CP violation in semileptonic tau lepton decays

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The leading order contribution to the direct *CP* asymmetry in $\tau^{\pm} \to K^{\pm}\pi^0\nu_\tau$ decay rates is evaluated within the standard model. The weak phase required for *CP* violation is introduced through an interesting mechanism involving second order weak interactions, which is also responsible for tiny violations of the $\Delta S = \Delta Q$ rule in K_{l3} decays. The calculated *CP* asymmetry turns out to be of order 10^{-12}, leaving a large window for studying effects of nonstandard sources of *CP* violation in this observable.

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Experimental searches for *CP* violating asymmetries in tau lepton semileptonic decays have been carried out in the $\tau \to \pi\pi\nu_\tau$ [1] and $\tau \to K_s\pi\nu_\tau$ [2] modes. Motivation for these searches in the context of beyond the standard model approaches were provided in Refs. [3,4]. In Ref. [2], the missing evidence for a nonzero *CP* asymmetry was interpreted in terms of a (*CP*-violating) coupling Λ due to a charged scalar exchange and the limit $-0.172 < \mathrm{Im}(\Lambda) < 0.067$ (at 90% C.L.) has been derived. The *CP*-odd observable studied in [2] depends upon two variables of a particular kinematical distribution of semileptonic tau decays as long as this effect is assumed to have its origin in the interference of scalar and vector form factors.

Motivated by these searches and the possibility of further improvements at a super-B-factory and taus produced in W and Z decays at the LHC, in the present paper we compute the leading order standard model (SM) contribution to the *CP* decay rate asymmetry between the two $\tau^{\pm} \to K^{\pm}\pi^0\nu_\tau$ decay channels. In order to have a nonzero *CP* asymmetry at the level of the decay rate, one requires that the *CP*-odd terms arise from the interference of terms in the same (vector or scalar) angular configuration of the $K\pi$ system. Although the leading contribution to this *CP* asymmetry is a second order weak process, it is interesting to estimate its actual magnitude to be sure that any eventual observation of *CP* violation in these tau decay experiments will have its origin beyond the SM framework.

For the sake of clearness, first we keep our discussion as general as possible. In the SM the total amplitude for $\tau^-(p) \to K^-(k)\pi^0(k')\nu_\tau(p')$ arising from tree level and the leading higher order terms in Figs. 1(a) and 1(b) can be written in the following general form:[1]

$$\mathcal{M} = \frac{G_F V_{us}}{\sqrt{2}}\Big\{\bar{u}(p')\gamma^\mu(1-\gamma_5)u(p) \times F_+(t)\Big[(k-k')_\mu - \frac{\Delta^2}{t}q_\mu\Big] + \bar{u}(p')(1+\gamma_5)u(p)m_\tau F_0(t)\frac{\Delta^2}{t}\Big\}, \quad (1)$$

where $q = k + k'$ ($t = q^2$) is the momentum transfer to the hadronic system, $\Delta^2 \equiv m_K^2 - m_\pi^2$ and $F_{+,0}(t)$ are the *effective* form factors describing the hadronic matrix elements.

The effective vector and scalar form factors can be written as follows:

$$F_+(t) = f_+(t) + a(t), \quad (2)$$

$$F_0(t) = f_0(t) + b(t), \quad (3)$$

where $f_i(t)$ are the usual tree-level contributions and $a(t)$, $b(t)$ denote the higher order terms arising from Fig. 1 [as we will see later, Fig. 1(b) does not induce a *CP*-violating phase]. For the purposes of numerical estimates of the *CP* asymmetry, we will choose a simple model where the form factors at the tree level are dominated by a single vector or scalar strange resonance as follows [5]:

$$f_i(t) = \frac{f_i(0)m_i^2}{m_i^2 - t - im_i\Gamma_i}, \qquad i = +, 0, \quad (4)$$

where (m_i, Γ_i) denote the mass and width of the resonance in the corresponding vector or scalar configuration [respectively the $K^*(892)$ or $K_0^*(1430)$]. Thus, the strong phase corresponding to the tree-level amplitudes is determined by the decay width of these resonances, while the weak phase is absent at the tree level.

The decay rate for the processes under consideration can be written in the following form [5]:

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[1] Note that the exchange of a charged scalar boson at the tree level such as the one advocated in Ref. [2] can be absorbed into the definition of the effective form factor $F_0(t)$ if neutrino masses can be neglected.

