

Decays of nonstrange positive parity excited baryons in the $1/N_c$ expansionJ. L. Goity^{1,2,*} and N. N. Scoccola^{3,4,5,†}¹*Department of Physics, Hampton University, Hampton, Virginia 23668, USA*²*Thomas Jefferson National Accelerator Facility, Newport News, Virginia 23606, USA*³*Physics Department, Comisión Nacional de Energía Atómica, (1429) Buenos Aires, Argentina*⁴*CONICET, Rivadavia 1917, (1033) Buenos Aires, Argentina*⁵*Universidad Favaloro, Solís 453, (1078) Buenos Aires, Argentina*

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The decays of nonstrange positive parity excited baryons via emission of a pseudoscalar meson are studied in the framework of the $1/N_c$ expansion to order $1/N_c$. In particular, the pionic decays of the $\ell = 0$ Roper baryons and of the $\ell = 2$ baryons in the mass interval 1680–1950 MeV are analyzed using the available partial decay widths. Decay widths by emission of an η meson are shown to be suppressed by a factor $1/N_c^2$ with respect to the pionic ones.

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I. INTRODUCTION

The $1/N_c$ expansion has been applied successfully to excited baryons as several studies of the baryon spectrum [1–7] and decays [2,5,8,9] have shown. The analysis of baryon masses has shown that there is an approximate $O(3) \times SU(2N_f)$ symmetry that serves to frame the $1/N_c$ approach to excited baryons. Although this is not a symmetry that holds strictly in the large N_c limit, its zeroth order in $1/N_c$ breaking turns out to be small. This facilitates the analysis of excited baryons in general, as one can work with a basis of states provided by the multiplets of $O(3) \times SU(2N_f)$.

The strong decays represent one of the most important problems where the $1/N_c$ expansion can be applied. Unfortunately the experimental information on partial widths is neither precise nor complete, and for these reasons the conclusions that can be drawn from the $1/N_c$ analyses are somewhat limited. The mesonic decays that have been so far addressed are those that proceed via the emission of a single pseudoscalar meson. The first complete analysis at $\mathcal{O}(1/N_c)$ was carried out for the decays of the nonstrange negative-parity baryons [9]. From that analysis, the main conclusion drawn is that the dominant decay mechanism is provided by the 1-body operator where a pion or an η meson couples to the axial current as in the chiral quark model. The deviations from that simple picture are due to a number of operators whose effective couplings are determined rather poorly because of the mentioned poor quality of the partial width inputs.

The framework used in the present work is similar to the one already developed for the negative-parity baryon decays [9], which is based on an operator analysis. A different approach has been recently discussed where the $1/N_c$ expansion is applied at the level of scattering amplitudes, and the baryonic resonances determined from the analytic

structure of these amplitudes [10]. In the present work the analysis is extended to the known nonstrange positive parity excited states that fit into the symmetric representation of $SU(4)$. As pointed out later, the decays via emission of an η meson are subleading in $1/N_c$ giving partial widths $\mathcal{O}(1/N_c^2)$. Consistent with this, there is virtually no experimental input available for a meaningful analysis of the η channels. Thus, only the decays via emission of a single pion are explicitly considered. The general motivation for the present analysis is to compare the results with those obtained in the negative-parity baryons in order to find out common features that may underlay the dynamics of the decays in general.

This paper is organized as follows: Sec. II contains the framework for the operator analysis of the decays, Sec. III presents the numerical results and their analysis, and finally the conclusions are given in Sec. IV.

II. OPERATOR ANALYSIS FOR DECAYS

In the present application of the $1/N_c$ expansion to excited baryons we assume that they can be classified in multiplets of the $O(3) \times SU(2N_f)$ group. $O(3)$ corresponds to spatial rotations and $SU(2N_f)$ is the spin-flavor group, where in this work the number of flavors is $N_f = 2$. The ground state baryons, namely, the N and Δ states, belong to the $(\mathbf{1}, \mathbf{20}_S)$ representation, where the $\mathbf{20}_S$ is the totally symmetric representation of $SU(4)$. The positive parity baryons considered here belong instead to the $(\mathbf{2\ell} + \mathbf{1}, \mathbf{20}_S)$ representation where $\ell = 0, 2, 4$. For general N_c , the excited baryon states are obtained by coupling the orbital angular momentum state carrying angular momentum ℓ with the spin-flavor symmetric states according to

$$|J, J_3; I, I_3; S\rangle_{\text{exc}} = \sum_m \langle \ell, m; S, J_3 - m | J, J_3 \rangle | \ell, m \rangle \times | S, J_3 - m; I = S, I_3 \rangle_S. \quad (1)$$

The states for $N_c = 3$ are displayed in Table I along with

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TABLE I. Positive parity nonstrange baryons and their decay widths and branching ratios from the Particle Data Group [13]. Channels not explicitly indicated are forbidden. Question marks imply that the empirical value is not known. Channels for which only one decay is allowed are not considered.

