

Does the neutrino magnetic moment have an impact on solar neutrino physics?

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Solar neutrino observations coupled with the recent KamLAND data suggest that spin-flavor precession scenario does not play a major role in neutrino propagation in the solar matter. We provide approximate analytical formulas and numerical results to estimate the contribution of the spin-flavor precession, if any, to the electron neutrino survival probability when the magnetic moment and magnetic field combination is small.

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I. INTRODUCTION

In the earlier days of the solar neutrino research activities one of the more speculative solutions proposed to resolve the puzzle of missing neutrinos invoked the interaction of the neutrino magnetic moment with the solar magnetic fields. Although initial attempts [1] ignored matter effects, eventually the combined effect of matter and magnetic fields was brought out [2,3]. Simultaneous presence of a large neutrino magnetic moment, magnetic field combination, and neutrino flavor mixing can give rise to two additional resonances besides the MSW resonance [4,5]. Initial numerical calculations [6] using the resonant spin-flavor precession scheme were carried out before the gallium experiments were completed. These calculations hinted a solution of the solar neutrino problem with the neutrino parameters in the LOW region (i.e., neutrino parameter space with $\delta m^2 \sim 10^{-7} \text{ eV}^2$) provided that there is transition magnetic moment as large as $10^{-11} \mu_B$ and a magnetic field of the order of 10^5 G . With this solution count rate at the gallium detectors would be significantly reduced. Even though gallium experiments ruled out this particular solution, variants of the spin-flavor precession solution to the solar neutrino problem continue to be investigated by many researchers. (A representative set of the recent work is given in Refs. [7–10]).

In the meantime experimental data has increasingly disfavored the spin-flavor precession solution to the solar neutrino problem. Earlier reports of an anticorrelation between the solar magnetic activity and solar neutrino capture rate at the Homestake detector [11] was a prime motivation for considering the magnetic field effects. An analysis of the Super-Kamiokande data rules out such an anticorrelation [12]. (For a counter argument, however, see Ref. [13]). Also, if the neutrinos are of Majorana type, such

a scenario would produce solar antineutrinos [14,15]. Both the Super-Kamiokande [16] and KamLAND collaborations [17] failed to find any evidence for solar antineutrinos. Finally a global fit of all solar neutrino experiments *without* using the spin-flavor precession solution (see e.g. Ref. [18]) was confirmed by the KamLAND experiment.

On the other hand our knowledge of both the solar magnetic fields and neutrino magnetic moments has been improving. (At the very least we now know that neutrinos, since they are definitely massive, have magnetic moments the magnitude of which depends on the physics beyond the standard model). Hence it may be worthwhile to revisit the spin-flavor precession mechanism and knowing it is not the dominant mechanism, explore what implications it may still have.

II. STATUS ON SOLAR MAGNETIC FIELDS AND μ_ν

Neither the magnetic pressure in the core nor the impact of the structure of the magnetic field on the stellar dynamics are taken into account usually in the standard solar model [19,20]. Within the uncertainties of the nuclear physics input [21] standard solar model agrees well not only with the neutrino observations but also with the helioseismological observations of the sound speed profile [22–24]. Direct measurements of the magnetic fields in the radiative zone with acoustic modes are not possible, however, even with a one-dimensional solar model a sizable magnetic field would contribute additional pressure. An analysis found that a magnetic field greater than $\sim 10^7 \text{ G}$ localized at about $0.2R_\odot$ would cause the sound speed profile to deviate from the observed values [24]. However this particular calculation does not place restrictions on the magnetic field exactly at the center of the Sun or even at $0.1 R_\odot$. Similarly, magnetic field strengths greater than $\sim 7 \times 10^6 \text{ G}$ are not allowed since they could cancel the observed oblateness of the Sun, making it spherical and even prolate [25].

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Helioseismology provides more detailed information about the magnetic fields in the convective zone. The rotation profile of the Sun is presently known down to about $0.2R_{\odot}$: the rotation of the solar radiative zone is like that of a solid body rotating at a constant rate [26]. Such a rotation profile suggests the existence of a magnetic field in the radiative zone. The observations of the splittings of the solar oscillation frequencies can be used to infer the magnetic field. The odd terms in the azimuthal order are determined only by the rotation rate in the solar interior. The even terms may receive additional contributions from the magnetic fields. An analysis of the helioseismic data indicates that rotation alone is not sufficient to explain the observed even splitting coefficients. Other helioseismic observations are consistent with a magnetic field of ~ 20 G at a depth of 30 000 km below the solar surface [27]. Similar arguments limit the toroidal magnetic field to < 300 kG at the bottom of the convective zone. On the contrary, at present, there is no direct helioseismic evidence for the presence or absence of sizable magnetic fields in the radiative zone.

