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Microstates of the D1-D5–Kaluza-Klein monopole system

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We find supergravity solutions corresponding to all $U(1) \times U(1)$ invariant chiral primaries of the D1-D5-KK system. These solutions are 1/8 BPS, carry angular momentum, and are asymptotically flat in the $3 + 1$ dimensional sense. They can be thought of as representing the ground states of the four-dimensional black hole constructed from the D1-D5-KK-P system. Demanding the absence of unphysical singularities in our solutions determines all free parameters, and gives precise agreement with the quantum numbers expected from the CFT point of view. The physical mechanism behind the smoothness of the solutions is that the D1 branes and D5 branes expand into a KK-monopole supertube in the transverse space of the original KK monopole.

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I. INTRODUCTION

The D1-D5-KK system (KK—Kaluza-Klein monopole) is a 1/8 Bogomol'nyi-Prasad-Sommerfield (BPS) configuration in type IIB string theory. For large brane charges there is a large microscopic degeneracy of states, corresponding to an entropy $S = 2\pi\sqrt{N_1 N_5 N_K}$. At low energies, the system is described by a $1 + 1$ dimensional conformal field theory (CFT) with $(4, 0)$ supersymmetry (SUSY) and central charge $c = 6N_1 N_5 N_K$, and the microscopic states correspond to the chiral primaries of this CFT.

In this paper we will find the supergravity geometries dual to a large class of these microstates. In particular, we will find all the geometries which preserve a $U(1) \times U(1)$ symmetry. These geometries are of interest for a number of reasons:

(1) Four-dimensional BPS black holes with macroscopic event horizons can be constructed by wrapping the D1-D5-KK system on T^6 and adding momentum along the intersection of the branes [1]. For large N_P, the corresponding black hole has a Bekenstein-Hawking entropy $S = 2\pi\sqrt{N_1 N_5 N_K N_P}$, which can be accounted for microscopically in the CFT [1,2]. Upon setting $P = 0$ so as to obtain the "ground states" of the black hole, one finds that the geometry develops a naked singularity. A related fact is that this naive $P = 0$ limit yields no trace of the microscopic degeneracy $S = 2\pi\sqrt{N_1 N_5 N_K}$ which we know to be present from CFT considerations. Our new solutions resolve this puzzle since, at least in the $U(1) \times U(1)$ invariant case, they provide the correct geometries which replace the singular limit of the black hole. This part of our story is directly parallel to the story involving the zero momentum limit [3–5] of the rotating D1-D5-P system [6,7], which has been much discussed recently (see [8] for a review). In that case the nonsingular geometries are due to the expansion of the D1 and D5 branes into a KK-monopole supertube [9]; the smoothness of the KK monopole ensures the smoothness of the full geometry. Our solutions will display a more intricate version of the same phenomenon.

(2) As argued by Mathur and collaborators [5,8], if it could be shown that the microstates of the D1-D5-P system are dual to individual bulk solutions,[1] this would give a bulk accounting of the black hole entropy and lead to a solution of the black hole information paradox. Some smooth solutions carrying these charges have indeed been found [10–13]. After a chain of dualities, the D1-D5-KK system can be transformed into the D1-D5-P system, and so our solutions can be thought of as providing dual versions of these bulk solutions. A subtlety, which we discuss more at the end of the paper, is that for this to be seen explicitly one must go beyond the supergravity approximation in performing the T duality along the KK-monopole fiber. Assuming that this in principle can be done, and assuming that we can eventually relax the condition of $U(1) \times U(1)$ invariance, it may be possible to account for all the entropy of the D1-D5-P black hole in this way. The key point is that we are finding microstates of a genuine three-charge system, which up to dualities, corresponds to a black hole with macroscopic event horizon. See [14] for another recent discussion of the relationship between these two systems.

(3) Studies of three-charge supertubes [15,16] recently led to the prediction and subsequent discovery of new BPS black ring solutions [17–21]. In the type IIB duality frame, these black rings carry the charges of the D1-D5-P system. Furthermore, they have a macroscopic entropy that can be accounted for (modulo some assumptions which remain to be fully understood) via two microscopic routes. In one approach [22] (see also [23]), one notes that the

[1]Not all of these solutions are expected to be smooth semiclassical geometries.

branes that make the BPS black ring are the same as the branes that make the 4D black hole, and so one can map the microscopic computation of the black ring entropy to that of the 4D black hole discussed above. This indeed yields agreement with the black ring entropy formula. As above, it is interesting to consider the limit in which the macroscopic entropy of the black ring is taken to zero; if smooth geometries result then these will yield smooth microstates of the D1-D5-P system. However, this limit yields singular geometries, which is in fact expected since the geometry near the ring is dual to the "zero entropy" limit of the 4D black hole, whose naive geometry is singular. The solutions we find resolve the singularity of the naive zero momentum limit of the D1-D5-KK-P 4D black hole, and it is likely that they can similarly be thought of as resolving the singularities of the black rings in this limit. To show this explicitly one must "glue" our solutions into the BPS black ring geometry, but we leave that for the future.

We now turn to a summary of our results. The CFT of the D1-D5-KK system is similar in many respects to that of the more familiar D1-D5 system; see [24–27] for discussion. In particular, at the orbifold point one can think of an effective string of length $N_1N_5N_K$ which can be broken up into any number of integer length component strings. Each component string carries 1/2 unit of four-dimensional angular momentum via fermion zero modes. Therefore, the microstates carry angular momentum in the range $\frac{1}{2} \leq |J| \leq \frac{1}{2}N_1N_5N_K$. The $U(1) \times U(1)$ invariant microstates whose geometries we find in this paper correspond to collections of component strings of equal length. Our solutions will thus be labeled by the number of component strings n, and will carry angular momentum $J = \pm\frac{1}{2} \times [(N_1N_5N_K)/n]$ with $1 \leq n \leq N_1N_5N_K$.

Our solutions are asymptotically flat in four dimensions, and have mass $M \propto Q_1 + Q_5 + Q_K$ as follows from BPS considerations. For $n = 1$ the solutions are completely smooth in the ten-dimensional sense, while for general n they have $\mathbb{Z}_n$ singularities caused by the presence of n coincident KK monopoles; from the point of view of string theory these are familiar and harmless orbifold singularities.

It will turn out that in addition to carrying D1-D5-KK charge, our solutions will also carry an electric charge with respect to the gauge field under which the KK monopole is magnetically charged. The solutions will carry N_e units of electric charge subject to the condition

$$J = \frac{1}{2}N_eN_K. \tag{1.1}$$

This is the same angular momentum that results in ordinary electromagnetism from having separated electric and magnetic charges; the angular momentum is generated by the crossed electric and magnetic fields. Although we thus have an additional charge as measured at infinity, this charge disappears after taking the near-horizon limit of our solutions. In this limit, our solutions reduce to certain BPS conical defect orbifolds of $\mathrm{AdS}_3 \times S^3/\mathbb{Z}_{N_K} \times T^4$, closely related to similar conical defects in the D1-D5 system [3–5,28–30]. Since these conical defect geometries are known to correspond to the CFT microstates this confirms that we have indeed constructed microstates of the D1-D5-KK CFT. In fact, a useful method of constructing our solutions is to start from the conical defects and then to try to extend them to the asymptotically flat region, although we will see that this is much more involved than simply inserting 1's in harmonic functions.

The essential mechanism that renders our solutions smooth is the expansion of the D1 and D5 branes into a KK-monopole supertube, as in [9]. This is seen in the 10D metric by the fact that one has harmonic functions sourced on a ring rather than just at a point, as is the case in the naive singular geometry. A novel feature in our case is that we will have harmonic functions sourced both on a ring and at the origin. The latter corresponds to the original KK monopole. Thus we have a separation of the D1 and D5 charges from that of the KK monopole. Hence, from a string theory perspective the singularity is resolved because the D1-D5 system expands into a supertube in the Taub-NUT geometry of the KK monopole. From a 4D perspective this singularity resolution does not appear to come from an expansion of the branes (the supertube formed by the D1 and D5 branes reduces to a point when compactified to 4D), but from the separation of the branes that form the D1-D5-KK system into two separated stacks. We believe that this separation of charges is a more generic phenomenon which will play a crucial role in providing smooth geometries for other multicharge systems. For instance, it is a basic aspect of the split attractor flows studied in [31–33], as is the angular momentum formula (1.1).

In Sec. II we use the D1-D5-KK CFT to find the near-horizon limit of and motivate an *ansatz* for the asymptotically flat solutions which are then constructed in Sec. III and summarized in Sec. IV. In Sec. V we explore the properties of these solutions, and in Sec. VI we discuss our results. Details of the singularity analysis are found in the Appendix.

II. D1-D5-KK CFT AND NEAR-HORIZON GEOMETRIES

A. Naive geometry of the D1-D5-KK system

The naive geometry of the D1-D5-KK system is obtained by assembling the three individual brane solutions according to the harmonic function rule [1]. We first review the KK-monopole metric by itself, since it plays a distinguished role in our construction, and is perhaps slightly less familiar than the D-brane metrics.

The KK monopole is obtained by replacing four spatial dimensions by the Euclidean Taub-NUT metric [34,35]:

$$ds^2 = -dt^2 + \sum_{i=5}^{9} dx_i^2 + ds_K^2,$$
$$ds_K^2 = Z_K(dr^2 + r^2 d\theta^2 + r^2\sin^2\theta d\phi^2) + \frac{1}{Z_K}(R_K d\psi + Q_K(1+\cos\theta)d\phi)^2 \qquad (2.1)$$

with

$$Q_K = \frac{1}{2}N_K R_K, \qquad Z_K = 1 + \frac{Q_K}{r}. \qquad (2.2)$$

The angular coordinates have the identifications $(\psi, \phi) \cong (\psi + 2\pi, \phi) \cong (\psi, \phi + 2\pi)$. The ψ circle stabilizes at large r at size $2\pi R_K$, and so the Taub-NUT metric is asymptotically $\mathbb{R}^3 \times S^1$. The KK-gauge field obtained from reduction on the S^1 is equal to $A = -Q_K(1 + \cos\theta)d\phi$. This is singular at $\cos\theta = 1$ where the ϕ coordinate breaks down. However, this is just a coordinate singularity, as it is removed by the shift $\psi \to \psi - N_K\phi$. This shift preserves the coordinate identifications, and it is this requirement which underlies the quantization of the magnetic charge in (2.2) with N_K an integer (which we will take to be positive).

