Double beta decay versus cosmology: Majorana *CP* phases and nuclear matrix elements

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We discuss the relation between the absolute neutrino mass scale, the effective mass measured in neutrinoless double beta decay, and the Majorana *CP* phases. Emphasis is placed on estimating the upper bound on the nuclear matrix element entering calculations of the double beta decay half-life. Combining the claimed evidence for neutrinoless double beta decay with the neutrino mass bound from cosmology, one of the Majorana *CP* phases can be constrained.

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I. INTRODUCTION

Over the past years much effort has been invested to probe leptonic mixing with increasing accuracy, and, in fact, a unique picture is evolving from the precise measurements of neutrino oscillation probabilities. For a full construction of the mixing matrix, however, knowledge about *CP* violating phases is necessary.

To describe leptonic mixing one can always work in a basis where the charged lepton Yukawa matrix is diagonal. In this case the neutrino mass matrix m^ν can be written in the flavor basis as

$$m^\nu = U^* m^{\mathrm{diag}} U^\dagger, \tag{1}$$

with $m^{\mathrm{diag}} = \mathrm{diag}(m_1, m_2, m_3)$, and the Maki-Nakagawa-Sakata (MNS) matrix U, which in general contains three mixing angles and three *CP* violating phases. The determination of the mixing angles is subject to neutrino oscillation experiments, as is (at least in principle) the determination of the Dirac phase—the leptonic analog of the Cabibbo-Kobayashi-Maskawa (CKM) phase in the quark sector. The three mixing angles can be identified with the maximal, large and small observables measured in atmospheric, solar and reactor neutrino oscillations, respectively [1]. Important information on the Dirac phase can be expected from future long-baseline experiments [2], and ultimately, from a neutrino factory [3].

If the neutrino is of Majorana type, two additional phases enter, though [4]. The determination of these Majorana phases is, in fact, the most challenging task in the reconstruction of the fundamental parameters of the standard model particle content. In general, U can be written as

$$U = V \cdot \mathrm{diag}(1, e^{i\phi_{12}/2}, e^{i\phi_{23}/2}), \tag{2}$$

where V is parametrized in the standard CKM form,

$$V = \begin{pmatrix} c_{13}c_{12} & c_{13}s_{12} & s_{13}e^{-i\delta} \\ -c_{23}s_{12} - s_{23}s_{13}c_{12}e^{i\delta} & c_{23}c_{12} - s_{23}s_{13}s_{12}e^{i\delta} & s_{23}c_{13} \\ s_{23}s_{12} - c_{23}s_{13}c_{12}e^{i\delta} & -s_{23}c_{12} - c_{23}s_{13}s_{12}e^{i\delta} & c_{23}c_{13} \end{pmatrix}, \tag{3}$$

and ϕ_{ij} are the Majorana phases under discussion. A viable possibility to obtain information on the Majorana phases is to compare measurements of the absolute neutrino masses m_i with the elements of the neutrino mass matrix m^ν in the flavor basis. While absolute neutrino masses are most stringently constrained from cosmology, only the ee element of m^ν is experimentally accessible, being the effective mass m_{ee} measured in neutrinoless double beta decay. Several studies have discussed the relations of m_{ee}, absolute neutrino masses and Majorana phases [5]. In [6] it was pointed out that one could restrict the Majorana phase ϕ_{12} by using the recently claimed evidence for neutrinoless double beta decay [7] and the cosmological neutrino mass bound derived by the WMAP Collaboration [8], if a certain nuclear matrix element (NME) calculation [9] is assumed. An important issue is, however, the uncertainty in the neutrino mass determination within the double beta decay framework due to systematical limitations in such NME calculations. In this work we focus on what can be learned about Majorana phases from recent double beta decay and cosmological structure formation data, in view of a theoretical upper bound on the NME. In the following, we first review the claimed evidence for neutrinoless double beta decay and discuss the upper bound on the NME. Finally, we compare the resulting lower bound on m_{ee} with the upper bounds on neutrino masses from cosmology.

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II. NEUTRINOLESS DOUBLE BETA DECAY

The half-life of neutrinoless double beta decay is given by

$$[T_{1/2}^{0\nu\beta\beta}]^{-1} = \left|\frac{m_{ee}}{m_e}\right|^2 G_1^{(0\nu)}|\mathcal{M}^{(0\nu)}|^2, \tag{4}$$

where m_e denotes the electron rest mass, $G_1^{(0\nu)}$ is a phase space factor, and the NME is given by $\mathcal{M}^{(0\nu)} = \mathcal{M}_{GT} - \mathcal{M}_F$, being a combination of Gamov-Teller and Fermi transitions.