B_J^ℓ	State	Mass [MeV]	Total width [MeV]		Branching ratios [%]		
					P-wave	F-wave	H-wave
$N_{1/2}^0$	$N(1440)$	1450 ± 20	350 ± 100	πN	65 ± 5		
				$\pi \Delta$	25 ± 5		
$\Delta_{3/2}^0$	$\Delta(1600)$	1625 ± 75	350 ± 100	πN	17.5 ± 7.5	Not considered	
				$\pi \Delta$	51.5 ± 24		
$N_{3/2}^2$	$N(1720)$	1700 ± 50	150 ± 50	πN	15.5 ± 5		
				$\pi \Delta$?	?	
$N_{5/2}^2$	$N(1680)$	1683 ± 8	130 ± 10	πN		65 ± 5	
				$\pi \Delta$	10 ± 4	1 ± 1	
$\Delta_{1/2}^2$	$\Delta(1910)$	1895 ± 25	230 ± 40	πN	22.5 ± 7.5		
				$\pi \Delta$?		
$\Delta_{3/2}^2$	$\Delta(1920)$	1935 ± 35	225 ± 75	πN	12.5 ± 7.5		
				$\pi \Delta$?	?	
$\Delta_{5/2}^2$	$\Delta(1905)$	1895 ± 25	360 ± 80	πN		10 ± 5	
				$\pi \Delta$?	?	
$\Delta_{7/2}^2$	$\Delta(1950)$	1950 ± 10	320 ± 30	πN		37.5 ± 2.5	
				$\pi \Delta$		24 ± 4	Not considered
$N_{7/2}^4$	$N(?)$?	?	πN		?	
				$\pi \Delta$?
$N_{9/2}^4$	$N(2220)$	2245 ± 65	450 ± 100	πN			15 ± 5
				$\pi \Delta$?	?
$\Delta_{5/2}^4$	$\Delta(?)$?	?	πN		?	?
				$\pi \Delta$?	?
$\Delta_{7/2}^4$	$\Delta(2390)^*$	$2390 \pm ?$?	πN		?	?
				$\pi \Delta$?	?
$\Delta_{9/2}^4$	$\Delta(2300)^{**}$	$2300 \pm ?$?	πN		?	?
				$\pi \Delta$?	?
	$\Delta(2420)$	2400 ± 100	400 ± 100	πN			10 ± 5
				$\pi \Delta$?

their quantum numbers, masses, decay widths and branching ratios.

The ℓ_P partial wave decay width into a GS baryons and a pseudoscalar meson with isospin I_P is given by

$$\Gamma^{[\ell_P, I_P]} = \frac{k_P}{8\pi^2} \frac{M_{B^*}}{M_B} \frac{|B(\ell_P, I_P, S, I, J^*, I^*, S^*)|^2}{\sqrt{(2J^* + 1)(2I^* + 1)}}, \quad (2)$$

where $B(\ell_P, I_P, S, I, J^*, I^*, S^*)$ are the reduced matrix elements of the baryonic operator. Such operator admits an expansion in $1/N_c$ that has the general form [9]:

$$B_{[\mu, \alpha]}^{[\ell_P, I_P]} = \left(\frac{k_P}{\Lambda}\right)^{\ell_P} \sum_q C_q^{[\ell_P, I_P]}(k_P) (B_{[\mu, \alpha]}^{[\ell_P, I_P]})_q, \quad (3)$$

where

$$(B_{[\mu, \alpha]}^{[\ell_P, I_P]})_q = \sum_m \langle \ell, m; j, j_z | \ell_P, \mu \rangle \xi_m^\ell (\mathcal{G}_{[j, \alpha]}^{[j, I_P]})_q. \quad (4)$$

The factor $(\frac{k_P}{\Lambda})^{\ell_P}$ is included to take into account the chief meson momentum dependence of the partial wave. For

definiteness, in the following the scale Λ is taken to be equal to 200 MeV. The operator ξ_m^ℓ drives the transition from the $(2\ell + 1)$ -plet to the singlet $O(3)$ state, and the spin-flavor operators $(\mathcal{G}_{[j, \alpha]}^{[j, I_P]})_q$ give transitions within the symmetric $SU(4)$ representation in which both the excited and GS baryons reside. The label j denotes the spin of the spin-flavor operator while its isospin obviously coincides with the isospin of the emitted meson. The dynamics of the decays is encoded in the effective dimensionless coefficients $C_q^{[\ell_P, I_P]}(k_P)$. The reduced matrix elements of the operators $(B_{[\mu, \alpha]}^{[\ell_P, I_P]})_q$ appearing in Eq. (3) can be easily calculated in terms of the reduced matrix elements of the spin-flavor operators. Note that in the present case, where ℓ_P can be 1, 3, or 5 only (i.e. P-, F-, or H-waves)¹, the spin-flavor operators can carry spin $j = 1, \dots, 5$.