From now on, we will find it convenient to take the KK-gauge field to be $A = -Q_K\cos\theta d\phi$ to simplify some algebra. With this choice of gauge, the angular identifications are $(\psi, \phi) \cong (\psi + 2\pi, \phi) \cong (\psi + N_K\pi, \phi + 2\pi)$.

At $r = 0$ the ψ circle shrinks to zero size. For $N_K = 1$ it does so smoothly, and in fact the Taub-NUT metric for $N_K = 1$ is completely smooth. However, for general N_K there is a $\mathbb{Z}_{N_K}$ singularity at $r = 0$.

The harmonic function rule yields the D1-D5-KK metric as

$$ds^2 = \frac{1}{\sqrt{Z_1 Z_5}}(-dt^2 + dx_5^2) + \sqrt{Z_1 Z_5}ds_K^2 + \sqrt{\frac{Z_1}{Z_5}}ds_{T^4}^2 \qquad (2.3)$$

with

$$Z_{1,5} = 1 + \frac{Q_{1,5}}{r}. \qquad (2.4)$$

$ds_{T^4}^2$ describes a four-torus of volume V_4, and x_5 is periodic: $x_5 \cong x_5 + 2\pi R_5$. The quantization conditions on the charges are

$$Q_1 = \frac{(2\pi)^4 g\alpha'^3 N_1}{2R_K V_4}, \qquad Q_5 = \frac{g\alpha' N_5}{2R_K}. \qquad (2.5)$$

The D1 branes are wrapped on the x_5 circle and smeared on T^4, while the D5 branes are wrapped on both spaces. Both branes are also smeared along the KK direction ψ. The solution also has a nontrivial dilaton and Ramond-Ramond (RR) potentials, which we have suppressed. The solution is 1/8 BPS, and the BPS mass formula is

$$M = \frac{Q_1 + Q_5 + Q_K}{4G_4}. \qquad (2.6)$$

B. Near-horizon limit

To take the near-horizon limit relevant for AdS/CFT we drop the 1's from the harmonic functions $Z_{1,5,K}$. We also change coordinates as

$$r = \frac{4Q_1Q_5Q_K}{z^2}, \qquad \phi = \tilde{\phi} - \tilde{\psi},$$
$$\psi = \frac{1}{2}N_K(\tilde{\psi} + \tilde{\phi}), \qquad \theta = 2\tilde{\theta} \qquad (2.7)$$

which brings the metric to the form

$$ds^2 = \frac{\ell^2}{z^2}(-dt^2 + dx_5^2 + dz^2) + \ell^2(d\tilde{\theta}^2 + \sin^2\tilde{\theta}d\tilde{\psi}^2 + \cos^2\tilde{\theta}d\tilde{\phi}^2) + \sqrt{\frac{Q_1}{Q_5}}ds_{T^4}^2, \qquad (2.8)$$

with

$$\ell^2 = 4\sqrt{Q_1Q_5Q_K^2}. \qquad (2.9)$$

Since the new angular coordinates have the identifications $(\tilde{\psi}, \tilde{\phi}) \cong (\tilde{\psi}, \tilde{\phi} + 2\pi) \cong (\tilde{\psi} + \frac{2\pi}{N_K}, \tilde{\phi} + \frac{2\pi}{N_K})$ we identify the geometry (2.8) as $AdS_3 \times S^3/\mathbb{Z}_{N_K} \times T^4$ [25].

For our purposes, a central feature is that the metric (2.8) is singular at $z = \infty$, since the compact x_5 circle shrinks to zero size. It is precisely this singularity which our new solutions will "resolve."

C. D1-D5-KK CFT

By the standard reasoning, there is a 1 + 1 dimensional CFT dual to string theory on the background (2.8). Only a few aspects of this CFT will be relevant for us. The central charge is determined from the Brown-Henneaux formula [36] $c = \frac{3\ell}{2G_3}$. For the more familiar D1-D5 system this gives $c = 6N_1N_5$. As we have seen, the KK monopoles reduce the volume of the S^3 by a factor of N_K, and hence decrease G_3 by this same amount, and so now $c = 6N_1N_5N_K$.

The $\mathbb{Z}_{N_K}$ identification of the sphere breaks the $SU(2)_L \times SU(2)_R$ isometry group down to $SU(2)_L$, and this becomes the R-symmetry of the CFT. The CFT has a corresponding chiral (4, 0) supersymmetry. For the asymptotically flat geometries, the $SU(2)$ R-symmetry will be identified with the four-dimensional angular momentum.

We will be interested in the Ramond ground states of the SUSY side of the CFT, or equivalently, the NS-sector chiral primaries. These states are conveniently summarized in the orbifold CFT language, exactly like in the case of the D1-D5 system. In particular, one considers an effective string of length $N_1N_5N_K$, which can be broken up into component strings of integer length. In the Ramond vacua, each component string carries $J = \pm\frac{1}{2}$, where J is the diagonal $SU(2)$ generator normalized to have half-integer eigenvalues. We therefore find that the complete system can carry R charge, or equivalently angular momentum, in the range

$$-\frac{1}{2}N_1N_5N_K \leq J \leq \frac{1}{2}N_1N_5N_K. \tag{2.10}$$

A particular subclass of states corresponds to taking all component strings to have equal length and equal R charge. These states are therefore labeled by n, the length of component strings, and their R charges are

$$J = \pm\frac{1}{2}\frac{N_1N_5N_K}{n}, \qquad 1 \leq n \leq N_1N_5N_K. \tag{2.11}$$

We will find the supergravity duals to this class of states. What makes these states simpler is that their geometries preserve a $U(1) \times U(1)$ symmetry corresponding to translation in ψ and ϕ.

D. Spectral flow of near-horizon geometry

As we have seen, the near-horizon geometry based on the metric (2.3) is singular because it yields AdS_3 in Poincaré coordinates with a compact spatial direction. Our new geometries will, by contrast, reduce to global AdS_3 in the near-horizon limit, and so be free of singularities.[2]

Indeed, we want the near-horizon limit of our geometries to be dual to the Ramond vacua of the CFT, which can be mapped into the NS-sector chiral primaries by spectral flow. Furthermore, the $U(1) \times U(1)$ invariant chiral primaries are dual to conical defect orbifolds of global AdS_3, and in the bulk spectral flow is just a coordinate transformation. Our strategy is therefore to start from global AdS_3 and "undo" the spectral flow to obtain the near-horizon limit of the geometries dual to the Ramond vacua. We then write these near-horizon solutions in a coordinate system adapted to the BPS equations, which can be used to extend these solutions to the asymptotically flat region.

With this in mind, we will now transform the metric of global $AdS_3 \times S^3/\mathbb{Z}_{N_K} \times T^4$ into this preferred coordinate system. This procedure will then suggest a natural *ansatz* for constructing the full asymptotically flat solutions.

We therefore start from

$$ds^2 = -\left(1 + \frac{\tilde{r}^2}{\ell^2}\right)d\tilde{t}^2 + \frac{d\tilde{r}^2}{1 + \frac{\tilde{r}^2}{\ell^2}} + \tilde{r}^2 d\chi^2 + \ell^2(d\tilde{\theta}^2 + \sin^2\tilde{\theta}d\tilde{\psi}^2 + \cos^2\tilde{\theta}d\tilde{\phi}^2). \tag{2.12}$$

We have omitted the T^4 since it plays no role in what follows.

We now perform the following chain of coordinate transformations:

Step 1:

$$\chi = \frac{R_K}{\ell^2}x_5, \qquad \tilde{\psi} = \hat{\psi} + \frac{R_K}{\ell^2}\hat{t},$$
$$\tilde{\phi} = \hat{\phi} + \frac{R_K}{\ell^2}x_5, \qquad \tilde{t} = \frac{R_K}{\ell}\hat{t}. \tag{2.13}$$

Step 2:

$$\rho = \sqrt{\tilde{r}^2 + R_K^2\sin^2\tilde{\theta}}, \qquad \cos\bar{\theta} = \frac{\tilde{r}\cos\tilde{\theta}}{\sqrt{\tilde{r}^2 + R_K^2\sin^2\tilde{\theta}}}. \tag{2.14}$$

Step 3:

$$r = \frac{\rho^2}{Q_K}, \qquad \phi = \hat{\phi} - \hat{\psi},$$
$$\psi = \frac{1}{2}N_K(\hat{\psi} + \hat{\phi}), \qquad \theta = 2\bar{\theta}, \tag{2.15}$$

with Q_K defined as in (2.2). The final angular coordinates have periodicities $(\psi, \phi) \cong (\psi + 2\pi, \phi) \cong (\psi, \phi + 2\pi)$, and $x_5 \cong x_5 + 2\pi\frac{\ell^2}{R_K}$. The metric takes the form

$$ds^2 = \frac{1}{\sqrt{Z_1Z_5}}[-(dt + k)^2 + (dx_5 - k - s)^2] + \sqrt{Z_1Z_5}ds_K^2, \qquad k = \frac{\ell^2}{4Q_K}\frac{\Sigma - r - \tilde{R}}{\Sigma}d\psi - \frac{\ell^2}{4R_K}\frac{\Sigma - r - \tilde{R}}{\Sigma}d\phi,$$
$$s = -\frac{\ell^2}{2Q_K}\frac{\Sigma - r}{\Sigma}d\psi - \frac{\ell^2}{2R_K}\frac{\tilde{R}}{\Sigma}d\phi, \qquad Z_K = \frac{Q_K}{\Sigma}, \qquad Z_{1,5} = \frac{Q_{1,5}}{\Sigma}, \qquad \Sigma = \sqrt{r^2 + \tilde{R}^2 + 2\tilde{R}r\cos\theta}, \qquad \tilde{R} = \frac{R_K^2}{4Q_K}. \tag{2.16}$$

Note that if we insert $\tilde{R} = 0$ and restore the 1 in $Z_{1,5}$ and Z_K then we revert back to the metric of (2.3).

