

Light deflection in the postlinear gravitational field of bounded pointlike masses

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Light deflection in the postlinear gravitational field of two bounded pointlike masses is treated. Both the light source and the observer are assumed to be located at infinity in an asymptotically flat space. The equations of light propagation are explicitly integrated to the second order in G/c^2 . Some of the integrals are evaluated by making use of an expansion in powers of the ratio of the relative separation distance to the impact parameter (r_{12}/ξ). A discussion of which orders must be retained to be consistent with the expansion in terms of G/c^2 is given. It is shown that the expression obtained in this paper for the angle of light deflection is fully equivalent to the expression obtained by Kopeikin and Schäfer up to the order given there. The deflection angle takes a particularly simple form for a light ray originally propagating orthogonal to the orbital plane of a binary with equal masses. Application of the formulas for the deflection angle to the double pulsar PSR J0737-3039 for an impact parameter 5 times greater than the relative separation distance of the binary's components shows that the corrections to the Epstein-Shapiro light deflection angle of about 10^{-6} arcsec lie between 10^{-7} and 10^{-8} arcsec.

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I. INTRODUCTION

Light deflection by a gravitational field [1] is one of the observational cornerstones of general relativity. The observational confirmation of Einstein's prediction that light would be deflected by the gravitational field of the Sun [2] brought general relativity to the attention of the general public in the 1920's. Today, technology has reached a level at which the extremely high precision of current ground-based radio interferometric astronomical observations approaches $1 \mu\text{arcsec}$ and within the next decade the accuracy of space-based astrometric positional observations is also expected to reach this accuracy. At this level of accuracy we can no longer treat the gravitational field of a system of moving bodies as static and spherically symmetric. This fact is one of the principal reasons for the necessity of a more accurate solution to the problem of the propagation of electromagnetic waves in nonstationary gravitational fields of celestial bodies. To reach the accuracy of $1 \mu\text{arcsec}$, many subtle relativistic effects must be taken into account in the treatment of light propagation in nonstationary gravitational fields of moving bodies. One of the most intricate problems is the computation of the effects of translational motion of the gravitating bodies on light propagation.

This question was treated for the first time by Hellings in 1986 [3]. In 1989 Klioner [4] solved the problem completely to the first post-Newtonian (1PN) order (i.e. to the order $1/c^2$) for the case of bodies moving with constant velocity. The complete solution of the problem for arbitrarily moving bodies in the first post-Minkowskian approximation (linear in the gravitational constant G) was found by Kopeikin and Schäfer in 1999 [5]. They succeeded in integrating analytically the post-Minkowskian equations of light propagation in the field of arbitrarily moving masses. In Ref. [6], Le Poncin-Lafitte *et al.* have

recently developed an alternative approach to the problem of light deflection and time/frequency transfer in post-Minkowskian gravitational fields based on an expansion of the Synge world function for null geodesics. In that paper the world function and time transfer function were computed for a static spherically symmetric body to the second post-Minkowskian approximation.

In this paper we treat light deflection in the postlinear gravitational field of two bounded pointlike masses (binary system). We assume that the light source as well the observer are located at infinity in an asymptotically flat space. To compute the light deflection we integrate the equations of light propagation explicitly to the second order in G/c^2 , i.e. to the order G^2/c^4 .

The assumption that the gravitational field is weak along the light path allows us to consider the metric as a perturbation of a flat metric represented by a power series in the gravitational constant G

$$g_{\mu\nu}[x^\sigma, G] \equiv g_{\mu\nu}^{(0)} + \sum_{n=1}^{\infty} G^n g_{\mu\nu}^{(n)}(x^\sigma), \quad (1)$$

with

$$g_{\mu\nu}^{(0)} = \eta_{\mu\nu} = \text{diag}(-1, 1, 1, 1).$$

For the same reason we can consider the light trajectory as a perturbation of its trajectory in flat space (a straight line) represented by a power series in G

$$\vec{z}(t) = \vec{z}_{(0)}(t) + \sum_{n=1}^{\infty} G^n \delta \vec{z}_{(n)}(t). \quad (2)$$

It follows from Eq. (2) that the vector tangent to the light trajectory takes the form

$$\vec{l}(t) \equiv \frac{d\vec{z}(t)}{dt} = \vec{l}_{(0)} + \sum_{n=1}^{\infty} G^n \delta\vec{l}_{(n)}(t), \quad (3)$$

where $\vec{l}_{(0)}$ is the constant vector tangent to the unperturbed light trajectory.