As far as the neutrino magnetic moment is concerned, best present direct upper limits come from reactor experiments, i.e. $\mu_{\nu} < 1.0 - 1.3 \times 10^{-10} \mu_B$ at 90% C.L. [28,29] which improve previous bounds [30–33], as well as from the recent Super-Kamiokande solar data which has given the limit of $< 1.5 \times 10^{-10} \mu_B$ at 90% C.L. [34]. Combining recent solar neutrino experiments with the KamLAND data yields a limit of $< 1.1 \times 10^{-10} \mu_B$ at 90% C.L. [35]. From astrophysical considerations, an indirect upper limit in the range of $10^{-11} - 10^{-12} \mu_B$ has been obtained [36], the exact limits being model dependent. New experiments are now under study which would lower the direct limits down to the level of a few $\times 10^{-12}$ using a static tritium source [37–39] while the use of low energy beta beams might lower it by about 1 order of magnitude [39,40].

III. SPIN-FLAVOR PRECESSION FORMALISM AND RESULTS

The evolution of the chiral components of two flavors of neutrinos is described by [2]

$$i \frac{d}{dt} \begin{pmatrix} \nu_e^{(L)} \\ \nu_{\mu}^{(L)} \\ \nu_e^{(R)} \\ \nu_{\mu}^{(R)} \end{pmatrix} = \begin{pmatrix} H^{(L)} & BM^{\dagger} \\ BM & H^{(R)} \end{pmatrix} \begin{pmatrix} \nu_e^{(L)} \\ \nu_{\mu}^{(L)} \\ \nu_e^{(R)} \\ \nu_{\mu}^{(R)} \end{pmatrix}. \quad (1)$$

For the Dirac neutrinos one has

$$H^{(L)} = \begin{pmatrix} \frac{\delta m^2}{2E} \sin^2 \theta + V_e & \frac{\delta m^2}{4E} \sin 2\theta \\ \frac{\delta m^2}{4E} \sin 2\theta & \frac{\delta m^2}{2E} \cos^2 \theta + V_{\mu} \end{pmatrix}, \quad (2)$$

and $H^{(R)}$ is given by setting V_e and V_{μ} equal to zero in Eq. (2). For the Majorana neutrinos in Eq. (1) one writes down for the left-handed component

$$H^{(L)} = \begin{pmatrix} V_e & \frac{\delta m^2}{4E} \sin 2\theta \\ \frac{\delta m^2}{4E} \sin 2\theta & \frac{\delta m^2}{2E} \cos 2\theta + V_{\mu} \end{pmatrix}. \quad (3)$$

For the Majorana neutrinos the right-handed part of the Hamiltonian, $H^{(R)}$, is given by replacing V_e and V_{μ} in Eq. (3) by $-V_e$ and $-V_{\mu}$, respectively. In these equations the matter potentials are

$$V_e = \frac{G_F}{\sqrt{2}} (2N_e - N_n), \quad (4)$$

and

$$V_{\mu} = -\frac{G_F}{\sqrt{2}} N_n, \quad (5)$$

where G_F is the Fermi constant of the weak interactions, N_e is the electron density, and N_n is the neutron density. In the above equations for the Dirac neutrinos a general magnetic moment matrix is possible:

$$M = \begin{pmatrix} \mu_{ee} & \mu_{e\mu} \\ \mu_{\mu e} & \mu_{\mu\mu} \end{pmatrix}. \quad (6)$$

For the Majorana neutrinos the diagonal components of Eq. (6) vanish and the off-diagonal components are related by $-\mu_{e\mu} = \mu_{\mu e} \equiv \mu$.