The neutrinoless double beta decay is sensitive to the ee element of the mass matrix m^ν (1) in flavor space,

$$\begin{aligned} |m_{ee}| &= \left|\sum_i |V_{ei}|^2 e^{i\phi_i} m_i\right| \\ &= |m_1|V_{e1}|^2 + \sqrt{m_1^2 + \Delta m_{12}^2}|V_{e2}|^2 e^{i\phi_{12}} \\ &\quad + \sqrt{m_1^2 + \Delta m_{12}^2 \pm \Delta m_{23}^2}|V_{e3}|^2 e^{i\phi_{23}}|. \end{aligned} \tag{5}$$

Here the mass eigenstates are expressed as m_1, $m_2 = \sqrt{m_1^2 + \Delta m_{12}^2}$, $m_3 = \sqrt{m_2^2 \pm \Delta m_{23}^2}$, where the plus (minus) sign applies in the normal (inverted) mass hierarchy case.

From the Chooz and Palo Verde experiments we know that $|V_{e3}^2| \ll |V_{e1}^2|, |V_{e2}^2|$. Moreover, in the quasidegenerate mass range explored by the present experiments, one has $m_1^2 \gg \Delta m_{12}^2, \Delta m_{23}^2$. Employing the above approximations we arrive at a simplified expression for m_{ee} [10]:

$$|m_{ee}|^2 \approx \left[1 - \sin^2(2\theta_{12})\sin^2\left(\frac{\phi_{12}}{2}\right)\right] m_1^2, \tag{6}$$

illustrating that m_{ee} is mostly sensitive to θ_{12} and ϕ_{12}.

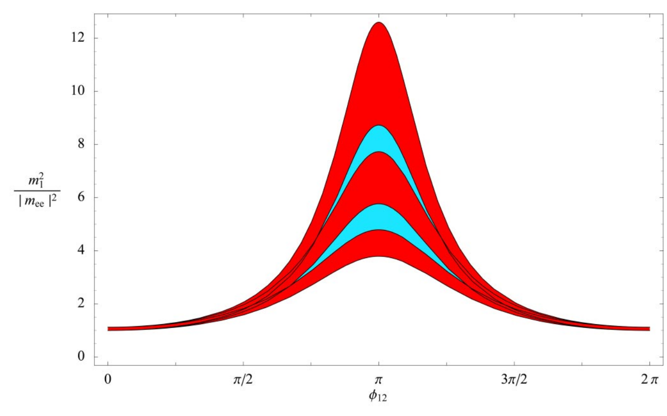

FIG. 1 (color online). $m_1^2/|m_{ee}|^2$ as a function of the Majorana phase ϕ_{12}. The full colored region is the allowed range defined by present 2σ limits on θ_{12}, θ_{23}, θ_{13} Δm_{12}^2, Δm_{23}^2. The dark/red bands correspond to a fixed maximal (upper band), best-fit (middle band) and minimal (lower band) value for θ_{12}, varying all other parameters. The second Majorana phase ϕ_{23} has been varied in the full range $[0, 2\pi]$.

In Figs. 1 and 2, the quantity $m_1^2/|m_{ee}|^2$ is shown as a function of ϕ_{12}, using the exact relation (5). The full shaded/colored region indicates the allowed range according to the combination of 2σ (Fig. 1) and 3σ (Fig. 2) limits on the neutrino oscillation observables θ_{12}, θ_{23}, θ_{13}, Δm_{12}^2, Δm_{23}^2 [1]. The remaining Majorana phase ϕ_{23} is varied in its full range, $\phi_{23} \in [0, 2\pi]$. The dark/red bands correspond to a fixed maximal (upper band), best-fit (middle band) and minimal (lower band) value for θ_{12}, varying all other parameters. The minimum values for $m_1^2/|m_{ee}|^2$ at $\phi_{12} = \pi$ are 3.81 and 3.23 for 2σ and 3σ oscillation limits, respectively. Thus, if the experimental value for $m_1^2/|m_{ee}|^2$ turns out to be smaller, a Majorana CP phase $\phi_{12} = \pi$ is excluded. If, finally $m_1^2/|m_{ee}|^2 < 1$, the bound from unitarity of the MNS matrix U would be violated, resulting in the conclusion that either the limit on m_1 or the limit on $|m_{ee}|$ is not applicable.