In order to obtain the postlinear equations for light propagation, we introduce into the differential equations for the null geodesics the metric as given in (1) and the expression for the tangent vector $\vec{l}(t)$ as given in (3). As a result we get a set of ordinary coupled differential equations of first order for the perturbation terms $\delta\vec{l}_{(n)}(t)$. Each term $\delta\vec{l}_{(n)}$ is given in the form of a line integral along a straight line in the fictitious metric $g_{\mu\nu}^{(0)}$, i.e. along the original unperturbed light trajectory. We get the postlinear light deflection to the order G^2/c^4 after computing the perturbation terms $\delta\vec{l}_{(1)}(t)$, $\delta\vec{l}_{(2)}(t)$ and the corrections arising from introducing the linear perturbation of the light trajectory $\delta\vec{z}_{(1)}(t)$, the motion of the masses and the shift of the 1PN–center of mass with respect to the Newtonian center of mass in the expression for linear light deflection. The final result we obtain is the expression for light deflection in the postlinear gravitational field of two bounded pointlike masses. The deflection angle takes a particularly simple form for a light ray originally propagating orthogonal to the orbital plane of a binary with equal masses.

This paper is organized as follows. In Sec. II we derive the postlinear light propagation equations. We set out an approximation scheme to integrate these equations. The deflection angle as a function of the perturbations of the vector tangent to the light ray is introduced. In Sec. III the postlinear and linear metric for two bounded pointlike masses in harmonic coordinates are given. The coordinate frame is chosen so that the 1PN–center of mass is at rest at the origin. In Sec. IV we compute the perturbation of the vector tangent to the unperturbed light ray and the corresponding light deflection in the linear gravitational field. In Sec. V we compute the light deflection in the postlinear gravitational field. To facilitate the computations we separate the light deflection terms which are functions of the postlinear metric coefficients from the terms that are functions of the linear metric coefficients and the perturbations of the first order in G of the vector tangent to the unperturbed light ray. The resulting integrals are given in Appendices A and B. In Sec. VI we calculate the additional linear and postlinear light deflection terms arising from the introduction of the motion of the masses into the expression for the linear perturbation. In Sec. VII we compute the corrections to the postlinear light deflection arising from the introduction of the linear perturbed light trajectory into the expression for the linear light deflection. The resulting integrals are given in Appendix C. Section VIII is devoted to the computation of the corrections to the linear and postlinear light deflection arising from the introduction of the shift of the 1PN–center of mass with respect to the Newtonian center of mass into the expression for the linear

light deflection. The resulting expressions for the total linear perturbation, linear and postlinear light deflection by the gravitational field of two bounded pointlike masses are given in an explicit form in Secs. IX and X. In Sec. XI we present our results and give in an explicit form the light deflection expression for some simple cases. The derived formulas for the angle of light deflection are applied to the double pulsar PSR J0737-3039. In Appendix D we compute the linear light deflection terms arising from the acceleration terms in the metric coefficients $h_{00}^{(1)}$ and $h_{pq}^{(1)}$. In Appendix E we show that the expression for the linear light deflection given in this paper is fully equivalent to the expression for the light deflection computed by Kopeikin and Schäfer [5] in the event that the velocities of the masses are small with respect to the velocity of light and the retarded times in the expression given by Kopeikin and Schäfer are close to the time of closest approach of the unperturbed light ray to the origin of the coordinate system. Finally, Sec. XII is devoted to the discussion of the results.

A. Notation

Let us summarize the notation and symbols used in this paper:

