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Effect of $D - \bar{D}$ mixing on the measurement of γ in $B \to DK$ decays

Yuval Grossman,[1,2,3] Abner Soffer,[4] and Jure Zupan[5,6]

[1]*Department of Physics, Technion-Israel Institute of Technology, Technion City, 32000 Haifa, Israel*
[2]*Physics Department, Boston University, Boston, Massachusetts 02215, USA*
[3]*Jefferson Laboratory of Physics, Harvard University, Cambridge, Massachusetts 02138, USA*
[4]*Department of Physics, Colorado State University, Fort Collins, Colorado 80523, USA*
[5]*Department of Physics, Carnegie Mellon University, Pittsburgh, Pennsylvania 15213, USA*
[6]*J. Stefan Institute, Jamova 39, P.O. Box 3000,1001 Ljubljana, Slovenia*

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$D - \bar{D}$ mixing is the source of the largest theoretical uncertainty in the extraction of γ from $B \to DK$ decays. In the standard model, the mixing can have a rate close to its current experimental upper bound and is CP conserving to an excellent approximation. We show that neglecting CP-conserving $D - \bar{D}$ mixing leads to an error in the determination of γ only at second order in the small parameters $\Delta m_D/\Gamma_D$ and $\Delta\Gamma_D/\Gamma_D$ and is therefore very small and can be safely neglected.

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The most precise determination of the standard model CKM phase γ will, in the long run, be provided by methods based on the interference between $b \to c\bar{u}s$ and $b \to u\bar{c}s$ decays [1]. In the case of charged B decays, the interference is between $B^- \to DK^-$ followed by a $D \to f$ decay and $B^- \to \bar{D}K^-$ followed by $\bar{D} \to f$, where f is any final state common to both D and $\bar{D}$. What makes this method theoretically powerful is that there are no penguin contributions, and all the hadronic unknowns are in principle obtainable from experiment.

For our purpose, it is useful to group the different methods according to the choice of the final state f, which can be (i) a CP eigenstate (e.g. $K_S\pi^0$, $K_S\phi$) [1], (ii) a flavor state ($K^+\pi^-$) [2], or (iii) a multibody final state (e.g. $K_S\pi^+\pi^-$, $\pi^+\pi^-\pi^0$) [3]. Additional variations of the basic method involve using multibody B decays (e.g. $B^+ \to DK^+\pi^0$) [4], use of D^{0*} in addition to D^0, self tagging D^{0**} states [5] and neutral B decays (both time dependent and time integrated) [6,7]. Since these measurements are statistically limited, an eventual combination of all the modes will be needed in order to minimize the overall γ measurement error [8,9]. So far, the most precise direct information on γ comes from $B^\pm \to (K_S\pi^+\pi^-)_D K^\pm$, where we use the notation f_D to indicate a D meson decaying into the final state f. Both Belle [10,11] and *BABAR* [12] have used D^{*0} and D^0 decays, where a subtlety of a sign difference between the two D^{*0} decay modes has been pointed out only recently [13]. These measurements use a sum of Breit-Wigner resonances to model the Dalitz plot distribution of $D^0 \to K_S\pi^+\pi^-$. It is possible to remove the associated modeling error by carrying out the measurements with a model-independent treatment of the Dalitz plot [3,14].

In all of the above methods, the standard model (SM) is assumed. (Indeed, these methods involve only tree-level amplitudes and, therefore, are unlikely to be affected by new physics.) Within the SM, the largest theoretical uncertainty is due to $D - \bar{D}$ mixing. The parameters that describe the mixing are

$$x \equiv \frac{\Delta m_D}{\Gamma_D}, \qquad y \equiv \frac{\Delta\Gamma_D}{2\Gamma_D}, \qquad \theta \equiv \arg\left(\frac{q}{p}\right), \tag{1}$$

where $\Delta\Gamma_D$ (Δm_D) is the decay width (mass) difference between the two neutral D mass eigenstates, Γ_D (m_D) is the average decay width (mass) of the mass eigenstates, and q and p are the elements of the rotation matrix between the interaction and mass eigenstate bases [15]. [Here, and in what follows, we choose a phase convention such that the tree-level D decay amplitudes are real, see Eq. (2).] The parameters x and y cannot be calculated reliably in the SM. In particular, one cannot rule out the possibility that they are as large as $x \sim y \sim O(10^{-2})$ [16], which is the range experiments are beginning to probe [17,18]. A robust SM prediction, however, is that $D - \bar{D}$ mixing is CP conserving to a very high accuracy, with a CP-violating phase of order $\theta \sim O(10^{-4})$ [18].

