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Coupled-channel model for charmonium levels and an option for $X(3872)$

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The effects of charmed meson loops on the spectrum of charmonium are considered, with special attention paid to the levels above open-charm threshold. It is found that the coupling to charmed mesons generates a structure at the $D\bar{D}^*$ threshold in the 1^{++} partial wave. The implications for the nature of the $X(3872)$ state are discussed.

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I. INTRODUCTION

The charmonium spectroscopy has again become a very interesting field. On one hand, the 2^1S_0 η_c' state was discovered by Belle [1] and confirmed by *BABAR* [2] and CLEO [3], and the 1^1P_1 h_c was observed in Fermilab [4] and by CLEO [5]. The masses of these long missing states are in perfect agreement with the predictions of quark model. At the same time, a new state $X(3872)$ was found by Belle [6] and CDF [7].

This discovery has attracted much attention. As the state is just at the $D\bar{D}^*$ threshold, it was immediately suggested [8] that it might be a $D\bar{D}^*$ molecule bound by pion exchange (''deuson''), considered long ago in [9] and, much earlier, in [10,11]. This requires 1^{++} quantum numbers, and this assignment seems to be favored by the data [12–14]. The discovery channel is $\pi^+\pi^-J/\psi$ with dipion most probably originating from the ρ. Together with the observation of X in the $\omega J/\psi$ channel [15] this opens fascinating possibilities for strong isospin violation, which is also along the lines of the deuson model.

Other options for $X(3872)$ are under discussion in the literature, see e.g. [16–19]. The most obvious possibility of X being a $c\bar{c}$ state seems to be ruled out by its mass: the state is too high to be a $1D$ charmonium, and too low to be a $2P$ one [17]. This assumes that we do know the spectrum of higher charmonia, namely, the fine splittings and the role of coupling to D-meson pairs. It is the latter issue which is addressed in the present paper.

The mechanism of open-flavor strong decay is not well-understood. The simplest model for light-quark pair creation is the so-called $^3P_0$ model, suggested many years ago [20]. It assumes that the pair is created with vacuum ($^3P_0$) quantum numbers uniformly in space. The application of this model has a long history [21–23]. Systematic studies [24,25] of the decays of light and strange quarkonia show that with a $^3P_0$ -type amplitude calculated widths agree with data to within 25%–40%. Recently the charmonia decays [26] and decays of D and D_s mesons [27] were considered in the framework of the $^3P_0$ model.

There exist also microscopic models of strong decays, which relate the pair-creation interaction to the interaction responsible for the formation of the spectrum, by constructing the current-current interaction due to confining force and one-gluon exchange. Among these is the Cornell model [28] which assumes that confinement has Lorentz vector nature. The model [29] assumes that the confining interaction is the scalar one, while one-gluon-exchange is, of course, Lorentz vector. Possible mechanisms of strong decays were studied in the framework of Field Correlator Method (FCM) [30], and an effective $^3P_0$ operator for open-flavor decay has emerged from this study, with the strength computed in terms of FCM parameters (string tension and gluonic correlation length).

Most of the above-mentioned papers are devoted to computing the widths, and only a few consider the effects of virtual hadronic loops on the spectra. The Cornell model [28] has presented a detailed analysis of charmonia with coupling to D mesons taken into account. The recent update [31] of the Cornell model has presented splittings caused by coupling to mesonic channels for $1D$ and $2P$ $c\bar{c}$ levels, confirming the previous result: $X(3872)$ is well above the range of $1D$ levels and well below the range of $2P$ ones. The paper [32] has reported first results for hadronic shifts of lower charmonia due to mixing with D meson pairs, calculated within the $^3P_0$ model. The shifts appear to be alarmingly large.

Meanwhile, phenomenological coupled-channel models like [33–36] accumulate experience on the possibilities to generate nontrivial effects due to the coupling to hadronic channels. As a recent example one should mention the analyses [37] of new D_{sJ} states with masses considerably lower than quark model predictions, and coupling to mesonic channels being responsible for these anomalously low masses. It is interesting to note that the coupled-channel calculations performed in the framework of chiral Lagrangian approach [38] has arrived at the same conclusions.

In this paper the coupled-channel model for charmonia levels is presented, based on the nonrelativistic quark model for $c\bar{c}$ spectrum and $^3P_0$-type model for pair-creation. In Sec. II the dynamics of coupled channels is briefly outlined. Section III introduces the quark model. Sections IV and V contain the results which are discussed in Sec. VI. The paper ends with a short summary.

II. DYNAMICS OF COUPLED CHANNELS

The details of coupled-channel model can be found e.g. in [28,39]. Here I review the essentials.

In what follows the simplest version of coupled-channel model is employed. Namely, it is assumed that the hadronic state is represented as

$$|\Psi\rangle = \begin{pmatrix} \sum_\alpha c_\alpha|\psi_\alpha\rangle \\ \sum_i \chi_i|M_1(i)M_2(i)\rangle \end{pmatrix}, \quad (1)$$

where the index α labels bare confined states $|\psi_\alpha\rangle$ with the probability amplitude c_α, and χ_i is the wave function in the i-th two-meson channel $|M_1(i)M_2(i)\rangle$. The wave function $|\Psi\rangle$ obeys the equation

$$\hat{\mathcal{H}}|\Psi\rangle = M|\Psi\rangle, \qquad \hat{\mathcal{H}} = \begin{pmatrix} \hat{H}_c & \hat{V} \\ \hat{V} & \hat{H}_{M_1M_2} \end{pmatrix}, \quad (2)$$

where $\hat{H}_c$ defines the discrete spectrum of bare states, with $\hat{H}_c|\psi_\alpha\rangle = M_\alpha|\psi_\alpha\rangle$. The part $\hat{H}_{M_1M_2}$ includes only the free-meson Hamiltonian, so that the direct meson-meson interaction (e.g., due to t or u channel exchange forces) is neglected. The term $\hat{V}$ is responsible for dressing of the bare states.

Consider one bare state $|\psi_0\rangle$ (the generalization to multilevel case is straightforward). The interaction part is given by the transition form factor $f_i(\boldsymbol{p})$,

$$\langle\psi_0|\hat{V}|M_{i1}M_{i2}\rangle = f_i(\boldsymbol{p}_i), \quad (3)$$

where $\boldsymbol{p}_i$ is the relative momentum in i-th mesonic channel. Then (2) leads to the system of coupled equations for $c_0(M)$ and $\chi_{i,M}(\boldsymbol{p}_i)$:

$$\begin{cases} c_0(M)M_0 + \sum_i \int f_i(\boldsymbol{p})\chi_{i,M}(\boldsymbol{p})d^3p = Mc_0(M), \\ \left(m_{i1} + m_{i2} + \frac{p^2}{2\mu_i}\right)\chi_{i,M}(\boldsymbol{p}_i) + c_0(M)f_i(\boldsymbol{p}_i) = M\chi_{i,M}(\boldsymbol{p}_i). \end{cases} \quad (4)$$

Here $\mu_i = m_{i1}m_{i2}/(m_{i1} + m_{i2})$ is the reduced mass in the system of mesons with the masses m_{i1} and m_{i2}, and M_0 is the mass of the bare state. In what follows the formalism will be applied to the system of charmed mesons, so the nonrelativistic kinematics is employed in (4). The fully relativistic version of coupled-channel model is presented in [34].

With the help of (4) one easily calculates the t matrix in the mesonic system:

$$t_{ik}(\boldsymbol{p}_i, \boldsymbol{p}'_k, M) = \frac{f_i(\boldsymbol{p}_i)f_k(\boldsymbol{p}'_k)}{M - M_0 + g(M)},$$
$$g(M) = \sum_i g_i(M), \quad g_i(M) = \int \frac{f_i(\boldsymbol{p})f_i(\boldsymbol{p})}{\frac{p^2}{2\mu_i} - E_i - i0}d^3p, \quad (5)$$

where $E_i = M - m_{i1} - m_{i2}$. The quantity $g(M)$ is often called the hadronic shift of the bare state $|\psi_0\rangle$, as the masses of physical states are defined, in accordance with Eq. (5), from the equation

$$M - M_0 + g(M) = 0. \quad (6)$$

Let the Eq. (6) have the solution M_B with M_B smaller than the lowest mesonic threshold, so there is a bound state with the wave function

$$|\Psi_B\rangle = \begin{pmatrix} \cos\theta|\psi_0\rangle \\ \sin\theta\sum_i \chi_{iB}(\boldsymbol{p}_i)\rangle \end{pmatrix}, \qquad \langle\Psi_B|\Psi_B\rangle = 1,$$
$$\cos\theta = \langle\psi_0|\Psi_B\rangle. \quad (7)$$

Here $\sum_i \chi_{iB}(\boldsymbol{p}_i)$ is normalized to unity, and $\cos\theta$ defines the admixture of the bare state $|\psi_0\rangle$ in the physical state $|\Psi_B\rangle$. The explicit expression for this admixture reads

$$Z \equiv \cos^2\theta = \left(1 + \sum_i \int \frac{f_i(\boldsymbol{p})f_i^*(\boldsymbol{p}')d^3p}{(\frac{p^2}{2\mu_i} + \epsilon_i)^2}\right)^{-1}$$
$$= \left(1 + \frac{\partial g(M)}{\partial M}|_{M=M_B}\right)^{-1}, \quad (8)$$

$\epsilon_i = m_{i1} + m_{i2} - M_B$, $\epsilon_i > 0$. As far as I know, this Z factor was first introduced by S. Weinberg in [40] many years ago as the field renormalization factor which defines the probability to find the physical deuteron $|d\rangle$ in a bare elementary-particle state $|d_0\rangle$, $Z = |\langle d_0|d\rangle|^2$.