- (1) G is the Newtonian constant of gravitation;
- (2) c is the velocity of light;
- (3) the Greek indices $\alpha, \beta, \gamma, \dots$, are space-time indices and run from 0 to 3;
- (4) the Latin indices i, j, k, \dots , are spatial indices and run from 1 to 3;
- (5) $g_{\mu\nu}$ is a metric tensor of curved, four-dimensional space-time, depending on spatial coordinates and time;
- (6) the signature adopted for $g_{\mu\nu}$ is $(- + + +)$;
- (7) we suppose that space-time is covered by a harmonic coordinate system $(x^\mu) = (x^0, x^i)$, where $x^0 = ct$, t being the time coordinate;
- (8) the three-dimensional quantities (3-vectors) are denoted by $\vec{a} = a^i$;
- (9) the three-dimensional unit vector in the direction of \vec{a} is denoted by $\vec{e}_a = e_a^i$;
- (10) the Latin indices are lowered and raised by means of the unit matrix $\delta_{ij} = \delta^{ij} = \text{diag}(1, 1, 1)$;
- (11) by $_{,\sigma}$ we denote the partial derivative with respect to the coordinate x^σ ;
- (12) the scalar product of any two 3-vectors \vec{a} and \vec{b} with respect to the Euclidean metric δ_{ij} is denoted by $\vec{a} \cdot \vec{b}$ and can be computed as $\vec{a} \cdot \vec{b} = \delta_{ij} a^i b^j = a^i b^i$;
- (13) the Euclidean norm of a 3-vector \vec{a} is denoted by $a = |\vec{a}|$ and can be computed as $a = [\delta_{mij} a^m a^n]^{1/2}$;
- (14) by $l_{(0)}$ we denote the vector tangent to the unperturbed light ray $\vec{z}(t)$ and the unit vector $\vec{e}_{(0)}$ is defined by $\vec{e}_{(0)} = l_{(0)}/|l_{(0)}|$.

II. LIGHT PROPAGATION AND LIGHT DEFLECTION IN THE POSTLINEAR GRAVITATIONAL FIELD

A. The light propagation equation

In this paper we calculate the light deflection in the postlinear gravitational field of two bounded masses for the case when the impact parameter $|\vec{\xi}|$ is much larger (5 times or more) than the distance r_{12} between the two accelerating masses, so that we can suppose that the gravitational field is weak along the light path.

For weak gravitational fields we can assume that the light propagation is very well governed by the laws of geometric optics, whereby light rays (photons) move in curved space-time along null geodesics. The equations of null geodesics with the time coordinate as a parameter are given by (e.g. see [7])

$$\frac{dl^i}{dt} + \Gamma_{\alpha\beta}^i l^\alpha l^\beta = c^{-1} \Gamma_{\nu\sigma}^0 l^\nu l^\sigma l^i, \quad (4)$$

where

$$\Gamma_{\rho\sigma}^\mu = \frac{1}{2} g^{\mu\lambda} [g_{\rho\lambda,\sigma} + g_{\sigma\lambda,\rho} - g_{\rho\sigma,\lambda}] \quad (5)$$

are the Christoffel symbols of the second kind and $l^\mu = dz^\mu/dt$ denotes the 4-vector $l^\mu = (c, l^i)$. Here, it is important to recall that $z^\mu = (z^0, z^i)$, where $z^0 = ct$. Notice that l^μ is not exactly a 4-vector because we differentiate with respect to the time coordinate t . So l^μ is a 4-vector up to a

factor. The space component of l^μ given by $l^i = dz^i/dt$ is the 3-vector tangent to the light ray $z^i(t)$. In the present case of null geodesics, l^μ has to fulfill the condition

$$l^2 \equiv g_{\mu\nu}[z^0, z^i(t), G] l^\mu l^\nu = 0. \quad (6)$$

Now we consider a light ray $z^i(t)$ that is propagating in a curved space-time $g_{\mu\nu}[z^0, z^i(t), G]$ with the signature $(-+++)$. If the gravitational field is weak, we can write the fundamental metric tensor $g_{\mu\nu}[z^0, z^i(t), G]$ as a power series in the gravitational constant G

$$g_{\mu\nu}[z^0, z^i(t), G] \equiv \eta_{\mu\nu} + \sum_{n=1}^{\infty} h_{\mu\nu}^{(n)}[z^0, z^i(t), G], \quad (7)$$

where $\eta_{\mu\nu}$ is the Minkowski metric and $h_{\mu\nu}^{(n)}[z^0, z^i(t), G]$ is a perturbation of the order n in the gravitational constant G equivalent to $G^n g_{\mu\nu}^{(n)}(z^0, z^i(t))$ of Eq. (1) (physically, this means an expansion in the dimensionless parameter $Gm/c^2 d$ which is usually very small, d being the characteristic length of the problem and m a characteristic mass).