¹For $\ell = 4$ there is, in addition, a pion J-wave decay which we are not going to consider here.

The terms on the right hand side of Eq. (3) are ordered in powers of $1/N_c$. The order in $1/N_c$ is determined by the spin-flavor operator, where for an n -body operator the power is given by [11]

$$\nu = n - 1 - \kappa, \quad (5)$$

where the power $n - 1$ results from the number of gluon exchanges needed to generate an n -body operator at the quark level. The coherence factor κ is equal to zero for incoherent operators and can be equal to one or even larger (up to n) for coherent operators.

The effective operators are defined such that all coefficients $C_q^{[\ell_p, I_p]}(k_p)$ in Eq. (3) are of zeroth order in N_c . This is due to the fact that the decay amplitudes via pion emission are in general $\mathcal{O}(N_c^0)$ [9,12]. The spin-flavor breaking in the masses, in both excited and GS baryons, give rise to different values of the momenta k_p . In this work, we adopt a scheme where the only momentum dependence assigned to the coefficients is the explicitly shown factor $(k_p/\Lambda)^{\ell_p}$ that takes into account the chief momentum dependence of the partial wave, while the rest of the dependence is absorbed into the coefficients of the subleading operators.

The construction of transition operators follows similar steps as in the decays of negative-parity baryons [9]. In the present case, however, there is an important simplification because the transitions only involve states in symmetric spin-flavor representations. The simplification is that there is no need to distinguish between excited and core spin-flavor operators, and thus an arbitrary transition operator can be constructed simply in terms of appropriate products of the generators of the spin-flavor group

$$\Lambda_\gamma = S^{[1,0]}, \quad T^{[0,1]}, \quad G^{[1,1]}, \quad (6)$$

where the notation $O^{[j,t]}$ is used to indicate the spin j and the isospin t of the operators.

A composite spin-flavor operator of spin j must be at least j -body. For arbitrary N_c , only n -body operators with $n \leq N_c$ are allowed. However, since in the present analysis one takes in the end $N_c = 3$, only composite operators up to at most 3-body are needed. The composite operators that provide the basis for the decay amplitudes are constructed by coupling the composite spin-flavor operators to the $\xi^{[\ell,0]}$ operator that acts on the $O(3)$ degrees of freedom of the excited quark. The composite spin-flavor operators up to 3-body ones are of the form:

$$\frac{1}{N_c} \Lambda_\gamma, \quad \frac{1}{N_c^2} \{\Lambda_{\gamma_1}, \Lambda_{\gamma_2}\}, \quad \frac{1}{N_c^3} \{\Lambda_{\gamma_1}, \{\Lambda_{\gamma_2}, \Lambda_{\gamma_3}\}\}, \quad (7)$$

where the factors $1/N_c^n$ take into account the usual gluon exchange rules mentioned earlier. For decays in the pion channels these products transform as $[j, 1]$, where $j = 1, 2, 3$ (for general N_c our analysis would have included

TABLE II. Basis operators. Note that in the case $\ell = 0$ the operator $O_3^{[\ell_p, 1]}$ is absent.

	Name	Operator	Order in $1/N_c$
1B	$O_1^{[\ell_p, 1]}$	$\frac{1}{N_c} (\xi^\ell G)^{[\ell_p, 1]}$	0
2B	$O_2^{[\ell_p, 1]}$	$\frac{1}{N_c^2} (\xi^\ell ([S, G])^{[1, 1]})^{[\ell_p, 1]}$	1
	$O_3^{[\ell_p, 1]}$	$\frac{1}{N_c^2} (\xi^\ell (\{S, G\})^{[2, 1]})^{[\ell_p, 1]}$	1

also $j = 4, 5$ that involve n -body operators with $n > 3$). For decays in the η channel they transform instead as $[j, 0]$.

