PHYSICAL REVIEW D **72,** 034014 (2005)

Diquarks in nonaquark states

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(Received 27 June 2005; published 18 August 2005)

We study the nonaquark states $S^0(3115)$ and $S^+(3140)$ which are reported by [T. Suzuki (KEK-PS), Phys. Lett. B **597**, 236 (2004)M. Iwasaki *et al.*, nucl-ex/0310018] by means of the quark model with diquark correlation. The nonaquark states form $\mathbf{1}, \mathbf{8}, \mathbf{10}, \overline{\mathbf{10}}, \mathbf{27}, \overline{\mathbf{35}}$ SU(3) multiplets. The flavor wave functions of all the nonaquark states are constructed through the standard tensor technique. The mass spectrum is studied by using the Gell-Mann-Okubo mass formula. Some nonaquark mass sum rules are obtained. We further investigate the decay of $S^0(3115)$ and $S^+(3140)$ under the assumption of the ''fall-apart'' mechanism. It has been found that the main decay mode is ΣNN rather than ΛNN which is consistent with experiment. Also we have uniquely determined the flavor wave function of $S^0(3115)$ which belongs to the **27**-plet with the quantum number $Y = 2, I = 1, I_z = -1$. Whereas the exotic states $S^+(3140)$ can belong to either the **27**-plet or the $\overline{\mathbf{35}}$-plet. In the exact $SU(3)^{\rm flavor} \times SU(3)^{\rm color} \times SU(2)^{\rm spin}$ limit, both $S^0(3115)$ and $S^+(3140)$ belong to the **27**-plet with negative parity. We predict that its flavor structure can be determined by measuring the branch fractions of its decay channels. The experiments to check this prediction are as expected.

DOI: 10.1103/PhysRevD.72.034014 PACS numbers: 12.38.Lg, 12.39.Mk, 13.30.Eg

I. INTRODUCTION

There is growing interest in exotic hadrons, which may open new windows for understanding the hadronic structures and QCD at low energies. Recently, the KEK-PS reported a tribaryon state $S^0(3115)$ [1] in the reaction

$$K^- + {}^4\text{He} \rightarrow S^0 + p. \quad (1)$$

The mass of the state is $3117^{+1.5}_{-4.4}$ MeV , the decay width $\Gamma_{S^0} < 21$ MeV, and the main decay mode is ΣNN rather than ΛNN. The peak in the proton spectrum is over the background with a significance level 13σ. A strange tribaryon $S^+(3140)$ of charge $+1$ was also reported in the reaction $K^- + {}^4\text{He} \rightarrow S^+ + n$ [2]. The mass and decay width of this exotic state are $M_{S^+} = 3141 \pm 3(\text{stat.})^{+4}_{-1} \times$ (sys.) MeV and $\Gamma_{S^+} \leq 23$ MeV , which is about 25 MeV higher than $S^+(3115)$, and its significance is 3.7σ. It also dominantly decays into ΣNN rather than ΛNN.

The $S^0(3115)$ was first predicted by Akaishi and Yamazaki [3] as a deeply-bound kaonic state. Since the discovery of $S^0(3115)$ and $S^+(3140)$, there has been some theoretical discussion [4–6], in Ref. [4] these exotic states are mainly analyzed from the MIT bag model, and they are identified as kaonic bound states in Ref. [5,6]. Since the quark dynamics could be regarded as a cornerstone for hadron physics, it is interesting to investigate the nonaquark states by means of the quark models. Various quark models have been used and proposed in studying the pentaquark baryon state [7–9]. Here we draw the spirit of Jaffe–Wilczek's work [7], since there are evidences for strong diquark correlation in the baryon spectrum [10], and, especially, in the light nonet-scalar ($J^{PC} = 0^{++}$) meson spectrum. Their masses are generally below 1000 MeV [$f_0(600)$, $f_0(980)$, $a_0(980)$, $\kappa(800)$], and they do not favor the predictions of the $q\bar{q}$-models, but favor the diquark-antidiquark's quite well. Diquark is a boson with color $\bar{3}_c$, flavor $\bar{3}_f$, and spin zero. Diquark correlation is also the basis of color superconductivity in dense quark matter, which has not being observed experimentally. This configuration is favored by one gluon exchange [11,12] and by instanton interactions [13,14]. It may play an important role in the exotic hadron physics. In this paper we try to investigate nonaquark baryons by means of the diquarks model, and to learn what happens in the nonaquark case due to the strong diquark correlation. Meanwhile, in order to understand the decay of nonaquark states, we suggest a decay mechanism which can qualitatively explain the experiments and give us new predictions. This decay mechanism is quite intuitive. To understand the structure of the nonaquark, its mass spectrum and the decay mechanism are the main aims of this paper.

The paper is organized as follows, in Sec. II we study the direct products of two diquarks states, four diquarks states, and of four diquarks plus one quarks. The irreducible tensors of the allowed nonaquark states are derived. The flavor wave functions are given by identifying the SU(3) tensors with the physical tribaryon states. In Sec. III, the mass spectrum is derived by using the Gell-Mann–Okubo mass formula. Section IV is devoted to study the decays of $S^0(3115)$ and $S^+(3140)$ under the assumption that the decays are caused by a ''fall-apart'' mechanism. We find when $S^0(3115)$ only belongs to a certain **27**-plet, its main decay mode is ΣNN rather than ΛNN, and $S^+(3140)$ can belong to either the **27**-plet or the $\overline{\mathbf{35}}$-plet. In Sec. V, we briefly summarize the results and give some discussions.

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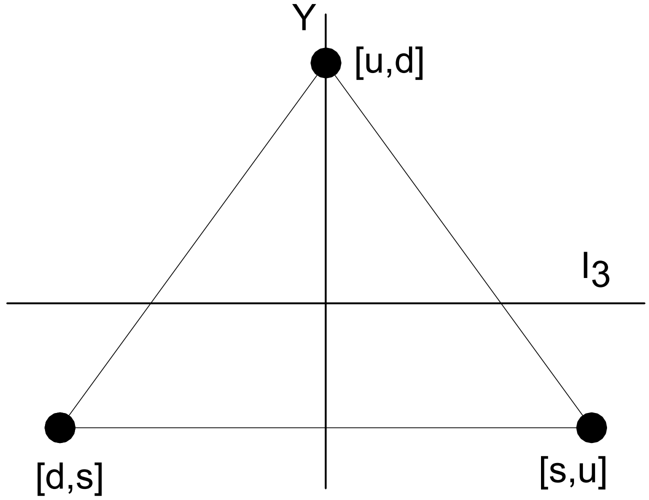

FIG. 1. $\overline{\mathbf{3}}$ diquark.

II. THE FLAVOR WAVE FUNCTION OF NONAQUARK STATES

Since the diquark is in the $\bar{3}_f$, it has three configurations in flavor space, which are shown in Fig. 1. We denote them as

$$\mathcal{Q}^1 = \frac{1}{\sqrt{2}}[d, s] \qquad \mathcal{Q}^2 = \frac{1}{\sqrt{2}}[s, u] \qquad \mathcal{Q}^3 = \frac{1}{\sqrt{2}}[u, d] \quad (2)$$

where u, d, s are, respectively, up quark, down quark, and strange quark. It is obvious that there are the following correspondences: $\mathcal{Q}^1 \leftrightarrow \bar{u}$, $\mathcal{Q}^2 \leftrightarrow \bar{d}$, $\mathcal{Q}^3 \leftrightarrow \bar{s}$.

The tensors $T^{j_1,j_2,\cdots j_q}_{i_1,i_2\cdots i_p}$, which are the bases for irreducible representations of SU(3), are totally symmetric to both all q upper indices and all p low indices, and also are traceless.

$$T^{j_1,j_2,\cdots j_q}_{i_1,i_2\cdots i_p} = T^{j_2,j_1,\cdots j_q}_{i_1,i_2\cdots i_p} = T^{j_1,j_2,\cdots j_q}_{i_2,i_1\cdots i_p} \qquad T^{i_1,j_2,\cdots j_q}_{i_1,i_2\cdots i_p} = 0. \quad (3)$$

Since δ^i_j, ε^{ijk}, and $\varepsilon_{\mathrm{ijk}}$ are tensors, we can use them to raise, lower, or contract indices when we construct new tensors that are bases of irreducible representation from the direct product tensor. The direct product of two diquarks is

$$\mathcal{Q}^i\mathcal{Q}^j = \frac{1}{\sqrt{2}}S^{ij} + \frac{1}{2\sqrt{2}}\varepsilon^{ijk}T_k, \quad (4)$$

with $S^{ij} = \frac{1}{\sqrt{2}}(\mathcal{Q}^i\mathcal{Q}^j + \mathcal{Q}^j\mathcal{Q}^i)$, $A^{ij} = \frac{1}{\sqrt{2}}(\mathcal{Q}^i\mathcal{Q}^j - \mathcal{Q}^j\mathcal{Q}^i)$, and $T_k = \varepsilon_{ijk}A^{ij}$. So the decomposition of the direct product of two diquarks is $\bar{3} \otimes \bar{3} = \bar{6} \oplus 3$, which is shown in Fig. 2 in the Young tabular.