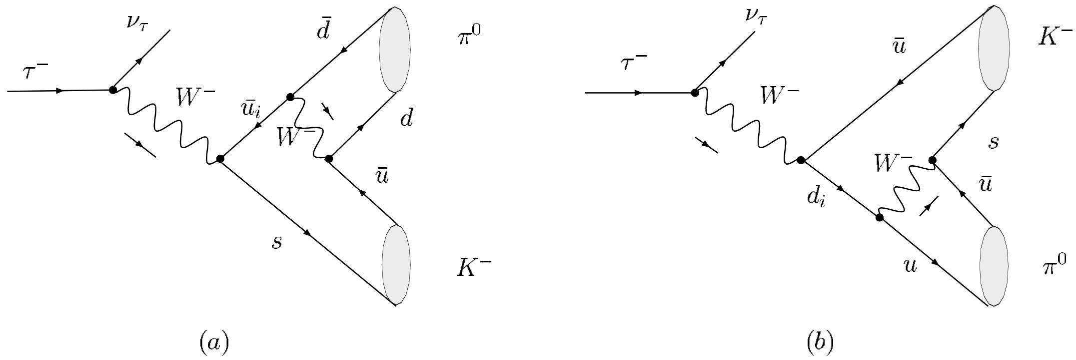

FIG. 1. Higher order terms contributing to the $\tau^- \to K^-\pi^0\nu_\tau$ decay.

$$\Gamma(\tau \to K\pi\nu) = \frac{G_F^2 m_\tau^5}{768\pi^3}|V_{us}|^2 I, \tag{5}$$

where

$$I = \frac{1}{m_\tau^6}\int_{(m_K+m_\pi)^2}^{m_\tau^2}\frac{dt}{t^3}(m_\tau^2 - t)^2 \times\left[|F_+(t)|^2\left(1+\frac{2t}{m_\tau^2}\right)\lambda^{3/2}(t, m_K^2, m_\pi^2) + 3|F_0(t)|^2\Delta^4\lambda^{1/2}(t, m_K^2, m_\pi^2)\right], \tag{6}$$

and $\lambda(x, y, z) = x^2 + y^2 + z^2 - 2xy - 2xz - 2yz$.

Note that the effective scalar and vector form factors do not interfere at the level of the hadronic spectrum or in the integrated rate because they correspond to two different angular momentum configurations ($l = 0$, and 1) of the hadronic system. This is interesting because the different strong and weak phases required for CP violation should be present in the same angular configuration of the two amplitudes. This mechanism is different from the one used in Ref. [2], where the CP asymmetry vanishes at the level of the total integrated rate and the hadronic spectrum, but it survives at the level of a double kinematical distribution. On the other hand, since the contribution due to the scalar form factor is suppressed by powers of the SU(3) breaking Δ^2 parameter, one expects that the dominant contribution to the CP asymmetry in the SM is given by the interference of the vector form factors in Eq. (2).

Next, we proceed to evaluate the amplitudes corresponding to the diagrams in Fig. 1. The weak vertices include the couplings of fermions to W and would-be Goldstone bosons, and we compute our amplitudes in the ‘t Hooft-Feynman gauge $\xi = 1$. Note that Fig. 1(a) [respectively 1(b)] involves the exchange of an intermediate up-type quark $u_i = u, c, t$ (down-type quark $d_i = d, s, b$). Note also that the amplitudes corresponding to Fig. 1(b) are proportional to $|V_{ud_i}|^2$ and will not induce a (CP-violating) weak phase in $a(t)$ or in $b(t)$.

The hadronic matrix elements that correspond to Fig. 1 can be evaluated in a similar way as perturbative QCD techniques allow one to evaluate hadronic matrix elements relevant to B decays [6]. We will use the distribution amplitudes for pseudoscalar mesons of momentum p and mass m_P [6]

$$\Psi_P(x, p) = \frac{-iI_C}{\sqrt{2N_C}}\phi_P(x)(\not{p} + m_P)\gamma_5, \tag{7}$$

where I_C is the identity in color space, N_C is the number of colors and f_P the pseudoscalar decay constant ($f_\pi = 130.7$ MeV and $f_K = 159.8$ MeV) [7]. The wave function of the pseudoscalar meson is given by $\phi_P(x) = \sqrt{3/2}\, f_P x(1-x)$ [6].