From the transformation properties of the generators given in Eq. (6) it is straightforward to construct all possible products of the forms given in Eqs. (7) with $[j, 1]$, where $j = 1, 2, 3$. The rather long list of operators obtained in this way can be reduced by using the reduction rules that apply to matrix elements of products of generators in the symmetric representation [11]. The reduced list that results from keeping contributions that are at most order $1/N_c$ is

$$\frac{1}{N_c} G, \quad \frac{1}{N_c^2} ([S, G])^{[1, 1]}, \quad \frac{1}{N_c^2} ([S, G])^{[2, 1]}, \quad (8)$$

$$\frac{1}{N_c^3} (G(GG)^{[2, 2]})^{[j, 1]},$$

where $([S, G])^{[1, 1]}$ denotes $(SG)^{[1, 1]} - (GS)^{[1, 1]} = \langle 1\alpha 1\beta | 1\delta \rangle (S^\alpha G^{\beta\alpha} - G^{\alpha\alpha} S^\beta)$, etc. Since G is a coherent operator, one can see that the 1-body operator is $\mathcal{O}(N_c^0)$, while the rest are $\mathcal{O}(1/N_c)$.

It is rather straightforward to build the composite spin-flavor operators that are needed to describe decays in the η channel. For instance, the 1-body operator is S/N_c , which is $\mathcal{O}(1/N_c)$. One can immediately establish that in fact all the operators are suppressed, and therefore the η channel is in amplitude suppressed by a factor $1/N_c$ with respect to the pion channel.

Further reductions in the number of operators result from the fact that not all of the operators are linearly independent at order $1/N_c$. The determination of the final set of independent operators for each particular decay channel is more laborious. This is achieved by coupling the spin-flavor operators with the $\xi^{[\ell, 0]}$ orbital transition

TABLE III. Reduced matrix element of basis operators for the P-wave decays of $\ell = 0$ excited baryons.

Pion P-waves	$O_1^{[1, 1]}$	$O_2^{[1, 1]}$	Overall factor
$N_{1/2}^0 \rightarrow \pi N$	1	0	$-\frac{N_c + 2}{2N_c}$
$N_{1/2}^0 \rightarrow \pi \Delta$	1	$-\frac{3}{\sqrt{2}N_c}$	$-\frac{\sqrt{(N_c + 5)(N_c - 1)}}{\sqrt{2}N_c}$
$\Delta_{3/2}^0 \rightarrow \pi N$	1	$\frac{3}{\sqrt{2}N_c}$	$-\frac{\sqrt{(N_c + 5)(N_c - 1)}}{\sqrt{2}N_c}$
$\Delta_{3/2}^0 \rightarrow \pi \Delta$	1	0	$-\frac{N_c + 2}{N_c}$
α	$-\frac{3}{5}$	$\frac{1}{2}$	

operator to the corresponding total spin and isospin and by explicitly calculating all the matrix elements. In this way, operators that are linearly dependent at the corresponding order in the $1/N_c$ expansion can be eliminated. The basis of independent operators $(O_{[m_p, I_{P_3}]}^{[\ell_p, I_P]})_n$ is displayed in Table II, where for the sake of simplicity the spin and isospin projections have been omitted. It is interesting to observe that in the end only 1- and 2-body operators are left. Note also that in the case of the $\ell = 0$ baryons $O_3^{[\ell_p, 1]}$ is absent. The corresponding reduced matrix elements are given in Tables III, IV, and V. In the bottom rows of these tables normalization coefficients $\alpha_n^{[\ell, I_P]}$ are given. These coefficients are used to define normalized basis operators such that, for $N_c = 3$, their largest reduced matrix element is equal to one for order N_c^0 operators and equal to $1/3$ for order $1/N_c$ operators. Thus,

TABLE IV. Reduced matrix element of basis operators for the decays of $\ell = 2$ excited baryons.