Even more detailed information is contained in the continuum counterpart of the factor Z, the spectral density $w(M)$ of the bare state, given by

$$w(M) = \sum_i w_i(M),$$
$$w_i(M) = 4\pi\mu_i p_i|c(M)|^2\Theta(M - m_{i1} - m_{i2}), \quad (9)$$

where $c(M)$ is the probability amplitude to find the bare state in the continuum wave function $|\Psi\rangle_M$. With $c(M)$ found from the system of Eqs. (4), one can calculate $w(M)$:

$$w(M) = \frac{1}{2\pi i}\left(\frac{1}{M - M_0 + g^*(M)} - \frac{1}{M - M_0 + g(M)}\right). \quad (10)$$

As shown in [41], the normalization condition for the distribution $w(M)$ follows from the completeness relation for the total wave function (1) projected onto bare state channel, and reads:

$$\int_{m_{01}+m_{02}}^{\infty} w(M)dM = 1 - Z, \quad (11)$$

if the system possesses a bound state, and

$$\int_{m_{01}+m_{02}}^{\infty} w(M)dM = 1, \quad (12)$$

if there is no bound state ($m_{01} + m_{02}$ is the lowest threshold). In the case of bound state present, all the information on the factor Z is encoded, due to Eq. (11), in the $w(M)$ too. On the other hand, the analysis in terms of $w(M)$ can be performed in the case of resonance as well, as exemplified in [42].

In the latter case the t matrix poles are situated in the complex plane. While the positions of the poles are the fundamental quantities, another quantities are useful for practical purposes. Namely, one defines the visible resonance mass M_R from the equation

$$M_R - M_0 + \mathrm{Re}\, g(M_R) = 0, \tag{13}$$

and calculates the visible width as

$$\Gamma = 2\Re \,\mathrm{Im}\, g(M_R), \quad \Re = \left(1 + \frac{\partial \,\mathrm{Re}\, g(M)}{\partial M}|_{M=M_R}\right)^{-1}. \tag{14}$$

Clearly this brings the t matrix into Breit-Wigner form, i.e. in the form in which experimental data are usually delivered. In what follows the factor $\Re$ will be called the renormalization factor.

There are some limitations of course, as not the every peak has the Breit-Wigner shape. In the case of overlapping resonances the formulas (13) and (14) do not work. The special case of near-threshold S-wave resonance is not described by Breit-Wigner or Flattè formula, and the scattering length parametrization is more appropriate [43].

The quantities Z and $w(M)$ are the ones of immediate relevance. Indeed, there is no hope that, say, the elastic $D\bar{D}$ scattering will be measured some time. Our knowledge on mesonic resonances comes from external reactions, like e^+e^- annihilation, $\gamma\gamma$ collisions, B-meson decays etc. Assuming that such reactions proceed via intermediate $q\bar{q}$ states, one obtains that the cross section is proportional to $w(M)$:

$$\sigma(\rightarrow \text{mesons}) \propto \Gamma_{0r} w(M), \tag{15}$$

where Γ_{0r} is the width of the bare state corresponding to the external reaction. Such formulas were used in [28] to describe the e^+e^- annihilation into charmed mesons. In the limit of narrow resonance Eq. (15) is reduced to the standard Breit-Wigner formula

$$\sigma(\rightarrow \text{mesons}) \propto \frac{1}{2\pi} \frac{\Gamma_r^0 \Gamma}{(M - M_0)^2 + \frac{1}{4}\Gamma^2}, \tag{16}$$

where $\Gamma = 2\,\mathrm{Im}\, g(M_0)$ is the (small) width of the resonance. Similarly, for the bound state case the width Γ_r for a given reaction is renormalized as

$$\Gamma_r = Z\Gamma_r^0. \tag{17}$$

III. THE QUARK MODEL

This section specifies the form factors $f_i(\boldsymbol{p})$. The pair-creation model employed is the $^3P_0$ one, that is the pair-creation Hamiltonian is the nonrelativistic reduction of

$$H_q = g_q \int d^3x \bar{\psi}_q \psi_q, \tag{18}$$

for a given flavor q, but two important points make it different from the model used in [24–26].

The approach [24–26] assumes that the pair creation is flavor-independent, which yields for the constant g_q the form

$$g_q = \gamma 2m_q, \tag{19}$$

where γ is the effective strength of pair-creation. The factor $2m_q$ implies enhancement of strange quarks creation comparing to light quarks one. There are no fundamental reasons to have such enhancement. Moreover, such factor is absent in microscopical models of pair creation, like [28,29]. So, throughout the present study, I use the effective strength γ for the creation of light (u and d) flavors, while for strange quarks the effective strength $\gamma_s = (m_q/m_s)\gamma$ is used, where m_q and m_s are the constituent masses of light and strange quarks correspondingly.

The authors of [24–26] argue that the assumption of flavor-independence gives a reasonably accurate description of known decays. One should have in mind, however, that the calculations [24–26] are performed with the so-called SHO wavefunctions, i.e. with the wave functions of harmonic oscillator, and with the same oscillator parameter β for all states. This assumption looks implausible, as the behavior of form factors is defined by scales of wavefunctions, which, in turn, are defined by quark model.

In what follows the standard nonrelativistic potential model is introduced, with the Hamiltonian

$$H_0 = \frac{p^2}{m_c} + V(r) + C, \qquad V(r) = \sigma r - \frac{4}{3}\frac{\alpha_s}{r}, \tag{20}$$

m_c is the mass of charmed quark. This Hamiltonian should be supplied by Fermi-Breit-type relativistic corrections, including spin-spin, spin-orbit and tensor force, which cause splittings in the $^{2S+1}L_J$ multiplets. In the first approximation these splitting should be calculated as perturbations, using the eigenfunctions of the Hamiltonian (20). The same interaction $V(r)$ should be used in spectra and wavefunction calculations of D (D_s) mesons.

In the first approximation the pair-creation amplitude is to be calculated with the eigenfunctions of the zero-order Hamiltonian. Use of the SHO wavefunctions simplify these calculations drastically. So the procedure adopted is to find the SHO wavefunctions (of the form $\exp(-\frac{1}{2}\beta^2 r^2)$ multiplied by appropriate polynomials) for each orbital momentum L and radial quantum number n, with the effective value of oscillator parameter β for each L and n, which reproduces the mean square radii of the states.

I use the following set of potential model parameters:

$$\alpha_s = 0.55, \quad \sigma = 0.175\ \text{GeV}^2, \quad m_c = 1.7\ \text{GeV},$$
$$C = -0.271\ \text{GeV}, \quad m_q = 0.33\ \text{GeV}, \quad m_s = 0.5\ \text{GeV}. \tag{21}$$

The spin-dependent force is taken in the form

$$V_{sd} = V_{\rm HF} + \frac{2\alpha_s}{m_c^2 r^3}\boldsymbol{L}\cdot\boldsymbol{S} - \frac{\sigma}{2m_c^2 r}\boldsymbol{L}\cdot\boldsymbol{S} + \frac{4\alpha_s}{m_c^2 r^3}T, \tag{22}$$

where $\boldsymbol{L}\cdot\boldsymbol{S}$ and T are spin-orbit and tensor operators correspondingly, and $V_{\rm HF}$ is the contact hyperfine interaction,

$$V_{\rm HF}(r) = \frac{32\pi\alpha_s}{9m_c^2}\tilde{\delta}(r)\boldsymbol{S}_q\cdot\boldsymbol{S}_{\bar{q}}, \tag{23}$$

where, following the lines of [26], Gaussian-smearing of the hyperfine interaction is introduced,

$$\tilde{\delta}(r) = \left(\frac{\kappa}{\sqrt{\pi}}\right)^3 e^{-\kappa^2 r^2}, \tag{24}$$

with $\kappa = 1.45$ GeV.

The masses and effective values of oscillator parameters β for the model (21) are listed in the Table I. The effective values of oscillator parameter for D mesons are $\beta_D = 0.385$ GeV and $\beta_{D_s} = 0.448$ GeV.