In this scenario there are several resonances. In addition to the standard MSW resonance ($\nu_e^{(L)} \rightarrow \nu_{\mu}^{(L)}$) that takes place where the condition

$$\sqrt{2} G_F N_e = \frac{\delta m^2}{2E_{\nu}} \cos 2\theta \quad (7)$$

is satisfied in the Sun, for the left-handed electron neutrinos that are produced in the core of the Sun a second, spin-flavor precession, resonance ($\nu_e^{(L)} \rightarrow \nu_{\mu}^{(R)}$) is possible. For the Dirac neutrinos it takes place where the condition

$$\frac{G_F}{\sqrt{2}} (2N_e - N_n) = \frac{\delta m^2}{2E_{\nu}} \cos 2\theta \quad (8)$$

is satisfied whereas for Majorana neutrinos it is where the condition

$$\sqrt{2} G_F (N_e - N_n) = \frac{\delta m^2}{2E_{\nu}} \cos 2\theta \quad (9)$$

is satisfied. This resonance converts a left-handed electron neutrino into a right-handed (sterile) muon neutrino for the Dirac case and into a muon antineutrino in the Majorana case. (In principle there is another resonance possible for Dirac neutrinos, converting the chirality of the electron neutrino but keeping its flavor the same through a diagonal moment. But for the neutrinos that also go through the MSW resonance—as higher energy solar neutrinos do—this requires a very high neutron density, $N_n = 2N_e$, which is not realized in the Sun). Clearly the resonances of (8) and (9) that flip both the chirality and the flavor of the electron neutrino produced in the nuclear reactions at the

TABLE I. The location of the MSW and spin-flavor precession resonances for Majorana neutrinos. Neutrino energies are given in MeV. The r/R_\odot value for the location of the resonances is shown.

E_ν	SFP	MSW
2.50	0	0.07
3.35	0.05	0.10
5.00	0.10	0.13
8.00	0.15	0.18
13.00	0.20	0.22

solar core take place at a higher electron density than the MSW resonance density. Even though their locations are different for the Dirac and Majorana cases, neutrinos need to go through them first. To estimate its location we use the approximate expression for the solar electron density [41]

$$N_e(r) = 245 \exp(-10.54r/R_\odot) N_A \text{ cm}^{-3}, \quad (10)$$

where N_A is the Avogadro's number. For the solar neutron density we use a spline fit to the values given in Ref. [24]. Using the values of $\delta m^2 = 8.2 \times 10^{-5} \text{ eV}^2$ and $\tan^2 \theta = 0.4$, obtained from a global analysis of the solar neutrino and most recent KamLAND data [42], we calculate the location of both the spin-flavor and MSW resonances. Results for the Majorana neutrinos are given in Table I. One can see that since the magnetic field should be present at the location of the spin-flavor precession resonance only fields at and very near the core play a role.

In this paper we present several approximate formulas for the electron neutrino survival probability in several limiting cases. We first consider the case of a small mixing angle (nonadiabatic limit). In this case the spin-flavor precession (SFP) and MSW resonances are well separated. The derivation of the reduction of the electron neutrino survival probability is presented in detail in the appendix. Following Eq. (A31) the electron neutrino oscillation probability in presence of a magnetic field can be rewritten as

$$P(\nu_{eL} \rightarrow \nu_{eL}, \mu B \neq 0) = P(\nu_{eL} \rightarrow \nu_{eL}, \mu B = 0) \times \exp[-\pi(\mu B)\delta r], \quad (11)$$

where the width of the SFP resonance [see Eq. (A27) of the appendix] is given as

$$\delta r = \left| \left(\frac{N'_e - N'_n}{N_e - N_n} \right)_{\text{at res}} \right|^{-1} \frac{4\mu B}{\delta m^2 \cos 2\theta}. \quad (12)$$

Clearly, even for the rather large values of the magnetic field $B \sim 10^6 \text{ G}$ and the magnetic moment of $10^{-11} \mu_B$ for a 10 MeV neutrino the width of the spin-flavor resonance would be very small, i.e. $(\delta r/R_\odot) \sim 0.002$. It is worthwhile to reiterate that the value of the magnetic field at the close vicinity of the solar core is not restricted by helioseismology as our approximations are no longer valid for very large values of the magnetic field.