Recently, a new publication of data of the Heidelberg-Moscow experiment [11], searching for the double beta decay of ^{76}Ge, has appeared. In this work the authors have analyzed the data taken in the period 1990–2003, and applied a new energy calibration, which increased the previous claim for evidence [7] to 4.2σ statistical significance. The allowed range corresponds to [12]

$$|m_{ee}| = \xi \cdot (0.39\text{–}0.49\ \text{eV}) \quad [1\sigma], \tag{7}$$

$$|m_{ee}| = \xi \cdot (0.32\text{–}0.54\ \text{eV}) \quad [2\sigma], \tag{8}$$

$$|m_{ee}| = \xi \cdot (0.24\text{–}0.58\ \text{eV}) \quad [3\sigma]. \tag{9}$$

Here, $\xi = \mathcal{M}^{\text{SMK}}/\mathcal{M}^{(0\nu)}$ denotes the normalization to the NME $\mathcal{M}^{\text{SMK}} = 4.2$, calculated by Staudt, Muto, and Klapdor-Kleingrothaus in the proton-neutron quasiparticle random-phase approximation (pn-QRPA) model [9]. While the initial claim caused a critical debate [13] and was not confirmed by an independent analysis of the data [14], several of these issues have been clarified in [15]. Remaining criticism concerns the exact peak position in

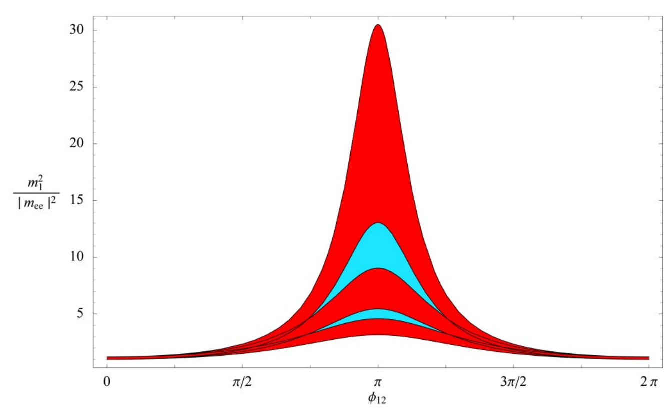

FIG. 2 (color online). As Fig. 1, but with 3σ limits on θ_{12}, θ_{23}, θ_{13} Δm_{12}^2, Δm_{23}^2.

the energy spectrum and the relative strength of measured background lines, which could suggest that the observed signal is due to an unidentified background line or that the statistical significance of the signal is overestimated [16]. In any case, we feel motivated to take the evidence claim at face value and discuss possible consequences, although we stress that an independent test of the claimed evidence is essential, if possible with a different double beta emitter isotope. Such a test could be realized by the recently started CUORICINO [17] and NEMO experiments [18] and the recent MPI proposal [19] which revived the GENIUS proposal of the Heidelberg group [20].

III. NUCLEAR MATRIX ELEMENT CALCULATIONS

Any conclusions about relations of the absolute neutrino mass m_1 and the double beta decay observable m_{ee} depend crucially on the magnitude of the calculated NME. Typically the uncertainty of such calculations has been estimated to be a factor 2–3, assumed around a given central value such as the calculation in [9], i.e. $\xi \in [0.5, 2]$. In the following we will argue in favor of a more stringent *upper* bound on ξ and that it constrains the allowed range of the *CP* Majorana phase ϕ_{12}. In Table I a scan of all the available matrix element calculations has been performed. Most of the used models are based on the pn-QRPA, like the renormalized pn-QRPA, denoted by RQRPA in the table, the self-consistent pn-QRPA (SQRPA), the self-consistent RQRPA (SRQRPA), and the fully self-consistent RQRPA (full-RQRPA). In addition, the pn-QRPA has been improved by performing a particle-number projection on it in Ref. [23]. This theory has been denoted by projected pn-QRPA in the table. The line with pn-QRPA + pn pairing in the table denotes a theory where proton-neutron pairing has been added to the RQRPA framework.

In these calculations various sizes of the proton and neutron single-particle valence spaces have been used. They range from rather modest to very extensive single-particle bases. Also, the single-particle energies have been obtained either from the experimental data, or, more frequently, from a Coulomb-corrected phenomenological Woods-Saxon potential where the parameters have been adjusted to reproduce spectroscopic properties of nuclei close to the beta-stability line. In addition, in some calculations the Woods-Saxon single-particle energies have been varied close to the proton and neutron Fermi levels to reproduce low-energy spectra of the neighboring nuclei with an odd number of protons or neutrons. Hence, remarkably diversified starting points have been used for the calculations.