The form factors arising from the four diagrams in Fig. 1 (both W^- gauge bosons and ϕ^- would-be Goldstones are understood in wavy lines) are given by

$$a(t) = -\frac{G_F}{\sqrt{2}}\frac{V_{ud}}{V_{us}}f_K f_\pi \sum_{u_i=u,c,t} V_{u_i d}^* V_{u_i s}\left[\left((t - m_\pi^2)\left(2 + \frac{m_{u_i}m_u}{m_W^2}\right) + \frac{m_d m_{u_i}}{m_W^2}m_K m_\pi\right)I_i^1 + \left(\frac{m_{u_i}m_u}{m_W^2}m_\pi(m_\pi + m_d) + 2m_\pi^2 - m_K(m_\pi + m_d)\frac{m_{u_i}m_d}{m_W^2}\right)I_i^0\right], \tag{8}$$

$$b(t) = -\frac{1}{\sqrt{2}}G_F\frac{V_{ud}}{V_{us}}f_Kf_\pi\sum_{u_i=u,c,t}V^*_{u_id}V_{u_is}\bigg[\bigg((t-m_\pi^2-2m_K^2)\bigg(2+\frac{m_{u_i}m_u}{m_W^2}\bigg)+\frac{m_dm_{u_i}}{m_W^2}m_Km_\pi\bigg)I_i^1\frac{t}{\Delta^2}$$
$$+\bigg(\frac{m_{u_i}m_u}{m_W^2}m_\pi(m_\pi+m_d)+2m_\pi^2+m_K(m_\pi+m_d)\frac{m_{u_i}m_d}{m_W^2}\bigg)I_i^0\frac{t}{\Delta^2}+a(t)+\bigg(\frac{m_Km_\pi^2}{m_W^2}\bigg(4m_{u_i}+\frac{m_d^3-m_um_{u_i}^2}{m_W^2}\bigg)$$
$$-\frac{m_s}{2m_W^2}(t-m_K^2-m_\pi^2)\bigg(4m_{u_i}+\frac{m_d}{m_s}\frac{m_u(m_d+m_{u_i})(m_\pi+m_{u_i})}{m_W^2}\bigg)+\frac{m_{u_i}^2}{m_W^4}m_dm_\pi m_K(m_u+m_d)\bigg)I_i^0$$
$$+\bigg(\frac{m_K}{2m_W^2}(t-m_K^2-m_\pi^2)\bigg(4m_{u_i}+\frac{(m_d^3-m_um_{u_i}^2)}{m_W^2}\bigg)+m_K^2\frac{m_um_d}{m_W^4}m_\pi(m_d+m_{u_i})\bigg)I_i^1\bigg], \quad (9)$$

where we have defined the integral functions ($n = 0, 1$):

$$I_i^n \equiv \int_0^1 dx\frac{x^{n+1}(1-x)}{x^2m_K^2+x(t-m_K^2-m_\pi^2)+m_\pi^2-m_{u_i}^2+i\epsilon}. \quad (10)$$

Using the unitarity of the Cabibbo-Kobayashi-Maskawa mixing matrix, one gets

$$\sum_{u_i=u,c,t}V^*_{u_id}V_{u_is}I_i^n = V^*_{ud}V_{us}(I_u^n-I_c^n)+V^*_{td}V_{ts}(I_t^n-I_c^n)$$
$$\approx V^*_{ud}V_{us}(I_u^n-I_c^n)-V^*_{td}V_{ts}I_c^n, \quad (11)$$

where the last line is obtained using the fact that m_t is much larger than any mass of the particles involved in the process. The first term in the last equation, being proportional to $V^*_{ud}V_{us}$, will only contribute to the total rate but not to the *CP* asymmetry. As a consequence, the *CP* asymmetry will only depend on the I_c^n functions. In the kinematic region allowed for t, I_c^n has a pole when the intermediary c quark is produced on its mass shell in Fig. 1. Since the decay width Γ_c of the charm quark is much smaller than its mass, it is possible to treat this pole through the $i\epsilon$ prescription for the quark propagator [8]. Integrating the $I_c^{0,1}$ functions, one gets

$$I_c^0 = \frac{1}{m_K^2}\bigg\{\theta(m_c^2-t)\bigg[-1+\frac{(x^+-1)x^+}{x^+-x^-}\ln\bigg(\frac{x^+-1}{x^+}\bigg)+\frac{(x^--1)x^-}{x^+-x^-}\ln\bigg(\frac{x^--1}{x^-}\bigg)\bigg]-i\pi\theta(t-m_c^2)\frac{x^+(1-x^+)}{|x^+-x^-|}\bigg\}, \quad (12)$$