It is not possible to find an expression similar to Eq. (11) for larger mixing angles or the adiabatic limit in which case the SFP and MSW resonances are no longer well separated. However it may be beneficial to investigate the unrealistic, but pedagogically instructive, limit where the neutron density vanishes. In this limit the SFP and MSW resonances overlap for any values of the neutrino parameters and magnetic field. From Eq. (A1) one can calculate

$$\frac{d^2}{dt^2} \nu_e^{(L)} + \left(\phi^2 + i \frac{d\phi}{dt} + \Delta^2 + (\mu B)^2 \right) \nu_e^{(L)} + \mu B \sqrt{2} G_F N_n \nu_\mu^{(R)} = 0. \quad (13)$$

In writing the above equation we assumed that the magnetic field is a constant. If the limit N_n goes to zero, the above equation becomes an equation for $\nu_e^{(L)}$ only and easily can be solved using the semiclassical methods of Ref. [43]. For large initial electron densities we obtain

$$P(\nu_e \rightarrow \nu_e) = \frac{1}{2} - \frac{1}{2} \cos 2\theta_\nu (1 - 2P_{\text{hop}}), \quad (14)$$

where the hopping probability is given by

$$P_{\text{hop}} = \exp(-\pi\Omega), \quad (15)$$

with

$$\Omega = \frac{i}{\pi} \int_{r_0}^{r_0^*} dr \frac{\delta m^2}{2E} [(\zeta^2(r) - 2\zeta(r) \cos 2\theta_\nu + 1) + (\mu B)^2]^{1/2}, \quad (16)$$

where r_0^* and r_0 are the turning points (zeros) of the integrand. In this expression we introduced the scaled density

$$\zeta(r) = \frac{2\sqrt{2}G_F N_e(r)}{\delta m^2/E}. \quad (17)$$

Using the Taylor series expansion of Eq. (16) for small values of μB we can write

$$P_{\text{hop}}(\mu B \neq 0) = P_{\text{hop}}(\mu B = 0) \exp \left\{ \frac{i}{\pi} \int_{r_0}^{r_0^*} dr \frac{\delta m^2}{2E} \left[\frac{(\mu B)^2}{\sqrt{\zeta^2(r) - 2\zeta(r) \cos 2\theta_\nu + 1}} \right] \right\}. \quad (18)$$

For an exponential density, $N_e(r) = n_0 e^{-\alpha r}$, the integral above can be calculated to give a hopping reduction factor

$$\exp\left[-\frac{\pi}{\alpha} \frac{(\mu B)^2 2E}{\delta m^2}\right]. \quad (19)$$

For a 10^5 G magnetic field, a magnetic moment of $10^{-12} \mu_B$, and $E \sim 10$ MeV, this hopping reduction factor is very small— $\sim 10^{-3}$. For the near-adiabatic limit, not only the hopping reduction factor is very small, but also the hopping probability itself is significantly reduced. Consequently one expects the change in the electron neutrino survival probability due to a near-allowed value of the neutrino magnetic moment to be very small. We do not expect using a realistic, nonzero value of the neutron density to change this conclusion. For $N_n \neq 0$, however, an analytic expression does not exist. Therefore, for the case of the observed large mixing angle, we present results of a numerical calculation, obtained by solving directly Eq. (A1). Figure 1 shows the electron survival probability as a function of the electron neutrino energy in presence of a magnetic field of 10^5 G and a magnetic moment of $10^{-11} \mu_B$. In particular a Gaussian profile has been taken for the magnetic field. Note that such results are identical (at a level at least of 10^5) to those obtained with the MSW effect only. This result is not changed if a width twice or five times larger than Eq. (12) is taken.

It was argued recently in Ref. [44] that random magnetic fields [45] can increase the reduction factor by enhancing solar antineutrino flux. It is possible to show that this conclusion also follows from our approximate expressions. We assume a random magnetic field with the correlation function [45]

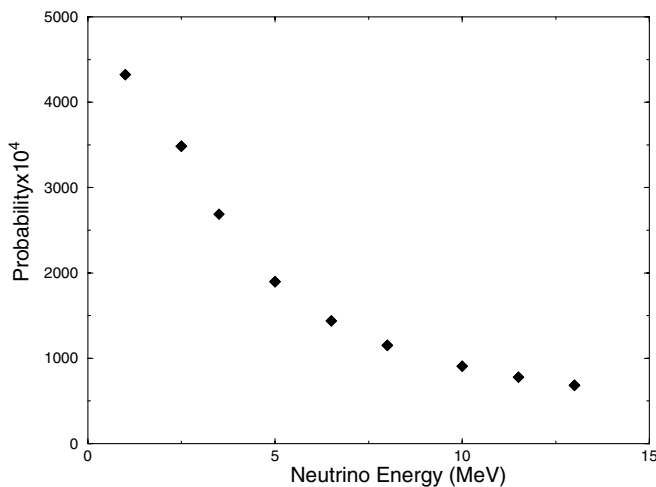


FIG. 1. Electron neutrino survival probability as a function of neutrino energy in presence of both the MSW and the SFP resonances. In particular, a Gaussian magnetic field profile is taken, with a magnetic field of 10^5 G and a magnetic moment of $10^{-11} \mu_B$. The location of the resonances are given in Table I. The results shown are indistinguishable from those due to the MSW resonance only, at least at the level of 10^5 .