One should not take the numbers given in last column too seriously, especially for higher states, as the fine splittings are not well-known, and the expression (22) is surely too naive. Moreover, various much more sophisticated approaches, which reproduce the splittings in $1P$ multiplet, give different predictions for higher multiplets, as discussed in detail in [44].

TABLE I. Masses and effective values β (in units GeV).

nL	$\beta(nL)$	State	Mass
$1S$	0.676	1^3S_1	3.264
		1^1S_0	3.135
$2S$	0.485	2^3S_1	3.905
		2^1S_0	3.850
$1P$	0.514	1^3P_2	3.773
		1^3P_1	3.718
		1^3P_0	3.631
		1^1P_1	3.732
$2P$	0.435	2^3P_2	4.230
		2^3P_1	4.181
		2^3P_0	4.108
		2^1P_1	4.192
$1D$	0.461	1^3D_3	4.051
		1^3D_2	4.043
		1^3D_1	4.026
		1^1D_2	4.043

The D-meson masses taken are $M_D = 1.867$ GeV, $M_{D^*} = 2.008$ GeV, $M_{D_s} = 1.969$ GeV, $M_{D_s^*} = 2.112$ GeV, so that the mass difference between neutral and charged D mesons is not taken into account. The pair-creation strength for light quarks $\gamma = 0.322$ is used. The $^3P_0$ amplitudes are listed in the Appendix A.

IV. LOWER CHARMONIA AND D LEVELS

The single-level version of the coupled-channel model is used in what follows, with the exception of $2^3S_1 - 1^3D_1$ levels.

All the physical charmonium masses below threshold are known. The hadronic shifts and bare masses were calculated from the Eq. (6),

$$M_0 = M_{\rm phys} + \delta, \quad \delta = g(M_{\rm phys}) = \sum_i g_i(M_{\rm phys}), \tag{25}$$

where the sum is over mesonic channels $D\bar{D}$, $D\bar{D}^*$, $D^*\bar{D}^*$, $D_s\bar{D}_s$, $D_s\bar{D}_s^*$ and $D_s^*\bar{D}_s^*$ (an obvious shorthand notation is used here and in what follows: $D\bar{D}^* \equiv D\bar{D}^* + \bar{D}D^*$, and $D_s\bar{D}_s^* \equiv D_s\bar{D}_s^* + \bar{D}_sD_s^*$).

Besides, as the position of the 1^{--} $\psi(3770)$ state is well-established, the bare mass of 1^3D_1 state was reconstructed by means of Eq. (13), and the visible width was calculated as (14).

The parameters of the underlying quark model (21) and the pair-creation strength $\gamma = 0.322$ were chosen to reproduce, with reasonable accuracy, the model masses of $1S$, $1P$ and $2S$ states, and the width of $\psi(3770)$.

The results for bound states are given in Table II together with corresponding values of Z factors. The shifts are much smaller than in [26], but still substantial.

Let me now discuss the 1^3D_1 level, lying above $D\bar{D}$ threshold. The bare mass is calculated to be 4.018 GeV. The calculated width of the $\psi(3770)$ is 25.5 MeV, which compares well with the PDG value of 23.6 ± 2.7 MeV [45]. Note that it is the visible width, while naive calculations would give

$$\Gamma_0 = 2\,{\rm Im}\,g(M_R) = 34.3\ \text{MeV}. \tag{26}$$

TABLE II. Hadronic shifts (in units MeV) of charmonium states below DD threshold due to individual channels, total shifts δ, bare masses and Z factors. The results for the 2^3S_1 state do not take into account mixing with the 1^3D_1 state.

$n^{2S+1}L_J$	DD	DD^*	D^*D^*	D_sD_s	$D_sD_s^*$	$D_s^*D_s^*$	δ	M_0	Z
1^3S_1	11	42	69	5	19	31	177	3274	0.899
1^1S_0	0	59	55	0	26	25	165	3145	0.913
1^3P_2	25	64	82	8	22	27	228	3784	0.804
1^3P_1	0	70	91	0	22	32	215	3726	0.820
1^3P_0	29	0	118	8	0	43	198	3613	0.841
1^1P_1	0	87	76	0	29	27	219	3744	0.817
2^3S_1	21	60	87	45	16	27	216	3902	0.743
2^1S_0	0	83	71	0	24	22	200	3838	0.802

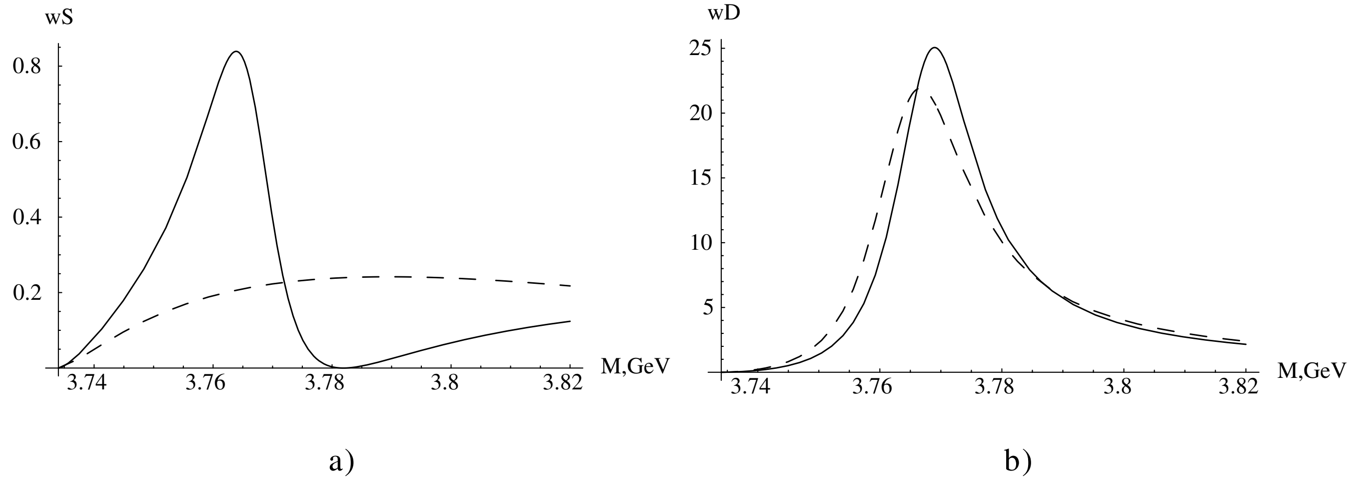

FIG. 1. a) Spectral density of 2^3S_1 bare state with (solid line) and without (dashed line) mixing. b) The same for the bare 1^3D_1 state.

So the effect of coupling to mesonic channels on the width of $\psi(3770)$ is not small, $\Re = 0.743$.

The mass of $\psi(3770)$ is less than 100 MeV higher than the mass of $\psi'(3686)$, with $D\bar{D}$ threshold opening in between, so one could in principle expect that these states are mixed due to coupling to mesonic channels. This mixing is not large in the given model, as the mixing disappears if the mass difference between various D mesons is neglected (see the corresponding spin-orbit recoupling coefficients listed in Appendix A). The relevant formulas for two-level mixing scheme are given in the Appendix B, and here I quote the results. With physical masses of $\psi'(3686)$ and $\psi(3770)$ the bare masses are reconstructed as $M(2^3S_1) =$ 3.899 GeV, and $M(1^3D_1) = 4.016$ GeV, so the masses are shifted only by a few MeV due to the mixing. The width of the $\psi(3770)$ becomes only 18.4 MeV, but it is the visible width, and the deviation from the value 25.5 MeV obtained without mixing is mainly due to the illegitimate attempt to fit the system of two overlapping states with a single-Breit-Wigner lineshape. The states indeed do overlap, as shown at Fig. 1, where the spectral densities of bare 2^3S_1 and 1^3D_1 bare states are plotted. The lineshape of 1^3D_1 is distorted due to the mixing, and the lineshape of the 2^3S_1 is drastically changed, displaying, instead of small smooth background, a peak at the mass of about 3750 MeV. The 10 MeV difference between the peak position and the mass of $\psi(3770)$ is again due to the prescription of visible width: single-Breit-Wigner approximation is not appropriate both for $^3S_1$ and $^3D_1$ lineshapes. As to the lower state $\psi'(3686)$, the admixture Z_1 of the 2^3S_1 bare state is 0.742, while the admixture Z_2 of the bare 1^3D_1 state is 0.000 63.