In order to obtain from Eq. (4) the equations of light propagation for the metric given in (7) we substitute the Christoffel symbols into the Eq. (4). To save writing we denote the metric coefficients $h_{pq}^{(1)}[z^0, z^i(t), G]$, $h_{pq}^{(2)}[z^0, z^i(t), G]$ by $h_{pq}^{(1)}$ and $h_{pq}^{(2)}$. Then the resulting equation of light propagation to the second order in G/c^2 is given by

$$\begin{aligned} \frac{dl^i}{dt} = & \frac{1}{2} c^2 h_{00,i}^{(1)} - c^2 h_{0i,0}^{(1)} - c h_{0i,m}^{(1)} l^m + c h_{0m,i}^{(1)} l^m - c h_{mi,0}^{(1)} l^m - h_{mi,n}^{(1)} l^m l^n + \frac{1}{2} h_{mn,i}^{(1)} l^m l^n - \frac{1}{2} c h_{00,0}^{(1)} l^i - h_{00,k}^{(1)} l^k l^i \\ & + \left(\frac{1}{2} c^{-1} h_{mp,0}^{(1)} - c^{-1} h_{op,m}^{(1)} \right) l^m l^p l^i + \frac{1}{2} c^2 h_{00,i}^{(2)} - \frac{1}{2} c^2 h^{(1)ik} h_{00,k}^{(1)} - h_{00,k}^{(2)} l^k l^i - \left(h_{mi,n}^{(2)} - \frac{1}{2} h_{mn,i}^{(2)} \right) l^m l^n \\ & + h^{(1)ik} \left(h_{mk,n}^{(1)} - \frac{1}{2} h_{mn,k}^{(1)} \right) l^m l^n - h_{00}^{(1)} h_{00,k}^{(1)} l^k l^i, \end{aligned} \quad (8)$$

where by $,0$ and $,i$ we denote $\partial/\partial z^0$ and $\partial/\partial z^i$ respectively. To calculate the light deflection we need to solve Eq. (8) for l^i . In order to solve this complicated, nonlinear differential equation, we turn to approximation techniques.

B. The approximation scheme

We can write the 3-vector $l^i(t)$ as

$$l^i(t) = l_{(0)}^i + \sum_{n=1}^{\infty} \delta l_{(n)}^i(t), \quad (9)$$

where $l_{(0)}^i$ denotes the constant incoming tangent vector $l^i(-\infty)$ and $\delta l_{(n)}^i(t)$ the perturbation of the constant tangent vector $l_{(0)}^i$ of order n in G equivalent to $G^n \delta \vec{l}_{(n)}^i(t)$. After

introducing the expression for $l^i(t)$ given by (9) into the Eq. (8), we obtain differential equations for the perturbations $\delta l_{(1)}^i$ and $\delta l_{(2)}^i$. These are given by:

$$\begin{aligned} \frac{d\delta l_{(1)}^i}{dt} = & \frac{1}{2} c^2 h_{00,i}^{(1)} - c^2 h_{0i,0}^{(1)} - c h_{0i,m}^{(1)} l_{(0)}^m + c h_{0m,i}^{(1)} l_{(0)}^m \\ & - c h_{mi,0}^{(1)} l_{(0)}^m - h_{mi,n}^{(1)} l_{(0)}^m l_{(0)}^n + \frac{1}{2} h_{mn,i}^{(1)} l_{(0)}^m l_{(0)}^n \\ & - \frac{1}{2} c h_{00,0}^{(1)} l_{(0)}^i - h_{00,k}^{(1)} l_{(0)}^k l_{(0)}^i \\ & + \left(\frac{1}{2} c^{-1} h_{mp,0}^{(1)} - c^{-1} h_{op,m}^{(1)} \right) l_{(0)}^m l_{(0)}^p l_{(0)}^i \end{aligned} \quad (10)$$