If the $D - \bar{D}$ mixing parameters are known, their effect can be corrected for in the measurement of γ [19,20]. Without knowing their values, assuming $x = y = 0$ introduces an error in the extracted value of γ. Making that assumption can introduce an error in the determination of the branching ratios used in the ADS method [2] of the order of $x/r_f \sim y/r_f \lesssim 20\%$, where r_f is defined in Eq. (2) below. Naïvely, one may conclude that this introduces a similar error in the extracted value of γ. It is the purpose of this paper to explain why this is not the case. We find that the effect is at most quadratic in x and y, thus it is very small and can be safely neglected.

Let us review the approach of extracting γ neglecting $D - \bar{D}$ mixing. We choose the phase convention in which the D meson decay amplitudes have the form

$$A(D^0 \to f) \equiv A_f, \qquad A(\bar{D}^0 \to f) \equiv \bar{A}_f = A_f r_f e^{-i\delta_f}, \tag{2}$$

such that A_f and r_f are positive. Since in the SM there is

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essentially no CP violation in the D decays, δ_f is a strong phase difference. Because of the abundance of flavor-tagged D decays at the B factories, the values of A_f and r_f can be measured very precisely from the decay rates

$$\Gamma_f \equiv \Gamma(D^0 \to f) = A_f^2, \qquad \bar{\Gamma}_f \equiv \Gamma(\bar{D}^0 \to f) = A_f^2 r_f^2. \tag{3}$$

In the absence of $D - \bar{D}$ mixing, the amplitude for the cascade decay $B^+ \to f_D K^+$ is

$$A(B^+ \to f_D K^+) = A_B A_f [r_f e^{-i\delta_f} + r_B e^{i(\delta_B+\gamma)}], \tag{4}$$

where A_B and r_B are positive parameters, δ_B is a strong phase difference, and we have defined

$$\begin{aligned} A_B &\equiv A(B^+ \to \bar{D}^0 K^+), \\ A_B r_B e^{i(\delta_B+\gamma)} &\equiv A(B^+ \to D^0 K^+). \end{aligned} \tag{5}$$

The sensitivity to γ comes from the interference term in the decay width

$$\begin{aligned} &\Gamma(B^+ \to f_D K^+) \\ &\quad = A_B^2 A_f^2 [r_f^2 + r_B^2 + 2 r_B r_f \cos(\delta_B + \gamma + \delta_f)]. \end{aligned} \tag{6}$$

A similar expression for the $B^- \to \bar{f}_D K^-$ decay width is obtained by making use of the absence of direct CP violation in the D decay:

$$\begin{aligned} &\Gamma(B^- \to \bar{f}_D K^-) \\ &\quad = A_B^2 A_f^2 [r_f^2 + r_B^2 + 2 r_B r_f \cos(\delta_B - \gamma + \delta_f)]. \end{aligned} \tag{7}$$

Each final state f introduces two new observables, $\Gamma(B^- \to \bar{f}_D K^-)$ and $\Gamma(B^+ \to f_D K^+)$, and a single new unknown, δ_f. The unknowns describing the $B \to DK$ part of the cascade decay, r_B, δ_B, and γ, are the same for all D decay final states. Therefore, with enough D decay modes, there are more observables than unknowns, and the values of all the unknowns can be determined.