The calculated mass of bare 1^3D_1 state is 4.016–4.018 GeV. So the value of 4.026 GeV given in the Table I looks quite acceptable. No other $1D$ states are known, so in what follows I take, with reservations mentioned in the previous section, the values of bare masses from the Table I. With the bare 1^1D_2 state having mass of 4.043 GeV, the physical state is a bound state,

$$M(1^1D_2) = 3.800 \text{ GeV}, \qquad Z(1^1D_2) = 0.712. \tag{27}$$

Similarly,

$$M(1^3D_2) = 3.806 \text{ GeV}, \qquad Z(1^3D_2) = 0.689 \tag{28}$$

for the mass 4.043 GeV of the bare 1^3D_2 state. The 1^3D_3 state is allowed to decay into $D\bar{D}$, but the width is extremely small, as the $D\bar{D}$ system is in the F wave:

$$\begin{gathered} M(1^3D_3) = 3.812 \text{ GeV}, \\ \Gamma(1^3D_3) \approx 0.7 \text{ MeV}, \quad \Re(1^3D_3) = 0.717. \end{gathered} \tag{29}$$

V. 2*P*-LEVELS

Coupled-channel effects do not cause dramatic changes for the charmonia states discussed in the previous section. The situation with $2P$ levels promises more, as $2P$-charmonia are expected to populate the mass range of 3.90–4.00 GeV, where more charmed meson channels start to open, and some of these channels are the S-wave ones.

The importance of S-wave channels follows from the Eq. (5). The form factor $f(p)$ of the S-wave mesonic channel behaves as some constant at small p, so the derivative of the hadronic shift $g(M)$ with respect to the mass is large for the masses close to the S-wave threshold. As the result, hadronic shift due to the coupling to S-wave channel displays rather vivid cusplike near-threshold behavior.

The physical masses and widths of $2P$ states were calculated with bare masses given by Table I, and the results are listed in Table III. Two different values are given for the mass and width of the 2^3P_0 state, for two different choices of the bare mass, see below. Looking at the numbers one would say that nothing dramatic has happened due to S-wave thresholds. Indeed, all the shifts

TABLE III. The masses and widths (in units of MeV), and renormalization factors $\Re$ of $2P$ charmonium states. The results for the 2^3P_0 state are given for two different values of bare mass as described in the text.

State	Bare mass	Mass	Mode	Width	$\Re$
2^1P_1	4200	3980	$D\bar{D}^*$	50	0.615
2^3P_2	4230	3990	$D\bar{D}$	22	0.603
			$D_s\bar{D}_s$	1	
			$D\bar{D}^*$	45	
			total	68	
2^3P_1	4180	3990	$D\bar{D}^*$	27	0.543
2^3P_0	4108	3918	$D\bar{D}$	7	0.622
2^3P_0	4140	3937	$D\bar{D}$	8	0.393

are about 200 MeV, and the renormalization factors are about 0.5–0.6. It is the behavior of spectral density which reveals the role of S-wave thresholds.

For the 2^1P_1 case the S-wave thresholds are $D\bar{D}^*$ with the spin-orbit recoupling coefficient $C = -1/\sqrt{2}$ and multiplicity 4, the $D_s\bar{D}_s^*$ with $C = -1/\sqrt{2}$ and multiplicity 2, the $D^*\bar{D}^*$ with $C = 1$ and multiplicity 2, and $D_s^*\bar{D}_s^*$ with $C = 1$ and multiplicity 1. If the resonance is in the mass range 3.90–4.00 GeV (and it appears to be so), then the relevant thresholds are $D\bar{D}^*$ and $D^*\bar{D}^*$. The spectral density of the bare 2^1P_1 state is shown at Fig. 2. It displays the relatively steep rise near $D\bar{D}^*$ threshold, and a beautiful well-pronounced cusp due to the opening of $D^*\bar{D}^*$ channel.

In the 2^1P_1 case the S-wave strength is shared equally between $D\bar{D}^*$ and $D^*\bar{D}^*$ channels, while for the 2^3P_2 case all S-wave strength is concentrated in the $D^*\bar{D}^*$, channel. As the result, the cusp due to the opening of $D^*\bar{D}^*$ channel is more spectacular in the 2^3P_2 case, as shown at Fig. 3.

The case of 2^3P_1 is even more interesting. Here, similarly to the 2^3P_2 case, all the S-wave strength is concentrated in two channels, $D\bar{D}^*$, $D_s\bar{D}_s^*$, and the multiplicity of the former is 4. So the strongest S-wave threshold is the $D\bar{D}^*$ one, well below the resonance. The behavior of the 2^3P_1 bare state spectral density is shown at Fig. 4, and is very peculiar: together with a clean and relatively narrow resonance, there is a near-threshold peak, rising at the flat background. The $D\bar{D}^*$ scattering length appears to be negative and large,

$$a_{D\bar{D}^*} = -8 \text{ fm}, \tag{30}$$

signalling the presence of virtual state very close to the $D\bar{D}^*$ threshold, with the energy $\epsilon = 0.32$ MeV. So the coupling to mesonic channels has generated not only the resonance, but, in addition, a virtual state very close to physical region.

The near-threshold peak should fade with the increase of bare state mass, and strengthen otherwise. There are uncertainties in the fine splitting estimates, so the mass of the bare 2^3P_1 state could easily be about 10 MeV larger or smaller. The dependence of spectral density behavior on the bare mass is shown at Fig. 5. The peak becomes less pronounced for the bare mass of 4.190 GeV, but the scattering length remains rather large, $a \approx -5.2$ fm, which corresponds to the energy of virtual state of about 0.76 MeV (compare this with the scattering length in the 1^{+-} channel, $|a| \approx 1$ fm). 10 MeV decrease of the bare state mass leads to incredibly large scattering length, $a \approx -17.8$ fm, and virtual state with the energy 0.065 MeV.

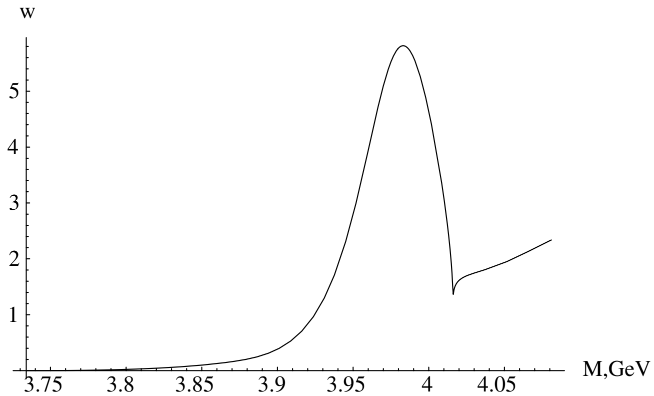

FIG. 3. Spectral density of the bare 2^3P_2 state.

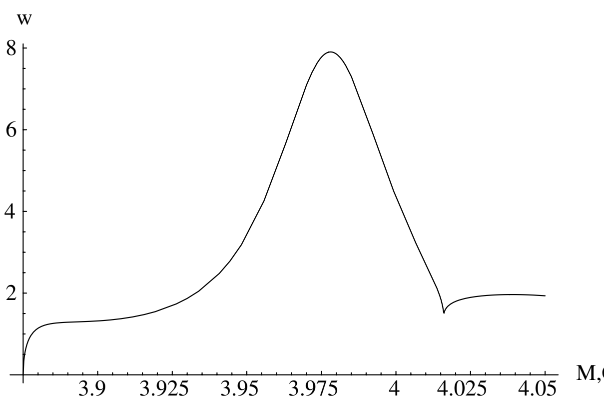

FIG. 2. Spectral density of the bare 2^1P_1 state.

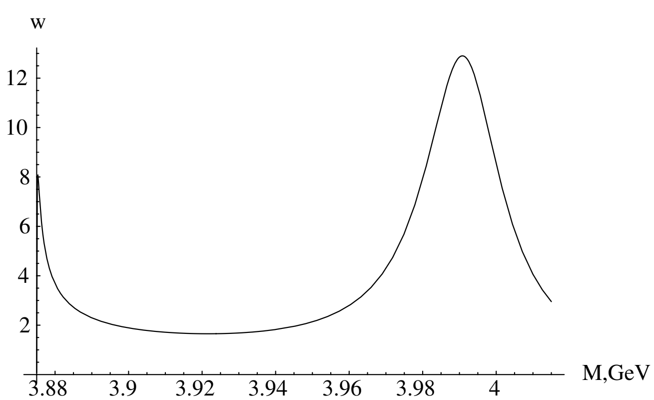

FIG. 4. Spectral density of the bare 2^3P_1 state.

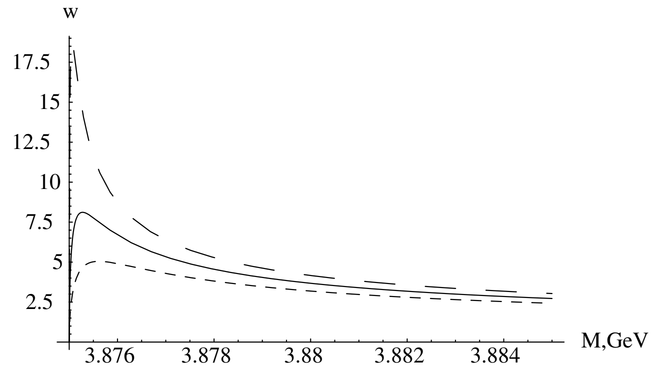

FIG. 5. Near-threshold 2^3P_1 spectral density for the mass of bare state $M_0 = 4.180$ GeV (solid line), $M_0 = 4.190$ GeV (long-dashed line), $M_0 = 4.170$ GeV (dashed line).