and

$$\begin{aligned}
\frac{d\delta l_{(2)}^i}{dt} = & \frac{1}{2}c^2 h_{00,i}^{(2)} - \frac{1}{2}c^2 h^{(1)ik} h_{00,k}^{(1)} - h_{00,k}^{(2)} l_{(0)}^k l_{(0)}^i - \left(h_{mi,n}^{(2)} - \frac{1}{2} h_{mn,i}^{(2)} \right) l_{(0)}^m l_{(0)}^n + h^{(1)ik} \left(h_{mk,n}^{(1)} - \frac{1}{2} h_{mn,k}^{(1)} \right) l_{(0)}^m l_{(0)}^n \\
& - h_{00}^{(1)} h_{00,k}^{(1)} l_{(0)}^k l_{(0)}^i - c h_{0i,m}^{(1)} \delta l_{(1)}^m + c h_{0m,i}^{(1)} \delta l_{(1)}^m - c h_{mi,0}^{(1)} \delta l_{(1)}^m - h_{mi,n}^{(1)} \delta l_{(1)}^m l_{(0)}^n - h_{mi,n}^{(1)} l_{(0)}^m \delta l_{(1)}^n + h_{mn,i}^{(1)} \delta l_{(1)}^m l_{(0)}^n \\
& - \frac{1}{2} c h_{00,0}^{(1)} \delta l_{(1)}^i - h_{00,k}^{(1)} \delta l_{(1)}^k l_{(0)}^i - h_{00,k}^{(1)} l_{(0)}^k \delta l_{(1)}^i + c^{-1} h_{mp,0}^{(1)} \delta l_{(1)}^m l_{(0)}^p l_{(0)}^i - c^{-1} h_{op,m}^{(1)} \delta l_{(1)}^m l_{(0)}^p l_{(0)}^i \\
& - c^{-1} h_{op,m}^{(1)} l_{(0)}^m \delta l_{(1)}^p l_{(0)}^i + \left(\frac{1}{2} c^{-1} h_{mp,0}^{(1)} - c^{-1} h_{op,m}^{(1)} \right) l_{(0)}^m l_{(0)}^p \delta l_{(1)}^i.
\end{aligned} \tag{11}$$

In order to calculate the perturbations $\delta l_{(1)}^i(t)$ and $\delta l_{(2)}^i(t)$ we have to integrate Eqs. (10) and (11) along the light ray trajectory to the appropriate order.

Before performing the integration it is convenient to introduce a new independent parameter τ along the photon's trajectory as defined by Kopeikin and Schäfer [5]. The relationship between the parameter τ and the time coordinate t is given by

$$\tau = t - t^*, \tag{12}$$

where t^* is the time of closest approach of the unperturbed trajectory of the photon to the origin in an asymptotically flat harmonic coordinate system. Then the equation of the unperturbed light ray can be represented as

$$z^i(\tau) = \tau l_{(0)}^i + \xi^i, \tag{13}$$

where ξ^i is a vector directed from the origin of the coordinate system towards the point of closest approach. The vector ξ^i is often called the impact parameter and is orthogonal to the vector $l_{(0)}^i$. The distance $r(\tau) = |\vec{z}(\tau)|$ of the photon from the origin of the coordinate system reads

$$r(\tau) = \sqrt{c^2 \tau^2 + \xi^2}. \tag{14}$$

It follows from Eq. (12) that the differential identity $dt = d\tau$ is valid, so that we can always replace the integration along the unperturbed light ray with respect to t by the integration with respect to the variable τ .

Then the resulting expression for $\delta l_{(1)}^i$ is given by

$$\begin{aligned}
\delta l_{(1)}^i(\tau) = & \frac{1}{2} \int_{-\infty}^{\tau} d\sigma l_{(0)}^\alpha l_{(0)}^\beta h_{\alpha\beta,i}^{(1)}|_{(\leftarrow)} - c h_{0i}^{(1)} - h_{mi}^{(1)} l_{(0)}^m - h_{00}^{(1)} l_{(0)}^i + \frac{1}{2} c \int_{-\infty}^{\tau} d\sigma h_{00,0}^{(1)} l_{(0)}^i|_{(\leftarrow)} \\
& + \int_{-\infty}^{\tau} d\sigma l_{(0)}^m l_{(0)}^p \left[\frac{1}{2} c^{-1} h_{mp,0}^{(1)} - c^{-1} h_{op,m}^{(1)} \right] l_{(0)}^i|_{(\leftarrow)}.
\end{aligned} \tag{15}$$

On the right-hand side of Eq. (15) after evaluating the partial derivatives of the metric coefficients with respect to the photon's coordinates [i.e. $(z^0, z^i(t))$], we replace in the integrals the photon trajectory by its unperturbed approximation $z^i(\sigma)_{\text{unpert.}} = \sigma l_{(0)}^i + \xi^i$ and the time coordinate z^0 by $\sigma + t^*$. In this paper we denote this operation by the symbol $|_{(\leftarrow)}$. Then we perform the integration with respect to σ . After substituting the expression obtained for $\delta l_{(1)}^i$ into Eq. (11) we can integrate it to get $\delta l_{(2)}^i$. To calculate the perturbation $\delta l_{(2)}^i$ we separate the part of $\delta l_{(2)}^i$ that depends on the postlinear metric coefficients from the part that depends on the linear metric coefficients. We denote these parts of $\delta l_{(2)}^i$ by $\delta l_{(2)\text{I}}^i$ and $\delta l_{(2)\text{II}}^i$ respectively. As in the case of Eq. (15) we replace the photon trajectory by its unperturbed approximation and the time coordinate z^0 by $\sigma + t^*$ after evaluating the partial derivatives of the metric coefficients with respect to the photon coordinates. The expressions for $\delta l_{(2)\text{I}}^i$ and $\delta l_{(2)\text{II}}^i$ are given by