TABLE I. Compilation of calculated nuclear matrix elements for the neutrinoless double beta decay of ^{76}Ge. The first column gives the value(s) of the NME, the second column the theory used to evaluate the NME, and the last column the works where the quoted theory was used to evaluate the NME.

NME	Theory	References
5.00, 1.74	Shell model	[21,22]
1.53–4.59	pn-QRPA	[9,23–31]
1.50	pn-QRPA + pn pairing	[26]
3.45	Projected pn-QRPA	[23]
6.76	VAMPIR	[32]
1.87–2.81	RQRPA	[30,31,33,34]
0.59–0.65	SRQRPA	[30]
2.40	full-RQRPA	[31]
3.21	SQRPA	[31]

Some nuclear matrix elements of this table can be discarded due to various deficiencies in the theoretical frameworks used to evaluate them. This concerns the shell-model matrix element $\mathcal{M}^{(0\nu)} = 5.00$ of Haxton and Stephenson [21], who used the weak-coupling approximation in evaluation of it. This is quite a rough approximation, and the more recent matrix element $\mathcal{M}^{(0\nu)} = 1.74$, obtained by performing a large-scale shell-model calculation with realistic two-body forces, should be more reliable, although some doubts concerning the adequacy of the size of the used single-particle basis have been voiced. The reliability of the pn-QRPA calculation including the proton-neutron pairing has been questioned due to the way the pairing is introduced to the theory. One can ignore the corresponding matrix element if one wants, without changing our final conclusions concerning the upper limit of the computed NME. The largest matrix element was calculated by Tomoda *et al.* in [32] by using a quasiparticle mean-field based VAMPIR (variation after mean-field projection in realistic model spaces) approach with particle-number and angular-momentum projections included. The shortcoming of this approach is that it does not include the proton-neutron interaction in its framework, being essential in the description of charge-changing nuclear transitions, which occur also in the double beta decay.

The above considerations lead to the range

$$0.59 \leq \mathcal{M}^{(0\nu)} \leq 4.59 \tag{10}$$

for the acceptable nuclear matrix elements. Since the upper limit of the above range comes from a pn-QRPA calculation, one may ask how does the "g_{pp} problem" of the pn-QRPA affect this value. This problem concerns the calculated matrix element of the two-neutrino double beta decay $\mathcal{M}^{(2\nu)}$, which turns out to depend strongly on the parameter g_{pp}, used as a scaling parameter of the particle-particle part of the proton-neutron two-body interaction [35,36]. However, while the uncertainty in g_{pp} affects the *lower bound* on NME calculations dramatically ($\mathcal{M}^{(2\nu)}$ can even become zero for a large value of g_{pp}) towards lower values of g_{pp} the value for the NME enters a plateau, making the *upper bound* more stable against variations in g_{pp}. Moreover, even though the NME corresponding to the two-neutrino mode depends strongly on g_{pp} within its physical range, the NME corresponding to the neutrinoless mode depends only very weakly on this parameter. This can be clearly seen in Figs. 1 and 2 of Ref. [23], where the relevant double Gamow-Teller and double Fermi matrix elements have been plotted as functions of g_{pp} for the pn-QRPA and projected pn-QRPA calculations of the ^{76}Ge double beta decay. The variation of these matrix elements around the physical value of $g_{pp} \simeq 1$ is less than 20%. Adding an uncertainty of 20% to the above range of acceptable values of NMEs leads us to the upper limit

$$\mathcal{M}^{(0\nu)} < 5.5 \tag{11}$$

of the NME.

Consequently, the limits (7)–(9) read as

$$|m_{ee}| > 0.30 \text{ eV} \quad [1\sigma], \tag{12}$$

$$|m_{ee}| > 0.24 \text{ eV} \quad [2\sigma], \tag{13}$$

$$|m_{ee}| > 0.18 \text{ eV} \quad [3\sigma]. \tag{14}$$

IV. COSMOLOGICAL BOUNDS ON $\sum_i m_i$

These lower bounds on m_{ee} have to be compared to the most stringent upper bounds on the absolute neutrino mass scale, m_1, which are presently provided by data on cosmological structure formation. According to big bang cosmology, the masses of nonrelativistic neutrinos are related to the neutrino fraction of the closure density by $\sum_i m_i = 40\Omega_\nu h_{65}^2$ eV, where h_{65} is the present Hubble parameter in units of 65 km/(s Mpc). In the currently favored cold dark matter cosmology with a nonvanishing cosmological constant Λ (ΛCDM), there is scant room left for the neutrino component. The free-streaming relativistic neutrinos suppress the growth of fluctuations on scales below the horizon [approximately the Hubble size $c/H(z)$] until they become nonrelativistic at redshifts $z \sim m_i/3T_0 \sim 1000\,(m_i/\text{eV})$.