Further decrease of bare state mass leads to moving the state to the physical sheet, i.e. to appearance of the bound state. This happens at the bare mass of about 4.160 GeV, which seems, in the present model, to be beyond acceptable range for fine splitting. Similarly, the bound state appears if the pair-creation strength is increased by several per cent.

The 2^3P_0 level is a disaster, as it always happens with scalars. The bare mass is considerably lower that the center-of-gravity, and the uncertainty in the fine splitting estimate is large. The S-wave $D^*\bar{D}^*$ and $D_s^*\bar{D}_s^*$ channels are too high. The S-wave $D\bar{D}$ channel is too low. The only relevant S-wave channel is $D_s\bar{D}_s$ ($C = \sqrt{\frac{3}{2}}$, multiplicity 1), and corresponding threshold is at 3.938 GeV, i.e. around the region where the resonance is expected. So, depending on the position of the bare state, variety of spectral density behavior can be achieved, as shown at Fig. 6, where spectral density is plotted for $M_0(2^3P_0) = 4.108$ GeV and $M_0(2^3P_0) = 4.140$ GeV. Note that two curves of Fig. 6 correspond to two completely different situations. The curve for $M_0(2^3P_0) = 4.108$ GeV displays the normal resonance behavior with a tiny cusp due to the opening of the $D_s\bar{D}_s$ channel. The curve for $M_0(2^3P_0) = 4.140$ GeV is not resonancelike at all. The formal exercise of calculating the $\Re$ factor and visible width does not make much sense, and, as suggested in [43], such excitation curve should be analyzed in terms of scattering length approximation, and not in terms of Breit-Wigner or Flattè distributions.

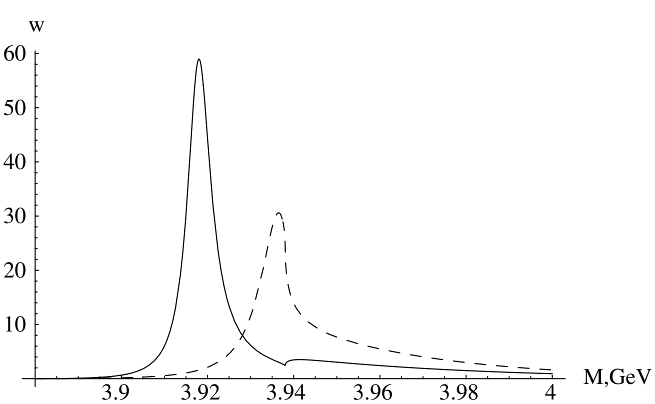

FIG. 6. Spectral density of the bare 2^3P_0 state for the mass of bare state $M_0 = 4.110$ GeV (solid line), $M_0 = 4.140$ GeV (dashed line).

VI. DISCUSSION

The quark model (21) is not the result of the sophisticated fit, it is rather a representative example. A serious fit should include proper treatment of fine and hyperfine splittings, as well as the mixing of bare 2^3S_1 and 1^3D_1 states. The calculations should be performed with more realistic wavefunctions, and not with the SHO ones. Relativistic corrections should be taken into account in calculations of bare spectra, and more realistic model should be used for D-meson wavefunctions. More QCD-motivated model should be employed for the pair-creation Hamiltonian, and loop integrals $g(M)$ are to be treated relativistically. Nevertheless, there are gross features which are model-independent.

In various pair-creation models, the shifts within the each orbital multiplet are approximately the same, and differ only due to the mass difference of bare states in the multiplet and different masses of charmed mesons. This is model-independent, as the charmed quark is heavy, and the wavefunctions within each nL multiplet do not differ much. The same is true for the wavefunctions of D and D^* (and D_s and D_s^*) mesons due to heavy-quark spin symmetry.

In addition, the shifts for all states are more or less the same. There are no *a priori* reasons for this, but the numbers given in Table II are stable up to overall multiplier γ as soon as the scales β behave as expected from quark model, and the value of γ is constrained by experimental value of the $\psi(3770)$ width.

Indeed, the present results appear to be very similar to the ones given in [28,31]. One might suggest that, from phenomenological point of view, the effect of coupling to mesonic channels can be approximated by adding a negative constant to the potential. But it is not the whole story, as the coupling to charmed mesons generates the admixture of D mesons in the wavefunctions of bound states, which affects such quantities as e^+e^- widths (in accordance with Eq. (17)) and the rates of radiative transitions, as discussed in detail in [28].

The e^+e^- widths of J/ψ and $\psi'(3686)$ are more or less accurately described by Van Royen-Weisskopf formula with QCD correction, so the renormalization (17) is not harmless, even if it is as mild as 10% reduction required for J/ψ by the results of Table II. For $\psi'(3686)$ Z factor is 0.743, so renormalization is larger.

In the nonrelativistic quark model the e^+e^- width of the $^3D_1$ state is zero. The mixing communicates the e^+e^-

width of the 2^3S_1 state to the $\psi(3770)$ region, with the $e^+e^- \to D\bar{D}$ cross section given by Eq. (15) where the spectral density of 2^3S_1 state $w_S(M)$ is substituted. It is reasonable to estimate the e^+e^- width of the $\psi(3770)$ using the peak value of $w_S(M)$, which yields less than 1/3 of the measured value 0.26 ± 0.04 keV [45]. Similar result was obtained in [28]. The bare 1^3D_1 state has the small e^+e^- width of its own as relativistic correction, and the bare 2^3S_1 and 1^3D_1 states are to be mixed by tensor force. The scale of the admixture required to reproduce the relatively large e^+e^- width of the $\psi(3770)$ is not small, as shown in [46]. The coupling to charmed mesons is able to explain only about 1/3 of the observed e^+e^- width of $\psi(3770)$, so direct mixing between bare 2^3S_1 and 1^3D_1 bare levels is still needed, which would reduce the e^+e^- width of the ψ' further. While the problem of leptonic widths is an open problem for coupled-channel model, it is clear that the values of Z factors considerably smaller than given in Table II would destroy fragile agreement with the data on e^+e^- widths achieved by quark model practitioners.

The 2^3S_1–1^3D_1 mixing due to coupling to meson channels is small, and this is also model-independent, as it vanishes when the masses and wavefunctions of D-mesons are taken to be the same. Note, however, that both $\psi(3770)$ and ψ' states contain considerable admixture of four-quark component of their own, in the form of various D-meson pairs. Thus, if the mass difference between charged and neutral D mesons is taken into account, some isospin violation could be generated. It is argued in [47] that a small admixture of $I = 1$ component is needed to explain some discrepancies in the observed properties of $\psi(3770)$ and ψ'. The question of whether such admixture can be generated by coupled-channel effects certainly deserves attention.

The shifts of D levels are more or less the same, the relevant thresholds are P-wave ones, and the 2^3S_1–1^3D_1 mixing does not affect the shift of the bare $^3D_1$ level drastically. So the combined effect of fine splitting and splitting due to coupled-channel effects on other D-levels is not large. In particular, it means that, with the mass of $\psi(3770)$ as experimental input, the physical $^3D_2$ or $^3D_3$ state cannot be placed as high as 3.872 GeV and, therefore, cannot be identified as $X(3872)$, unless something drastic happens with fine splittings in the $1D$ multiplet.

Because of the presence of S-wave thresholds, the situation with $2P$ levels is more interesting. The coupling to $D^*\bar{D}^*$ channels generates pronounced cusps in the spectral densities of bare 2^1P_1 and 2^3P_2 levels, and the coupling to $D\bar{D}^*$ channel generates the strong threshold effect in the 1^{++} wave. Within the given model, it is a virtual state with the energy less than 1 MeV. The mechanism of generating such a state is quite peculiar: it is one and a same bare 2^3P_1 state, which gives rise both to the 1^{++} resonance with the mass of about 3990 MeV, and a virtual state at the $D\bar{D}^*$ threshold, i.e. where the $X(3872)$ is observed. Because of the presence of strong S-wave threshold, the hadronic shift appears to be large enough to destroy the one-to-one correspondence between bare and physical states, as widely discussed in connection with light scalar mesons [34,36].

To what extent this prediction is robust? Changes of the underlying quark model parameters or of the value of pair-creation strength can shift this extra state either to the physical sheet or away from the physical region. In the latter case, however, the $D\bar{D}^*$ scattering length remains large. One should have in mind that if such dynamical generation of extra state at $D\bar{D}^*$ threshold is possible in the charmonia, the 1^{++} channel is the most appropriate place for this phenomenon. The latter statement is model-independent.