Pion P-waves	$O_1^{[1,1]}$	$O_2^{[1,1]}$	$O_3^{[1,1]}$	Overall factor
$N_{3/2}^2 \rightarrow \pi N$	1	0	0	$\frac{N_c+2}{2N_c}$
$N_{3/2}^2 \rightarrow \pi\Delta$	$\frac{1}{\sqrt{2}}$	$-\frac{3}{2N_c}$	$\frac{3\sqrt{3}}{\sqrt{10}N_c}$	$-\frac{\sqrt{(N_c+5)(N_c-1)}}{\sqrt{10}N_c}$
$N_{5/2}^2 \rightarrow \pi\Delta$	$-\frac{1}{2\sqrt{2}}$	$\frac{3}{4N_c}$	$\frac{1}{4\sqrt{3}N_c}$	$3\sqrt{\frac{2}{5}}\frac{\sqrt{(N_c+5)(N_c-1)}}{N_c}$
$\Delta_{1/2}^2 \rightarrow \pi N$	$\frac{1}{\sqrt{2}}$	$\frac{3}{2N_c}$	$\frac{\sqrt{3}}{2N_c}$	$-\frac{\sqrt{(N_c+5)(N_c-1)}}{\sqrt{2}N_c}$
$\Delta_{1/2}^2 \rightarrow \pi\Delta$	$-\frac{1}{\sqrt{3}}$	0	$-\frac{2\sqrt{2}}{N_c}$	$-\frac{\sqrt{\frac{3}{5}}\frac{N_c+2}{2N_c}}{\sqrt{2}N_c}$
$\Delta_{3/2}^2 \rightarrow \pi N$	$-\frac{1}{\sqrt{2}}$	$-\frac{3}{2N_c}$	$\frac{\sqrt{3}}{2N_c}$	$\frac{\sqrt{(N_c+5)(N_c-1)}}{\sqrt{2}N_c}$
$\Delta_{3/2}^2 \rightarrow \pi\Delta$	$\frac{1}{\sqrt{2}}$	0	$\frac{\sqrt{3}}{N_c}$	$\frac{4}{5}\frac{N_c+2}{N_c}$
$\Delta_{5/2}^2 \rightarrow \pi\Delta$	-3	0	$\frac{2\sqrt{6}}{N_c}$	$-\frac{\sqrt{7}}{10}\frac{N_c+2}{N_c}$
α	$\frac{2}{\sqrt{7}}$	$\frac{\sqrt{10}}{6}$	$\frac{\sqrt{3}}{4}$	

Pion F-waves	$O_1^{[3,1]}$	$O_2^{[3,1]}$	$O_3^{[3,1]}$	Overall factor
$N_{3/2}^2 \rightarrow \pi\Delta$	1	$-\frac{3}{\sqrt{2}N_c}$	$\frac{1}{N_c}$	$-\frac{\sqrt{7}}{10}\frac{\sqrt{(N_c+5)(N_c-1)}}{N_c}$
$N_{5/2}^2 \rightarrow \pi N$	1	0	0	$-\frac{\sqrt{\frac{7}{3}}\frac{N_c+2}{2N_c}}{\sqrt{2}N_c}$
$N_{5/2}^2 \rightarrow \pi\Delta$	1	$-\frac{3}{\sqrt{2}N_c}$	$-\frac{3}{2N_c}$	$-\frac{\sqrt{7}}{15}\frac{\sqrt{(N_c+5)(N_c-1)}}{N_c}$
$\Delta_{3/2}^2 \rightarrow \pi\Delta$	1	0	$\frac{4}{N_c}$	$-\frac{\sqrt{7}}{5}\frac{N_c+2}{N_c}$
$\Delta_{5/2}^2 \rightarrow \pi N$	1	$\frac{3}{\sqrt{2}N_c}$	$\frac{3}{N_c}$	$-\frac{1}{\sqrt{6}}\frac{\sqrt{(N_c+5)(N_c-1)}}{N_c}$
$\Delta_{5/2}^2 \rightarrow \pi\Delta$	1	0	$\frac{3}{2N_c}$	$-\frac{8}{5\sqrt{3}}\frac{N_c+2}{N_c}$
$\Delta_{7/2}^2 \rightarrow \pi N$	1	$\frac{3}{\sqrt{2}N_c}$	$-\frac{1}{2N_c}$	$-\frac{\sqrt{(N_c+5)(N_c-1)}}{N_c}$
$\Delta_{7/2}^2 \rightarrow \pi\Delta$	1	0	$-\frac{2}{N_c}$	$-\frac{\sqrt{6}}{5}\frac{N_c+2}{N_c}$
α	$-\frac{\sqrt{3}}{10}$	$-\frac{1}{2\sqrt{2}}$	$\frac{1}{2}\sqrt{\frac{3}{10}}$	

$$(B_{[m_p, I_{P_3}]}^{[\ell_p, I_P]})_n = \alpha_n^{[\ell, I_P]} (O_{[m_p, I_{P_3}]}^{[\ell_p, I_P]})_n \quad (9)$$

furnishes the list of basis operators normalized according to the $1/N_c$ power counting.

TABLE V. Reduced matrix element of basis operators for the decays of $\ell = 4$ excited baryons.