Since the metric (2.16) is, by construction, smooth, our goal is to extend it to the asymptotically flat region. In the analogous case of the D1-D5 system this can be done simply by adding 1's to the Z functions. In our case it turns out to be much more involved. Therefore, we will just use (2.16) as a guide for writing down an appropriate asymptotically flat *ansatz*, but then analyze the equations of motion independently of the preceding near-horizon construction. The asymptotically flat metric we eventually find

[2] To be precise, the only singularities that will be acceptable are $\mathbb{Z}_n$ singularities.

will, however, turn out to have (2.16) as its near-horizon limit.

III. CONSTRUCTION OF ASYMPTOTICALLY FLAT SOLUTIONS

For the purposes of writing a supergravity *ansatz* it is preferable to work in the M-theory frame, where there is more symmetry between the types of branes. However, the IIB duality frame is distinguished by the fact that the resulting geometry is free of singularities. This follows from the fact that in this frame the branes expand into a KK-monopole supertube, and the KK monopole is a smooth solution. We will therefore work in the IIB frame, so that we can fix all free coefficients by demanding smoothness, and be confident that we have constructed a legitimate physical solution.

In the M-theory frame, a background that preserves the same supersymmetries as three sets of M2 branes can be written as [18,37]

$$\begin{aligned} ds_{11}^2 &= -\left(\frac{1}{Z_1Z_2Z_3}\right)^{2/3}(dt+k)^2 + (Z_1Z_2Z_3)^{1/3}h_{mn}dx^m dx^n + \left(\frac{Z_2Z_3}{Z_1^2}\right)^{1/3}(dx_1^2+dx_2^2) + \left(\frac{Z_1Z_3}{Z_2^2}\right)^{1/3}(dx_3^2+dx_4^2) \\ &\quad + \left(\frac{Z_1Z_2}{Z_3^2}\right)^{1/3}(dx_5^2+dx_6^2), \\ \mathcal{A} &= A^1\wedge dx_1\wedge dx_2 + A^2\wedge dx_3\wedge dx_4 + A^3\wedge dx_5\wedge dx_6 \end{aligned} \tag{3.1}$$

where A^I and k are one-forms in the five-dimensional space transverse to the T^6. h_{mn} is a four-dimensional hyper-Kahler metric.

To obtain the solutions in the type IIB frame with D1, D5, and momentum charges, we KK reduce along x_6, and then perform T dualities along $x_{3,4,5}$. The three types of M2 branes become D1 branes, D5 branes, and momentum ($Z_1\to Z_5$, $Z_2\to Z_1$, $Z_3\to Z_p$ and similarly for the A^I) and the resulting string frame background is

$$\begin{aligned} ds_{10}^2 &= -Z_1^{-1/2}Z_5^{-1/2}Z_p^{-1}(dt+k)^2 + Z_1^{1/2}Z_5^{1/2}h_{mn}dx^m dx^n \\ &\quad + Z_1^{1/2}Z_5^{-1/2}(dx_1^2+dx_2^2+dx_3^2+dx_4^2) \\ &\quad + Z_1^{-1/2}Z_5^{-1/2}Z_p^{-1}(dx_5+A^p)^2, \\ e^{2\phi} &= \frac{Z_1}{Z_5}, \\ F_{(3)} &= (Z_5^{5/4}Z_1^{-3/4}Z_p^{-1/2})\star_5 dA^5 - dA^1\wedge(dx_5+A^p) \end{aligned} \tag{3.2}$$

where $\star_5$ is taken with respect to the five-dimensional metric that appears in the first line of (3.2).

When written in terms of the "dipole field strengths" Θ^I,

$$\Theta^I \equiv dA^I + d\left(\frac{dt+k}{Z_I}\right), \tag{3.3}$$

the BPS equations simplify to [18,37]:

$$\begin{aligned} \Theta^I = \star_4\Theta^I, \qquad \nabla^2 Z^I = \frac{1}{2}|\epsilon_{IJK}|\star_4(\Theta^J\wedge\Theta^K), \\ dk + \star_4 dk = Z_I\Theta^I \end{aligned} \tag{3.4}$$

where $\star_4$ is the Hodge dual taken with respect to the four-dimensional metric h_{mn}. We are looking for a solution describing the dual of microstates of the D1-D5-KK system, so we take the momentum to zero. This furthermore implies the absence of dipole charges associated to the momentum charge. Hence $Z_p = 1$, and $\Theta^1 = \Theta^2 = 0$. Moreover, we are interested in a solution that has KK-monopole charge, so we take the transverse metric $h_{mn}dx^m dx^n = ds_K^2$, the Euclidean Taub-NUT metric of (2.1).

It will also be convenient to define s as

$$s \equiv -A^p - (dt+k), \tag{3.5}$$

such that $ds = -\Theta^p$. With these simplifications the metric is

$$\begin{aligned} ds_{10}^2 &= \frac{1}{\sqrt{Z_1Z_5}}[-(dt+k)^2 + (dx_5-k-s)^2] \\ &\quad + \sqrt{Z_1Z_5}ds_K^2 + \sqrt{\frac{Z_1}{Z_5}}ds_{T^4}^2 \end{aligned} \tag{3.6}$$

where we took $dx_5\to dx_5 - dt$ to impose $g_{tt} = -1$ asymptotically. Note that the metric takes the same form as in (2.16). The dilaton is

$$e^{\phi} = \sqrt{\frac{Z_1}{Z_5}}, \tag{3.7}$$

and the RR fields have an "electric" component given by

$$C_e^2 = Z_1^{-1}(dt+k)\wedge(dx^5 - s - k) \tag{3.8}$$

and a "magnetic" component given implicitly by

$$dC_m^2 = -\star_4(dZ_5) \tag{3.9}$$

where $\star_4$ is now the Hodge dual on the Taub-NUT metric (2.1).

With these definitions, the BPS equations become simply

$$ds = \star_4 ds = -(dk + \star_4 dk), \qquad \nabla^2 Z_{1,5} = 0. \tag{3.10}$$

To simplify further, we define

$$a = k + \frac{1}{2}s \tag{3.11}$$

so that the full set of equations is

$$ds = \star_4 ds, \qquad da = -\star_4 da, \qquad \nabla^2 Z_1 = \nabla^2 Z_5 = 0. \tag{3.12}$$

Of course, strictly speaking these equations only hold away from the brane sources which we also need to specify. If we replace the Taub-NUT space by $\mathbb{R}^4$, we recover the solutions of [9,38].

From (3.12) we see that the problem has been reduced to finding (anti) self-dual 2-forms and harmonic functions on Taub-NUT. In fact, we can further reduce the problem of finding the 2-forms to that of finding harmonic functions, as we now discuss.

A. (Anti) self-dual 2-forms and harmonic functions

As explained above, we need to find closed, (anti) self-dual 2-forms on the Taub-NUT space. We are restricting ourselves to $U(1) \times U(1)$ invariant solutions, where the $U(1)$'s correspond to shifts in ψ and ϕ, and so we demand this of our 2-forms and harmonic functions.

We can approach the problem in the following way. Write Taub-NUT in Cartesian coordinates as

$$ds^2 = Z_K d\vec{x}^2 + \frac{1}{Z_K}(R_K d\psi + \vec{A} \cdot d\vec{x})^2 \tag{3.13}$$

with orientation $\epsilon_{\psi 123} > 0$. We have

$$\epsilon_i{}^{jk} \partial_j A_k = \partial_i Z_K \tag{3.14}$$

where the i indices refer to the flat metric $d\vec{x}^2$.

Then, any self-dual, closed 2-form Θ^+ takes the form

$$\Theta^+_{\psi i} = R_K B_i, \qquad \Theta^+_{ij} = A_i B_j - B_i A_j + Z_K \epsilon_{ij}{}^k B_k \tag{3.15}$$

where

$$B_i = \partial_i P^+, \qquad \partial_i^2 (Z_K P^+) = 0 \tag{3.16}$$

and ∂_i^2 is the Laplacian with respect to $d\vec{x}^2$.

Similarly, any anti-self-dual, closed 2-form Θ^- takes the form

$$\Theta^-_{\psi i} = R_K B_i, \qquad \Theta^-_{ij} = A_i B_j - B_i A_j - Z_K \epsilon_{ij}{}^k B_k \tag{3.17}$$

where

$$B_i = \partial_i P^-, \qquad \partial_i^2 (P^-) = 0. \tag{3.18}$$

In our case, we make the identifications

$$\Theta^+ = ds, \qquad \Theta^- = da. \tag{3.19}$$

We can specify harmonic functions $Z_K P^+$ and P^-, work out the 2-forms $\Theta^\pm$, and then integrate to find the 1-forms s and a. We have therefore shown that our full solution is specified by the four harmonic functions Z_1, Z_5, $Z_K P^+$, and P^-.

IV. ASYMPTOTICALLY FLAT SOLUTION: RESULTS

Using our previous near-horizon solution (2.16) as a guide, we now look for a nonsingular asymptotically flat solution. As we have discussed, the solution is specified by four harmonic functions. Writing Taub-NUT as in (2.1), our requirement of $U(1) \times U(1)$ symmetry means that the harmonic functions should only depend on r and θ. It is then easy to check that Laplace's equation for such functions is the same as on $\mathbb{R}^3$. So we just have to specify the locations of our sources, and then our harmonic functions will be of the form $\sum_i [q_i/(|\vec{x} - \vec{x}_i|)]$.