First, note that the scalar charmonium does not decay into $D\bar{D}^*$ at all, and the tensor one decays into $D\bar{D}^*$ in the D wave. As to 1^{++} and 1^{+-} levels, they both have the desired S-wave decay mode. Apply now the heavy-quark spin selection rule [48], which suggests that the spin of a heavy-quark pair is conserved in the decay. The S-wave decay mode comes from the four-quark state $c\bar{c}q\bar{q}$ with all relative angular momenta equal to zero. Then the total angular momentum $J = 1$ can result only from the quark spins. The combination of $S_{c\bar{c}} = 1$ and $S_{q\bar{q}} = 1$ is C-even, while the combination of $S_{c\bar{c}} = 0$ and $S_{q\bar{q}} = 1$ is C-odd. It is a simple algebra exercise to show that, symbolically,

$$(c\bar{c})_{S=1} \otimes (q\bar{q})_{S=1} \to \frac{1}{\sqrt{2}}(D\bar{D}^* + \bar{D}D^*), \tag{31}$$

and

$$(c\bar{c})_{S=0} \otimes (q\bar{q})_{S=1} \to -\frac{1}{2}(D^*\bar{D} - D\bar{D}^*) + \frac{1}{\sqrt{2}}D^*\bar{D}^*, \tag{32}$$

and, independently of the pair-creation model, all the S-wave strength of the $3P_1$ decay is concentrated in the $D\bar{D}^*$ channel, while in the $1P_1$ decay it is shared equally between $D\bar{D}^*$ and $D^*\bar{D}^*$. Thus the threshold attraction in the 1^{++} $D\bar{D}^*$ channel is always much stronger than in the 1^{+-} one. It would be interesting to see if the pair-creation model of [28] is able to generate large $D\bar{D}^*$ scattering length.

Note, in passing, that the possibility to get large scattering length in the $D\bar{D}^*$ channel is dictated by specific form of spin-recoupling coefficients (31), which follows from the conservation of heavy-quark pair spin. Thus, the same phenomenon might happen in the $B\bar{B}^*$ system too, as argued in the recent paper [49] in the framework of the approach of effective Lagrangian consistent with heavy-quark and chiral symmetry.

The model [50] contains the detailed analysis of the $X(3872)$ as a state bound both by pion exchange and quark exchange in the form of transitions $D\bar{D}^* \to J/\psi\rho, J/\psi\omega$, with the latter contributions being important for the binding. In fact, this model has predicted the $J/\psi\omega$ decay mode of the $X(3872)$, and, after observation [15] of the decay $X(3872) \rightarrow \pi^+\pi^-\pi^0 J/\psi$, it has become almost official model of $X(3872)$. This is challenged by preliminary data from Belle [14] on large $D^0 D^0 \pi^0$ rate, more than 10 times larger than $\pi^+\pi^- J/\psi$ one, while the model [50] claims the opposite.

One could question validity of the naive quark-exchange model. Besides, the pion exchange is definitely attractive in the 1^{++} channel, but there are uncertainties in the actual calculations; the details of binding depend on the cutoff scale Λ, as recognized in [50]. The attraction found in the coupled-channel model is large, and could help binding without large quark-exchange kernels, and, correspondingly, without large $\pi^+\pi^- J/\psi$ rate.

From practical point of view, the wavefunctions of both models are not very distinguishable. Indeed, the near-threshold virtual state of the coupled-channel model owes its existence to the bare 1^{++} state, but the near-threshold admixture of the bare state in the wavefunction is extremely small, as seen from Fig. 4 (recall that the spectral density is normalized to unity). So the decays like $D\bar{D}\pi$ and $D\bar{D}\gamma$ would proceed via D^* decays, as described in [51]. As to short-distance decays and exclusive production, the rates of these are governed by large scattering length. This phenomenon was called low-energy universality in [52]. Consider, for example, the near-threshold production of $D\bar{D}^*$ pairs in the reaction $B \rightarrow D\bar{D}^* K$. As explained in Sec. II, the $D\bar{D}^*$ invariant mass distribution is proportional to $w(M)$ in the coupled-channel model, with the lineshape plotted at Fig. 5. Now compare these curves with the ones presented in [53,54] with the scattering length approximation for the $D\bar{D}^*$ amplitude, and observe that the low-energy universality indeed takes place.

The lineshape for $D\bar{D}^*$ production depends only on the modulus of scattering length, and the cases of bound state and virtual state are not distinguishable. If some inelastic channel like $\pi\pi J/\psi$ is present, then, as shown in [54], the lineshape for this channel depends on whether there is a bound state or virtual state (cusp). The area under the cusp is much smaller than the area under the resonance, so, in principle, the data could distinguish between $X(3872)$ as a bound state or a virtual state.

To conclude the discussion of coupled-channel virtual state as $X(3872)$ I note that, with 7 MeV difference between $D^0\bar{D}^{*0}$ and $D^+\bar{D}^{*-}$ thresholds, the coupled-channel model is able to generate considerable isospin violation. The coupled-channel calculations which resolve the mass difference between charged and neutral D mesons are in progress now.

Last year yet another new state was reported [55] as an enhancement in $\omega J/\psi$ mode, with the mass of 3941 MeV and the width of about 90 MeV ($Y(3940)$). As there are two 1^{++} physical states per one bare 2^3P_1, it is tempting to identify the 1^{++} resonance with the $Y(3940)$ state, following the suggestion of [19]. Nevertheless, with the given set of quark model parameters the resonance is 40–50 MeV higher than 3940 MeV, and is narrow.

One might suggest the 0^{++} assignment for $Y(3940)$, as the coupled-channel model places the scalar just at the right place. However, as the $D_s\bar{D}_s$ channel is opening at 3938 MeV, the width of the state cannot be as large as observed 90 MeV independently of what channel saturates it, unless, for some strange reasons, the state couples weakly to $D_s\bar{D}_s$. It is similar to $a_0(980)/f_0(980)$ case, where, in spite of large coupling to light pseudoscalars ($\pi\pi$ or $\pi\eta$), the visible width is small due to strong affinity to $K\bar{K}$ threshold.

VII. CONCLUSIONS

The spectrum of charmonium states below 4 GeV is calculated taking into account coupling to the pairs of lowest D and D_s mesons. The analysis is performed within the $^3P_0$ model for light-quark pair-creation. It appears that quite moderate modification of quark model parameters is needed to describe charmonia below $D\bar{D}$ threshold, while coupling to open charm does not cause drastic effects on $1D$ levels. This is in contrast to $2P$ levels, where opening of strong S-wave channels leads to pronounced threshold effects.

In particular, coupling of the bare 2^3P_1 state to $D\bar{D}^*$ channel generates, together with the 1^{++} resonance with the mass of 3990 MeV, a near-threshold virtual state with the energy of about 0.3 MeV, which corresponds to the extremely large $D\bar{D}^*$ scattering length $a \approx -8$ fm, with the possibility to identify this state with $X(3872)$. The admixture of the bare $c\bar{c}$ state in the near-threshold wavefunction is very small, so it is essentially the $D\bar{D}^*$ state. The 1^{++} channel appears to be the only one where such state can be formed.

ACKNOWLEDGMENTS

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APPENDIX A

The $^3P_0$ form factors are defined following the lines of [24,29], adapted for charmonium transitions. Let the $c\bar{c}$ meson A decay to $q\bar{c}$ meson B and $c\bar{q}$ meson C. Then the spin-space part of the amplitude in the c.m. frame of the initial meson A is given by

$$I(\boldsymbol{p}) = \int d^3k \phi_A(\boldsymbol{k} - \boldsymbol{p})\langle s_q s_{\bar{q}}|\hat{O}(\boldsymbol{k})|0\rangle \phi_B^*(\boldsymbol{k} - r_q\boldsymbol{p}) \times \phi_C^*(\boldsymbol{k} - r_q\boldsymbol{p}), \tag{A1}$$

ϕ_A is the wavefunction of the initial meson in the momentum space, ϕ_B and ϕ_C are the wavefunctions of the final mesons B and C, $\boldsymbol{p} = \boldsymbol{p}_B$, $r_q = m_q/(m_q + m_c)$, m_c is the mass of charmed quark, m_q is the mass of light quark. $\hat{O}(\boldsymbol{k})$ is the $^3P_0$ operator:

$$\hat{O}(\boldsymbol{k}) = -2\gamma\sigma \cdot \boldsymbol{k}. \tag{A2}$$

The form factors for the transition $A \to BC$ are given below in the lS basis, where l is the orbital momentum in the final meson system, and $\vec{S} = \vec{J}_B + \vec{J}_C$ is the total spin of the mesons B and C. In the narrow width approximation these form factors f_{lS} define the partial widths $\Gamma_{A\to BC}$ as

$$\Gamma_{A\to BC} = 2\,\mathrm{Im}\, g_{BC}(M_A) = 2\pi p_{BC}\mu_{BC}\sum_{lS}|f_{lS}|^2. \tag{A3}$$