$$I_c^1 = \frac{1}{m_K^2}\bigg\{\theta(m_c^2-t)\bigg(\frac{1}{2}-(x^++x^-)+\frac{x^{+2}(x^+-1)}{x^+-x^-}\ln\bigg(\frac{x^+-1}{x^+}\bigg)+\frac{x^{-2}(x^--1)}{x^+-x^-}\times\ln\bigg(\frac{x^--1}{x^-}\bigg)\bigg)-i\pi\theta(t-m_c^2)\frac{x^{+2}(1-x^+)}{|x^+-x^-|}\bigg\}, \quad (13)$$

where $x^\pm = (-(t-m_K^2-m_\pi^2)\pm\sqrt{\lambda(t,m_K^2,m_\pi^2)+4m_K^2m_c^2})/(2m_K^2)$.

The following remarks are important. A simple inspection of Fig. 1 tell us that the strong phases in $f_+(t)$ [respectively $f_0(t)$] and $a(t)$ [respectively $b(t)$] are necessarily different. Indeed, the higher order *CP*-violating contributions with $\bar{c}s$ and $\bar{t}s$ intermediate states cannot produce the same resonance as the tree-level contribution does in the $\bar{u}s$ channel. Second, the presence of the pole in the I_c^n function will produce a *CP* conserving phase which could interfere with the tree-level contribution. Thus, the tree-level and higher order contributions have different weak and strong phases and will induce a direct violation of *CP*. On the other hand, note that the higher order contributions $b(t)$ to the scalar form factors are suppressed compared to the $a(t)$ contribution as it will only interfere with the f_0 form factor which is itself suppressed compared to f_+ [5]. So, we can safely neglect its contribution to the *CP* asymmetry.

The *CP* asymmetry can be written as follows:

$$A_{CP} = \frac{\Gamma(\tau^+\to K^+\pi^0\bar{\nu}_\tau)-\Gamma(\tau^-\to K^-\pi^0\nu_\tau)}{\Gamma(\tau^+\to K^+\pi^0\bar{\nu}_\tau)+\Gamma(\tau^-\to K^-\pi^0\nu_\tau)}, \quad (14)$$

$$\approx -\frac{\sqrt{2}G_F^3m_\tau^5\,\mathrm{Im}(V_{us}V^*_{ud}V_{td}V^*_{ts})f_Kf_\pi}{768\pi^3\Gamma(\tau^+\to K^+\pi^0\bar{\nu}_\tau)}\times I_{CP}, \quad (15)$$

where we have neglected the F_0 contribution to the *CP* asymmetry and have kept the dominant contributions from the amplitudes,

$$I_{CP} = \frac{1}{m_\tau^6}\int_{(m_K+m_\pi)^2}^{m_\tau^2}\frac{dt}{t^3}(m_\tau^2-t)^2h(t)\bigg(1+\frac{2t}{m_\tau^2}\bigg)\times\lambda^{3/2}(t,m_K^2,m_\pi^2), \quad (16)$$

where $h(t)$ receives two dominant contributions according to the values of t in its kinematical domain:

$$h(t) = \frac{2f_+(0)m_*^2}{(m_*^2-t)^2+m_*^2\Gamma_*^2}\{m_*\Gamma_*((t-m_\pi^2)\,\mathrm{Re}[I_c^1]+m_\pi^2\,\mathrm{Re}[I_c^0])-(m_*^2-t)((t-m_\pi^2)\,\mathrm{Im}[I_c^1]+m_\pi^2\,\mathrm{Im}[I_c^0])\}, \quad (17)$$

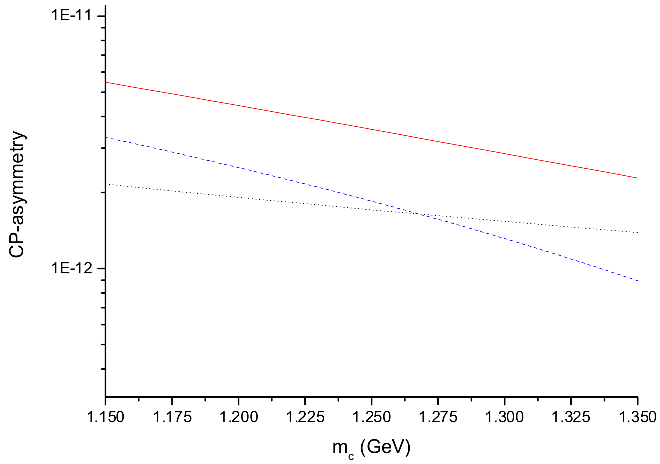

FIG. 2 (color online). Contributions to the absolute value of the CP rate asymmetry as a function of the charm quark mass m_c: contributions coming from the pole of the c quark propagator (dotted line) and coming from the interference with the strong phase of the $K^*(892)$ (dashed line), and their sum (solid line).