As in (2.16) (but now including the 1 for asymptotic flatness) we will take

$$Z_{1,5} = 1 + \frac{Q_{1,5}}{\Sigma}, \qquad \Sigma = \sqrt{r^2 + \tilde{R}^2 + 2\tilde{R}\cos\theta} \tag{4.1}$$

corresponding to charges $Q_{1,5}$ placed at a distance $\tilde{R}$ along the negative z-axis. The charges are quantized as in (2.5).

Next, we need to specify the harmonic functions P^- and $Z_K P^+$. A natural ansatz is

$$P^- = c_1 + \frac{c_2}{r} + \frac{c_3}{\Sigma}, \qquad Z_K P^+ = d_1 + \frac{d_2}{r} + \frac{d_3}{\Sigma}. \tag{4.2}$$

We now need to solve (3.19) to find s and a. They have the structure

$$\begin{aligned} s &= s_\psi(r, \theta) d\psi + s_\phi(r, \theta) d\phi, \\ a &= a_\psi(r, \theta) d\psi + a_\phi(r, \theta) d\phi. \end{aligned} \tag{4.3}$$

From (3.15), (3.16), and (3.17) we can immediately read off

$$\begin{aligned} a_\psi &= -R_K P^- = -R_K\left(c_1 + \frac{c_2}{r} + \frac{c_3}{\Sigma}\right), \\ s_\psi &= -R_K P^+ + d_5 = -\frac{R_K}{Z_K}\left(d_1 + \frac{d_2}{r} + \frac{d_3}{\Sigma}\right) + d_5. \end{aligned} \tag{4.4}$$

To determine a_ϕ and s_ϕ we solve the second equations in (3.15) and (3.17), which read

$$\begin{aligned} (da)_{ij} &= A_i \partial_j P^- - \partial_i P^- A_j - Z_K \epsilon_{ij}{}^k \partial_k P^-, \\ (ds)_{ij} &= A_i \partial_j P^+ - \partial_i P^+ A_j + Z_K \epsilon_{ij}{}^k \partial_k P^+. \end{aligned} \tag{4.5}$$

Solving the $r\phi$ and $\theta\phi$ components of these equations yields

$$a_\phi = -P^- Q_K \cos\theta + c_3\left(\frac{Q_K}{\tilde{R}}\frac{(r+\tilde{R}\cos\theta)}{\Sigma} - \frac{(r\cos\theta+\tilde{R})}{\Sigma}\right) + (Q_K c_1 - c_2)\cos\theta + c_4,$$
$$s_\phi = -P^+ Q_K \cos\theta + d_3 \frac{r\cos\theta + \tilde{R}}{\Sigma} + d_2\cos\theta + d_4. \tag{4.6}$$

Finally, from $k = a - \frac{1}{2}s$ we have (we omit the d_5 term from k_ψ corresponding to redefining constants)

$$k_\psi = -R_K\left(P^- - \frac{1}{2}P^+\right),$$
$$k_\phi = -\left(P^- - \frac{1}{2}P^+\right)Q_K\cos\theta + c_3\frac{Q_K}{\tilde{R}}\frac{(r+\tilde{R}\cos\theta)}{\Sigma} - \left(c_3 + \frac{1}{2}d_3\right)\frac{(r\cos\theta+\tilde{R})}{\Sigma} + \left(Q_K c_1 - c_2 - \frac{1}{2}d_2\right)\cos\theta + c_4 - \frac{1}{2}d_4. \tag{4.7}$$

A. Result of singularity analysis

We have now specified all quantities appearing in the metric (3.6) in terms of the constants c_i and d_i and the radius $\tilde{R}$. Although we have a solution for any choice of constants, we want to further demand that we have a smooth solution, free of any singularities.

There are potential singularities at $r = 0$ and $\Sigma = 0$ where the harmonic functions diverge. Furthermore, there are potential Dirac-Misner string singularities at $\sin\theta = 0$ where the ϕ coordinate breaks down. In the Appendix we analyze all the conditions for smoothness, and find that all of the coefficients c_i and d_i are uniquely fixed, along with the ring radius $\tilde{R}$. The values obtained are

$$c_1 = \frac{1}{2Q_K}\sqrt{\frac{Q_1Q_5}{\tilde{Z}_K}} + \frac{1}{2R_K}d_5, \qquad c_2 = 0,$$
$$c_3 = \frac{1}{2}\sqrt{\frac{Q_1Q_5}{\tilde{Z}_K}}, \qquad c_4 = -\frac{Q_K}{2\tilde{R}}\sqrt{\frac{Q_1Q_5}{\tilde{Z}_K}},$$
$$d_1 = \frac{1}{Q_K}\sqrt{\frac{Q_1Q_5}{\tilde{Z}_K}} + \frac{1}{R_K}d_5, \tag{4.8}$$
$$d_2 = \sqrt{Q_1Q_5\tilde{Z}_K} + \frac{Q_K}{R_K}d_5, \qquad d_3 = -\sqrt{Q_1Q_5\tilde{Z}_K},$$
$$d_4 = 0,$$

with

$$\tilde{Z}_K = 1 + \frac{Q_K}{\tilde{R}}. \tag{4.9}$$

The value of d_5 is not determined by the singularity analysis. However, it turns out that with the constants given by (2.12) there ends up being no dependence on d_5 in the solution, so we now set $d_5 = 0$.

The ring radius $\tilde{R}$ is determined from

$$R_5 = \frac{2\sqrt{Q_1Q_5\tilde{Z}_K}}{n}. \tag{4.10}$$

Here n is any positive integer. As discussed in the Appendix, complete smoothness of the geometry requires that we take $n = 1$. Other values of n correspond to allowing $\mathbb{Z}_n$ singularities due to the presence of n coincident KK-monopole supertubes. These more general solutions, while singular in supergravity, are nonsingular from the point of view of string theory.

B. Summary of solution

Now that we have worked out all the free parameters we can write down the explicit solution. For convenience, we collect all the relevant formulas here. The type IIB string frame metric, dilaton, and RR three-form field strength are

$$ds_{10}^2 = \frac{1}{\sqrt{Z_1Z_5}}[-(dt+k)^2 + (dx_5 - k - s)^2] + \sqrt{Z_1Z_5}ds_K^2 + \sqrt{\frac{Z_1}{Z_5}}ds_{T^4}^2,$$
$$e^\phi = \sqrt{\frac{Z_1}{Z_5}}, \tag{4.11}$$
$$F^{(3)} = d[Z_1^{-1}(dt+k)\wedge(dx^5 - s - k)] - \star_4(dZ_5)$$

where $\star_4$ is taken with respect to the metric ds_K^2, and

$$ds_K^2 = Z_K(dr^2 + r^2d\theta^2 + r^2\sin^2\theta d\phi^2) + \frac{1}{Z_K}(R_K d\psi + Q_K\cos\theta d\phi)^2, \tag{4.12}$$

$$Z_K = 1 + \frac{Q_K}{r}, \qquad Z_{1,5} = 1 + \frac{Q_{1,5}}{\Sigma},$$
$$\Sigma = \sqrt{r^2 + \tilde{R}^2 + 2\tilde{R}r\cos\theta}, \tag{4.13}$$

$$x_5 \cong x_5 + 2\pi R_5, \qquad R_5 = \frac{2\sqrt{Q_1Q_5\tilde{Z}_K}}{n},$$
$$\tilde{Z}_K = 1 + \frac{Q_K}{\tilde{R}}. \tag{4.14}$$

The 1-forms s and k have the structure $s = s_\psi d\psi + s_\phi d\phi$ (and analogously for k) with components

$$s_\psi = -\frac{\sqrt{Q_1 Q_5 \tilde{Z}_K} R_K}{Z_K r \Sigma}\left[\Sigma - r + \frac{r\Sigma}{Q_K \tilde{Z}_K}\right],$$

$$s_\phi = -\frac{\sqrt{Q_1 Q_5 \tilde{Z}_K}}{\Sigma}\left[\tilde{R} - \frac{(\Sigma - \frac{1}{\tilde{Z}_K}\Sigma - r)}{Z_K}\cos\theta\right],$$

$$k_\psi = \frac{\sqrt{Q_1 Q_5 \tilde{Z}_K} R_K Q_K}{2\tilde{R}\tilde{Z}_K r Z_K \Sigma}\left[\Sigma - r - \tilde{R} - \frac{2\tilde{R}r}{Q_K}\right], \tag{4.15}$$

$$k_\phi = -\frac{\sqrt{Q_1 Q_5 \tilde{Z}_K} Q_K}{2\tilde{R}\tilde{Z}_K \Sigma} \times\left[\Sigma - r - \tilde{R} + \frac{(\Sigma - r + \tilde{R})}{Z_K}\cos\theta\right].$$

The charges Q_K and $Q_{1,5}$ are quantized according to (2.2) and (2.5).

The free parameters in the solution are the moduli R_5, R_K, V_4, and g_s; the quantized charges N_K, N_1, and N_5; and the quantized dipole charge n. As we explain in the Appendix, it is also possible to add two constant parameters to s_ψ and s_ψ; after compactifying to four dimensions, one of these constants corresponds to shifting the modular parameter of the T^2 at infinity, and the other is a trivial gauge transformation of one of the potentials that are obtained after the reduction.