We now study the effect of CP-conserving $D - \bar{D}$ mixing. Specifically, we ask what error, $\Delta\gamma$, is introduced in the extracted value of γ when the analysis is done assuming no $D - \bar{D}$ mixing. A crucial ingredient in the approach described above for extracting γ is that δ_f is a pure phase, i.e., a single real parameter. Specifically, we assumed that the absolute magnitude of the interference term

$$|A_f \bar{A}_f^*| = |A_f^2 r_f e^{i\delta_f}| = A_f^2 r_f, \tag{8}$$

which is used in Eqs. (6) and (7) is already measured in flavor-tagged D decays. However, due to $D - \bar{D}$ mixing, time evolution dilutes the absolute magnitude of the interference term (8), which becomes another unknown. It is the deviation of the magnitude of the interference term from its naïve value that introduces the error in the extracted value of γ.

In the presence of CP-conserving $D - \bar{D}$ mixing, the time-dependent D decay amplitudes are [15]

$$\begin{aligned} \mathcal{A}_f(t) &= \mathcal{A}(D^0(t) \to f) = g_+(t) A_f + g_-(t) \bar{A}_f, \\ \bar{\mathcal{A}}_f(t) &= \mathcal{A}(\bar{D}^0(t) \to f) = g_+(t) \bar{A}_f + g_-(t) A_f, \end{aligned} \tag{9}$$

where the time evolution functions are

$$\begin{aligned} g_+(t) &= \exp(-i m_D t - \tau/2)[\cosh(y\tau/2)\cos(x\tau/2) \\ &\quad + i \sinh(y\tau/2)\sin(x\tau/2)] \\ &\approx \exp(-i m_D t - \tau/2)[1 + (y + ix)^2 \tau^2/4], \\ g_-(t) &= \exp(-i m_D t - \tau/2)[-\sinh(y\tau/2)\cos(x\tau/2) \\ &\quad - i \cosh(y\tau/2)\sin(x\tau/2)] \\ &\approx \exp(-i m_D t - \tau/2)[(-ix - y)\tau/2], \end{aligned} \tag{10}$$

with $\tau \equiv \Gamma_D t$. The approximations in Eqs. (10) hold to second order in x and y. $D - \bar{D}$ mixing changes the time-integrated decay rates of Eq. (3) at leading order in x and y. The precise change is not important for our purpose. The key point is that the time-integrated decay rates

$$\Gamma_f = \int dt |\mathcal{A}_f(t)|^2, \qquad \bar{\Gamma}_f = \int dt |\bar{\mathcal{A}}_f(t)|^2, \tag{11}$$

which are measured in tagged D decays, are exactly the rates that enter the B decay rate, which now reads

$$\begin{aligned} &\Gamma(B^+ \to f_D K^+) \\ &\quad = A_B^2 \Big[\bar{\Gamma}_f + r_B^2 \Gamma_f \\ &\qquad + 2 r_B \, \mathrm{Re}\Big(e^{i(\delta_B+\gamma)} \int dt \mathcal{A}_f(t) \bar{\mathcal{A}}_f(t)^* \Big) \Big]. \end{aligned} \tag{12}$$

The impact of $D - \bar{D}$ mixing on the γ measurement occurs only in the interference term

$$\int dt \mathcal{A}_f(t) \bar{\mathcal{A}}_f(t)^* \equiv \sqrt{\Gamma_f \bar{\Gamma}_f} e^{i\tilde{\delta}_f} e^{-\epsilon_f}, \tag{13}$$

where $\tilde{\delta}_f$ is a pure strong phase[1] and

$$\epsilon_f = \frac{1}{8}(x^2 + y^2)\Big(\frac{1}{r_f^2} + r_f^2\Big) - \frac{1}{4}(x^2 \cos 2\delta_f + y^2 \sin 2\delta_f) \tag{14}$$

describes the dilution due to $D - \bar{D}$ mixing. The parameter ϵ_f gives the approximate magnitude of the shift $\Delta\gamma$ in the determination of γ. Since the leading term in ϵ_f is proportional to $(x^2 + y^2)/r_f^2$, $\Delta\gamma$ is larger for cases where r_f is smaller. Apart from the trivial case of no mixing ($x = y = 0$), ϵ_f vanishes only if $r_f = 1$ and either $y = 0$ and

[1]Note that this is not the case in the presence of CP-violating $D - \bar{D}$ mixing when the phase $\tilde{\delta}_f$ is a combination of a weak and a strong phase.