$$\langle B(r)B(r') \rangle = B_0^2(t)L\delta(r-r'), \quad (20)$$

where B_0 is the average value of the magnetic field and L is the correlation length. We rewrite the reduction factor given in Eq. (A30) in the form

$$\mathcal{R} = 2\mu^2 \int_0^T dt B(t) e^{iQ(t)} \int_0^t dt' B(t') e^{-iQ(t')}, \quad (21)$$

where we defined

$$Q(t) = \int_0^t dt' [\phi(t') + \kappa(t')]. \quad (22)$$

Using Eq. (20), Eq. (21) can be easily integrated to yield

$$\mathcal{R} = 2\mu^2 L \int_0^T dt B_0^2(t), \quad (23)$$

which clearly can be very large depending on the chosen average magnetic field profile.

IV. CONCLUSION

In conclusion, neutrinos emitted by the Sun undergo a spin-flavor precession resonance and then an MSW resonance. In this paper we have discussed the conditions to be met to encounter such resonances and derived analytical formulas for the reduction of the electron survival probability due to solar magnetic fields. Our results show that the coupling of the neutrino magnetic moment to the solar magnetic field have small effects on the neutrino fluxes. Such results indicate that future solar neutrino measurements could not easily reach the level of precision to pinpoint alternative solutions to the solar neutrino deficit than the oscillation one confirmed by the recent KamLAND data.

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APPENDIX: REDUCTION OF THE ELECTRON NEUTRINO SURVIVAL PROBABILITY IN THE PRESENCE OF MAGNETIC FIELDS

In order to obtain the reduction of the electron neutrino survival probability due to solar magnetic fields we implement the logarithmic perturbation theory (for a description see the appendix of Ref. [46]). Here we show the derivation

of the probability reduction in the case of Majorana neutrinos. A similar derivation can be given for the Dirac case. Note that the results presented here can also be used to explore the effect of the magnetic fields on the neutrino fluxes produced during the explosion of core-collapse supernovae. Since, as described above, there is no resonant production of $\nu_e^{(R)}$ in the Sun, we will set its amplitude to zero in Eq. (1) to obtain

$$i \frac{d}{dt} \begin{pmatrix} \nu_e^{(L)} \\ \nu_\mu^{(L)} \\ \nu_\mu^{(R)} \end{pmatrix} = \begin{pmatrix} \phi & \Delta & \mu B \\ \Delta & -\phi & 0 \\ \mu B & 0 & -\kappa \end{pmatrix} \begin{pmatrix} \nu_e^{(L)} \\ \nu_\mu^{(L)} \\ \nu_\mu^{(R)} \end{pmatrix}, \quad (\text{A1})$$

where we defined

$$\Delta = \frac{\delta m^2}{4E} \sin 2\theta, \quad (\text{A2})$$

$$\phi = \frac{1}{\sqrt{2}} G_F N_e - \frac{\delta m^2}{4E} \cos 2\theta, \quad (\text{A3})$$

and

$$\kappa = \phi - \sqrt{2} G_F N_n. \quad (\text{A4})$$

Introducing

$$z^{(L)} = \frac{\nu_\mu^{(L)}}{\nu_e^{(L)}}, \quad (\text{A5})$$

and

$$z^{(R)} = \frac{\nu_\mu^{(R)}}{\nu_e^{(L)}}, \quad (\text{A6})$$

Eq. (1) can be rewritten as two coupled nonlinear equations:

$$i \frac{dz^{(L)}}{dt} = \Delta - 2\phi z^{(L)} - \Delta [z^{(L)}]^2 - \mu B z^{(L)} z^{(R)}, \quad (\text{A7})$$

and

$$i \frac{dz^{(R)}}{dt} = \mu B - (\kappa + \phi) z^{(R)} - \Delta z^{(L)} z^{(R)} - \mu B [z^{(R)}]^2. \quad (\text{A8})$$

