corresponding vertex function is given by

$$\Gamma_\mu^{(1)}(\mathbf{p},\mathbf{q}) = \bar{u}_{q_1}(p_1)\gamma^\mu u_{q_1}(q_1)(2\pi)^3\delta(\mathbf{p}_2-\mathbf{q}_2) + (1\leftrightarrow 2), \qquad (24)$$

where [17]

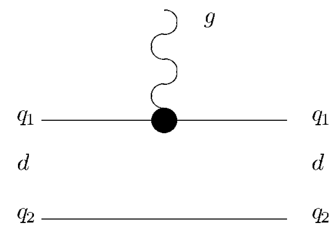

FIG. 1. The vertex function Γ in the impulse approximation. The gluon interaction only with one light quark is shown.

$$\begin{aligned} p_{1,2} &= \epsilon_{1,2}(p)\frac{P}{\mathcal{M}_d} \pm \sum_{i=1}^3 n^{(i)}(P)p^i, \\ \mathcal{M}_d &= \epsilon_1(p)+\epsilon_2(p), \\ q_{1,2} &= \epsilon_{1,2}(q)\frac{Q}{\mathcal{M}'_d} \pm \sum_{i=1}^3 n^{(i)}(Q)q^i, \\ \mathcal{M}'_d &= \epsilon_1(q)+\epsilon_2(q), \end{aligned}$$

and $n^{(i)}$ are three four vectors defined by

$$n^{(i)\mu}(P) = \left\{\frac{P^i}{\mathcal{M}_d}, \delta_{ij} + \frac{P^iP^j}{\mathcal{M}_d[E_d(P)+\mathcal{M}_d]}\right\},$$
$$E_d(P) = \sqrt{\mathbf{P}^2 + \mathcal{M}_d^2}.$$

After making necessary computations, the expression for Γ should be continued in $\mathcal{M}_d$ and $\mathcal{M}'_d$ to the diquark mass M_d.

Substituting the vertex function $\Gamma^{(1)}$ given by Eq. (24) in the matrix element (23) and comparing the resulting expressions with the form factor decompositions (21) and (22), we find

$$h_+(k^2) = h_1(k^2) = h_2(k^2) = F(\mathbf{k}^2), \qquad h_3(k^2) = 0,$$

$$\begin{aligned} F(\mathbf{k}^2) = &\frac{\sqrt{E_dM_d}}{E_d+M_d}\int\frac{d^3p}{(2\pi)^3}\bar{\Psi}_d\left(\mathbf{p} + \frac{2\epsilon_2(p)}{E_d+M_d}\mathbf{k}\right) \\ &\times\sqrt{\frac{\epsilon_1(p)+m_1}{\epsilon_1(p+k)+m_1}}\left[\frac{\epsilon_1(p+k)+\epsilon_1(p)}{2\sqrt{\epsilon_1(p+k)\epsilon_1(p)}}\right. \\ &\left.+\frac{\mathbf{pk}}{2\sqrt{\epsilon_1(p+k)\epsilon_1(p)}(\epsilon_1(p)+m_1)}\right] \\ &\times\Psi_d(\mathbf{p}) + (1\leftrightarrow 2), \end{aligned} \qquad (25)$$

where Ψ_d are the diquark wave functions. We calculated the corresponding form factors $F(r)/r$ which are the Fourier transforms of $F(\mathbf{k}^2)/\mathbf{k}^2$ using the diquark wave functions found by numerical solving the quasipotential equation. In Fig. 2 the functions $F(r)$ for the scalar $[u, d]$ and axial vector $\{u, d\}$ diquarks are shown as an example. Our estimates show that this form factor can be approximated with a high accuracy by the expression

$$F(r) = 1 - e^{-\xi r - \zeta r^2}, \qquad (26)$$

which agrees with previously used approximations [6]. The values of parameters ξ and ζ for light diquark scalar $[q, q']$

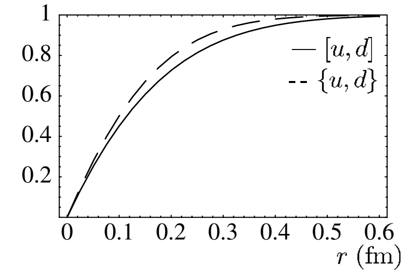

FIG. 2. The form factors $F(r)$ for the scalar $[u, d]$ (solid line) and axial vector $\{u, d\}$ (dashed line) diquarks.

TABLE II. Parameters ξ and ζ for ground state light diquarks.