$\delta = k\pi$, or $x = 0$ and $\delta = \pi/2 + k\pi$, where k is an integer. In all other cases, ϵ_f is positive.

In the special case $\epsilon_f = 0$, there is no change in the γ measurement. Each new mode f still introduces only one new parameter, $\tilde{\delta}_f$, which is obtained from the fit to the decay widths. Moreover, the form of the equations is unchanged, as can be seen by defining $\Gamma_f = \tilde{A}_f^2$, $\bar{\Gamma}_f = \tilde{A}_f^2 \tilde{r}_f^2$ and comparing (11) and (12), with (6). Therefore, for $\epsilon_f = 0$, the correct value of γ is measured, even if the formalism used in the analysis ignores $D - \bar{D}$ mixing.

Our first main point is that while in general $\epsilon_f \neq 0$, ϵ_f is of second order in the small parameters x and y. Therefore, its effect on the measurement of γ is small. Moreover, given measurements of, or upper limits on, x and y, the impact of $D - \bar{D}$ mixing can be accounted for without the need to perform a time-dependent analysis of the B decay. This can be done by using (14) and (13) in (12).

That ϵ_f is of second order in x and y can be understood as follows: One can think of integration over time as a scalar product in the vector space of time-dependent complex functions

$$\langle \mathcal{A}_f, \bar{\mathcal{A}}_f \rangle = \int dt \mathcal{A}_f(t) \bar{\mathcal{A}}_f(t)^*. \tag{15}$$

Then

$$|\langle \mathcal{A}_f, \bar{\mathcal{A}}_f \rangle|^2 = \langle \mathcal{A}_f, \mathcal{A}_f \rangle \langle \bar{\mathcal{A}}_f, \bar{\mathcal{A}}_f \rangle |\cos\Delta|^2, \tag{16}$$

where Δ is a small angle linear in x, y. The difference that defines ϵ_f in Eq. (13) is then

$$\begin{aligned} \epsilon_f &\propto \langle \mathcal{A}_f, \mathcal{A}_f \rangle \langle \bar{\mathcal{A}}_f, \bar{\mathcal{A}}_f \rangle - |\langle \mathcal{A}_f, \bar{\mathcal{A}}_f \rangle|^2 \\ &\propto 1 - \cos^2\Delta \sim O(\Delta^2) \sim O(x^2, y^2). \end{aligned} \tag{17}$$

We provide an explicit expression for $\Delta\gamma$ in one specific case where γ is extracted from a combination of a doubly Cabibbo-suppressed decay width $\Gamma(B^\pm \to (K^\mp \pi^\pm)_D K^\pm)$ and a decay width $\Gamma(B^\pm \to (f_{CP})_D K^\pm)$ into a CP eigenstate. To first order in r_B and r_f (here $f = K\pi$), we get

$$\Delta\gamma = -\epsilon_f \frac{\cos\gamma \sin 2\gamma}{\cos\gamma[\cos 2(\delta_f + \delta_B) - \cos 2\delta_B] + (r_B/r_f)\cos(\delta_f + \delta_B)[\cos 2\gamma - \cos 2\delta_B]}, \tag{18}$$

where ϵ_f is taken from (14). For $x^2 + y^2 \sim 2\%$, $\gamma \sim 60^\circ$, $r_B \sim 0.2$, and $r_f \sim 6\%$ we find the typical range $\Delta\gamma \sim 0.1 - 1^\circ$, depending on the values of the strong phases δ_f and δ_B. By the time the precision of the γ measurement reaches this level, we will have either measurements or tighter upper limits on x and y, so that the measurement could be corrected for this shift.