Since the two diquarks can be decomposed into $\overline{\mathbf{6}}$ plus $\mathbf{3}$, the direct product of four diquarks raises $\overline{\mathbf{6}} \otimes \overline{\mathbf{6}}$, $\overline{\mathbf{6}} \otimes \mathbf{3}$, $\mathbf{3} \otimes \overline{\mathbf{6}}$, and $\mathbf{3} \otimes \mathbf{3}$. And the corresponding Young tabular is shown in Fig. 3. It is straightforward that

$$\begin{aligned}(\mathcal{Q}^i\mathcal{Q}^j)(\mathcal{Q}^m\mathcal{Q}^n) &= \frac{1}{2}S^{ij}S^{mn} + \frac{1}{4}(\varepsilon^{kij}T_kS^{mn} + \varepsilon^{kmn}S^{ij}T_k) \\ &\quad + \frac{1}{8}\varepsilon^{kij}\varepsilon^{lmn}T_kT_l = \frac{1}{2\sqrt{6}}T^{ijmn} \\ &\quad + \frac{1}{4\sqrt{2}}(\varepsilon^{ajm}\delta^n_b\delta^i_c + \varepsilon^{ain}\delta^m_b\delta^j_c)S^{bc}_a \\ &\quad + \frac{1}{2\sqrt{6}}(\varepsilon^{aim}\varepsilon^{bjn} + \varepsilon^{ajm}\varepsilon^{bin})T_{ab} \\ &\quad + \frac{1}{4}\varepsilon^{kij}\left[\tilde{T}^{mn}_k + \frac{1}{\sqrt{2}}(\delta^m_k\delta^n_a + \delta^n_k\delta^m_a)\tilde{Q}^a\right] \\ &\quad + \frac{1}{4}\varepsilon^{kmn}\left[T^{ij}_k + \frac{1}{\sqrt{2}}(\delta^i_k\delta^j_a + \delta^j_k\delta^i_a)Q^a\right] \\ &\quad + \frac{1}{4}\varepsilon^{kij}\varepsilon^{lmn}(\sqrt{2}S_{kl} + \varepsilon_{kla}T^a), \end{aligned} \quad (5)$$

where the tensors in the above formula are defined as follows.

$$T^i = \frac{1}{8}\varepsilon^{ijk}(T_jT_k - T_kT_j) \quad (6)$$

$$Q^i = \frac{1}{\sqrt{8}}S^{ij}T_j \quad (7)$$

$$\tilde{Q}^i = \frac{1}{\sqrt{8}}T_jS^{ji} \quad (8)$$

$$S_{ij} = \frac{1}{4\sqrt{2}}(T_iT_j + T_jT_i) \quad (9)$$

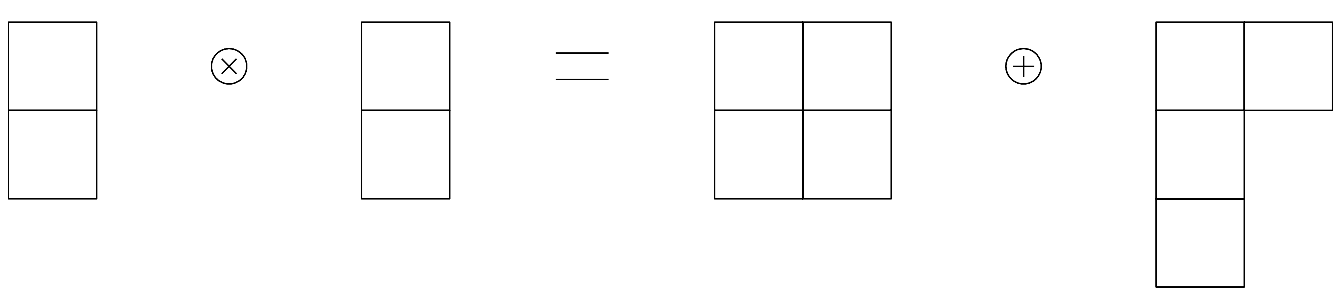

FIG. 2. Direct product of two diquarks $\overline{\mathbf{3}} \otimes \overline{\mathbf{3}} = \overline{\mathbf{6}} + \mathbf{3}$.

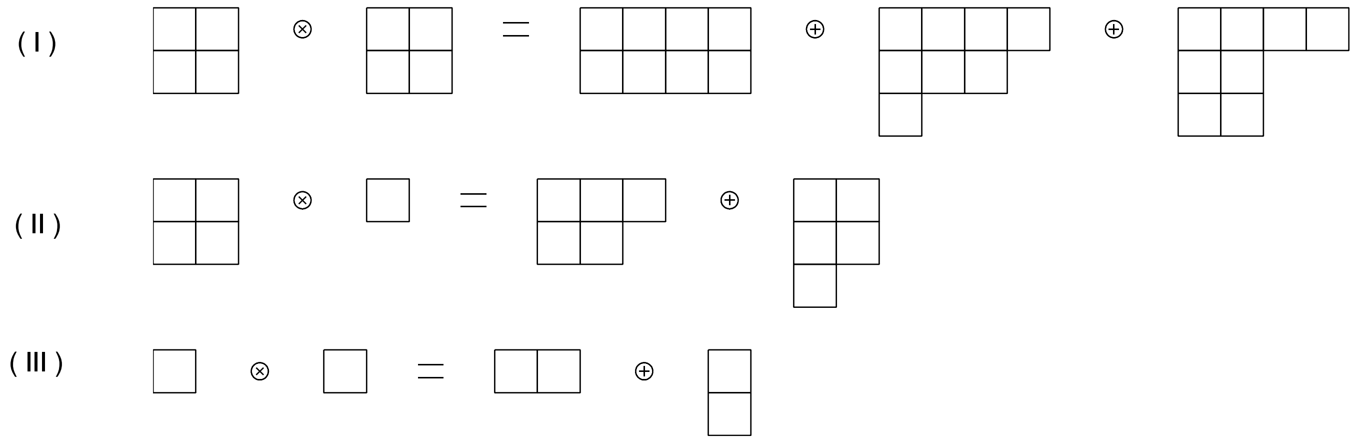

FIG. 3. The direct product of four diquarks: (I) $\overline{\mathbf{6}}\otimes\overline{\mathbf{6}} = \overline{\mathbf{15}}_1 + \overline{\mathbf{15}}_2 + \mathbf{6}$; (II) $\overline{\mathbf{6}}\otimes\mathbf{3} = \overline{\mathbf{15}}_2 + \overline{\mathbf{3}}$; (III) $\mathbf{3}\otimes\mathbf{3} = \mathbf{6}\oplus\overline{\mathbf{3}}$.

$$T^{ijmn} = \frac{1}{\sqrt{6}}(S^{ij}S^{mn} + S^{mj}S^{in} + S^{in}S^{jm} + S^{mi}S^{jn} + S^{jn}S^{im} + S^{mn}S^{ij}) \tag{10}$$

$$S_i^{jk} = \frac{1}{\sqrt{2}}\varepsilon_{imn}(S^{jm}S^{kn} + S^{km}S^{jn}) \tag{11}$$

$$\tilde{T}_i^{jk} = T_i S^{jk} - \frac{1}{\sqrt{2}}(\delta_i^j\delta_m^k + \delta_i^k\delta_m^j)\tilde{Q}^m \tag{12}$$

$$T_i^{jk} = S^{jk}T_i - \frac{1}{\sqrt{2}}(\delta_i^j\delta_m^k + \delta_i^k\delta_m^j)Q^m \tag{13}$$

So the four-diquarks product can be decomposed into $\overline{\mathbf{15}}_1 \oplus \overline{\mathbf{15}}_2(3) \oplus \overline{\mathbf{3}}(3) \oplus \mathbf{6}$, where the numbers in the parentheses denote the degeneracy in each multiplet. The tensors corresponding to $\overline{\mathbf{3}}$ are T^i, Q^i, $\tilde{Q}^i$, the tensor S_{ij} forms the basis of the irreducible representation $\mathbf{6}$, the tensor corresponding to $\overline{\mathbf{15}}_1$ is T^{ijmn}, and the tensors S_i^{jk}, $\tilde{T}_i^{jk}$, T_i^{jk} are, respectively, the bases of the irreducible representation $\overline{\mathbf{15}}_2$.