where (m_*, Γ_*) denote the mass and width of the $K^*(892)$ resonance. Using the approximate expression for the lifetime of the τ lepton $\tau^{-1} \approx 5G_F^2 m_\tau^5/192\pi^3$, one gets

$$A_{CP} \approx -\frac{\sqrt{2}\, G_F \, \mathrm{Im}(V_{us}V_{ud}^* V_{td} V_{ts}^*) f_K f_\pi}{20B(\tau^+ \to K^+ \pi^0 \bar{\nu}_\tau)} \times I_{CP}, \quad (18)$$

with $B(\tau^+ \to K^+\pi^0\bar{\nu}_\tau) = (4.5 \pm 0.3) \times 10^{-3}$ [7]. As we should have expected, the CP asymmetry becomes proportional to the invariant measure of CP violation: $J = \mathrm{Im}(V_{us}V_{ud}^* V_{td}V_{ts}^*) = (2.88 \pm 0.33) \times 10^{-5}$ [7].

If we insert the expressions for the form factors and use $f_+(0) = 0.982/\sqrt{2}$ [9] and $m_c = 1.35$ GeV, we obtain the following estimate for the decay rate asymmetry:

$$|A_{CP}| \approx 2.3 \times 10^{-12}. \quad (19)$$

In Fig. 2 we have plotted the absolute value of the CP asymmetry coming from the pole of the c quark propagator (dotted line) and the one coming from the interference with the strong phase of the $K^*(892)$ (dashed line) as a function of the charm quark mass within the m_c range recommended in Ref. [7]. The absolute value of the total CP rate asymmetry is represented by the solid line. As observed from Fig. 2, the CP asymmetry is not strongly sensitive on the value chosen for the charm quark mass, and the contribution from the pole term becomes smaller for larger values of m_c.

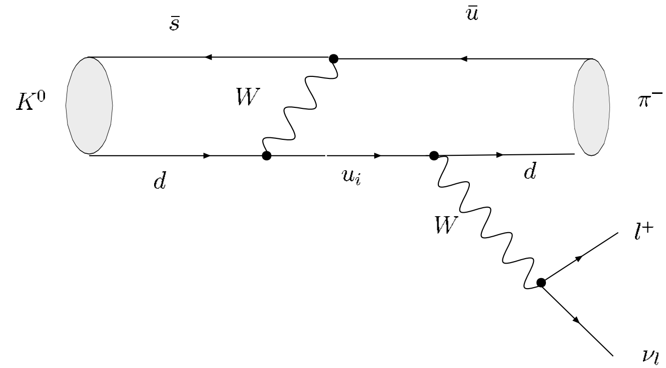

FIG. 3. Higher order contribution to the $\Delta S = \Delta Q$ violating $K^0 \to \pi^- l^+ \nu_l$ decay.

The calculated CP rate asymmetry in this τ lepton decay is small as expected from a CP violating effect that is generated by a second order weak interaction process. Thus, we can conclude that this decay mode opens a large window to study constraints on CP violation of a nonstandard origin.

The mechanism we have discussed here to generate the CP asymmetry in the SM is unusual in the sense that the weak phase does not arise from loop effects. The same mechanism can also generate a direct CP decay rate asymmetry for the isospin related $\tau^\pm \to K_S \pi^\pm \nu_\tau$ decays. Also, such a mechanism could induce tiny violations of the $\Delta S = \Delta Q$ rule. As is well known, this selection rule implies that only the decays $K^0 \to \pi^+ l^- \bar{\nu}_l$ and $\overline{K}^0 \to \pi^- l^+ \nu_l$ are allowed in the SM at the tree level. The second order processes shown in Fig. 3 would induce a very small $\Delta S = -\Delta Q$ component opening the possibility that the $K^0 \to \pi^- l^+ \nu_l$ and $\overline{K}^0 \to \pi^+ l^- \bar{\nu}_l$ can be allowed in the SM. Present experimental limits on $\Delta Q = \Delta S$ violating interactions in three-body semileptonic decays of K_L are at the 10^{-3} level, while the four-body $K^+ \to \pi^+\pi^+ e^- \bar{\nu}_e$ branching ratio has been bounded at the 10^{-8} level [7].

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