V. PROPERTIES OF THE SOLUTION

A. Near-horizon limit

As usual, to take the near-horizon decoupling limit we take $\alpha' \to 0$, while scaling coordinates and moduli such that the metric picks up an overall factor of α'. In our case, this is achieved by the scaling

$$r \sim (\alpha')^{3/2}, \qquad \tilde{R} \sim (\alpha')^{3/2}, \qquad V_4 \sim (\alpha')^2, \qquad R_K \sim (\alpha')^{1/2}. \tag{5.1}$$

This scaling effectively takes the large charge limit of the solution, and eliminates, for example, the 1 from $Z_{1,5,K}$ and $\tilde{Z}_K$. The 1-forms in (4.15) become

$$s_\psi = -\frac{\sqrt{Q_1 Q_5 \tilde{Z}_K} R_K}{Q_K}\frac{\Sigma - r}{\Sigma},$$
$$s_\phi = -\sqrt{Q_1 Q_5 \tilde{Z}_K}\tilde{R}\frac{1}{\Sigma},$$
$$k_\psi = \frac{\sqrt{Q_1 Q_5 \tilde{Z}_K} R_K}{2Q_K}\frac{\Sigma - r - \tilde{R}}{\Sigma},$$
$$k_\phi = -\frac{\sqrt{Q_1 Q_5 \tilde{Z}_K}}{2}\frac{\Sigma - r - \tilde{R}}{\Sigma}. \tag{5.2}$$

We can now compare with the solution of (2.16) obtained by spectral flow. First consider the case $n = 1$ corresponding to a singly wound KK-monopole supertube. We find complete agreement with (2.16) after performing the coordinate rescalings

$$r \to \frac{R_K^2}{4Q_K \tilde{R}} r, \qquad x_5 \to \sqrt{\frac{R_K^2}{4Q_K \tilde{R}}} x_5. \tag{5.3}$$

Recalling that (2.16) is just a diffeomorphism of (2.12), this demonstrates that the near-horizon limit of our asymptotically flat $n = 1$ geometries is just $AdS_3 \times S^3/\mathbb{Z}_{N_K} \times T^4$ with AdS_3 appearing in global coordinates. In particular, this makes the smoothness of the near-horizon geometry manifest.

Now consider the case of general n. We still get the metric (2.16), and hence (2.12), the only difference is that the periodicity in (2.12) is

$$(\chi, \hat{\phi}) \cong \left(\chi + \frac{2\pi}{n}, \hat{\phi} + \frac{2\pi}{n}\right). \tag{5.4}$$

In other words, we have a conical defect. This has a nice correspondence with what one expects from the CFT point of view. In the CFT, states with general n correspond to having component strings whose length is proportional to n. The energy gap above the vacuum is therefore of the form $\Delta E = \frac{\omega_0}{n}$. This is also the case for the conical defect geometries. To see this, perform the rescalings

$$\chi \to n\chi, \qquad r \to \frac{r}{n}, \qquad t \to \frac{t}{n} \tag{5.5}$$

to bring χ to the standard 2π periodicity while maintaining the asymptotically global AdS_3 form of the metric. The rescaling of t precisely accounts for the n dependence of the energy gap.

The fact that our asymptotically flat geometries reduce in the near-horizon limit to geometries with a clear CFT interpretation gives us confidence that we have correctly identified our solutions as the microstates of the D1-D5-KK system.

B. Smoothness

We now give a qualitative explanation for the absence of singularities in our solution. In the naive solution (2.3) the D1 branes, D5 branes, and KK monopoles can all be thought of as sitting at $r = 0$. The x_5 direction common to all the branes shrinks to zero size at the origin, yielding the singularity. In the nonsingular solution the D1 branes and D5 branes expand into a KK-monopole supertube, with the tube direction being the KK fiber direction of the original KK monopole.

This is roughly a supertube in Taub-NUT, to be contrasted with the usual supertube in $\mathbb{R}^4$ [43–45]. From the point of view of the $\mathbb{R}^3$ base of the Taub-NUT, the KK monopole sits at $r = 0$ while the supertube sits at $r = \tilde{R}$ and $\cos\theta = -1$. In this sense, the KK-monopole charge is separated from the D1-brane and D5-brane charges.

There is in fact a sort of symmetry between the two types of charges, as is most readily seen in the context of the near-horizon solution. In particular, consider the loci $r = 0$ and $\Sigma = 0$ corresponding to the "locations" of the charges. We ask for their locations in the global AdS metric of (2.12). Tracing back through the coordinate transformations (2.13), (2.14), and (2.15), we see that $r = 0$ corresponds to ($\tilde{r} = 0, \sin\tilde{\theta} = 0$), and $\Sigma = 0$ corresponds to ($\tilde{r} = 0, \cos\tilde{\theta} = 0$). These are identified as two nonintersecting circles on the S^3/Z_{N_K}, centered at the origin of global AdS_3. So the divergent loci of the two types of harmonic functions—Z_K and $Z_{1,5}$—are simply related by a redefinition of θ.

C. Kaluza-Klein reduction to four dimensions

In order to read off the mass, angular momentum, and charge of our solution it is convenient to reduce it to four dimensions. This will also demonstrate that the solution is asymptotically flat in the four-dimensional sense. The compact directions along which we reduce are T^4 and the asymptotic T^2 parametrized by ψ and x_5.

First reduce from $D = 10$ to $D = 6$. Writing the $D = 10$ metric as

$$ds_{10}^2 = ds_6^2 + e^{2\chi} ds_{T^4}^2 \tag{5.6}$$

the $D = 6$ metric is ds_6^2 and the $D = 6$ dilaton is

$$\phi_6 = \phi_{10} - 2\chi = 0. \tag{5.7}$$

To reduce to $D = 4$ we need to write the six-dimensional metric as

$$\begin{aligned} ds_6^2 = {}& ds_4^2 + G_{\psi\psi}(R_K d\psi - A_\mu^{(\psi)} dx^\mu)^2 \\ & + G_{55}(dx_5 - A_\mu^{(z)} dx^\mu)^2 \\ & + 2G_{\psi 5}(R_K d\psi - A_\mu^{(\psi)} dx^\mu)(dx_5 - A_\mu^{(5)} dx^\mu). \end{aligned} \tag{5.8}$$

Then the $D = 4$ action is (see, e.g. [39])

$$\begin{aligned} S_4 = {}& \frac{1}{16\pi G_4} \int d^4x \sqrt{-g} e^{-2\phi_4} \bigg\{ R + 4(\partial\phi_4)^2 \\ & + \frac{1}{4} \partial_\mu G_{\alpha\beta} \partial^\mu G^{\alpha\beta} - \frac{1}{4} G_{\alpha\beta} F_{\mu\nu}^{(\alpha)} F^{(\beta)\mu\nu} \bigg\}, \end{aligned} \tag{5.9}$$

where the indices α and β run over ψ and z_5, and R is the Ricci scalar of ds_4^2. The $D = 4$ dilaton is

$$e^{-2\phi_4} = \sqrt{\det G} e^{-2\phi_6} = \sqrt{\det G}. \tag{5.10}$$

Also, the $D = 4$ Einstein metric is

$$g_{\mu\nu}^E = e^{-2\phi_4} g_{\mu\nu} = \sqrt{\det G} g_{\mu\nu}. \tag{5.11}$$

Of most interest are the asymptotic formulas for the four-dimensional quantities. At $r = \infty$ the T^2 metric is

$$\begin{aligned} G_{\alpha\beta} dx^\alpha dx^\beta = {}& 4Q_1 Q_5 \tilde{Z}_K \bigg(dz^2 - \frac{\hat{s}_\psi}{\sqrt{Q_1 Q_5 \tilde{Z}_K}} dz d\psi \\ & + \frac{R_K^2 + \hat{s}_\psi^2}{4Q_1 Q_5 \tilde{Z}_K} d\psi^2 \bigg) \end{aligned} \tag{5.12}$$

where we defined the angular variable $z = x_5/R_5$. $\hat{s}_\psi$ denotes the asymptotic value following from (2.16):

$$\hat{s}_\psi = s_\psi|_{r=\infty} = -\frac{2\sqrt{Q_1 Q_5 \tilde{Z}_K}}{N_K \tilde{Z}_K}. \tag{5.13}$$

The T^2 metric corresponds to a torus with modular parameter

$$\tau = \frac{1}{2\sqrt{Q_1 Q_5 \tilde{Z}_K}} (-\hat{s}_\psi + i R_K). \tag{5.14}$$

By doing coordinate transformations preserving the periodicities we can transform τ by $SL(2, Z)$. However, the above τ depends on continuous moduli, and so we cannot generically transform it to a purely imaginary τ. In other words, we cannot transform away the mixed $dz d\psi$ terms in the metric (5.12).

The asymptotic string frame metric is

$$\begin{aligned} d\hat{s}^2 \approx {}& -\left(1 - \frac{Q_1 + Q_5}{2r}\right) dt^2 - \frac{2Q_1 Q_K Q_K}{nR_5} \frac{\sin^2\theta}{r} dt d\phi \\ & + dr^2 + r^2 d\theta^2 + r^2 \sin^2\theta d\phi^2. \end{aligned} \tag{5.15}$$

The $D = 4$ dilaton is

$$e^{-2\phi_4} = \sqrt{\det G} = \frac{1}{\sqrt{Z_K}} \approx 1 - \frac{Q_K}{2r}. \tag{5.16}$$

To read off the mass and angular momentum we need the following two components of the asymptotic Einstein metric:

$$\begin{aligned} g_{tt}^E &\approx -\left(1 - \frac{Q_1 + Q_5 + Q_K}{2r}\right), \\ g_{t\phi}^E &\approx -\frac{Q_1 Q_5 Q_K}{nR_5} \frac{\sin^2\theta}{r}. \end{aligned} \tag{5.17}$$

An asymptotically flat $D = 4$ metric has the terms

$$g_{tt}^E \approx -\left(1 - \frac{2G_4 M}{r}\right), \qquad g_{t\phi}^E \approx -2G_4 J \frac{\sin^2\theta}{r}. \tag{5.18}$$

We therefore read off the mass and angular momentum as

$$M = \frac{Q_1 + Q_5 + Q_K}{4G_4}, \qquad J = \frac{Q_1 Q_5 Q_K}{2nR_5 G_4}. \tag{5.19}$$

The $D = 4$ Newton constant is

$$G_4 = \frac{G_{10}}{V_6} = \frac{1}{8} \frac{(2\pi)^4 g^2 \alpha'^4}{R_K R_5 V_4}, \tag{5.20}$$

where we used

$$V_6 = (2\pi R_K)(2\pi R_5)V_4, \qquad G_{10} = \frac{1}{8}(2\pi)^6 g^2 \alpha'^4. \tag{5.21}$$

Also,

$$Q_1 Q_5 = \frac{(2\pi)^4 g^2 \alpha'^4}{4R_K^2 V_4} N_1 N_5. \tag{5.22}$$

This then gives

$$J = \frac{1}{2}\frac{N_1 N_5 N_K}{n}. \tag{5.23}$$

This is precisely what we expect from the CFT point of view. The solutions considered above have $J > 0$, but we can trivially get the solutions with $J < 0$ by time reversal.