A. Nonaquark states

Since quark is in the fundamental representation $\mathbf{3}$, when the four diquarks form the irreducible representation $\overline{\mathbf{3}}$, the nonaquark state must be in the representation $\overline{\mathbf{3}}\otimes\mathbf{3} = \mathbf{8} + \mathbf{1}$. This means the nonaquark state can be either in the octet or in the singlet. We use $\mathcal{T}^i$ to stand for T^i, Q^i, $\tilde{Q}^i$, then the tensor product $\mathcal{T}^i q_n$ can be decomposed as follows

$$\mathcal{T}^i q_n = \sqrt{2}\left(P_n^i + \frac{1}{\sqrt{3}}\delta_n^i S\right), \tag{14}$$

where $S = \frac{1}{\sqrt{6}}\mathcal{T}^m q_m$, $P_n^i = \frac{1}{\sqrt{2}}(\mathcal{T}^i q_n - \sqrt{\frac{2}{3}}\delta_n^i S)$. P_n^i stand for the nonaquark octet, and that S stands for the nonaquark singlet.

When the four diquarks are in the representation $\overline{\mathbf{6}}\otimes\overline{\mathbf{6}}$ [$= \overline{\mathbf{15}}_1 \bigoplus \overline{\mathbf{15}}_2 \bigoplus \mathbf{6}$ (see Fig. 3)], they can form the irreducible representative of $\mathbf{6}$. Since $\mathbf{6}\otimes\mathbf{3} = \mathbf{10}\oplus\mathbf{8}$, the nonaquark states can be in decuplet or octet.

$$S_{ij}q_n = \frac{1}{\sqrt{3}}[T_{ijk} + \varepsilon_{mjn}P_i^m + \varepsilon_{min}P_j^m] \tag{15}$$

with $T_{ijk} = \frac{1}{\sqrt{3}}[S_{ij}q_n + S_{in}q_j + S_{jn}q_i]$, $P_i^j = \frac{1}{\sqrt{3}}\varepsilon^{jab}S_{ia}q_b$ and T_{ijk}, P_i^j, respectively, correspond to the nonaquark decuplet and octet.

Again, for the four-diquarks states in $\overline{\mathbf{6}}\otimes\overline{\mathbf{6}}$, they can also form $\overline{\mathbf{15}}_1$. The direct product $\overline{\mathbf{15}}_1\otimes\mathbf{3}$ can be reduced as follows

$$T^{ijkl}q_n = T_n^{ijkl} + \frac{1}{\sqrt{6}}(\delta_n^i\delta_b^j\delta_c^k\delta_d^l + \delta_n^j\delta_b^i\delta_c^k\delta_d^l + \delta_n^k\delta_b^i\delta_c^j\delta_d^l + \delta_n^l\delta_b^i\delta_c^j\delta_d^k)D^{bcd} \tag{16}$$

where $T_n^{ijkl} = T^{ijkl}q_n - \frac{1}{\sqrt{6}}(\delta_n^i D^{jkl} + \delta_n^j D^{ikl} + \delta_n^l D^{ijk})$, $D^{ijk} = \frac{1}{\sqrt{6}}T^{ijkn}q_n$, T_n^{ijkl} and D^{ijk}, respectively, mean that the nonaquark state belongs to the $\overline{\mathbf{35}}$-plet and the $\overline{\mathbf{10}}$-plet.

Finally, we consider the case of that the four-diquarks states form $\overline{\mathbf{15}}_2$. The $\overline{\mathbf{15}}_2-$ four-diquarks states have three irreducible representatives: they can be in the product of $\overline{\mathbf{6}}\otimes\mathbf{3}$, $\mathbf{3}\otimes\overline{\mathbf{6}}$, or $\overline{\mathbf{6}}\otimes\overline{\mathbf{6}}$. The corresponding tensors are S_i^{jk},$\tilde{T}_i^{jk}$, or T_i^{jk}, respectively. Using $\mathcal{T}_i^{jk}$ to denote each one of them, then we have

$$\mathcal{T}_i^{jk}q_n = \sqrt{2}T_{in}^{jk} + \sqrt{\frac{2}{3}}\varepsilon_{inm}D^{mjk} + \frac{4}{\sqrt{15}}(\delta_n^k\delta_m^j + \delta_m^k\delta_n^j)P_i^m - \frac{1}{\sqrt{15}}(\delta_i^k\delta_m^j + \delta_m^k\delta_i^j)P_n^m, \tag{17}$$

where

$$P^i_j = \frac{1}{\sqrt{15}}\mathcal{T}^{ik}_j q_k D^{ijk} = \frac{1}{24}(\varepsilon^{jab}\mathcal{T}^{km}_a q_b + \varepsilon^{kab}\mathcal{T}^{jm}_a q_b + \varepsilon^{mab}\mathcal{T}^{jk}_a q_b)T^{jk}_{in} = \frac{1}{2\sqrt{2}}(\mathcal{T}^{jk}_i q_n + \mathcal{T}^{jk}_n q_i) - \frac{\sqrt{30}}{20}(\delta^k_i P^j_n + \delta^k_n P^j_i + \delta^j_i P^k_n + \delta^j_n P^k_i). \quad (18)$$

T^{jk}_{in} represents the nonaquark $\overline{35}$-plet, and D^{ijk}, P^i_j correspond to the nonaquark $\overline{10}$-plet and octet. The Young tabular of the tensor decomposition are shown in Fig. 4.

Now we take the color symmetry $SU(3)^{color}$ into account. When the $SU(3)^{flavor} \times SU(3)^{color} \times SU(2)^{spin}$ serves as an exact symmetry, then, due to the boson statistics, the full combined (flavor × color × spin × space) wave functions of four-diquarks states must be symmetric. Since the spin of the diquark is zero, the *spin* wave function is trivially to be symmetric. Thus, the (space × flavor × color) wave function must be symmetric. Under this constraint, only two choices are available:

(1) The *space* wave functions are symmetric: In this case, the relative angular momentum ℓ is even and starts from $\ell = 0$, and then the (flavor × color) wave functions are symmetric. They must, therefore, belong to the $\overline{495}$ dimensional irreducible representation of $SU(9) \supset SU(3)$flavor × $SU(3)$color. The reduction of the SU(9) irreducible representation $\overline{495}$ with respect to SU(3)flavor × SU(3)color [15] is

$$\overline{495} = (\overline{15}_1, \overline{15}_1) + (\overline{15}_2, \overline{15}_2) + (\bar{3}, \bar{3}) + (6, 6). \quad (19)$$

We can see only that the four-diquarks states (flavor, color) $= (\bar{3}, \bar{3})$ can combine with the ninth quark with (flavor, color) $= (3, 3)$ to form a color singlet. Because $\bar{3} \otimes 3 = 8 \oplus 1$, there may exist nonaquark nonet with positive parity. But the states $S^0(3115)$ and $S^+(3140)$ cannot belong to this nonet, because the hypercharge of these two states is $Y = 2$, while the maximum of hypercharge of the nonet is 1.

(2) The *space* wave functions are antisymmetric: This means that the relative angular momentum ℓ is odd and starts from $\ell = 1$, and then the (flavor × color) wave functions are antisymmetric. And they must belong to the 126 dimensional irreducible representation of $SU(9) \supset SU(3)$flavor × $SU(3)$color. The reduction of the SU(9) irreducible representation 126 with respect to SU(3)flavor × SU(3)color is

$$126 = (6, 6) + (\overline{15}_2, \bar{3}) + (\bar{3}, \overline{15}_2). \quad (20)$$

So only when the four diquarks in the state (flavor, color) $= (\overline{15}_2, \bar{3})$, they can combine with the ninth quark to form a color singlet hadron. Because $\overline{15}_2 \otimes 3 = 27 \oplus \overline{10} \oplus 8$, the nonaquark