Quark content	Diquark type	ξ (GeV)	ζ (GeV^2)
$[u, d]$	S	1.09	0.185
$\{u, d\}$	A	1.185	0.365
$[u, s]$	S	1.23	0.225
$\{u, s\}$	A	1.15	0.325
$\{s, s\}$	A	1.13	0.280

and axial vector $\{q, q'\}$ ground states are given in Table II. As we see, the functions $F(r)$ vanish at $r = 0$ and tend to unity for large values of r. Such a behavior can be easily understood intuitively. At large distances a diquark can be well approximated by a pointlike object and its internal structure cannot be resolved. When the distance to the diquark decreases the internal structure plays a more important role. As the distance approaches zero, the interaction weakens and turns to zero for $r = 0$. Thus the function $F(r)$ gives an important contribution to the short-range part of the interaction of the heavy quark with the light diquark in the baryon and can be neglected for the long-range (confining) interaction.

IV. MASSES OF HEAVY BARYONS

At a second step, we calculate the masses of heavy baryons as the bound state of a heavy quark and light diquark. For the potential of the heavy-quark–light-diquark interaction (6) we use the expansion in p/m_Q. Since the light diquark is not heavy enough for the applicability of a p/m_d expansion, it should be treated fully relativistically. To achieve this goal and simplify the potential we follow the same procedure, which was used for light quarks in a diquark, and replace the diquark energies $E_d(p) \equiv \sqrt{\mathbf{p}^2 + M_d^2} \to E_d \equiv (M^2 - m_Q^2 + M_d^2)/(2M)$ in Eqs. (6) and (11). This substitution makes the Fourier transform of the potential (6) local. At leading order in p/m_Q the resulting potential for the S-wave states ($\mathbf{L}^2 = 0, \mathbf{LS} = 0$) is the same for scalar and axial vector diquarks and is given by

$$V^{(0)}(r) = \hat{V}_{\rm Coul}(r) + V_{\rm conf}(r),$$
$$\hat{V}_{\rm Coul}(r) = -\frac{4}{3}\alpha_s \frac{F(r)}{r}, \qquad V_{\rm conf}(r) = Ar + B, \tag{27}$$

where $\hat{V}_{\rm Coul}(r)$ is the smeared Coulomb potential (which accounts for the diquark structure) and α_s is given by Eq. (20) with $N_f = 3$. The masses of baryons with spin $1/2$ and $3/2$, containing the axial vector diquark, are degenerate in this approximation since the spin-spin interaction arises only at first order in p/m_Q. Solving Eq. (1) numerically we get the spin-independent part of the baryon wave function Ψ_B. Then the total baryon wave function is a product of Ψ_B and the spin-dependent part U_B (for details see Eq. (43) of Ref. [18]).

The leading order degeneracy of heavy baryon states is broken by p/m_Q corrections. The ground-state quark-diquark potential (6) up to the second order of the p/m_Q expansion is given by the following expressions:

(a) scalar diquark

$$\delta V(r) = \frac{1}{E_d m_Q}\left\{\mathbf{p}[\hat{V}_{\rm Coul}(r) + V^V_{\rm conf}(r)]\mathbf{p} - \frac{1}{4}\Delta V^V_{\rm conf}(r)\right\} + \frac{1}{m_Q^2}\left\{\frac{1}{8}\Delta(\hat{V}_{\rm Coul}(r) + V^S_{\rm conf}(r) - [1 - 2(1+\kappa)]V^V_{\rm conf}(r)) - \frac{1}{2}\mathbf{p}V^S_{\rm conf}(r)\mathbf{p}\right\}, \tag{28}$$

(b) axial vector diquark

$$\delta V(r) = \frac{1}{E_d m_Q}\left\{\mathbf{p}[\hat{V}_{\rm Coul}(r) + V^V_{\rm conf}(r)]\mathbf{p} - \frac{1}{4}\Delta V^V_{\rm conf}(r) + \frac{2}{3}[\Delta\hat{V}_{\rm Coul}(r) + (1+\kappa)\Delta V^V_{\rm conf}(r)]\mathbf{S}_d\mathbf{S}_Q\right\} + \frac{1}{m_Q^2}\left\{\frac{1}{8}\Delta(\hat{V}_{\rm Coul}(r) + V^S_{\rm conf}(r) - [1 - 2(1+\kappa)]V^V_{\rm conf}(r)) - \frac{1}{2}\mathbf{p}V^S_{\rm conf}(r)\mathbf{p}\right\}, \tag{29}$$

where $\mathbf{S}_d$ and $\mathbf{S}_Q$ are the light diquark and heavy quark spins, respectively. It is necessary to note that the confining vector interaction gives a contribution to the spin-dependent part which is proportional to $(1 + \kappa)$. Thus it vanishes for the chosen value of $\kappa = -1$, while the confining vector contribution to the spin-independent part is nonzero.