Recent limits, obtained by combining cosmological microwave background radiation (CMB) measurements with data on the large-scale structure of the universe, imply an upper bound on the sum of the three neutrino mass eigenstates [37] (compare also [38])

$$\sum_i m_i < 0.42\text{–}1.80 \text{ eV} \quad [2\sigma]. \tag{15}$$

The exact cosmological bound depends on the data and specific priors on cosmological parameters used in the analysis: The weakest bound utilizes only data from WMAP and the Sloan Digital Sky Survey (SDSS). In particular, limits utilizing the Lyman-α forest, the absorption observed in quasar spectra by neutral hydrogen in the intergalactic medium, provide more stringent bounds [8,37,39]

$$\sum_i m_i < 0.42\text{–}0.69 \text{ eV} \quad [2\sigma], \tag{16}$$

as compared to the data sets without the Lyman-α forest [40],

$$\sum_i m_i < 0.6\text{–}1.8 \text{ eV} \quad [2\sigma]. \tag{17}$$

Two further analyses should only be mentioned here. A recent work using only the CMB data of WMAP and the assumption of a flat universe obtained an upper bound of 2.0 eV [41], and a fit with a free parametrization of the dark energy equation of state [42] has shown to yield a neutrino mass bound of 1.48 eV, being weakened by more than a factor of 2, compared to the bound assuming a cosmological constant. Finally it should be stressed, that within the present decade, the combination of SDSS data with CMB data of the PLANCK satellite will obtain a 2σ detection threshold on $\sum_i m_i$ close to 0.1–0.2 eV [37,43].

Since the effects of possible systematics in the Lyman-α forest data set need still to be explored, both observationally and theoretically, each set of analyses shall be discussed in the following.

Figure 3 shows the relation of the sum of neutrino masses with m_1,

$$\sum_i m_i = m_1\left(1 + \sqrt{1 + \frac{\Delta m_{12}^2}{m_1^2}} + \sqrt{1 + \frac{\Delta m_{12}^2}{m_1^2} \pm \frac{\Delta m_{23}^2}{m_1^2}}\right), \tag{18}$$

for normal and inverse hierarchy in the upper and lower panels, respectively. The different curves indicate the best fit and upper and lower 2σ and 3σ ranges.

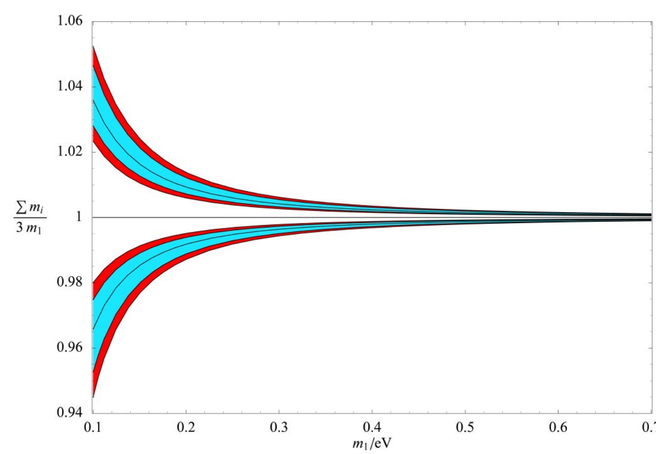

FIG. 3 (color online). Relation of the cosmologically relevant sum of neutrino masses, $\sum_i m_i$ and the lightest neutrino mass m_1. The upper and lower branches correspond to normal and inverse hierarchy, respectively. The light/blue and dark/red bands indicate the 2σ and 3σ ranges of Δm_{12}^2 and Δm_{23}^2, while the central line marks the best fit.