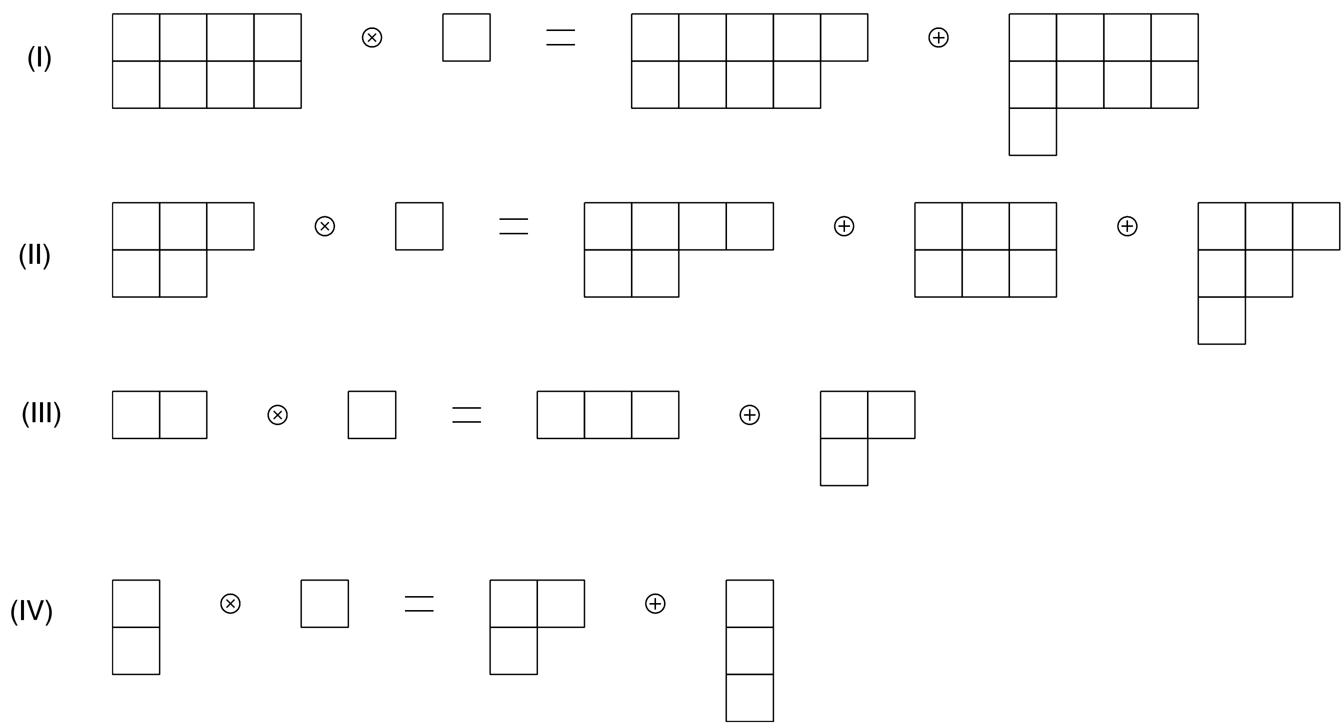

FIG. 4. direct product of four diquarks and a quark (I) $\mathbf{\overline{15}_1 \otimes 3 = \overline{35} + \overline{10}}$; (II) $\mathbf{\overline{15}_2 \otimes 3 = 27 + \overline{10} + 8}$; (III) $\mathbf{6 \otimes 3 = 10 \oplus 8}$; (IV)$\mathbf{\bar{3} \otimes 3 = 8 \oplus 1}$.

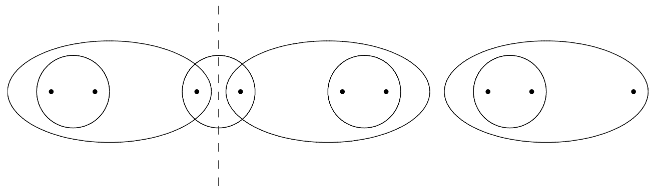

FIG. 5. The decay mechanism of nonaquark states.

can only be in flavor multiplet 27-plet, $\overline{10}$-plet, 8-plet, and the nonaquark state cannot be in flavor 35-plet. This is a rigorous result when the flavor symmetry is exact. Considering, however, that $SU(3)^{flavor}$ is an approximate symmetry which will lead to $SU(3)^{flavor} \times SU(3)^{color} \times SU(2)^{spin}$ to be approximate, so we cannot completely rule out the 35-plet.

From the quantum number of $S^0(3115)$ and $S^+(3140)$, they possibly belong to the **27**-plet or the $\overline{\mathbf{35}}$-plet. In the exact $SU(3)^{flavor} \times SU(3)^{color} \times SU(2)^{spin}$ limit, both $S^0(3115)$ and $S^+(3140)$ belong to the **27**-plet whose lowest angular momentum is $\ell = 1$ (they are P-wave states), and the corresponding weight diagram is Fig. 6. Figure 7 is the weight diagram for the 35-plet. In the figures, we show the names of these exotic states, with the subscripts that are the representation-dimensions and the isospin of the particle. Their superscripts are the charges of the state.

In tensor representations, the number of lower indices of $T^{j_1,\ldots,k_q}_{i_1,\ldots,i_p}$ is p and that of upper indices is q. Now we suppose that among its lower indices the numbers of 1, 2, and 3 are

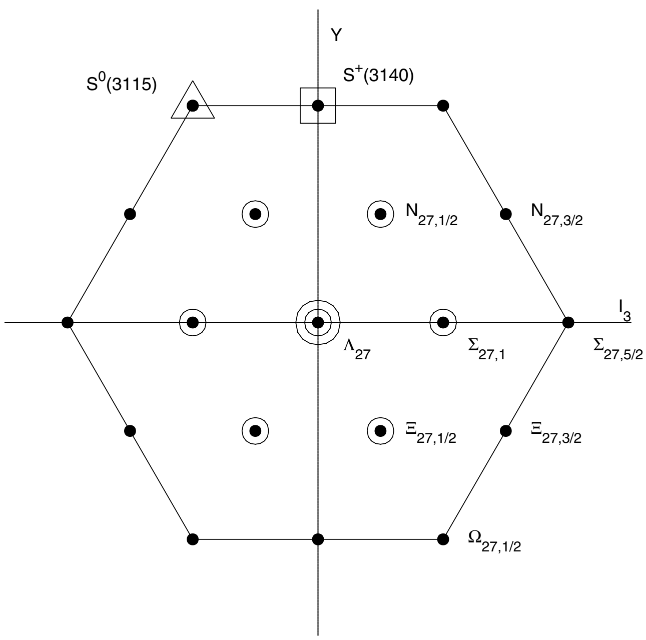

FIG. 6. The weight diagram of the nonaquark **27**-plet, and the location of $S^0(3115)$ and $S^+(3140)$ are expressly shown by the triangle and square, respectively.

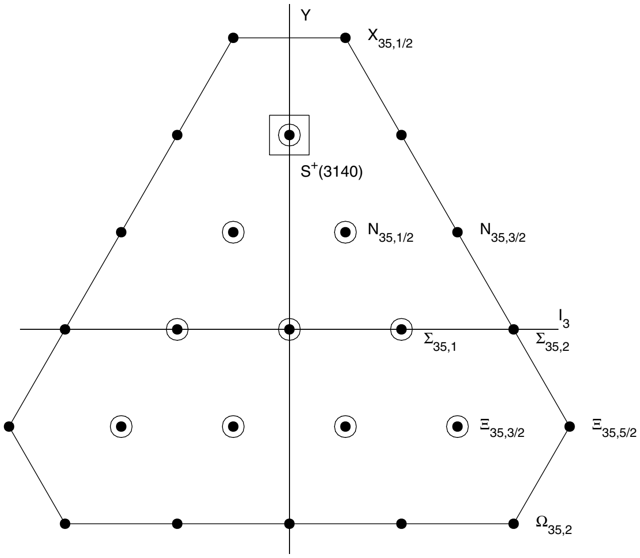

FIG. 7. The weight diagram of the nonaquark $\overline{\mathbf{35}}$-plet, only $S^+(3140)$ is specifically shown by the square, whereas $S^0(3115)$ is not shown because our analysis indicates that $S^0(3115)$ cannot belong to the $\overline{\mathbf{35}}$-plet.

p_1, p_2, and p_3, respectively, and that among upper indices it has $q_1 1$, $q_2 2$, and $q_3 3$. Then we have $p_1 + p_2 + p_3 = p$ and $q_1 + q_2 + q_3 = q$. The irreducible tensor is an eigenstate of hypercharge Y and the third component of isospin I_3 with the eigenvalues [16–18].

$$\begin{aligned} Y &= p_1 - q_1 + p_2 - q_2 - \frac{2}{3}(p - q) \\ I_3 &= \frac{1}{2}(p_1 - q_1) - \frac{1}{2}(p_2 - q_2). \end{aligned} \tag{21}$$

The charge of the particle is obtained from the Gell-Mann–Nishijima formula, $Q = I_3 + Y/2$. By this way, we can match the SU(3) tensors to the physical baryon states.