Pion F-waves	$O_1^{[3,1]}$	$O_2^{[3,1]}$	$O_3^{[3,1]}$	Overall factor
$N_{7/2}^4 \rightarrow \pi N$	1	0	0	$\sqrt{\frac{7}{3}}\frac{N_c+2}{2N_c}$
$N_{7/2}^4 \rightarrow \pi\Delta$	1	$-\frac{3}{\sqrt{2}N_c}$	$\frac{\sqrt{15}}{\sqrt{2}N_c}$	$-\frac{\sqrt{7}\sqrt{(N_c+5)(N_c-1)}}{6N_c}$
$N_{9/2}^4 \rightarrow \pi\Delta$	1	$-\frac{3}{\sqrt{2}N_c}$	$-\frac{\sqrt{3}}{\sqrt{10}N_c}$	$-\frac{\sqrt{35}\sqrt{(N_c+5)(N_c-1)}}{6N_c}$
$\Delta_{5/2}^4 \rightarrow \pi N$	1	$\frac{3}{\sqrt{2}N_c}$	$\frac{\sqrt{5}}{\sqrt{6}N_c}$	$-\frac{\sqrt{3}\sqrt{(N_c+5)(N_c-1)}}{2N_c}$
$\Delta_{5/2}^4 \rightarrow \pi\Delta$	1	0	$2\frac{\sqrt{10}}{\sqrt{3}N_c}$	$\sqrt{\frac{3}{2}}\frac{N_c+2}{2N_c}$
$\Delta_{7/2}^4 \rightarrow \pi N$	1	$\frac{3}{\sqrt{2}N_c}$	$-3\frac{\sqrt{3}}{\sqrt{10}N_c}$	$-\frac{\sqrt{5}\sqrt{(N_c+5)(N_c-1)}}{\sqrt{12}N_c}$
$\Delta_{7/2}^4 \rightarrow \pi\Delta$	1	0	$\frac{\sqrt{6}}{\sqrt{5}N_c}$	$\frac{2\sqrt{2}}{3}\frac{N_c+2}{N_c}$
$\Delta_{9/2}^4 \rightarrow \pi\Delta$	1	0	$-2\frac{\sqrt{6}}{\sqrt{5}N_c}$	$\sqrt{\frac{77}{18}}\frac{N_c+2}{2N_c}$
α	$\frac{18\sqrt{2}}{5\sqrt{77}}$	$\frac{3}{\sqrt{70}}$	$-\frac{3\sqrt{3}}{\sqrt{385}}$	

Pion H-waves	$O_1^{[5,1]}$	$O_2^{[5,1]}$	$O_3^{[5,1]}$	Overall factor
$N_{7/2}^4 \rightarrow \pi\Delta$	1	$-\frac{3}{\sqrt{2}N_c}$	$\frac{\sqrt{3}}{2N_c}$	$-\frac{\sqrt{11}}{3}\frac{\sqrt{(N_c+5)(N_c-1)}}{N_c}$
$N_{9/2}^4 \rightarrow \pi N$	1	0	0	$-\frac{\sqrt{11}}{3}\frac{N_c+2}{2N_c}$
$N_{9/2}^4 \rightarrow \pi\Delta$	1	$-\frac{3}{\sqrt{2}N_c}$	$-\frac{\sqrt{3}}{N_c}$	$-\frac{\sqrt{11}}{2}\frac{\sqrt{(N_c+5)(N_c-1)}}{3N_c}$
$\Delta_{7/2}^4 \rightarrow \pi\Delta$	1	0	$\frac{2\sqrt{3}}{N_c}$	$-\frac{\sqrt{154}}{15}\frac{N_c+2}{N_c}$
$\Delta_{9/2}^4 \rightarrow \pi N$	1	$\frac{3}{\sqrt{2}N_c}$	$\frac{3\sqrt{3}}{2N_c}$	$-\frac{1}{\sqrt{3}}\frac{\sqrt{(N_c+5)(N_c-1)}}{N_c}$
$\Delta_{9/2}^4 \rightarrow \pi\Delta$	1	0	$\frac{\sqrt{3}}{2N_c}$	$-\frac{8}{\sqrt{5}}\frac{N_c+2}{3N_c}$
$\Delta_{11/2}^4 \rightarrow \pi N$	1	$\frac{3}{\sqrt{2}N_c}$	$-\frac{1}{\sqrt{3}N_c}$	$-\frac{\sqrt{3}}{2}\frac{\sqrt{(N_c+5)(N_c-1)}}{N_c}$
$\Delta_{11/2}^4 \rightarrow \pi\Delta$	1	0	$-\frac{4}{\sqrt{3}N_c}$	$-\frac{\sqrt{39}}{5}\frac{N_c+2}{N_c}$
α	$-\frac{\sqrt{3}}{13}$	$-\frac{1}{2\sqrt{3}}$	$\frac{3}{4\sqrt{13}}$	

TABLE VI. Fit parameters and partial widths corresponding to the pion P-wave decays of $\ell = 0$ excited baryons.