Combining the resulting bound on m_1 with the range for the effective mass m_{ee} given in [11], one obtains

$$m_1^2/|m_{ee}|^2 < 0.36\text{–}0.93\ (0.64\text{–}1.7) \quad \text{at } 2\sigma \quad (3\sigma) \qquad (19)$$

for data sets using the Lyman-α forest and

$$m_1^2/|m_{ee}|^2 < 0.71\text{–}6.3\ (1.3\text{–}11.1) \quad \text{at } 2\sigma \quad (3\sigma) \qquad (20)$$

without the Lyman-α forest.

It is obvious, that the 2σ range for the double beta decay observable is in conflict with all cosmological fits including the Lyman-α forest and with some without the Lyman-α forest. This may indicate that either the double beta decay signal is due to a statistical fluctuation or due to a mechanism involving exchange of other particles besides light massive Majorana neutrinos. Examples for the latter case include, e.g. sparticles in R-parity violating supersymmetry, leptoquarks, or right-handed neutrinos and W bosons (for an overview see [44]). Alternatively, the cosmological neutrino mass bound may not be applicable, e.g. by introducing broken scale invariance in the primordial power spectrum [45], or due to fast decays of the relic neutrino background [46].

The 3σ range for m_{ee}, however, is still compatible with most of the cosmological bounds although in most cases it is smaller than the minimum value, $m_1^2/|m_{ee}|^2 = 3.81$ for $\phi_{12} = \pi$. If a value $1 < m_1^2/|m_{ee}|^2 < 3.81$ will be confirmed in future cosmological data fits or in the upcoming tritium beta decay spectrometer KATRIN [47], Majorana CP phases around $\phi_{12} = \pi$ can be excluded.

If applied conversely, the analysis yields an experimental lower bound on the NME $\mathcal{M}^{(0\nu)}$, using the relations (8) and (9) and assuming that all theoretical and experimental inputs discussed are applicable. Figure 4 shows the lower bound on $\mathcal{M}^{(0\nu)}$ as a function of ϕ_{12} for 2σ and 3σ measurements of m_{ee} and the cosmological bounds (16) and (17) with and without the Lyman-α forest data. As in (19), one finds again that most of the Lyman-α data set is not compatible with the theoretical upper bound on the NME, $\mathcal{M}^{(0\nu)} < 5.5$.

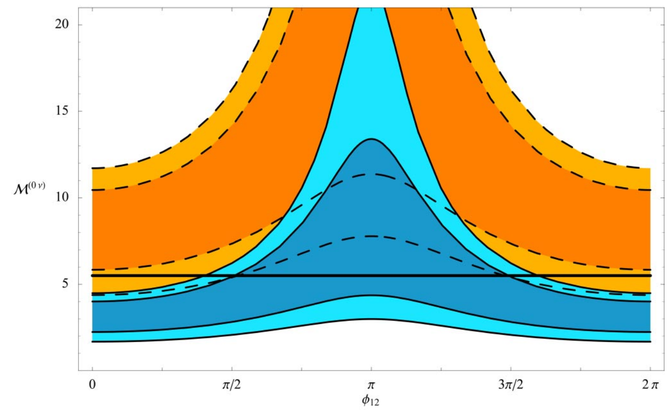

FIG. 4 (color online). Lower bound on $\mathcal{M}^{(0\nu)}$ as a function of ϕ_{12} using various data sets, obtained from the relation (8) (2σ) and (9) (3σ). The dashed boundings (red bands) include the various cosmological bounds (16) utilizing the Lyman-α forest, whereas the solid boundings (blue bands) correspond to the bounds (17) without the Lyman-α forest data. Darker and light colors refer to a combination of 2σ and 3σ neutrino constraints, respectively. The straight line denotes the theoretical upper limit (11) on $\mathcal{M}^{(0\nu)}$.

V. CONCLUSION

In conclusion, we discussed upper bounds on nuclear matrix elements and their implications for the Majorana CP phase ϕ_{12}, when the recent evidence claim for neutrinoless double beta decay and the cosmological neutrino mass bound within the standard ΛCDM cosmology with a nonvanishing cosmological constant are combined. The result depends crucially, both on the confidence region assumed for neutrino data as well as on the data used in the cosmological analyses. We deduced that for a combination of 2σ experimental limits in most analyses the mass mechanism interpretation of the double beta decay evidence is incompatible with the cosmological neutrino mass bound. On the other hand, the range of the Majorana phase ϕ_{12} can be constrained, when combining 3σ experimental limits with reasonable upper bounds for the nuclear matrix elements. Assuming CP conservation, the CP phase factors $\exp(i\phi_{ij})$ are reduced to CP parities $\eta_{ij} = \pm 1$. In this case the neutrino CP parity is fixed to $\eta_{12} = +1$.