B. The wave function of nonaquark $\overline{\mathbf{27}}$-plets

It is straightforward to write out the wave functions of nonaquark $\overline{\mathbf{27}}$-plets in the flavor space by means of the irreducible representation tensors. The $S^+_{27,1}$ and $S^0_{27,1}$ read

$$S^+_{27,1} = \frac{1}{\sqrt{2}} T^{33}_{12} = \frac{1}{4}(\mathcal{T}^{33}_{1} q_2 + \mathcal{T}^{33}_{2} q_1) \tag{22}$$

$$S^0_{27,1} = \frac{1}{2} T^{33}_{22} = \frac{1}{2\sqrt{2}} \mathcal{T}^{33}_{2} q_2, \tag{23}$$

where $\mathcal{T}^{ij}_{k}$ stands for the tensors of four-diquarks states. We now provide the explicit expressions of $S^+_{27,1}$ and $S^0_{27,1}$ for the each irreducible representatives of $\overline{\mathbf{15}}_2$ in order:

(1) The case of $\overline{\mathbf{15}}_2 \subset \overline{\mathbf{6}} \otimes \overline{\mathbf{6}}$: In this case $\mathcal{T}^{jk}_{i}$ is S^{jk}_{i} which has been defined in Eq. (11). So,

$$S_1^{33} = \frac{1}{2\sqrt{2}}([u,d][s,u][u,d][u,d] + [s,u][u,d][u,d][u,d] - [u,d][u,d][u,d][s,u] - [u,d][u,d][s,u][u,d]), \quad (24)$$

$$S_2^{33} = \frac{1}{2\sqrt{2}}([u,d][u,d][u,d][d,s] + [u,d][u,d][d,s][u,d] - [u,d][d,s][u,d][u,d] - [d,s][u,d][u,d][u,d]), \quad (25)$$

the wave function of $S^+_{27_1,1}$ and $S^0_{27_1,1}$ are

$$\begin{aligned} S^+_{27_1,1} &= \frac{1}{4}(S_1^{33}q_2 + S_2^{33}q_1) \\ &= \frac{1}{8\sqrt{2}}([u,d][s,u][u,d][u,d]d + [s,u][u,d][u,d][u,d]d - [u,d][u,d][u,d][s,u]d - [u,d][u,d][s,u][u,d]d \\ &\quad + [u,d][u,d][u,d][d,s]u + [u,d][u,d][d,s][u,d]u - [u,d][d,s][u,d][u,d]u - [d,s][u,d][u,d][u,d]u); \end{aligned} \quad (26)$$

$$\begin{aligned} S^0_{27_1,1} &= \frac{1}{2\sqrt{2}}S_2^{33}q_2 \\ &= \frac{1}{8}([u,d][u,d][u,d][d,s]d + [u,d][u,d][d,s][u,d]d - [u,d][d,s][u,d][u,d]d - [d,s][u,d][u,d][u,d]d). \end{aligned} \quad (27)$$

(2) The case of $\overline{\mathbf{15}}_2 \subset \mathbf{3} \otimes \overline{\mathbf{6}}$: The $\mathcal{T}_i^{jk}$ is $\tilde{T}_i^{jk}$ which is defined in Eq. (12), and

$$\tilde{T}_1^{33} = T_1 S^{33} = \frac{1}{2}([s,u][u,d][u,d][u,d] - [u,d][s,u][u,d][u,d]) \quad (28)$$

$$\tilde{T}_2^{33} = T_2 S^{33} = \frac{1}{2}([u,d][d,s][u,d][u,d] - [d,s][u,d][u,d][u,d]) \quad (29)$$

then, the wave functions of $S^+_{27_2,1}$ and $S^0_{27_2,1}$ are

$$\begin{aligned} S^+_{27_2,1} &= \frac{1}{4}(\tilde{T}_1^{33}q_2 + \tilde{T}_2^{33}q_1) \\ &= \frac{1}{8}([s,u][u,d][u,d][u,d]d - [u,d][s,u][u,d][u,d]d + [u,d][d,s][u,d][u,d]u - [d,s][u,d][u,d][u,d]u); \end{aligned} \quad (30)$$

$$S^0_{27_2,1} = \frac{1}{2\sqrt{2}}\tilde{T}_2^{33}q_2 = \frac{1}{4\sqrt{2}}([u,d][d,s][u,d][u,d]d - [d,s][u,d][u,d][u,d]d). \quad (31)$$

(3) The case of $\overline{\mathbf{15}}_2 \subset \overline{\mathbf{6}} \otimes \mathbf{3}$: In this case $\mathcal{T}_i^{jk}$ is the tensor T_i^{jk} defined in Eq. (13), obviously

$$T_1^{33} = S^{33}T_1 = \frac{1}{2}([u,d][u,d][s,u][u,d] - [u,d][u,d][u,d][s,u]) \quad (32)$$

$$T_2^{33} = S^{33}T_2 = \frac{1}{2}([u,d][u,d][u,d][d,s] - [u,d][u,d][d,s][u,d]) \quad (33)$$

The wave functions of the two states are

$$\begin{aligned} S^+_{27_3,1} &= \frac{1}{4}(T_1^{33}q_2 + T_2^{33}q_1) \\ &= \frac{1}{8}([u,d][u,d][s,u][u,d]d - [u,d][u,d][u,d][s,u]d + [u,d][u,d][u,d][d,s]u - [u,d][u,d][d,s][u,d]u) \end{aligned} \quad (34)$$

$$S^0_{27_3,1} = \frac{1}{2\sqrt{2}} T_2^{33} q_2 = \frac{1}{4\sqrt{2}}([u,d][u,d][u,d][d,s]d - [u,d][u,d][d,s][u,d]d). \tag{35}$$

C. The wave function of the nonaquark $\overline{35}$-plet

Since $\overline{\mathbf{35}}$ cannot be completely excluded (see Sec. II A) , we should also discuss its wave function for completeness. It is easy to identify

$$T_2^{1333} = -\sqrt{6}S^0_{\overline{35},1}, \qquad T_1^{1333} = -\sqrt{3}S^+_{\overline{35},1} - \sqrt{2}S^+_{\overline{35}} \qquad T_2^{2333} = \sqrt{3}S^+_{\overline{35},1} - \sqrt{2}S^+_{\overline{35}}, \qquad T_3^{3333} = 2\sqrt{2}S^+_{\overline{35}},$$
$$T_1^{2333} = -\sqrt{6}S^{++}_{\overline{35},1} \tag{36}$$

then

$$\begin{cases} S^0_{\overline{35},1} = -\frac{1}{\sqrt{6}} T_2^{1333} \\ S^+_{\overline{35},1} = \frac{1}{2\sqrt{3}}(T_2^{2333} - T_1^{1333}) \\ S^+_{\overline{35}} = \frac{1}{2\sqrt{2}} T_3^{3333} \end{cases} \tag{37}$$

and T_n^{ijkl} is defined in Eq. (16). It is easy to see

$$T^{1333} = \frac{1}{\sqrt{6}}(S^{13}S^{23} + S^{33}S^{13} + S^{13}S^{33} + S^{31}S^{33} + S^{33}S^{13} + S^{33}S^{13}) = \frac{3}{\sqrt{6}}(S^{13}S^{33} + S^{33}S^{13})$$
$$= \frac{3}{4\sqrt{6}}([d,s][u,d][u,d][u,d] + [u,d][d,s][u,d][u,d] + [u,d][u,d][d,s][u,d] + [u,d][u,d][u,d][d,s]) \tag{38}$$

$$T^{2333} = \frac{3}{\sqrt{6}}(S^{23}S^{33} + S^{33}S^{23})$$
$$= \frac{3}{4\sqrt{6}}([s,u][u,d][u,d][u,d] + [u,d][s,u][u,d][u,d] + [u,d][u,d][s,u][u,d] + [u,d][u,d][u,d][s,u]) \tag{39}$$

$$T^{3333} = \sqrt{6}S^{33}S^{33} = \frac{\sqrt{6}}{2}[u,d][u,d][u,d][u,d] \tag{40}$$

and the wave function of $S^0_{\overline{35},1}$, $S^+_{\overline{35},1}$, $S^+_{\overline{35}}$ are as follows:

$$S^0_{\overline{35},1} = -\frac{1}{\sqrt{6}} T_2^{1333} = -\frac{1}{\sqrt{6}} T^{1333} q_2$$
$$= -\frac{1}{8}([d,s][u,d][u,d][u,d] + [u,d][d,s][u,d][u,d] + [u,d][u,d][d,s][u,d] + [u,d][u,d][u,d][d,s])d \tag{41}$$

$$S^+_{\overline{35},1} = \frac{1}{2\sqrt{3}}(T_2^{2333} - T_1^{1333}) = \frac{1}{2\sqrt{3}}(-T^{1333}q_1 + T^{2333}q_2)$$
$$= \frac{1}{8\sqrt{2}}(-[d,s][u,d][u,d][u,d]u - [u,d][d,s][u,d][u,d]u - [u,d][u,d][d,s][u,d]u - [u,d][u,d][u,d][d,s]u$$
$$+ [s,u][u,d][u,d][u,d]d + [u,d][s,u][u,d][u,d]d + [u,d][u,d][s,u][u,d]d + [u,d][u,d][u,d][s,u]d) \tag{42}$$

$$S^+_{\overline{35}} = \frac{1}{2\sqrt{2}} T_3^{3333} = \frac{1}{6\sqrt{2}}(-2T^{1333}q_1 - 2T^{2333}q_2 + T^{3333}q_3)$$
$$= -\frac{1}{8\sqrt{3}}([d,s][u,d][u,d][u,d]u + [u,d][d,s][u,d][u,d]u + [u,d][u,d][d,s][u,d]u + [u,d][u,d][u,d][d,s]u$$
$$+ [s,u][u,d][u,d][u,d] + [u,d][s,u][u,d][u,d] + [u,d][u,d][s,u][u,d] + [u,d][u,d][u,d][s,u]$$
$$- 2[u,d][u,d][u,d][u,d]s). \tag{43}$$