$$\delta l_{(2)\text{I}}^i(\tau) = \int_{-\infty}^{\tau} d\sigma \left[\frac{1}{2} c^2 h_{00,i}^{(2)} - h_{00,k}^{(2)} l_{(0)}^k l_{(0)}^i \right] |_{(\leftarrow)} + \int_{-\infty}^{\tau} d\sigma \left[\frac{1}{2} h_{mn,i}^{(2)} - h_{mi,n}^{(2)} \right] l_{(0)}^m l_{(0)}^n |_{(\leftarrow)} \tag{16}$$

and

$$\begin{aligned}
 \delta l_{(2)\text{II}}^i(\tau) = & - \int_{-\infty}^{\tau} d\sigma \left[\frac{1}{2} c^2 h^{(1)ik} h_{00,k}^{(1)} + h_{00}^{(1)} h_{00,k}^{(1)} l_{(0)}^k \right] \Big|_{(\leftarrow)} + \int_{-\infty}^{\tau} d\sigma \left[h^{(1)ik} \left(h_{mk,n}^{(1)} - \frac{1}{2} h_{mn,k}^{(1)} \right) \right] l_{(0)}^m l_{(0)}^n \Big|_{(\leftarrow)} \\
 & + c \int_{-\infty}^{\tau} d\sigma [h_{0m,i}^{(1)} - h_{0i,m}^{(1)} - h_{mi,0}^{(1)}] \delta l_{(1)}^m(\sigma) \Big|_{(\leftarrow)} \\
 & + \int_{-\infty}^{\tau} d\sigma [h_{mn,i}^{(1)} \delta l_{(1)}^m(\sigma) l_{(0)}^n - h_{mi,n}^{(1)} \delta l_{(1)}^m(\sigma) l_{(0)}^n - h_{mi,n}^{(1)} l_{(0)}^m \delta l_{(1)}^n(\sigma)] \Big|_{(\leftarrow)} \\
 & - \int_{-\infty}^{\tau} d\sigma \left[\frac{1}{2} c h_{00,0}^{(1)} \delta l_{(1)}^i(\sigma) + h_{00,k}^{(1)} \delta l_{(1)}^k(\sigma) l_{(0)}^i + h_{00,k}^{(1)} l_{(0)}^k \delta l_{(1)}^i(\sigma) \right] \Big|_{(\leftarrow)} \\
 & + c^{-1} \int_{-\infty}^{\tau} d\sigma [h_{mp,0}^{(1)} \delta l_{(1)}^m(\sigma) l_{(0)}^p - h_{0p,m}^{(1)} \delta l_{(1)}^m(\sigma) l_{(0)}^p - h_{0p,m}^{(1)} l_{(0)}^m \delta l_{(1)}^p(\sigma)] l_{(0)}^i \Big|_{(\leftarrow)} \\
 & + c^{-1} \int_{-\infty}^{\tau} d\sigma \left[\frac{1}{2} h_{mp,0}^{(1)} - h_{0p,m}^{(1)} \right] l_{(0)}^m l_{(0)}^p \delta l_{(1)}^i(\sigma) \Big|_{(\leftarrow)}. \tag{17}
 \end{aligned}$$

C. Light deflection

The dimensionless vector $\alpha_{(n)}^i$ of order n in G describing the angle of total deflection of the light ray measured at the point of observation and calculated with respect to the vector $l_{(0)}^i$ (see [5]) is given by

$$\alpha_{(n)}^i(\tau) = P_q^i \frac{\delta l_{(n)}^q(\tau)}{|\vec{l}_{(0)}|}, \tag{18}$$

where $\delta l_{(n)}^i$ is the perturbation of the constant tangent vector of order n in G . Here, $P_q^i = \delta_q^i - e_{(0)}^i e_{(0)q}$ is the projection tensor onto the plane orthogonal to the vector $l_{(0)}^i$. In the case of light rays (photons) $|\vec{l}_{(0)}| = c$.