We now work out the gauge charges. The asymptotic gauge fields are

$$\begin{aligned} A_\phi^{(\psi)} &= -Q_K \cos\theta + O\left(\frac{1}{r}\right), \\ A_t^{(\psi)} &= -\frac{1}{R_K}\hat{k}_\psi = \frac{2R_K Q_1 Q_5}{nR_5}\frac{1}{r} + O\left(\frac{1}{r^2}\right), \\ A_\phi^{(5)} &= \hat{s}_\phi - \frac{Q_K}{R_K}\cos\theta \hat{s}_\psi = O\left(\frac{1}{r}\right), \\ A_t^{(5)} &= \frac{\hat{s}_\psi \hat{k}_\psi}{R_K^2} = \frac{Q_1 Q_5}{Q_K \tilde{Z}_K}\frac{1}{r} + O\left(\frac{1}{r^2}\right). \end{aligned} \tag{5.24}$$

We need to take into account that in (5.9) the gauge fields mix via $G_{\alpha\beta}$. To read off the charges we write the magnetic potentials with upper indices, and electric ones with lower indices (since the electric field corresponds to a canonical momentum).

We immediately read off that $A^{(\psi)}$ has magnetic charge Q_K, and corresponding quantized charge $N_m = N_K$. $A^{(5)}$ has vanishing magnetic charge.

The electric potentials are then

$$\begin{aligned} A_{(\psi)t} &= G_{\psi\psi}A_t^{(\psi)} + G_{\psi 5}A_t^{(5)} = \left(1 + \frac{s_\psi^2}{R_K^2}\right)A_t^{(\psi)} - \frac{s_\psi}{R_K}A_t^{(5)} \\ &= A_t^{(\psi)}, \\ A_{(5)t} &= G_{55}A_t^{(5)} + G_{5\psi}A_t^{(\psi)} = A_t^{(5)} - \frac{s_\psi}{R_K}A_t^{(\psi)} = 0. \end{aligned} \tag{5.25}$$

Therefore, the electric charge with respect to $A_{(\psi)}$ is nonvanishing, while it is vanishing for $A_{(5)}$. Next, we use the fact that N_e units of quantized electric charge gives rise to the long range potential

$$A_{(\psi)t} = (16\pi G_4)\frac{N_e}{4\pi r}, \tag{5.26}$$

where we took into account the normalization factor of $\frac{1}{16\pi G_4}$ in (5.9). We therefore read off

$$N_e = \frac{R_K Q_1 Q_5}{2nR_5 G_4} = \frac{R_K}{Q_K}J = \frac{2J}{N_K} = N_1 N_5. \tag{5.27}$$

We find the relation

$$J = \frac{1}{2}N_e N_m. \tag{5.28}$$

As mentioned previously, this is the same angular momentum as arises in ordinary electromagnetism from separated electric and magnetic charges.

D. Features of the singularity resolution

As we have seen, the solutions containing a D1-D5-KK supertube wrapped n times have

$$R_5 = \frac{2\sqrt{Q_1 Q_5 (1 + \frac{Q_K}{\tilde{R}})}}{n}. \tag{5.29}$$

One can think of this relation as determining $\tilde{R}$ in terms of R_5; i.e. the separation between the "location" of the KK monopole and the location of the D1 and D5 branes as a function of the compactification radius. This formula is analogous to the radius formula for supertubes in flat space. Another feature similar to the flat space case is that the "radius" $\tilde{R}$ of the nonsingular configuration decreases with increased dipole charge.

A more unexpected feature of Eq. (5.29) is that as R_5 approaches $(2\sqrt{Q_1 Q_5})/n$ we find that $\tilde{R}$ goes to infinity. Hence, for fixed charges it is possible to change the moduli of the solution only within some range. Although this behavior is perhaps unexpected for the asymptotically flat geometry, if it persisted in the near-horizon limit it would be truly peculiar, with no obvious CFT interpretation. Fortunately, after taking the near-horizon limit, formula (5.29) becomes

$$R_5 = \frac{2\sqrt{Q_1 Q_5}\sqrt{\frac{Q_K}{\tilde{R}}}}{n} \tag{5.30}$$

and there is no longer a lower bound on R_5.

Physically, the reason why the supertube disappears from the spectrum for sufficiently small R_5 is that the space at infinity is not $\mathbb{R}^4$ but $\mathbb{R}^3 \times S^1$. If we think about Taub-NUT space as a cigar, then small supertubes sit near the tip of the cigar. As the supertube radius increases (this can be done by changing moduli), the tubes become larger and slide away from the tip, while wrapping the cigar. Since the radius of the cigar is finite, the tubes will eventually slide off to infinity and disappear.

It is interesting to note that although the D1 branes and D5 branes are smeared along the Kaluza Klein monopole (KKM) fiber in both the naive and correct geometries, at $r = 0$ this fiber shrinks to zero size. Therefore, the fact that the D1 and D5 branes move away from the origin and acquire a KKM dipole moment is indeed an expansion into a supertube. However, from the 4D point of view this expansion is not easily seen, since both the unexpanded branes and the supertube reduce to a point when compactified to four dimensions.

Hence, from a four-dimensional point of view our solutions contain just two sets of charges separated by a certain distance. If one tries to take this separation to zero the solution is the naive singular geometry (2.3). Hence, from a four-dimensional perspective the singularity of the naive geometry is resolved by the splitting of the brane sources. Moreover, for certain values of the separation the resulting configuration is a bound state with a clear CFT dual description, while for other values it is not.

We see that not any splitting of the branes into distinct stacks will resolve the singularity, but only a special type of split (with the KK monopoles in one stack and the D1 and D5 branes in the other). From a supergravity perspective it is not always clear when a given solution is physically acceptable or not; it depends on the duality frame chosen. The IIB duality frame employed here admits manifestly smooth geometries, which are thus physically allowed. But in other duality frames these geometries will be singular, and one needs other criteria to determine their physical relevance. One such method is to give an open string description of the corresponding object, as can be done for the original supertube $D(p-2) + F1 \rightarrow Dp$. Then the microscopic description will yield the necessary constraints on the splitting.

VI. DISCUSSION

In this paper we found asymptotically flat solutions representing the $U(1) \times U(1)$ invariant chiral primaries of the D1-D5-KK system. We found that these solutions are either completely smooth or have acceptable orbifold singularities due to coincident KK monopoles. These solutions have several novel features. One is the separation of the D1-D5 charges from that of the KK monopole, in the sense that the corresponding harmonic functions are sourced at different locations in $\mathbb{R}^3$. Another feature is that the solution carries an electric charge with respect to the same gauge field that is magnetically charged. The charges combined together to obey the relation $J = \frac{1}{2} N_e N_m$, which also appears in pure electromagnetism.

It would clearly be desirable to relax the condition of $U(1) \times U(1)$ symmetry so as to be able to find the full set of chiral primaries. It is likely that the corresponding supergravity solutions will have the Taub-NUT metric replaced by a less symmetric hyper-Kahler manifold, since there would be no obvious reason for the four-dimensional base metric to preserve more symmetry than the full solution.

By a chain of dualities we can transform our D1-D5-KK solution into one carrying charges D1-D5-P, corresponding to the canonical five-dimensional black hole. Therefore, it is appropriate to ask to what extent our solutions can be thought of as the microstates of the D1-D5-P system. The main issue is that our solution is asymptotically $\mathbb{R}^{(3,1)} \times T^6$, while the finite entropy black hole of the D1-D5-P system is asymptotically $\mathbb{R}^{(4,1)} \times T^5$. If we perform duality transformations at the level of supergravity there is no possibility of transforming between these two types of solutions. The dualities would instead produce a D1-D5-P system smeared over a transverse circle.

Nevertheless, it is possible that a more accurate duality transformation would avoid this problem. The key step in the duality chain is the T duality along ψ, the fiber direction of the KK monopole. The D1 branes and D5 branes are both delocalized in this direction, but let us ignore them for the moment, so that we are just considering the T duality of a KK monopole. The T duality produces an NS5 brane, and the question is whether this NS5 brane is smeared or localized over the dual circle. The standard Buscher rules [40] certainly produce a smeared solution. However, as discussed in [41] and shown explicitly in [42], an exact CFT treatment of the T duality yields a localized NS5 brane. One way to see this is that the CFT derivation of T duality involves gauging the translational isometry of the original circle. This requires introducing a corresponding $U(1)$ gauge field, which is subsequently integrated out. The point is that there are instantons in this gauge field which violate the translational isometry of the dual circle.

We might expect a similar phenomenon to occur in our case, leading to a solution which is asymptotic to the standard D1-D5-P solution. Of course, since we have a much more complicated setup than just a KK monopole, involving RR fields and the 1-forms k and s, it is hard to give a direct argument for this. One indirect way to see that this phenomenon is likely to occur in our case is to recall that T duality interchanges winding and momentum modes. Since winding number is not conserved in our backgrounds (due to the contractibility of the S^1 fiber of Taub-NUT), the resulting T-dual background will not preserve momentum, and hence it will not be a smeared collection of branes, but a localized one. While it would certainly be desirable to directly write down solutions for the microstates of D1-D5-P, if the above reasoning is correct we may at least be able to extract some of the physics of these microstates by studying our dual D1-D5-KK solutions.