III. THE MASS SPECTRUM OF NONAQUARK STATES

Since all the particles belonging to an irreducible representation of SU(3) are degenerate in the SU(3) symmetry limit, it is necessary to introduce the SU(3) symmetry breaking terms into the Hamiltonian in order to obtain the mass splitting. The Hamiltonian that breaks SU(3) symmetry but still preserves the isospin symmetry and hypercharge is proportional to the Gell-Mann matrix λ_8, and the baryon mass can be obtained by constructing the SU(3) singlet term including the hypercharge tensor, in this way we obtain the Gell-Mann-Okubo mass formula:

$$M = M_0 + \alpha Y + \beta D_3^3, \tag{44}$$

where M_0 is a common mass of a given multiplet and $D_3^3 = I(I+1) - \frac{Y^2}{4} - \frac{C}{6}$ with $C = 2(p+q) + \frac{2}{3}(p^2 + pq + q^2)$ for the (p, q) representation. α and β are mass constants that are in principle different for different multiplets. Using these constants, we can obtain the masses of all the baryons within the multiplet. Note that in this picture the isospin is conserved.

A. The mass spectrum of the nonaquark 27-plet

In the case of the **27**-plet, $p = q = 2$ and the corresponding weight diagram is Fig. 6. By using the Gell-Mann-Okubo mass formula Eq. (44) we can get all the masses of these states.

$$\begin{aligned} &M_{S_{27,1}} = M_{27} + 2\alpha_{27} - \frac{5}{3}\beta_{27}, \quad M_{N_{27,3/2}} = M_{27} + \alpha_{27} + \frac{5}{6}\beta_{27}, \quad M_{N_{27,1/2}} = M_{27} + \alpha_{27} - \frac{13}{6}\beta_{27},\\ &M_{\Sigma_{27,2}} = M_{27} + \frac{10}{3}\beta_{27}, \quad M_{\Sigma_{27,1}} = M_{27} - \frac{2}{3}\beta_{27}, \quad M_{\Lambda_{27}} = M_{27} - \frac{8}{3}\beta_{27}, \quad M_{\Xi_{27,3/2}} = M_{27} - \alpha_{27} + \frac{5}{6}\beta_{27},\\ &M_{\Xi_{27,1/2}} = M_{27} - \alpha_{27} - \frac{13}{6}\beta_{27}, \quad M_{\Omega_{27,1}} = M_{27} - 2\alpha_{27} - \frac{5}{3}\beta_{27}. \end{aligned} \tag{45}$$

The Gell-Mann-Okubo mass relation for **27**-plet nonaquark baryons is

$$3M_{\Lambda_{27}} + M_{\Sigma_{27,1}} = 2(M_{N_{27,1/2}} + M_{\Xi_{27,1/2}}) = 4M_{27} - \frac{26}{3}\beta_{27}. \tag{46}$$

The equal mass space relations also exist in the two separate sectors: $(\Omega_{27,1}, \Xi_{27,\frac{3}{2}}, \Sigma_{27,2})$ and $(\Sigma_{27,2}, N_{27,\frac{3}{2}}, S_{27,1})$. Their mass relations are as follows

$$M_{\Omega_{27,1}} - M_{\Xi 27,3/2} = M_{\Xi 27,3/2} - M_{\Sigma_{27,2}} = -\alpha_{27} - \frac{5}{2}\beta_{27}, \tag{47}$$

$$M_{\Sigma_{27,2}} - M_{N_{27,3/2}} = M_{N_{27,3/2}} - M_{S_{27,1}} = -\alpha_{27} + \frac{5}{2}\beta_{27}. \tag{48}$$

From Eq. (45) we can further obtain some relation between the masses of these states

$$\begin{aligned} &M_{\Lambda_{27}} + M_{S_{27,1}} = 2M_{N_{27,1/2}}, \quad M_{\Sigma_{27,2}} + M_{S_{27,1}} = 2M_{N_{27,3/2}}, \quad 3M_{\Sigma_{27,1}} + 3M_{S_{27,1}} = 4M_{N_{27,1/2}} + 2M_{N_{27,3/2}},\\ &3M_{\Xi_{27,3/2}} + 6M_{S_{27,1}} = 5M_{N_{27,1/2}} + 4M_{N_{27,3/2}}, \quad 3M_{\Xi_{27,1/2}} + 6M_{S_{27,1}} = 8M_{N_{27,1/2}} + M_{N_{27,3/2}},\\ &3M_{\Omega_{27,1}} + 9M_{S_{27,1}} = 10M_{N_{27,1/2}} + 2M_{N_{27,3/2}}. \end{aligned} \tag{49}$$

Both the masses of $\overline{\mathbf{35}}$-plets and the masses of **27**-plets contain three parameters: $M_{\overline{35}}$, $\alpha_{\overline{35}}$, $\beta_{\overline{35}}$ or M_{27}, α_{27}, β_{27}, but we only know experimentally the mass of $S^0(3115)$ and $S^+(3140)$. So we cannot fix the masses of other nonaquark states which are predicted by us.

It mostly seems that $S^0(3115)$ and $S^+(3140)$ would belong to the same isospin multiplet and the mass difference between them is mainly due to electromagnetic interaction and the mass differences between u quarks and d quarks.

B. The mass spectrum of the nonaquark $\overline{\mathbf{35}}$-plet

By means of the Gell-Mann–Okubo mass formula Eq. (44), and noting the $\overline{\mathbf{35}}$-plet with $p = 1, q = 4$ whose weight diagram is shown in Fig. 7, the masses of all the exotic nonaquark states are as follows.

$$
\begin{aligned}
&M_{X_{\overline{35},1/2}} = M_{\overline{35}} + 3\alpha_{\overline{35}} - \frac{11}{2}\beta_{\overline{35}}, \qquad M_{S_{\overline{35},1}} = M_{\overline{35}} + 2\alpha_{\overline{35}} - 3\beta_{\overline{35}}, \qquad M_{S_{\overline{35}}} = M_{\overline{35}} + 2\alpha_{\overline{35}} - 5\beta_{\overline{35}},\\
&M_{N_{\overline{35},3/2}} = M_{\overline{35}} + \alpha_{\overline{35}} - \frac{\beta_{\overline{35}}}{2}, \qquad M_{N_{\overline{35},1/2}} = M_{\overline{35}} + \alpha_{\overline{35}} - \frac{7}{2}\beta_{\overline{35}}, \qquad M_{\Sigma_{\overline{35},2}} = M_0 + 2\beta_{\overline{35}},\\
&M_{\Sigma_{\overline{35},1}} = M_{\overline{35}} - 2\beta_{\overline{35}}, \qquad M_{\Xi_{\overline{35},5/2}} = M_{\overline{35}} - \alpha_{\overline{35}} + \frac{9}{2}\beta_{\overline{35}}, \qquad M_{\Xi_{\overline{35},3/2}} = M_{\overline{35}} - \alpha_{\overline{35}} - \frac{1}{2}\beta_{\overline{35}},\\
&M_{\Omega_{\overline{35},2}} = M_{\overline{35}} - 2\alpha_{\overline{35}} + \beta_{\overline{35}}.
\end{aligned} \tag{50}
$$