Now we can calculate the mass spectra of heavy baryons with the account of all corrections of order p^2/m_Q^2. For this purpose we consider Eq. (1) with the quasipotential which is the sum of the leading order potential $V^{(0)}(r)$ (27) and the correction $\delta V(r)$ (28) and (29). We multiply this equation from the left by the quasipotential wave function of a bound state and integrate both sides over the relative momentum. Within the adopted accuracy of calculations, we can use for the resulting matrix elements the wave functions of Eq. (1) with the leading order potential

TABLE III. Masses of the ground state heavy baryons (in MeV).

Baryon	$I(J^P)$	Theory						Experiment
		this work	Ref. [19]	Ref. [20]	Ref. [21]	Ref. [22]	Ref. [23][a]	PDG [24]
Λ_c	$0(\frac{1}{2}^+)$	2297	2265	2285			2290	2284.9(6)
Σ_c	$1(\frac{1}{2}^+)$	2439	2440	2453			2452	2451.3(7)
Σ_c^*	$1(\frac{3}{2}^+)$	2518	2495	2520	2518		2538	2515.9(2.4)
Ξ_c	$\frac{1}{2}(\frac{1}{2}^+)$	2481		2468			2473	2466.3(1.4)
Ξ_c'	$\frac{1}{2}(\frac{1}{2}^+)$	2578		2580	2579	2580.8(2.1)	2599	2574.1(3.3)
Ξ_c^*	$\frac{1}{2}(\frac{3}{2}^+)$	2654		2650			2680	2647.4(2.0)
Ω_c	$0(\frac{1}{2}^+)$	2698		2710			2678	2697.5(2.6)
Ω_c^*	$0(\frac{3}{2}^+)$	2768		2770	2768	2760.5(4.9)	2752	
Λ_b	$0(\frac{1}{2}^+)$	5622	5585	5620			5672	5624(9)
Σ_b	$1(\frac{1}{2})+$	5805	5795	5820		5824.2(9.0)	5847	
Σ_b^*	$1(\frac{3}{2}^+)$	5834	5805	5850		5840.0(8.8)	5871	
Ξ_b	$\frac{1}{2}(\frac{1}{2}^+)$	5812		5810		5805.7(8.1)	5788	
Ξ_b'	$\frac{1}{2}(\frac{1}{2}^+)$	5937		5950		5950.9(8.5)	5936	
Ξ_b^*	$\frac{1}{2}(\frac{3}{2}^+)$	5963		5980		5966.1(8.3)	5959	
Ω_b	$0(\frac{1}{2}^+)$	6065		6060		6068.7(11.1)	6040	
Ω_b^*	$0(\frac{3}{2}^+)$	6088		6090		6083.2(11.0)	6060	

[a]Error estimates are about 50 MeV for charmed baryons and 100 MeV for bottom baryons.

$V^{(0)}(r)$. In this way we obtain the mass formula

$$\frac{b^2(M)}{2\mu_R} = \frac{\langle \mathbf{p}^2\rangle}{2\mu_R} + \langle V^{(0)}(r)\rangle + \langle \delta V(r)\rangle. \tag{30}$$

The contribution of the spin-spin interaction in (29) is proportional to

$$\langle \mathbf{S}_d \mathbf{S}_Q\rangle = \frac{1}{2}\left[J(J+1) - S_d(S_d+1) - \frac{3}{4}\right], \tag{31}$$

where $\mathbf{J} = \mathbf{S}_d + \mathbf{S}_Q$ is the spin of the ground state heavy baryon.[1]

The calculated values of the baryon masses are given in Table III in comparison with some theoretical predictions [19–23] and experimental data [24].