From Eqs. (A7) and (A8) it follows that

$$i \frac{d}{dt} \log(1 + |z^{(L)}|^2 + |z^{(R)}|^2) = \mu B (z^{(R)*} - z^{(R)}) + \Delta (z^{(L)*} - z^{(L)}). \quad (\text{A9})$$

Using the unitarity of the neutrino amplitudes (i.e. $1 + |z^{(L)}|^2 + |z^{(R)}|^2 = |\nu_e^{(L)}|^{-2}$) we obtain an *exact* expression for the electron neutrino amplitude at any location inside or outside the Sun:

$$|\nu_e^{(L)}(T)|^2 = \exp \left\{ i \int_0^T dt [\mu B(t) (z^{(R)*}(t) - z^{(R)}(t))] \right\} \times \exp \left\{ i \int_0^T dt [\Delta (z^{(L)*}(t) - z^{(L)}(t))] \right\}. \quad (\text{A10})$$

Equation (A10), which is an exact result, represents the formulation of the N -flavor (or antiflavor) propagation problem in the $SU(N)/SU(N-1) \times U(1)$ coset space instead of the usual $SU(N)$ one. To illustrate its utility in separating different contributions we define the perturbation parameter

$$g = \frac{\mu B_0}{\Delta} = \frac{4E \mu B_0}{\delta m^2 \sin 2\theta}, \quad (\text{A11})$$

which we will take to be small. In Eq. (A11) B_0 is the maximal value of the magnetic field. We write the magnetic field in dimensionless form

$$\beta = B/B_0. \quad (\text{A12})$$

Defining a new variable $\tau = \Delta t$, Eqs. (A7) and (A8) take the form

$$i \frac{dz^{(L)}}{d\tau} = 1 - 2\phi z^{(L)} - [z^{(L)}]^2 - g\beta z^{(L)} z^{(R)}, \quad (\text{A13})$$

and

$$i \frac{dz^{(R)}}{d\tau} = g\beta - \frac{\kappa + \phi}{\Delta} z^{(R)} - z^{(L)} z^{(R)} - g\beta [z^{(R)}]^2. \quad (\text{A14})$$

These equations need to be solved with the initial condition $z^{(L)} = 0 = z^{(R)}$ at $t = 0$. We consider a perturbative solution of the form

$$z^{(L)} = z_0^{(L)} + g z_1^{(L)} + g^2 z_2^{(L)} + \dots, \quad (\text{A15})$$

and

$$z^{(R)} = z_0^{(R)} + g z_1^{(R)} + g^2 z_2^{(R)} + \dots. \quad (\text{A16})$$

Clearly $z_0^{(L)}$, when substituted into Eq. (A10) gives the MSW solution and the terms proportional to various powers of g are the corrections due to the existence of the spin-flavor precession.

The quantity $z_0^{(L)}$ satisfies the equation

$$i \frac{dz_0^{(L)}}{d\tau} = 1 - 2\phi z_0^{(L)} - [z_0^{(L)}]^2, \quad (\text{A17})$$

whereas the evolution of the $z_1^{(L)}$ is given by

$$i \frac{dz_1^{(L)}}{d\tau} = -2 \frac{\phi}{\Delta} z_1^{(L)} - 2z_0^{(L)} z_1^{(L)}. \quad (\text{A18})$$

Equation (A18) implies that the quantity $z_1^{(L)}$ is a constant times an exponential. The only way to satisfy the initial condition is to set this multiplicative constant to zero. Hence the lowest order correction to $z^{(L)}$ is $z_2^{(L)}$, satisfying the equation

$$i \frac{dz_2^{(L)}}{d\tau} = -2 \frac{\phi}{\Delta} z_2^{(L)} - 2z_0^{(L)} z_2^{(L)} - \beta z_0^{(L)} z_1^{(R)}. \quad (\text{A19})$$

Similarly, as it is expected on physical grounds, $z_0^{(R)}$ vanishes. The lowest order correction to $z^{(R)}$ is given by $z_1^{(R)}$, which satisfies the equation

$$i \frac{dz_1^{(R)}}{d\tau} = \beta - \frac{\phi + \kappa}{\Delta} z_1^{(R)} - z_0^{(L)} z_1^{(R)}. \quad (\text{A20})$$