We can find that the equal space rule holds for two sectors of nonaquark baryons: ($X_{\overline{35},1/2}$, $S_{\overline{35},1}$, $N_{\overline{35},3/2}$, $\Sigma_{\overline{35},2}$, $\Xi_{\overline{35},5/2}$) and ($S_{\overline{35}}$, $N_{\overline{35},1/2}$, $\Sigma_{\overline{35},1}$, $\Xi_{\overline{35},3/2}$, $\Omega_{\overline{35},2}$). They satisfy the mass relations as follows:

$$M_{X_{\overline{35},1/2}} - M_{S_{\overline{35},1}} = M_{S_{\overline{35},1}} - M_{N_{\overline{35},3/2}} = M_{N_{\overline{35},3/2}} - M_{\Sigma_{\overline{35},2}} = M_{\Sigma_{\overline{35},2}} - M_{\Xi_{\overline{35},5/2}} = \alpha_{\overline{35}} - \frac{5}{2}\beta_{\overline{35}}, \tag{51}$$

$$M_{S_{\overline{35}}} - M_{N_{\overline{35},1/2}} = M_{N_{\overline{35},1/2}} - M_{\Sigma_{\overline{35},1}} = M_{\Sigma_{\overline{35},1}} - M_{\Xi_{\overline{35},3/2}} = M_{\Xi_{\overline{35},3/2}} - M_{\Omega_{\overline{35},2}} = \alpha_{\overline{35}} - \frac{3}{2}\beta_{\overline{35}}. \tag{52}$$

We can also derive the mass relation between these nonaquark states

$$
\begin{aligned}
&M_{X_{\overline{35},1/2}} + M_{N_{\overline{35},3/2}} = 2M_{S_{\overline{35},1}}, \qquad 3(M_{S_{\overline{35}}} - M_{S_{\overline{35},1}}) = 2(M_{N_{\overline{35},1/2}} - M_{N_{\overline{35},3/2}}), \qquad M_{\Sigma_{\overline{35},2}} + M_{S_{\overline{35},1}} = 2M_{N_{\overline{35},3/2}},\\
&M_{\Sigma_{\overline{35},1}} + 3M_{S_{\overline{35},1}} = 2(M_{S_{\overline{35}}} + M_{N_{\overline{35},3/2}}), \qquad M_{\Xi_{\overline{35},5/2}} + 2M_{S_{\overline{35},1}} = 3M_{N_{\overline{35},3/2}}, \qquad 2M_{\Xi_{\overline{35},3/2}} + 9M_{S_{\overline{35},1}} = 5M_{S_{\overline{35}}} + 6M_{N_{\overline{35},3/2}}\\
&M_{\Omega_{\overline{35},2}} + 6M_{S_{\overline{35},1}} = 3M_{S_{\overline{35}}} + 4M_{N_{\overline{35},3/2}}.
\end{aligned} \tag{53}
$$

IV. DECAY OF THE NONAQUARK STATES

We assume the nonaquark state decays arise by a "fall-apart" mechanism [19–22] without need for gluon exchange to trigger the decay. There are some discussions on this mechanism in the pentaquark spectrum studies. By this mechanism, a diquark in the pentaquark must be so clever that its two quarks are detached to two isolated quarks, and one of them enters into the adjacent diquarks to form a baryon separately, and another combines with the residual antiquark to form a meson, and then a pentaquark baryon decays into a usual baryon plus a meson. In this mechanism, the dynamics from the color coupling contributes a common factor to the decay amplitude for a certain flavor multiplet which is irrelevant to the discussions on its decay branch fractions. Extending this "fall-apart" mechanism to nonaquark state decays is natural and straightforward: (1) one diquark is detached into two quarks; (2) these two quarks enter the adjacent diquarks and form two baryons separately; (3) the ninth quark also enters a diquark to form a baryon, and then the nonaquark state consequently decays into three baryons. We show this mechanism in Fig. 5.

To the baryon octet, its quark content and the corresponding tensor are well known, which is listed in the Table I. The tensor basis of the baryon octet is

$$B^l_r = \frac{1}{\sqrt{3}}\varepsilon^{lmn}S_{rn}q_m \tag{54}$$

with $S_{rn} = \frac{1}{\sqrt{2}}(q_r q_n + q_n q_r)$. And the baryon singlet Λ^0_1 is given by

$$\Lambda^0_1 = \frac{1}{2}\frac{1}{\sqrt{6}}\varepsilon^{klm}(q_l q_m - q_m q_l)q_k = \frac{1}{\sqrt{6}}([d, s]u + [s, u]d + [u, d]s). \tag{55}$$

From these wave functions we can see:

$$\begin{cases} [d, s]u = \Sigma^0 + \frac{\sqrt{6}}{3}\Lambda^0_1 - \frac{\sqrt{3}}{3}\Lambda^0_8 \\ [s, u]d = -\Sigma^0 + \frac{\sqrt{6}}{3}\Lambda^0_1 - \frac{\sqrt{3}}{3}\Lambda^0_8 \\ [u, d]s = \frac{\sqrt{6}}{3}\Lambda^0_1 + \frac{2\sqrt{3}}{3}\Lambda^0_8. \end{cases} \tag{56}$$

A. The decay of the 27-plet

Since the **27**-plet is degenerate, i.e., $\mathbf{27_1}$, $\mathbf{27_2}$, and $\mathbf{27_3}$ are the three irreducible representatives of the **27**-dimension, we discuss the decays for each case following the method

TABLE I. Baryon octet and its flavor wave function.

State	Tensor	Quark content
p	B^3_1	$\frac{1}{\sqrt{2}}[u, d]u$
n	B^3_2	$\frac{1}{\sqrt{2}}[u, d]d$
Σ^+	B^2_1	$\frac{1}{\sqrt{2}}[s, u]u$
Σ^0	$\frac{1}{\sqrt{2}}(B^1_1 - B^2_2)$	$\frac{1}{2}([d, s]u + [u, s]d])$
Σ^-	B^1_2	$\frac{1}{\sqrt{2}}[d, s]d$
Λ^0	$-\frac{3}{\sqrt{6}}B^3_3$	$\frac{1}{\sqrt{12}}(2[u, d]s - [d, s]u - [s, u]d)$
Ξ^-	B^1_3	$\frac{1}{\sqrt{2}}[d, s]s$
Ξ^0	B^2_3	$\frac{1}{\sqrt{2}}[s, u]s$

of dealing with the "fall-apart" decay [23]. Starting with the wave function of $S^+_{27_1,1}$ (or $S^0_{27_1,1}$ etc.) which is given in Eq. (26) [Eq. (27), etc.], we rewrite the wave function in the form of $(qqq)(qqq)(qqq)$ which is similar to Ref. [23], then using Eq. (56) and Table I and considering fermion statistics, we can map $S^+_{27_1,1}$ (or $S^0_{27_1,1}$ etc.) into the ground state of three baryons, from which we can learn what particles are the decay products of the $S^+_{27_1,1}$ (or $S^0_{27_1,1}$ etc.), and derive further the ratios of the branch fractions of these channels. We discuss the decays for each case, respectively, by means of the previous method of dealing with the decays in the following:

(1) The first case: The wave function of $S^+_{27_1,1}$ and $S^0_{27_1,1}$ is given by Eqs. (26) and (27) separately, and they are mapped into

$$S^+_{27_1,1} \to \frac{1}{\sqrt{2}}\Sigma^0 pn, \tag{57}$$

$$S^0_{27_1,1} \to \frac{1}{2}\left(\sqrt{2}\Sigma^- pn + \frac{\sqrt{6}}{3}\Lambda^0_1 nn + \frac{2\sqrt{3}}{3}\Lambda^0_8 nn\right). \tag{58}$$

Then we obtain that the main decay channel of $S^+_{27_1,1}$ is ΣNN, and the ratio of coupling constants is

$$g(S^+_{27_1,1}\Sigma^- pn):g(S^+_{27_1,1}\Lambda^0_1 nn):g(S^+_{27_1,1}\Lambda^0_8 nn) = \sqrt{2}:\frac{\sqrt{6}}{3}:\frac{2\sqrt{3}}{3}. \tag{59}$$

Considering phase space effect, we get

$$\frac{BR[S^+_{27_1,1} \to \Sigma NN]}{BR[S^+_{27_1,1} \to \Lambda NN]} = \frac{2}{((\frac{\sqrt{6}}{3})^2 + \frac{2\sqrt{3}}{3})^2) \times 4.164} \approx 0.24. \tag{60}$$

Therefore, $S^0_{27_1,1}$ dominantly decays into ΛNN, and $S^0_{27_1,1}$ cannot be $S^0(3115)$.