The form factors calculated with SHO wavefunctions take the form

$$f_{lS} = \frac{\gamma}{\pi^{1/4}\beta_A^{1/2}} \exp\left(-\frac{p^2(r_q-1)^2}{\beta_B^2 + 2\beta_A^2}\right)\mathcal{P}_{lS}, \tag{A4}$$

where β_A and $\beta_B = \beta_C$ are the oscillator parameters of initial meson A and final mesons B and C, γ is the pair-creation strength. The polynomial $\mathcal{P}_{LS}$ is a channel-dependent one:

$$\mathcal{P}_{lS} = f_l C_{lS}, \tag{A5}$$

where C_{lS} are spin-orbit recoupling coefficients for specific mesonic channels, and f_l are:

$$f_P(1S \to 1S + 1S) = -\frac{2^3}{3^2}\frac{\lambda\beta^3}{\beta_B^3\beta_A}p \tag{A6}$$

$$f_P(2S \to 1S + 1S) = \frac{2^{5/2}}{3^{3/2}}\frac{\beta^3}{\beta_A\beta_B^3}p\left(-\lambda + \left(\frac{10}{9}\lambda - \frac{4}{9}\right)\frac{\beta^2}{\beta_A^2} + \frac{2p^2}{3\beta_A^2}\lambda(\lambda-1)^2\right) \tag{A7}$$

$$f_S(1P \to 1S + 1S) = \frac{2^4}{3^{5/2}}\frac{\beta^5}{\beta_B^3\beta_A^2}\left(1 + \lambda(\lambda-1)\frac{p^2}{\beta^2}\right) \tag{A8}$$

$$f_D(1P \to 1S + 1S) = -\frac{2^4}{3^2\cdot 5^{1/2}}\frac{\lambda(\lambda-1)\beta^3}{\beta_A^2\beta_B^3}p^2 \tag{A9}$$

$$f_P(1D \to 1S + 1S) = -\frac{2^{11/2}}{3^3}\frac{(\lambda-1)\beta^5}{\beta_A^3\beta_B^3}p\left(1 + \frac{3}{5}\lambda(\lambda-1)\frac{p^2}{\beta^2}\right) \tag{A10}$$

$$f_F(1D \to 1S + 1S) = -\frac{2^4}{3^{3/2}\cdot 5^{1/2}\cdot 7^{1/2}}\frac{\lambda(\lambda-1)^2\beta^3}{\beta_A^3\beta_B^3}p^3 \tag{A11}$$

$$f_S(2P \to 1S + 1S) = \frac{2^{7/2}\cdot 5^{1/2}}{3^{5/2}}\frac{\beta^5}{\beta_B^3\beta_A^2}\left(1 - \frac{2\beta^2}{3\beta_A^2} + \lambda(\lambda-1)\frac{p^2}{\beta^2} - \frac{2p^2}{3\beta_A^2}(\lambda-1)(2\lambda-1) - \frac{2p^4}{5\beta^2\beta_A^2}\lambda(\lambda-1)^3\right) \tag{A12}$$

$$f_D(2P \to 1S + 1S) = -\frac{2^{7/2}}{3^2}\frac{\beta^3}{\beta_A^2\beta_B^3}p^2\left(\lambda(\lambda-1) - \frac{2\beta^2}{15\beta_A^2}(\lambda-1) \times (7\lambda-2) - \frac{2p^2}{5\beta_A^2}\lambda(\lambda-1)^3\right). \tag{A13}$$

Here

$$\lambda = \frac{\beta_B^2 + 2r_q\beta_A^2}{\beta_B^2 + 2\beta_A^2}, \tag{A14}$$

and

$$\beta^2 = \frac{3\beta_A^2\beta_B^2}{\beta_B^2 + 2\beta_A^2}. \tag{A15}$$

For the transitions to strange mesons, one should replace r_q by $r_s = m_s/(m_c + m_s)$, and insert the multiplier m_q/m_s, as explained in the main text.

The decays of light mesons were considered in [24], which corresponds to $r_q = 1/2$. In the case of $\beta_A = \beta_B = \beta$, and $r_q = 1/2$ the expressions for the amplitudes listed above are equal to those of [24] up to the factor of 1/2. In general, there are two graphs with different topologies which contribute to the $^3P_0$ amplitude, and the sum of both is quoted in [24], while in actual calculations each graph contributes with the individual flavor factor. In the case of charmonia transitions only one graph contributes, so that the amplitude for the transition into given charge channel is equal to (A4) with the flavor factor of unity.

The mass difference between neutral and charged mesons is not taken into account, so the sum over charge states is equivalent to introducing the multiplicity factor 2 for $D\bar{D}$ and $D^*\bar{D}^*$ channels, and 4 for $D\bar{D}^*$ channel. The multiplicity factor for $D_s\bar{D}_s$ and $D_s^*\bar{D}_s^*$ is 1, and it is 2 for $D_s\bar{D}_s^*$ channel.

Spin-orbit recoupling coefficients are tabulated in the Appendix A of [24]. In the single-level coupled-channel calculations the squares of spin-orbit recoupling coefficients are needed, given in Table IV. Note that the multiplicity factors for $D\bar{D}^*$ channels are twice as large as of $D\bar{D}$ and $D^*\bar{D}^*$ ones.

TABLE IV. The sums $\sum_S |C_{lS}|^2$ for given initial and final states.

State	$D\bar{D}$		$D\bar{D}^*$		$D^*\bar{D}^*$	
	$l = L - 1$	$l = L + 1$	$l = L - 1$	$l = L + 1$	$l = L - 1$	$l = L + 1$
$3S_1$	...	1	...	2	...	7
$1S_0$	...	0	...	3	...	6
$1P_1$	0	0	1/2	5/3	1	10/3
$3P_2$	0	1	0	3/2	2	8/3
$3P_1$	0	0	1	5/6	0	5
$3P_0$	3/2	0	0	0	1/2	20/3
$1D_2$	0	0	1/4	7/5	1/2	14/5
$3D_3$	0	1	0	4/3	1	29/15
$3D_2$	0	0	3/8	14/15	1/4	56/15
$3D_1$	5/12	0	5/24	0	1/6	28/5

The case of 2^3S_1–1^3D_1 mixing requires explicit expressions for spin-orbit recoupling coefficients, as relative signs are important in the two-level mixing scheme. These are given below:

$$C_{10}(^3S_1\xrightarrow{1}S_0 + ^1S_0) = 1 \quad C_{11}(^3S_1\xrightarrow{3}S_1 + ^1S_0) = -\sqrt{2}$$
$$C_{10}(^3S_1\xrightarrow{3}S_1 + ^3S_1) = \sqrt{\frac{1}{3}} \quad C_{12}((^3S_1\xrightarrow{3}S_1 + ^3S_1) = -\sqrt{\frac{20}{3}} \tag{A16}$$

$$\begin{aligned} C_{10}(^3D_1\xrightarrow{1}S_0 + ^1S_0) &= -\sqrt{\frac{5}{12}} \\ C_{11}(^3D_1\xrightarrow{3}S_1 + ^1S_0) &= -\sqrt{\frac{5}{24}} \\ C_{10}(^3D_1\xrightarrow{3}S_1 + ^3S_1) &= -\frac{\sqrt{5}}{6} \\ C_{12}(^3D_1\xrightarrow{3}S_1 + ^3S_1) &= \frac{1}{6} \\ C_{32}(^3D_1\xrightarrow{3}S_1 + ^3S_1) &= -\sqrt{\frac{28}{5}}. \end{aligned} \tag{A17}$$

APPENDIX B

The formulas necessary to describe the 2^3S_1–1^3D_1 mixing are collected here.

Two sets of form factors, f_S and f_D, are introduced, which describe the transitions between mesons and 2^3S_1, 1^3D_1 levels. The system of coupled-channel equation similar to (4) leads to the $D\bar{D}$ t-matrix:

$$t(\boldsymbol{p}, \boldsymbol{p}', M) = \sum_{\mu,\nu} f_{\mu,D\bar{D}}(\boldsymbol{p})\tau_{\mu\nu}(M)f_{\nu,D\bar{D}}(\boldsymbol{p}'), \tag{B1}$$

where sum is over S and D states, and

$$\begin{aligned} \tau_{SS}(M) &= \frac{M - M_D + g_{DD}(M)}{\Delta(M)}, \\ \tau_{DD}(M) &= \frac{M - M_S + g_{SS}(M)}{\Delta(M)}, \\ \tau_{DS}(M) &= \tau_{SD}(M) = -\frac{g_{SD}(M)}{\Delta(M)}, \end{aligned} \tag{B2}$$

$$\Delta(M) = [M - M_S + g_{SS}(M)][M - M_D + g_{DD}(M)] - g_{SD}^2, \tag{B3}$$

M_S and M_D are the masses of bare states, and

$$\begin{aligned} g_{\mu\nu}(M) &= \sum_i g_{i,\mu\nu}(M), \\ g_{i,\mu\nu}(M) &= \int \frac{f_{\mu,i}(\boldsymbol{p})f_{\nu,i}(\boldsymbol{p})}{\frac{p^2}{2\mu_i} - E_i - i0} d^3p, \end{aligned} \tag{B4}$$

the index i labels mesonic channels.