As we have discussed, our solutions have a clear microscopic meaning in the D1-D5-KK CFT. They are the chiral primaries, or equivalently, the Ramond ground states. In the effective string language, these are described by $N_1 N_5 N_K / n$ effective strings, each of length n. On the other hand, the standard finite entropy black hole of the D1-D5-KK-P system corresponds to taking a single effective string of length $N_1 N_5 N_K$ and adding momentum to it. To preserve SUSY, the momentum is added to the nonsupersymmetric side of the $(4, 0)$ CFT; the excitations carry no R charge, and hence the black hole carries no angular momentum. It is natural to consider combining these two elements. That is, to consider dividing the full effective string into two parts, one of which is a collection of short effective strings in the Ramond ground state, and the other is a single effective string carrying momentum. In [22], this sort of configuration in the D1-D5 CFT was argued to correspond to five-dimensional BPS black rings. With this in mind, we expect that these configurations in the D1-D5-KK CFT will correspond to the solutions we have found in this paper, except that the two-charge supertube is replaced by the three-charge BPS black ring. That is, we would have a BPS black ring whose ring direction is wrapped around the nontrivial S^1 of Taub-NUT. From the four-dimensional point of view this would be a black hole, since the ring extends along a compact direction. So we are led to the prediction of a new four-dimensional BPS black hole solution carrying nonzero angular momentum.

Such a solution could also have been anticipated in another way. In [22] it was noted that the charges of the five-dimensional black ring correspond to the charges appearing in the quartic $E_{7(7)}$ invariant. However, one of the charges was actually vanishing for the black rings discussed there, and it was noted that it corresponded to a KK-monopole charge. Now we see that this missing charge is precisely that of the KK monopole discussed in this paper. The new solution we are conjecturing will combine all these charges together.

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APPENDIX: SINGULARITY ANALYSIS

In this appendix we analyze the potential singularities in

$$ds_{10}^2 = \frac{1}{\sqrt{Z_1 Z_5}}[-(dt+k)^2 + (dx_5 - k - s)^2] + \sqrt{Z_1 Z_5}\, ds_K^2 + \sqrt{\frac{Z_1}{Z_5}}\, ds_{T^4}^2 \tag{A1}$$

with

$$Z_{1,5} = 1 + \frac{Q_{1,5}}{\Sigma}, \qquad \Sigma = \sqrt{r^2 + \tilde{R}^2 + 2\tilde{R}r\cos\theta}, \tag{A2}$$

$$ds_K^2 = Z_K(dr^2 + r^2 d\theta^2 + r^2 \sin^2\theta d\phi^2) + \frac{1}{Z_K}(R_K d\psi + Q_K \cos\theta d\phi)^2, \tag{A3}$$

and with the 1-forms k and s given in (4.4), (4.6), and (4.7).

For generic choices of parameters c_i, d_i and ring radius $\tilde{R}$, our solution will have curvature singularities at $r = 0$ and $\Sigma = 0$, and Dirac-Misner string singularities at $\sin\theta = 0$. In this appendix we show that all the free parameters of the solution are fixed by demanding smoothness. We will then generalize to allow for $\mathbb{Z}_n$ singularities corresponding to n coincident KK-monopole supertubes.

In the following we suppress the trivial T^4 part of the metric, since it is manifestly nonsingular.

1. $r = 0$ singularities

Viewing k as a 1-form on the Taub-NUT metric, we demand that k is nonsingular at $r = 0$. Otherwise there will be singular terms in the metric of the form $dtd\psi$ and $dtd\phi$. Since the angular coordinates break down at $r = 0$, finiteness of k requires that k_ψ and k_ϕ vanish at $r = 0$.

We find the leading small r behavior

$$k_\psi \approx -\frac{c_2 R_K}{r} - c_1 R_K - \frac{c_3 R_K}{\tilde{R}} + \frac{d_2 R_K}{2Q_K},$$
$$k_\phi \approx \frac{Q_K}{R_K}\cos\theta k_\psi + \frac{Q_K}{\tilde{R}}c_3 + c_4 - \frac{1}{2}d_4 + \left(-\frac{1}{2}d_3 - c_3 + Q_K c_1 - c_2 - \frac{1}{2}d_2\right)\cos\theta \tag{A4}$$

and so we demand the following four conditions:

$$\begin{aligned} c_2 &= 0,\\ \frac{Q_K}{\tilde{R}}c_3 + c_4 - \frac{1}{2}d_4 &= 0,\\ -\frac{1}{2}d_3 - c_3 + Q_K c_1 - \frac{1}{2}d_2 &= 0,\\ c_1 + \frac{1}{\tilde{R}}c_3 - \frac{1}{2Q_K}d_2 &= 0. \end{aligned} \tag{A5}$$

Next, we focus on the small r behavior of s, which is

$$s \approx \bar{s}_\psi d\psi + \bar{s}_\phi d\phi, \tag{A6}$$

where $\bar{s}_\psi = (d_5 - d_2 \frac{R_K}{Q_K})$ and $\bar{s}_\phi = (d_3 + d_4)$. If we were to demand that s be a well-defined 1-form on Taub-NUT we would require $\bar{s}_\psi = \bar{s}_\phi = 0$, as for k. However, taking into account the nontrivial mixing of the angular Taub-NUT coordinates with x_5, we can in fact relax this condition and still obtain a nonsingular metric. This is most easily seen by transforming to the following new coordinates:

$$r = \frac{1}{4}\hat{r}^2, \qquad \theta = 2\hat{\theta}, \qquad x_5 = R_5\hat{\gamma},$$
$$\phi = \hat{\phi} - \hat{\psi} - \frac{1}{R_5}x_5, \qquad \psi = \frac{Q_K}{R_K}\left(\hat{\psi} + \hat{\phi} - \frac{1}{R_5}x_5\right). \tag{A7}$$

$\hat{\gamma}$ is 2π periodic, while $(\hat{\psi}, \hat{\phi})$ have periodicities

$$(\hat{\psi}, \hat{\phi}) \cong (\hat{\psi}, \hat{\phi} + 2\pi) \cong \left(\hat{\psi} + \frac{2\pi}{N_K}, \hat{\phi} + \frac{2\pi}{N_K}\right). \tag{A8}$$

Assuming (A5), and thereby solving for c_1, c_2, d_3, d_4, and also writing

$$d_2 = \frac{Q_K}{R_K} d_5 + \frac{1}{2} R_5, \qquad c_4 = c_3 - \frac{1}{4} R_5, \tag{A9}$$

the leading behavior of the metric is

$$ds^2 \approx -\frac{\tilde{R}}{\sqrt{Q_1 Q_5}} dt^2 + \frac{R_5^2 \tilde{R}}{\sqrt{Q_1 Q_5}} d\hat{\phi}^2 + \frac{\sqrt{Q_1 Q_5} Q_K}{\tilde{R}} \{d\hat{r}^2 + \hat{r}^2 (d\hat{\theta}^2 + \sin^2\hat{\theta} d\hat{\psi}^2 + \cos^2\hat{\theta} d\hat{\gamma}^2)\}. \tag{A10}$$

This metric is smooth given the identification in (A8). Note, in particular, that the $\mathbb{Z}_{N_K}$ identification includes a shift of the fixed size $\hat{\phi}$ circle; therefore there are no fixed points.

Besides the new coordinates (A7), there are other coordinate transformations that give smooth metrics. The first two coordinate changes are the same as in (A7), while the last three are modified. We choose one of the angles to be proportional to the combination of x^5, ψ, and ϕ that appears in the vielbein containing dx^5:

$$\frac{1}{R_5} dx_5 - \frac{\bar{s}_\psi}{R_5} d\psi - \frac{\bar{s}_\phi}{R_5} d\phi \equiv \frac{1}{R_5} dx_5 - n_1 d\psi + \left(n_1 \frac{N_K}{2} - n_2\right) d\phi \equiv d\hat{\phi}. \tag{A11}$$

If we further define

$$\hat{\psi} = \frac{\psi}{N_K} - \frac{\phi}{2}, \quad \hat{\gamma} = \left(\frac{n_1}{n_2} - \frac{1}{N_K}\right)\left(\psi - \frac{N_K}{2}\phi\right) - \frac{x^5}{n_2 R_5} \tag{A12}$$

then the metric becomes again (A10), except that the identifications of the coordinates are now

$$(\hat{\phi}, \hat{\psi}, \hat{\gamma}) \cong \left(\hat{\phi} + 2\pi n_1, \hat{\psi} + \frac{2\pi}{N_K}, \hat{\gamma} + 2\pi\left(\frac{n_1}{n_2} - \frac{1}{N_K}\right)\right) \cong \left(\hat{\phi} + 2\pi, \hat{\psi}, \hat{\gamma} - \frac{2\pi}{n_2}\right) \cong (\hat{\phi} + 2\pi n_2, \hat{\psi}, \hat{\gamma}). \tag{A13}$$

Of course, in order for these identifications to give a smooth space, both n_1 and n_2 must be rational. One can also see that the choice of constants in (A7) and (A9) is equivalent to taking

$$n_2 = -1, \qquad n_1 = -\frac{1}{N_K}. \tag{A14}$$

It is interesting to explore the physical meaning of the extra parameters n_1 and n_2. Their only effect is to add two constants to s_ψ and s_ϕ in Eq. (4.15). Adding a constant to s_ϕ is a trivial diffeomorphism transformation. Adding a constant to s_ψ changes the mixing of $d\psi$ and dx^5 at infinity, and affects the modular parameter of the torus that one uses to obtain the four-dimensional theory.