Pion P-waves	Emp. width MeV	LO MeV	NLO MeV
χ_{dof}^2		4.05	0.1
dof		3	2
$C_1^{[1,1]}$		18.7 ± 2.4	17.0 ± 1.6
$C_2^{[1,1]}$...	24.4 ± 6.3
$N(1440) \rightarrow \pi N$	227.5 ± 67.3	106	245
$N(1440) \rightarrow \pi\Delta$	87.5 ± 30.5	16.0	83.6
$\Delta(1600) \rightarrow \pi N$	61.25 ± 31.6	106	59.4
$\Delta(1600) \rightarrow \pi\Delta$	180 ± 99	63.5	146

TABLE VII. Fit parameters and partial widths corresponding to the pion P-wave decays of $\ell = 2$ excited baryons.

Pion P-waves	Emp. width	LO MeV	NLO MeV
χ_{dof}^2		0.15	0.44
dof		3	1
$C_1^{[1,1]}$		6.83 ± 0.77	4.09 ± 0.47
$C_2^{[1,1]}$...	0.11 ± 2.41
$C_3^{[1,1]}$...	0.43 ± 6.09
$N(1720) \rightarrow \pi N$	22.5 ± 11	28.5	28.4
$N(1720) \rightarrow \pi \Delta$	Unknown	2.2	2.4
$N(1680) \rightarrow \pi \Delta$	13 ± 5	11.9	11.5
$\Delta(1910) \rightarrow \pi N$	52 ± 20	46.2	48.1
$\Delta(1910) \rightarrow \pi \Delta$	Unknown	5.25	5.9
$\Delta(1920) \rightarrow \pi N$	28 ± 19	25.1	24.7
$\Delta(1920) \rightarrow \pi \Delta$	Unknown	19.3	20.3
$\Delta(1905) \rightarrow \pi \Delta$	Unknown	22.0	21.1

III. RESULTS

In the context of the $1/N_c$ expansion, the decays of the Roper multiplet were already studied in Ref. [5], where the whole $SU(6)$ 56-plet was analyzed using only the 1-body operator $O_1^{[0,1]}$ and the breaking of spin and flavor symmetries was included by means of a profile function conveniently chosen and adjusted by a best fit to the partial widths. Here we analyze the P-wave decays (for the F-wave decay of the $\Delta(1600)$ there is too little to be said). The analysis includes the $1/N_c$ corrections via the only operator $\ell_P = 0$ that appears at that order, namely, the 2-body operator $O_2^{[0,1]}$, and do not include a profile function. The results of the fits are shown in Table VI, where it is evident that at LO the fit turns out to be rather poor. At NLO on the other

hand the fit is excellent. The subleading operator is essential for inverting the ordering of the partial widths of the $\Delta(1600)$ obtained at LO. The NLO corrections are also essential for improving the ratio of the two $N(1440)$ partial widths, which is reproduced rather poorly at LO. The coefficient of the LO operator remains stable as one includes the NLO corrections, and the coefficient of the NLO operator is of natural size when compared with the coefficient of the LO operator.

In the case of the $\ell = 2$ states one has two nucleon and four Δ states, and the relevant decays can be P- and F-wave. In both channels there are three operators when the $1/N_c$ corrections are included. There are only 4 P-wave partial widths listed by the Particle Data Group [13], which is sufficient for the analysis to $\mathcal{O}(1/N_c)$. As Table VII shows, in this case one finds that an excellent fit is already obtained at LO. This means that the NLO operators are of marginal significance. In fact, in the NLO fit the coefficients are compatible with zero. For the F-wave decays there are five known widths, two of them ($N(1680) \rightarrow \pi N$ and $\Delta(1950) \rightarrow \pi N$) are known with an accuracy of about 10%, which is the magnitude of the $\mathcal{O}(1/N_c^2)$ corrections. Thus, in order to perform a consistent LO fit those empirical errors of 10% are replaced by 30% errors, which is the magnitude of the $\mathcal{O}(1/N_c)$ corrections. The results in Table VIII show that a reasonably good LO fit is obtained in this way. On the other hand, in the NLO fits the errors used are the ones given by the Particle Data Group [13]. Several of the errors remain around 30%, which is poor accuracy for inputs to a NLO fit. Two possible NLO fits are given in Table VIII. In the #1 NLO fit all five experimental inputs are included resulting in a very large value of χ^2 . It is found that the narrow partial width assigned to $N(1680) \rightarrow \pi \Delta$ is the chief contribution to the χ^2 in this

TABLE VIII. Fit parameters and partial widths corresponding to the pion F-wave decays of $\ell = 2$ excited baryons. Note that for the LO fit those empirical errors that are less than 30% have been increased up to that value. In the fit #2 NLO the empirical value of the decay $N(1680) \rightarrow \pi \Delta$ has not been considered.