The visible physical masses are defined, similarly to (13), from the equation

$$\mathrm{Re}\,\Delta(M_{\mathrm{phys}}) = 0. \tag{B5}$$

There are two solutions of this equation, M_b below $D\bar{D}$ threshold, corresponding to the bound state, and M_a above threshold corresponding to the resonance. The visible width is defined for the latter as

$$\begin{aligned} \Gamma &= 2\Re\,\mathrm{Im}\,\Delta(M_a), \\ \Re^{-1} &= [1 + d_{SS}(M_a)][M_a - M_D + \mathrm{Re}\,g_{DD}(M_a)] \\ &\quad + [1 + d_{DD}(M_a)][M_a - M_S + \mathrm{Re}\,g_{SS}(M_a)] \\ &\quad - 2\,\mathrm{Re}\,g_{SD}(M_A)d_{SD}(M_A), \\ d_{\mu\nu}(M) &= \frac{\partial\,\mathrm{Re}\,g_{\mu\nu}(M)}{\partial M}. \end{aligned} \tag{B6}$$

The probabilities to find bare states in the wavefunction of bound state are

$$Z_S = [M_b - M_D + g_{DD}(M_b)]Z^{-1},$$
$$Z_D = [M_b - M_S + g_{SS}(M_b)]Z^{-1}, \tag{B7}$$

and Z^{-1} is obtained from $\Re^{-1}$ given in (B6) with the replacement $M_a \to M_b$. The spectral densities of bare states are:

$$w_S(M) = \frac{1}{2\pi i}\left(\frac{M - M_D + g^*_{DD}}{\Delta^*(M)} - \frac{M - M_D + g_{DD}}{\Delta(M)}\right), \tag{B8}$$

and

$$w_D(M) = \frac{1}{2\pi i}\left(\frac{M - M_S + g^*_{SS}}{\Delta^*(M)} - \frac{M - M_S + g_{SS}}{\Delta(M)}\right), \tag{B9}$$

[1] S.-K. Choi *et al.* (Belle Collaboration), Phys. Rev. Lett. **89**, 102001 (2002).
[2] B. Aubert *et al.* (*BABAR* Collaboration), Phys. Rev. Lett. **92**, 142002 (2004).
[3] D. M. Asner *et al.* (CLEO Collaboration), Phys. Rev. Lett. **92**, 142001 (2004).
[4] C. Patrignani, hep-ex/0410085.
[5] A. Tomaradze (CLEO Collaboration), hep-ex/0410090.
[6] S. K. Choi *et al.* (Belle Collaboration), Phys. Rev. Lett. **91**, 262001 (2003).
[7] D. Acosta *et al.* (CDF II Collaboration), Phys. Rev. Lett. **93**, 072001 (2004).
[8] N. A. Tornqvist, Phys. Lett. B **590**, 209 (2004); hep-ph/0308277.
[9] N. A. Tornqvist, Phys. Rev. Lett. **67**, 556 (1991).
[10] M. B. Voloshin and L. B. Okun, JETP Lett. **23**, 333 (1976) [Pis'ma Zh. Eksp. Teor. Fiz. **23**, 369 (1976)].
[11] A. de Rujula, H. Georgi, and S. L. Glashow, Phys. Rev. Lett. **38**, 317 (1977).
[12] K. Abe, hep-ex/0505038.
[13] G. Bauer, hep-ex/0505083.
[14] S. L. Olsen, http://belle.kek.jp/belle/talks/aps05/olsen.pdf.
[15] K. Abe *et al.* (Belle Collaboration, hep-ex/0408116.
[16] F. E. Close and P. R. Page, Phys. Lett. B **578**, 119 (2003).
[17] T. Barnes and S. Godfrey, Phys. Rev. D **69** , 054008 (2004).
[18] S. Pakvasa and M. Suzuki, Phys. Lett. B **579**, 67 (2004).
[19] D. V. Bugg, Phys. Rev. D **71**, 016006 (2005).
[20] L. Micu, Nucl. Phys. **B10**, 521 (1969).
[21] A. Le Yaouanc, L. Oliver, O. Pene, and J. Raynal, Phys. Rev. D **8**, 223 (1973).
[22] G. Busetto and L. Oliver, Z. Phys. C **20**, 247 (1983).
[23] R. Kokoski and N. Isgur, Phys. Rev. D **35**, 907 (1987).
[24] T. Barnes, F. E. Close, P. R. Page, and E. S. Swanson, Phys. Rev. D **55**, 4157 (1997).
[25] T. Barnes, N. Black, and P. R. Page, Phys. Rev. D **68**, 054014 (2003).
[26] T. Barnes, S. Godfrey, and E. S. Swanson, hep-ph/0505002.
[27] F. E. Close and E. S. Swanson, hep-ph/0505206.
[28] E. Eichten, K. Gottfried, T. Kinoshita, K. D. Lane, and T. M. Yan, Phys. Rev. D **17**, 3090 (1978); **21**, 203 (1980).
[29] E. S. Ackleh, T. Barnes, and E. S. Swanson, Phys. Rev. D **54**, 6811 (1996).
[30] Yu. A. Simonov, Phys. At. Nucl. **66**, 2045 (2003) [Yad. Fiz. **66**, 2095 (2003)].
[31] E. J. Eichten, K. Lane, and C. Quigg, Phys. Rev. D **69**, 094019 (2004).
[32] T. Barnes, hep-ph/0412057.
[33] E. van Beveren, C. Dullemond, and G. Rupp, Phys. Rev. D **21**, 772 (1980); G. Rupp, E. van Beveren, and M. D. Scadron, Phys. Rev. D **65**, 078501 (2002);E. van Beveren and G. Rupp, hep-ph/0304105.
[34] N. A. Tornqvist, Z. Phys. C **68**, 647 (1995); N. A. Tornqvist and M. Roos, Phys. Rev. Lett. **76**, 1575 (1996).
[35] M. P. Locher, V. E. Markushin, and H. Q. Zheng, Eur. Phys. J. C **4**, 317 (1998); V. E. Markushin, Eur. Phys. J. A **8**, 389 (2000).
[36] M. Boglione and M. R. Pennington, Phys. Rev. D **65**, 114010 (2002).
[37] T. Barnes, F. E. Close, and H. J. Lipkin, Phys. Rev. D **68**, 054006 (2003); E. van Beveren and G. Rupp, Phys. Rev. Lett. **91**, 012003 (2003).
[38] Yu. A. Simonov and J. A. Tjon, Phys. Rev. D **70**, 114013 (2004).
[39] B. O. Kerbikov, Theor. Math. Phys. (Engl. Transl.) **65**, 379 (1985) [Teoreticheskaya i Matematicheskaya Fizika **65**, 379 (1985)].
[40] S. Weinberg, Phys. Rev. **130**, 776 (1963); **131**, 440 (1963); **137**, B672 (1965).
[41] L. N. Bogdanova, G. M. Hale, and V. E. Markushin, Phys. Rev. C **44**, 1289 (1991).
[42] V. Baru, J. Haidenbauer, C. Hanhart, Yu. Kalashnikova, and A. Kudryavtsev, Phys. Lett. B **586**, 53 (2004).
[43] V. Baru, J. Haidenbauer, C. Hanhart, A. Kudryavtsev, and Ulf-G. Meissner, Eur. Phys. J. A **23**, 523 (2005).
[44] A. M. Badalian, V. L. Morgunov, and B. L. G. Bakker, Phys. At. Nucl. **63**, 1635 (2000) [Yad. Fiz. **63**, 1722 (2000).

[45] S. Eidelman *et al.* (Particle Data Group), Phys. Lett. B **592**, 1 (2004).
[46] J.L. Rosner, Phys. Rev. D **64**, 094002 (2001).
[47] M.B. Voloshin, Phys. Rev. D **71**, 114003 (2005).
[48] M. B. Voloshin, Phys. Lett. B **604**, 69 (2004).
[49] M.T. AlFiky, F. Gabbiani, and A.A. Petrov, hep-ph/0506141.
[50] E.S. Swanson, Phys. Lett. B **588**, 189 (2004).
[51] M.B. Voloshin, Phys. Lett. B **579**, 316 (2004).
[52] E. Braaten and M. Kusunoki, Phys. Rev. D **69**, 074005 (2004).
[53] E. Braaten, M. Kusunoki, and S. Nussinov, Phys. Rev. Lett. **93**, 162001 (2004).
[54] E. Braaten and M. Kusunoki, Phys. Rev. D **72**, 014012 (2005).
[55] S.-K. Choi, and S.L. Olsen, *et al.* (Belle Collaboration), Phys. Rev. Lett. **94**, 182002 (2005).