The new value of s_ψ at infinity is

$$\hat{s}_\psi = n_1 R_5 + R_K \sqrt{\frac{Q_1 Q_5}{\tilde{R}(Q_K + \tilde{R})}}, \tag{A15}$$

and for some values of the moduli it is possible to find a rational n_1 that gives $\hat{s}_\psi = 0$. However, even if the torus is diagonal, the gauge potentials that appear in Eqs. (5.24) and (5.25) are modified, and, in particular, the four-dimensional KK-monopole charge does not point in the ψ direction but in a combination of x_5 and ψ. The gauge choice (A14) aligns the KK-monopole charge and the electric charge along the ψ direction, and we will be choosing it from now on.

2. Dirac-Misner string singularities

As written in (A3), the Taub-NUT metric has coordinate singularities at $\sin\theta = 0$. These can be removed by shifting ψ. In particular, at $\cos\theta = \pm 1$ the metric involves the combination $d\psi \pm \frac{1}{2} N_K d\phi$, and so the shift is $\psi \to \psi \mp \frac{1}{2} N_K \phi$.

If the 1-forms k and s are proportional to $d\psi \pm \frac{1}{2} N_K d\phi$ at $\cos\theta = \pm 1$, then the shift $\psi \to \psi \mp \frac{1}{2} N_K \phi$ will remove the offending ϕ components. Using (A5) and (A9), it is straightforward to verify that at $\cos\theta = 1$ both k and s are in fact proportional to $d\psi + \frac{1}{2} N_K d\phi$. At $\cos\theta = -1$ we find that k is proportional to $d\psi - \frac{1}{2} N_K d\phi$, as is s in the region $r > \tilde{R}$. But for $r < \tilde{R}$ this does not hold, and the situation is more involved. In particular, for $\cos\theta = -1$ and $r < \tilde{R}$ we find

$$k = -2k_\phi d\hat{\psi}, \qquad k + s = -2(k_\phi + s_\phi) d\hat{\psi} - R_5(d\hat{\psi} + d\hat{\phi}) + dx_5, \tag{A16}$$

with angular coordinates defined as in (A7). We then note that dx_5 appears in the metric via the combination $dx_5 - k - s$. This indicates that the contribution to g_{55} from this term vanishes. Moreover, as one can see from (A3) and (A7), the contribution to g_{55} from the Taub-NUT vanishes at $\cos\theta = -1$. In the hatted coordinates system and near $\cos\theta = -1$, dx_5 and $d\theta$ appear in the combination $d(\pi - \theta)^2 + (\pi - \theta)^2 (1/R_5^2) dx_5^2$, which shows that the x_5 circle smoothly shrinks to zero size. On the other hand, the $\hat{\psi}$ and $\hat{\phi}$ circles stabilize at finite size. Thus the complete metric is smooth at $\cos\theta = -1$ and $r < \tilde{R}$.

3. Checking the metric and forms at $\Sigma = 0$

The periodicities of the coordinates ψ, ϕ, x_5 appearing in (A3) are

$$(\psi, \phi, x_5) \cong (\psi, \phi, x_5 + 2\pi R_5) \cong (\psi + N\pi, \phi + 2\pi, x_5) \cong (\psi + 2\pi, \phi, x_5). \quad \text{(A17)}$$

In order to check the behavior of the metric and forms at the point $\Sigma = 0$ it is good to transform to a coordinate system in which this point is the origin of the $\mathbb{R}^3$ that forms the base of the Taub-NUT space (A3):

$$ds_K^2 = Z_K(d\Sigma^2 + \Sigma^2 d\theta_1^2 + \Sigma^2 \sin^2\theta_1 d\phi^2) + \frac{1}{Z_K}(R_K d\psi + Q_K \cos\theta d\phi)^2, \quad \text{(A18)}$$

where Σ is defined in (A2), $\cos\theta_1 = \frac{\tilde{R}+r\cos\theta}{\Sigma}$, and ϕ is unchanged After substituting s and k in the metric, and expanding the metric components around $\Sigma = 0$, the leading components of the metric are nondiagonal. Moreover, the leading components of the metric in the $\psi\psi$, $\phi\phi$, and $\psi\phi$ directions blow up like

$$\frac{Q_1 Q_5 - 4c_3^2 \tilde{Z}_K}{\Sigma}. \quad \text{(A19)}$$

To render these components finite we then must choose

$$c_3 = \frac{1}{2}\sqrt{\frac{Q_1 Q_5}{\tilde{Z}_K}}. \quad \text{(A20)}$$

After making this substitution one can make the leading metric diagonal by making the transformation:

$$t = l_0 \bar{t}, \qquad x_5 = R_5 \eta - l_0 \bar{t}, \qquad \phi = \phi, \qquad \psi = \frac{N_K}{2}(-\bar{t} - \gamma + \phi) \quad \text{(A21)}$$

where l_0 is a finite constant.[3] From (A17) and (A21) one can see that the period of ϕ and η is 2π, while the period of γ is $\frac{4\pi}{N}$.

After the diagonalization, the components of the metric g_{tt} and $g_{\gamma\gamma}$ are finite, while the leading metric in the $\Sigma, \theta_1, \eta, \phi$ directions is

$$\sqrt{Q_1 Q_5}\tilde{Z}_K\left[\frac{d\Sigma^2}{\Sigma} + \Sigma(d\theta_1^2 + \sin^2\theta_1 d\phi^2) + \frac{R_5^2 \Sigma}{Q_1 Q_5 \tilde{Z}_K}\left(d\eta + \frac{\sqrt{Q_1 Q_5 \tilde{Z}_K}}{R_5}(1 + \cos\theta_1)d\phi\right)^2\right]. \quad \text{(A23)}$$

[3]For the curious,

$$l_0 \equiv \frac{Q_K(Q_K Q_1 Q_5 + (Q_1 + Q_5)\tilde{R}(Q_K + \tilde{R}))}{\sqrt{Q_1 Q_5 \tilde{R}}(Q_K + \tilde{R})^{3/2}}. \quad \text{(A22)}$$

It is clear that when

$$R_5 = 2\sqrt{Q_1 Q_5 \tilde{Z}_K} \quad \text{(A24)}$$

this metric becomes the metric at the origin of a new Taub-NUT space, in which Σ plays the role of a radius and $d\eta$ that of a fiber. The fact that the metric near the location of the D1 branes and D5 branes can be written as a Taub-NUT space indicates that the D1 branes and D5 branes have formed a D1-D5-KK supertube at that location. Hence our solution contains a D1-D5-KK supertube at $\Sigma = 0$ and a KK monopole at $r = 0$. When (A24) is satisfied, the dipole charge of the KK monopole of the supertube is 1, and the metric (A23) is manifestly smooth.

We can also consider metrics in which the KKM supertube is wrapped n times. These metrics have a $\mathbb{Z}_n$ orbifold singularity at $\Sigma = 0$, and are obtained by simplify modifying (A24) to

$$R_5 = \frac{2\sqrt{Q_1 Q_5 \tilde{Z}_K}}{n}. \quad \text{(A25)}$$

From the point of view of string theory, these $\mathbb{Z}_n$ orbifolds are nonsingular, and so we should allow them in our class of solutions.

We now turn to checking the smoothness of the RR 2-form potential of our solutions. The only place where the RR fields might be divergent is at $\Sigma = 0$, where the D1 branes and the D5 branes are located. Both the smoothness of the metric, and the fact that the near-horizon region of these solutions can be mapped to AdS_3 in global coordinates clearly point to the absence of any divergences. However, it is instructive to see how this happens.

As we have discussed in Sec. III the electric RR potential of the solution is

$$C_e^2 = Z_1^{-1}(dt + k_\psi d\psi + k_\phi d\phi) \wedge (dx^5 - s_\psi d\psi - s_\phi d\phi - k_\psi d\psi - k_\phi d\phi) \quad \text{(A26)}$$

while the magnetic one is given implicitly by

$$dC_m^2 = - \star_4 (dZ_5) \quad \text{(A27)}$$

where $\star_4$ is the Hodge dual on the Taub-NUT metric (A3). After making the coordinate change (A21), the leading components of the electric potential near the point $\Sigma = 0$ are

$$C_e^2 \sim \Sigma\left(d\eta + \frac{1 + \cos\theta_1}{2}d\phi + \frac{c_1}{\Sigma}(d\gamma + d\bar{t})\right) \wedge \left(d\bar{t} + \frac{c_2}{\Sigma}(d\gamma + d\bar{t})\right) \quad \text{(A28)}$$

where c_1 and c_2 are constants that can be determined straightforwardly from the metric. The part proportional to $\frac{1}{\Sigma}$ cancels, and most of the constant forms are not dangerous because the angles they contain never shrink to zero size. The only possibly dangerous component of C_e^2

is

$$(d\gamma + d\bar{t}) \wedge \left(d\eta + \frac{1+\cos\theta_1}{2} d\phi\right). \qquad \text{(A29)}$$

However, since $g_{\bar{t}\bar{t}}$ and $g_{\gamma\gamma}$ are finite, and the second parenthesis is nothing but the fiber of the Taub-NUT space (A23), this component is also benign.

Since the harmonic function Z_5 is very simple when written in terms of Σ, the magnetic field strength can be easily evaluated to be

$$dC_m^2 = R_K Q_5 \sin\theta_1 d\theta_1 \wedge d\phi \wedge d\psi \qquad \text{(A30)}$$

and hence one can write the potential that gives rise to this field strength as

$$C_m^2 = Q_K Q_5 (2d\eta + (1+\cos\theta_1)d\phi) \wedge (d\gamma + d\bar{t}) \quad \text{(A31)}$$

which is again proportional to the fiber of the Taub-NUT space, and hence regular.

We have therefore verified that with suitable choice of parameters the metric is smooth everywhere. Solving for the parameters yields the values in (4.8).

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