Pion F-waves	Emp. width MeV	LO MeV	#1 NLO MeV	#2 NLO MeV
χ_{dof}^2		1.73	6.9	0.38
dof		4	2	1
$C_1^{[3,1]}$		1.01 ± 0.09	0.78 ± 0.04	0.84 ± 0.04
$C_2^{[3,1]}$...	-0.84 ± 0.18	-1.30 ± 0.21
$C_3^{[3,1]}$...	-0.81 ± 0.26	-0.26 ± 0.47
$N(1720) \rightarrow \pi \Delta$	unknown	6.86	19.9	25.1
$N(1680) \rightarrow \pi N$	84.5 ± 9	44.8	72.9	86.0
$N(1680) \rightarrow \pi \Delta$	1.3 ± 1.3	2.39	4.84	7.94
$\Delta(1920) \rightarrow \pi \Delta$	unknown	17.3	51.4	40.4
$\Delta(1905) \rightarrow \pi N$	36 ± 20	26.0	43.5	25.9
$\Delta(1905) \rightarrow \pi \Delta$	unknown	24.8	51.6	51.3
$\Delta(1950) \rightarrow \pi N$	120 ± 14	159	129	121
$\Delta(1950) \rightarrow \pi \Delta$	77 ± 15	42.0	46.3	72.3

fit. It should be noticed that the PDG only quotes an upper bound for the corresponding branching ratio. If one disregards that input (#2 NLO fit), an excellent fit is obtained where one of the NLO coefficients is of natural size and the other one associated with the operator $O_3^{[2,1]}$ is suppressed. Actually, an equally good fit can be obtained by disregarding the latter operator. Interestingly enough, the predicted $N(1680) \rightarrow \pi\Delta$ width is predicted to be several times smaller than the rest of the F-wave pion decay widths, which qualitatively agrees with observation despite violating the experimental bound slightly.

Finally, for the $\ell = 4$ baryons the available experimental information is unfortunately too incomplete to warrant any reasonable analysis. In fact, there is no data about the F-wave decays, and there is information about only two out of nine possible H-wave decays. In any case, as soon as more information will become available Eqs. (2) and (3) together with the expressions in Table V could be readily used to perform the corresponding analysis.

IV. CONCLUSIONS

The decays via pseudoscalar meson emission of the low lying positive parity excited baryons have been studied to $\mathcal{O}(1/N_c)$. For pionic decays of the $\ell = 0$ Roper baryons it is found that the $1/N_c$ corrections are essential for obtaining a good fit. The $1/N_c$ corrections are clearly required by the observed ordering of the two $\Delta(1600)$ partial widths, namely $\Gamma(\Delta(1600) \rightarrow \pi N) < \Gamma(\Delta(1600) \rightarrow \pi\Delta)$, which at LO cannot be accounted for. It is important to notice that these are the only decays where the LO 1-body operator O_1 (see Table II) does not give a qualitatively correct picture of the decays. The $\mathcal{O}(1/N_c)$ operators are 2-body operators; one of these operators, namely O_2 , provides the observed ordering of the $\Delta(1600)$ partial widths and its coefficient is a factor 1.4 larger than the expected natural size, while the operator O_3 turns out to be insignificant for the fit. The simple picture of decays based on a chiral quark model where the 1-body operator is dominant seems to work qualitatively well in all decays so far studied except the decays of the Roper multiplet. The meaning of this

observation for the nature of the Roper multiplet is an interesting issue to understand. It has been argued [14] that effective 2-body interactions carrying spin-flavor are essential to lower the Roper multiplet masses. It would be interesting to see whether this scenario would naturally lead to the significance of the operator O_2 for the decays.

The pionic decays of the $\ell = 2$ multiplet are qualitatively well described at LO, and the improvements at NLO are accomplished with essentially one extra operator whose coefficient is of natural magnitude. As in the case of the negative-parity baryons, the extension of the analyses to include the strange excited baryons seems to be the natural step to gain further insight on decays. The general point that one should emphasize is that so far the main limitation in the analyses is due to the poor quality of the input partial widths. To improve this it is necessary to have extensive experimental progress, which is in part being fulfilled by the present N^* program at Jefferson Lab [15].

One interesting observation that points to the consistency of the framework based on the approximate $O(3) \times SU(4)$ symmetry is that the predicted suppression of the η channels in the decays discussed here is clearly displayed by the observed decays. This represents a strong experimental confirmation that the excited baryons we analyzed do belong primarily into a symmetric representation of $SU(4)$. In the case of the negative-parity baryons the η channel is not suppressed and is indeed very important. This implies that these states belong primarily into the mixed-symmetric representation of $SU(4)$. Thus, η channels serve as a selector of the spin-flavor structure of non-strange excited baryons.

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