Next, we consider the effect of $D - \bar{D}$ mixing in the case of multibody D decays. For the Breit-Wigner treatment of Dalitz plot, the corrections due to $D - \bar{D}$ mixing arise at $O(x^2, y^2)$ as in the two-body case discussed above. Similar considerations apply in both the two-body and three-body cases, with the difference being that in the Breit-Wigner-based Dalitz plot analysis, r_f varies over the Dalitz plot. The lowest value of r_f, and hence the largest $\Delta\gamma$, is obtained in areas populated by doubly Cabibbo-suppressed decays. Specifically, for the final state $f = K_S \pi^+ \pi^-$ this is the region of the decay $D^0 \to K^{*+} \pi^-$, which contributes most to the γ measurement. Nonetheless, with r_f of order a few percent in this region, this still results in a small contribution to $\Delta\gamma$. Moreover, the overall value of $\Delta\gamma$ is smaller due to the contributions of other regions in the Dalitz plot in which r_f is larger. The shift $\Delta\gamma$ is significantly smaller for singly Cabibbo-suppressed multibody decays, in which $r_f \sim \mathcal{O}(1)$ [21].

Our second main point is that CP-conserving $D - \bar{D}$ mixing does not affect the determination of γ if the relevant Dalitz plot parameters are determined by binning the Dalitz plot according to the model-independent approach of Ref. [3]. The phase space integration over bin i of the Dalitz plot introduces two new real variables, $\hat{c}_i$ and $\hat{s}_i$:

$$\hat{c}_i + i\hat{s}_i \equiv \frac{c_i + i s_i}{T_i}, \tag{19}$$

where

$$\begin{aligned} c_i + i s_i &\equiv \int_i dp \int dt \mathcal{A}_f(t) \bar{\mathcal{A}}_f(t)^*, \\ T_i &\equiv \int_i dp \int dt |\mathcal{A}_f(t)|^2. \end{aligned} \tag{20}$$

The variables c_i and s_i are determined either from the binned Dalitz plot obtained from the B decay sample, or from time-integrated decays of entangled D states at a charm factory operating at the $\Psi(3770)$ [3]. The point is that measuring c_i and s_i already accounts for the dilution due to $D - \bar{D}$ mixing. This is demonstrated by the fact that in the two-body case, one can replace the two variables δ_f and ϵ_f of (13) with $\hat{c}_i$ and $\hat{s}_i$, which satisfy $\hat{c}_i^2 + \hat{s}_i^2 = 1 - \mathcal{O}(x^2, y^2)$. The method of Ref. [3] is already designed to handle $\hat{c}_i^2 + \hat{s}_i^2 < 1$, which in multibody decays arises due to the phase space integration over each bin.

We concentrated on the case of CP-conserving $D - \bar{D}$ mixing, since this is the case in the SM. With new physics, this may not be the case. Then, our results do not hold and larger effects are introduced. For example, consider the case where there is new physics in the mixing, with a CP-violating phase $\theta \sim O(1)$. Then the assumption of no $D - \bar{D}$ mixing introduces an error in the value of γ of order $\Delta\gamma \sim O(x\theta, y\theta)$ which is linear in the small parameters.

It is instructive to compare our results to those of [19]. The analysis in [19] corresponds to a situation in which the D decay amplitudes were determined from D decay data by taking $D-\bar{D}$ mixing into account but neglecting mixing in the B decay analysis. The error in the value of γ extracted in this way is linear in x and y, regardless of whether $D-\bar{D}$ mixing is CP conserving or not. Here, on the other hand, we show that when both the D and the B decay amplitudes are extracted ignoring $D-\bar{D}$ mixing, CP-conserving $D-\bar{D}$ mixing induces an error in the extracted value of γ that is of second order in x and y. This provides a simpler practical approach for solving the problem introduced by CP-conserving $D-\bar{D}$ mixing, which is the case in the SM.

To conclude, we show that within the SM, neglecting $D-\bar{D}$ mixing in the extraction of γ using $B \to DK$ type decays introduces at most an $O(x^2, y^2)$ effect. This is a very small effect that can be neglected for all practical purposes.

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