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[1] M. Maltoni, T. Schwetz, M. A. Tortola, and J. W. F. Valle, New J. Phys. **6**, 122 (2004).
[2] P. Huber, M. Lindner, M. Rolinec, T. Schwetz, and W. Winter, Phys. Rev. D **70**, 073014 (2004).
[3] M. Apollonio *et al.*, hep-ph/0210192.
[4] S. M. Bilenky, J. Hosek, and S. T. Petcov, Phys. Lett. **94B**, 495 (1980); J. Schechter and J. W. F. Valle, Phys. Rev. D **23**, 1666 (1981); M. Doi, T. Kotani, H. Nishiura, K. Okuda, and E. Takasugi, Phys. Lett. **102B**, 323 (1981).
[5] S. M. Bilenky, C. Giunti, C. W. Kim, and S. T. Petcov, Phys. Rev. D **54**, 4432 (1996); K. Matsuda, N. Takeda, T. Fukuyama, and H. Nishiura, Phys. Rev. D **63**, 077301 (2001); S. M. Bilenky, S. Pascoli, and S. T. Petcov, Phys.

Rev. D **64**, 053010 (2001); W. Rodejohann, hep-ph/0203214; V. Barger, S. L. Glashow, P. Langacker, and D. Marfatia, Phys. Lett. B **540**, 247 (2002); S. Pascoli, S. T. Petcov, and W. Rodejohann, Phys. Lett. B **549**, 177 (2002); A. Abada and G. Bhattacharyya, Phys. Rev. D **68**, 033004 (2003); S. h. Chang, S. K. Kang, and K. Siyeon, Phys. Lett. B **597**, 78 (2004); S. Pascoli and S. T. Petcov, Phys. Lett. B **580**, 280 (2004).
[6] K. Matsuda, T. Fukuyama, and H. Nishiura, Mod. Phys. Lett. A **18**, 1803 (2003).
[7] H. V. Klapdor-Kleingrothaus, A. Dietz, H. L. Harney, and I. V. Krivosheina, Mod. Phys. Lett. A **16**, 2409 (2001); H. V. Klapdor-Kleingrothaus, A. Dietz, and I. Krivosheina, Part. Nucl. **110**, 57 (2002); Found. Phys. **32**, 1181 (2002).
[8] D. N. Spergel *et al.*, Astrophys. J. Suppl. Ser. **148**, 175 (2003).
[9] A. Staudt, K. Muto, and H. V. Klapdor-Kleingrothaus, Europhys. Lett. **13**, 31 (1990); K. Muto, E. Bender, and H. V. Klapdor, Z. Phys. A **334**, 187 (1989); Z. Phys. A **334**, 177 (1989).
[10] H. Päs and T. J. Weiler, Phys. Rev. D **63**, 113015 (2001).
[11] H. V. Klapdor-Kleingrothaus, I. V. Krivosheina, A. Dietz, and O. Chkvorets, Phys. Lett. B **586**, 198 (2004).
[12] A. Dietz (private communication).
[13] C. E. Aalseth *et al.*, Mod. Phys. Lett. A **17**, 1475 (2002); F. Feruglio, A. Strumia, and F. Vissani, Nucl. Phys. **B637**, 345 (2002); H. V. Klapdor-Kleingrothaus, hep-ph/0205228.
[14] A. M. Bakalyarov, A. Y. Balysh, S. T. Belyaev, V. I. Lebedev, and S. V. Zhukov, Pis'ma Fiz. Elem. Chast. Atom. Yadra **2**, 21 (2005) [Phys. Part. Nucl. Lett. **2**, 77 (2005)].
[15] H. V. Klapdor-Kleingrothaus, A. Dietz, I. V. Krivosheina, C. Dorr, and C. Tomei, Phys. Lett. B **578**, 54 (2004).
[16] G. Heusser (private communication).
[17] C. Arnaboldi *et al.*, Phys. Lett. B **584**, 260 (2004).
[18] F. Piquemal *et al.* (NEMO Collaboration), hep-ex/0205006.
[19] I. Abt *et al.*, hep-ex/0404039.