The solution of Eq. (A20) is given by

$$gz_1^{(R)}(T) = -ie^{[i \int_0^T dt' [\phi(t') + \kappa(t') + \Delta z_0^{(L)}(t')]]} \times \int_0^T dt \mu B(t) e^{-i \int_0^t dt' [\phi(t') + \kappa(t') + \Delta z_0^{(L)}(t')]}, \quad (\text{A21})$$

and the solution of Eq. (A19) is given by

$$gz_2^{(L)}(T) = ie^{[i2 \int_0^T dt' [\phi(t') + \Delta z_0^{(L)}(t')]]} \times \int_0^T dt \mu B(t) z_0^{(L)}(t) z_1^{(R)}(t) \times e^{[-i2 \int_0^t dt' [\phi(t') + \Delta z_0^{(L)}(t')]]}. \quad (\text{A22})$$

When the magnetic field is set to zero Eq. (A10) gives the neutrino survival probability to be

$$|\nu_e^{(L)}(T)|^2 = \exp \left\{ i \int_0^T dt [\Delta(z_0^{(L)*}(t) - z_0^{(L)}(t))] \right\}. \quad (\text{A23})$$

It is easy to see that this result, along with Eq. (A17), represents a resonance. Introducing

$$\Psi(T) = \exp \left[-i \int_0^T dt (\Delta z_0^{(L)}(t) + \phi(t)) \right], \quad (\text{A24})$$

one observes that $|\nu_e^{(L)}(T)|^2 = |\Psi(T)|^2$. It follows from Eq. (A17) that Ψ satisfies the differential equation

$$\frac{d^2 \Psi}{d\tau^2} = - \left[1 + \frac{\phi^2(t)}{\Delta^2} + \frac{i}{\Delta} \frac{d\phi}{d\tau} \right] \Psi. \quad (\text{A25})$$

The rate of change of the probability is maximized when the right-hand side of Eq. (A25) is an extremum, which is achieved when $\phi = 0$. The width of this resonance (the

MSW resonance) is $\Gamma = \frac{d\phi}{dt} / \Delta$, which corresponds to a spatial width of $\Delta \delta r = 2/\Gamma$ or

$$\delta r_{\text{MSW}} = \frac{2\Delta}{(d\phi/dt)_{\phi=0}}. \quad (\text{A26})$$

A similar argument, applied to Eq. (A8), gives the width of the spin-flavor resonance to be

$$\delta r_{\text{SFP}} = 2 / \frac{d}{dt} \left(\frac{\phi + \kappa}{\mu B} \right). \quad (\text{A27})$$

As we mentioned earlier the spin-flavor precession resonance takes place before the MSW resonance. In most cases the quantity $z_0^{(L)}$ is very small at the SFP resonance zone and can be neglected in Eqs. (A18) and (A19). In this approximation $z_2^{(L)} = 0$ and

$$gz_1^{(R)}(T) = -ie^{[i \int_0^T dt' [\phi(t') + \kappa(t')]]} \times \int_0^T dt \mu B(t) e^{-i \int_0^t dt' [\phi(t') + \kappa(t')]} \quad (\text{A28})$$

Substituting these in Eq. (A10) we obtain

$$|\nu_e^{(L)}(T)|^2 = \exp \left\{ - \left| \int_0^T dt \mu B(t) e^{[i \int_0^t dt' [\phi(t') + \kappa(t')]]} \right|^2 \right\} \times \exp \left\{ i \int_0^T dt [\Delta(z_0^{(L)*}(t) - z_0^{(L)}(t))] \right\}, \quad (\text{A29})$$

or

$$P(\nu_{eL} \rightarrow \nu_{eL}, \mu B \neq 0) = P(\nu_{eL} \rightarrow \nu_{eL}, \mu B = 0) \times \exp \left\{ - \left| \int_0^T dt \mu B(t) \times e^{[i \int_0^t dt' [\phi(t') + \kappa(t')]]} \right|^2 \right\}. \quad (\text{A30})$$

If the SFP resonance width is rather small one can calculate the integral in Eq. (A30) rather accurately in the stationary phase approximation [47]. The stationary point is where the derivative of the argument of the exponent is zero, i.e. $\phi + \kappa = 0$, the SFP resonance point. One finally gets

$$P(\nu_{eL} \rightarrow \nu_{eL}, \mu B \neq 0) = P(\nu_{eL} \rightarrow \nu_{eL}, \mu B = 0) \times \exp \left\{ - \frac{2\pi(\mu B)^2}{|d(\phi + \kappa)/dt|_{(\phi + \kappa)=0}} \right\}. \quad (\text{A31})$$

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