In Ref. [19] the baryon masses are calculated in the framework of a relativized quark model, applying a variational approach to obtain the mass eigenvalues and bound state wave functions by using a harmonic-oscillator basis. In Ref. [20] the Feynman-Hellman theorem and semiempirical mass formulas are used to predict the masses of heavy baryons. The heavy-quark symmetry ($1/m_Q$ expansion) and $SU(3)$ flavor symmetry are applied in Refs. [21,22,25] to evaluate the masses of baryons with a single heavy quark. At lowest order in $SU(3)$ breaking these masses obey an equal-spacing rule:

$$\begin{aligned} J &= \frac{1}{2}, \qquad M_{\Sigma_Q} + M_{\Omega_Q} = 2M_{\Xi_Q'}, \\ J &= \frac{3}{2}, \qquad M_{\Sigma_Q^*} + M_{\Omega_Q^*} = 2M_{\Xi_Q^*}, \qquad Q = b, c. \end{aligned} \tag{32}$$

[1]It should be mentioned that $Q[sq] \leftrightarrow Q\{sq\}$ mixing can exist in conventional constituent quark models with three quarks, but it is absent for ground states in our approach.

The corrections to this rule, estimated on the basis of chiral perturbation theory (light meson loops) combined with heavy-quark symmetry, are found to be small [21]. The equal-spacing rule holds also for the hyperfine mass splittings [21]:

$$\begin{gathered} \delta_{\Sigma_Q} + \delta_{\Omega_Q} = 2\delta_{\Xi_Q}, \qquad Q = b, c; \\ \delta_{\Sigma_Q} = M_{\Sigma_Q^*} - M_{\Sigma_Q}; \qquad \delta_{\Xi_Q} = M_{\Xi_Q^*} - M_{\Xi_Q'}; \\ \delta_{\Omega_Q} = M_{\Omega_Q^*} - M_{\Omega_Q}. \end{gathered} \tag{33}$$

This relation is expected [22] to be more accurate than the relation (32).

The hyperfine splitting calculation is used in [25] to estimate the masses $M_{\Sigma_c^*} = 2514$ MeV and $M_{\Omega_c^*} = 2771$ MeV. The heavy-quark expansion and broken $SU(3)$ symmetry are combined in Ref. [22] with the $1/N_c$ expansion. As a result, some new mass relations are obtained, which allowed to predict accurately the heavy baryon masses. The accuracy of the mass relation (33) is estimated there to be of order 1 MeV for $Q = c$ and 0.3 MeV for $Q = b$.

In Ref. [23] the masses of charmed and bottom baryons are computed within quenched lattice nonrelativistic QCD (NRQCQ). The masses of baryons with b quark are also calculated in lattice NRQCD in Ref. [26]. The error bars of lattice calculations are usually of order 50–100 MeV at present.

From Tables IV and V it is evident that the values of baryon masses obtained in the present paper (see Table III) satisfy rather well both the mass relations (32) and (33). Note that these masses satisfy mass inequality $2M_{\Xi_Q} \geq M_{\Sigma_Q} + M_{\Omega_Q}$ found from analysis of the spectral properties of the Hamiltonians in Refs. [27]. This gives a strong

TABLE IV. Test of validity of the equal-spacing rule (32) for heavy baryon masses obtained in this paper (in MeV).

	$J=\frac{1}{2}$		$J=\frac{3}{2}$	
	$Q=c$	$Q=b$	$Q=c$	$Q=b$
$M_{\Sigma_Q}+M_{\Omega_Q}$	5137	11870	5286	11922
$2M_{\Xi_Q}$	5156	11874	5308	11926

TABLE V. Test of validity of the equal-spacing rule (33) for hyperfine mass splittings obtained in this paper (in MeV).

	$Q=c$	$Q=b$
$\delta_{\Sigma_Q}+\delta_{\Omega_Q}$	149	52
$2\delta_{\Xi_Q}$	152	52

additional support to our model, since it means that the model incorporates the important features of broken $SU(3)$ flavor symmetry and heavy-quark expansion of QCD (see also [13]) in a reasonable way.

V. CONCLUSIONS

It is important to emphasize that, in calculating the heavy baryon masses, we do not use any free adjustable parameters. Indeed, all parameters of the model (including quark masses and parameters of the quark potential) have fixed values which were determined from our previous considerations of heavy and light meson properties. Note that the light diquark in our approach is not considered as a pointlike object. Instead we use its wave functions to calculate diquark-gluon interaction form factors and, thus, take into account the finite (and relatively large) size of the light diquark. The other important advantage of our model is the completely relativistic treatment of the light quarks in the diquark and the light diquark in the heavy baryon. We use the v/c expansion only for heavy (b and c) quarks. The overall reasonable agreement of our model predictions given in Table III with both available experimental data and the results of significantly distinct theoretical approaches gives further grounds for the heavy-quark–light-diquark picture of heavy baryons.

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