(2) The second case: Eqs. (30) and (31) give the wave function of $S^+_{27_2,1}$ and $S^0_{27_2,1}$ in this case, and the map is as follows:

$$S^+_{27_2,1} \to \frac{1}{2\sqrt{2}}(\Sigma^+ nn + \Sigma^- pp + \sqrt{2}\Sigma^0 pn), \tag{61}$$

$$S^0_{27_2,1} \to \frac{1}{2\sqrt{2}}(-\Sigma^0 nn + \sqrt{2}\Sigma^- pn - \sqrt{6}\Lambda^0_1 nn - \sqrt{3}\Lambda^0_8 nn). \tag{62}$$

We note that in this case $S^+_{27_2,1}$ also can only decay into ΣNN, and the ratio of the branch fractions

$$BR[S^+_{27_2,1} \to \Sigma^+ nn]:BR[S^+_{27,1} \to \Sigma^- pp]:BR[S^+_{27,1} \to \Sigma^0 pn] = 1:1:2 \tag{63}$$

and the main decay modes of $S^0_{27_2,1}$ is ΛNN.

$$\frac{BR[S^0_{27_2,1} \to \Sigma NN]}{BR[S^0_{27_2,1} \to \Lambda NN]} \approx 0.08. \tag{64}$$

(3) The third case: The wave function of $S^+_{27_3,1}$ and $S^0_{27_3,1}$ is given by Eqs. (34) and (35), similarly

$$S^+_{27_3,1} \to \frac{3}{2}\Sigma^0 pn + \frac{1}{2\sqrt{2}}\Sigma^+ nn + \frac{1}{2\sqrt{2}}\Sigma^- pp, \tag{65}$$

$$S^0_{27_3,1} \to \frac{1}{4\sqrt{2}}\left(6\sqrt{2}\Sigma^- pn - 2\Sigma^0 nn - \frac{2\sqrt{6}}{3}\Lambda^0_1 nn + \frac{2\sqrt{3}}{3}\Lambda^0_8 nn\right). \tag{66}$$

From the above two formula, we know that in this case $S^+_{27_3,1}$ decay mainly into ΣNN not to ΛNN, and

$$BR[S^+_{27_3,1} \to \Sigma^0 pn]:BR[S^+_{27_3,1} \to \Sigma^+ nn]:BR[S^+_{27_3,1} \to \Sigma^- pp] = 18:1:1 \tag{67}$$

and the ratio of branch fractions

$$\frac{BR[S^0_{27_3,1} \to \Sigma NN]}{BR[S^0_{27_3,1} \to \Lambda nn]} \approx 2.8 \tag{68}$$

We note that in this case the main decay mode of $S^0_{27_3,1}$ is ΣNN, which is consistent with the decay of $S^0(3115)$.

B. The decay of the $\overline{35}$-plet

Starting with the wave function of $S^+_{\overline{35},1}$ which is given in Eq. (42), we discuss the decays by means of the method used in the previous subsection to deal with the decays. We map $S^+_{\overline{35},1}$ into the ground state of three baryons

$$S^+_{\overline{35},1} \to \frac{1}{2}(\Sigma^- pp + \Sigma^+ nn). \tag{69}$$

So, $S^+_{\overline{35},1}$ can only decay into $\Sigma^- NN$, but cannot decay into $\Lambda^- NN$ (N stands for nucleon, i.e., proton or neutron). And the ratio of branch fractions is

$$\frac{BR[S^+_{\overline{35},1} \to \Sigma^- pp]}{BR[S^+_{\overline{35},1} \to \Sigma^+ nn]} = \frac{1}{1} = 1. \tag{70}$$

Similarly $S^+_{\overline{35}}$ whose wave function is defined in Eq. (43) are mapped onto

$$S^+_{\overline{35}} \to \frac{1}{\sqrt{6}}(\Sigma^- pp - \Sigma^+ nn). \tag{71}$$

Also $S^+_{\overline{35}}$ decay only into $\Sigma^- NN$, can not decay into $\Lambda^- NN$, and the ratio of branch fractions is

$$\frac{BR[S^+_{\overline{35}} \to \Sigma^- pp]}{BR[S^+_{\overline{35}} \to \Sigma^+ nn]} = \frac{1}{1} = 1. \qquad (72)$$

We also obtain that the $S^+_{\overline{35}}\Sigma^- pp$ and $S^+_{\overline{35}}\Sigma^+ nn$ interactions have different phases, while the $S^+_{\overline{35},1}\Sigma^- pp$ and $S^+_{\overline{35},1}\Sigma^+ nn$ interactions have the same phase [see Eqs. (71) and (69)].

In the same way, starting from Eq. (41) which is the wave function of $S^0_{\overline{35},1}$, we have

$$S^0_{\overline{35},1} \to \frac{1}{2}(-\Sigma^0 + \sqrt{3}\Lambda^0_8)nn. \qquad (73)$$

So the main decay channel of $S^0_{\overline{35},1}$ is ΛNN, but not ΣNN, i.e., the ratio of the effective couplings reads

$$\frac{g(S^0_{\overline{35},1}\Sigma^0 nn)}{g(S^0_{\overline{35},1}\Lambda^0_8 nn)} = -\frac{1}{\sqrt{3}}. \qquad (74)$$

Considering the different three body phase space, we can obtain the ratio of branch fractions

$$\frac{BR[S^0_{\overline{35},1} \to \Sigma^0 nn]}{BR[S^0_{\overline{35},1} \to \Lambda^0_8 nn]} \approx 0.05. \qquad (75)$$

So the observed $S^0(3115)$ [1] cannot be $S^0_{\overline{35},1}$.

From the decay of the $\overline{\mathbf{35}}$-plet and the **27**-plet, we see that the nonaquark state $S^0(3115)$ can only possibly belong to the **27**-plet, and its flavor configuration is Eq. (35), or its main component is $S^0_{27_3,1}$ with small mixing of $S^0_{27_1,1}$, $S^0_{27_2,1}$, $S^0_{\overline{35},1}$. While $S^+(3140)$ may belong to the $\overline{\mathbf{35}}$-plet or the **27**-plet, it possibly is $S^+_{27_1,1}$, $S^+_{27_2,1}$, $S^+_{27_3,1}$, $S^+_{\overline{35},1}$ or the mixing of all of them, since these states have the same quantum numbers. From the discussion of Sec. II A, we know that in the exact $SU(3)^{\text{flavor}} \times SU(3)^{\text{color}} \times SU(2)^{\text{spin}}$ limit, $S^0(3115)$ and $S^+(3140)$ possibly belong to the **27**-plet. This further supports our suggestion that $S^0(3115)$ can only possibly belong to the **27**-plet. Furthermore, in this case the nonaquark states have negative parity. This is an unusual result, and since "standard" nonaquark states, which involve 9 quarks in relative S-wave, have positive parity, an exotic nonaquark **27**-plet may really exist. We can obtain useful information about $S^+(3140)$ by experimentally measuring the branch fractions of its decay channels.

V. DISCUSSION AND CONCLUSION

In summary, we have obtained the wave functions of the nonaquark states in the SU(3) quark model with diquark correlation using standard direct tensor decomposition, we predict the existence of other nonaquark states. It would be helpful for constructing the effective interaction Lagrangian to describe the nonaquark decays with rational SU(3) flavor structure. We obtain some interesting mass sum rules for the nonaquark $\overline{\mathbf{35}}$-plet and **27**-plet, but we are still open to fix the spectrum of the $\overline{\mathbf{35}}$-plet and **27**-plet due to the scarcity of experiments. More data are expected. Under the assumption of the "fall-apart" decay mechanism, which has been subtly used in studying pentaquark decays, we find out that the $S^0(3115)$ belongs to a **27**-plet. Its main component is $S^0_{27_3,1}$, and the mixing with $S^0_{27_1,1}$, $S^0_{27_2,1}$, $S^0_{\overline{35},1}$ must be few. It is possible that $S^0(3115)$ and $S^+(3140)$ belong to the same isospin multiplet, since their mass difference is about 25 MeV, which can be interpreted by electromagnetic interaction and u, d quark mass difference. Furthermore, in the exact $SU(3)^{\text{flavor}} \times SU(3)^{\text{color}} \times SU(2)^{\text{spin}}$ limit, $S^0(3115)$ and $S^+(3140)$ belong to the **27**-plet, and its parity is negative. We suggest the study of the decays of $S^+(3140)$ more in experiment, especially the ratios of the various decay modes branch fractions, through which we can learn the structure of $S^+(3140)$ and maybe discover a new nonaquark state.

There may be other types of quark correlation in the nonaquark state, such as (qqq) − (qqq) − (qqq), since either (qq$\bar{\text{q}}$) − (qq) quark correlation [8] or diquark correlation [7] is possible in pentaquark. They have different flavor and color structures from the diquark correlation, so the masses, the decay property, and the products etc. of the states may be different from the prediction of the quark model with diquark correlation.

It would be interesting to study the mixing between the nonaquark states with same quantum numbers. It is necessary to consider the mixing in order to compare theoretical predictions with experimental data more precisely. This would be highly nontrivial.

Finally, we might imagine that this multibaryon problem could be studied in other quark models, e.g., the model in [24], and in the chiral soliton model [25].

ACKNOWLEDGMENTS

We acknowledge Dr. J. Deng for his help in plotting the diagrams. This work is partially supported by National Natural Science Foundation of China under Grant Nos. 90403021, and by the PhD Program Funds of the Education Ministry of China and KJCX2-SW-N10 of the Chinese Academy.

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