[20] J. Hellmig and H. V. Klapdor-Kleingrothaus, Z. Phys. A **359**, 351 (1997); H. V. Klapdor-Kleingrothaus and M. Hirsch, Z. Phys. A **359**, 361 (1997); H. V. Klapdor-Kleingrothaus, L. Baudis, G. Heusser, B. Majorovits, and H. Päs, hep-ph/9910205; MPI Report No. MPI-H-V26-1999, 1999; G. Heusser, Annu. Rev. Nucl. Part. Sci. **45**, 543 (1995).
[21] W. C. Haxton and G. J. Stephenson, Prog. Part. Nucl. Phys. **12**, 409 (1984).
[22] J. Retamosa, E. Caurier, F. Nowacki, and A. Poves, Phys. Rev. C **55**, 1266 (1997).
[23] J. Suhonen, O. Civitarese, and A. Faessler, Nucl. Phys. **A543**, 645 (1992).
[24] T. Tomoda, Rep. Prog. Phys. **54**, 53 (1991).
[25] G. Pantis, J. D. Vergados, A. Faessler, and W. A. Kaminski, J. Phys. G **18**, 605 (1992).
[26] G. Pantis, F. Simkovic, J. D. Vergados, and A. Faessler, Phys. Rev. C **53**, 695 (1996).
[27] M. Aunola and J. Suhonen, Nucl. Phys. **A643**, 207 (1998).
[28] C. Barbero, F. Krmpotic, A. Mariano, and D. Tadic, Nucl. Phys. **A650**, 485 (1999).
[29] J. Suhonen, Phys. Lett. B **477**, 99 (2000).
[30] A. Bobyk, W. A. Kaminski, and F. Simkovic, Phys. Rev. C **63**, 051301 (2001).
[31] S. Stoica and H. V. Klapdor-Kleingrothaus, Phys. Rev. C **63**, 064304 (2001).
[32] T. Tomoda, A. Faessler, K. W. Schmid, and F. Grummer, Nucl. Phys. **A452**, 591 (1986).
[33] F. Simkovic, G. Pantis, J. D. Vergados, and A. Faessler, Phys. Rev. C **60**, 055502 (1999).
[34] V. A. Rodin, A. Faessler, F. Simkovic, and P. Vogel, Phys. Rev. C **68**, 044302 (2003).
[35] P. Vogel and M. R. Zirnbauer, Phys. Rev. Lett. **57**, 3148 (1986).
[36] O. Civitarese, A. Faessler, and T. Tomoda, Phys. Lett. B **194**, 11 (1987).
[37] S. Hannestad, hep-ph/0409108; New J. Phys. **6**, 108 (2004).
[38] S. Pastor, hep-ph/0505148.
[39] U. Seljak *et al.*, Phys. Rev. D **71**, 103515 (2005); G. L. Fogli, E. Lisi, A. Marrone, A. Melchiorri, A. Palazzo, P. Serra, and J. Silk, Phys. Rev. D **70**, 113003 (2004).
[40] O. Elgaroy *et al.*, Phys. Rev. Lett. **89**, 061301 (2002); O. Elgaroy and O. Lahav, J. Cosmol. Astropart. Phys. 04 (2003) 004; S. Hannestad, J. Cosmol. Astropart. Phys. 05 (2003) 004; S. W. Allen, R. W. Schmidt, and S. L. Bridle, Mon. Not. R. Astron. Soc. **346**, 593 (2003); M. Tegmark *et al.* (SDSS Collaboration), Phys. Rev. D **69**, 103501 (2004); V. Barger, D. Marfatia, and A. Tregre, Phys. Lett. B **595**, 55 (2004); P. Crotty, J. Lesgourgues, and S. Pastor, Phys. Rev. D **69**, 123007 (2004).
[41] K. Ichikawa, M. Fukugita, and M. Kawasaki, Phys. Rev. D **71**, 043001 (2005).
[42] S. Hannestad, astro-ph/0505551.
[43] J. Lesgourgues, S. Pastor, and L. Perotto, Phys. Rev. D **70**, 045016 (2004).
[44] H. V. Klapdor-Kleingrothaus and H. Päs, hep-ph/9808350.
[45] A. Blanchard, M. Douspis, M. Rowan-Robinson, and S. Sarkar, Astron. Astrophys. **412**, 35 (2003).
[46] J. F. Beacom, N. F. Bell, and S. Dodelson, Phys. Rev. Lett. **93**, 121302 (2004).
[47] K. Eitel, Nucl. Phys. B Proc. Suppl. **143**, 197 (2005); A. Osipowicz *et al.* (KATRIN Collaboration), hep-ex/0109033.