III. THE POSTLINEAR GRAVITATIONAL FIELD OF TWO BOUNDED POINTLIKE MASSES

In the computation of the metric generated by a system of two bounded pointlike masses we distinguish between 3 zones: the near zone, the intermediate zone and the wave zone or far zone. In Refs. [8,9] it was shown that leading order terms for the effect of light deflection in the case of small impact parameter depend neither on the radiative part ($\sim 1/\xi$) of the gravitational field nor on the intermediate ($\sim 1/\xi^2$) zone terms. The main effect rather comes from the near-zone ($\sim 1/\xi^3$) terms. Taking into account this property of strong suppression of the influence of gravitational waves on the light propagation, we can assume in the present work that the light deflection in the postlinear gravitational field of two pointlike masses is mainly determined by the near-zone metric.

A. The metric in the near zone

In Ref. [10], Blanchet *et al.* calculated the conservative 2PN harmonic-coordinate metric for the near zone of a system of two bounded pointlike masses as a function of the position \vec{z} and of the positions and velocities of the masses $\vec{x}_a(t)$ and $\vec{v}_a(t)$ respectively, with $a = 1, 2$. We shall use their metric in this paper. The postlinear metric for two

bounded pointlike masses, at the 2PN-order, is given by

$$\begin{aligned}
 h_{00}^{(2)} = & \frac{1}{c^4} \left\{ -2 \frac{G^2 m_1^2}{r_1^2} + G^2 m_1 m_2 \left(-\frac{2}{r_1 r_2} - \frac{r_1}{2r_{12}^3} + \frac{r_1^2}{2r_2 r_{12}^3} \right. \right. \\
 & \left. \left. - \frac{5}{2r_2 r_{12}} \right) \right\} + \frac{1}{c^4} (1 \leftrightarrow 2), \tag{19}
 \end{aligned}$$

$$\begin{aligned}
 h_{pq}^{(2)} = & + \frac{1}{c^4} \left\{ \delta^{pq} \left[\frac{G^2 m_1^2}{r_1^2} + G^2 m_1 m_2 \left(\frac{2}{r_1 r_2} - \frac{r_1}{2r_{12}^3} + \frac{r_1^2}{2r_2 r_{12}^3} \right. \right. \right. \\
 & \left. \left. - \frac{5}{2r_1 r_{12}} + \frac{4}{r_{12} S} \right) \right] + \frac{G^2 m_1^2}{r_1^2} n_1^p n_1^q \\
 & - 4G^2 m_1 m_2 n_{12}^p n_{12}^q \left(\frac{1}{S^2} + \frac{1}{r_{12} S} \right) \\
 & \left. + \frac{4G^2 m_1 m_2}{S^2} (n_1^p n_2^q + 2n_1^{(p} n_{12}^{q)}) \right\} + \frac{1}{c^4} (1 \leftrightarrow 2), \tag{20}
 \end{aligned}$$

where the symbol $(1 \leftrightarrow 2)$ refers to the preceding term in braces but with the labels 1 and 2 exchanged; by S we denote $S = r_1 + r_2 + r_{12}$, where $r_1 = |\vec{z} - \vec{x}_1(t)|$, $r_2 = |\vec{z} - \vec{x}_2(t)|$ and $r_{12} = |\vec{x}_1(t) - \vec{x}_2(t)|$. The vectors n_1^p , n_2^p and n_{12}^p are unit vectors defined by $n_1^p = r_1^p/r_1$, $n_2^p = r_2^p/r_2$ and $n_{12}^p = r_{12}^p/r_{12}$.

In our computations we also need a part of the linear gravitational field of two bounded pointlike masses. The part that is relevant to our calculation is given by

$$h_{00}^{(1)} = 2 \frac{G}{c^2} \sum_{a=1}^2 \frac{m_a}{r_a} + \frac{G}{c^4} \sum_{a=1}^2 \frac{m_a}{r_a} [-(\vec{n}_a \cdot \vec{v}_a)^2 + 4v_a^2], \tag{21}$$

$$h_{0p}^{(1)} = -4 \frac{G}{c^3} \sum_{a=1}^2 \frac{m_a}{r_a} v_a^p, \tag{22}$$

$$h_{pq}^{(1)} = 2 \frac{G}{c^2} \sum_{a=1}^2 \frac{m_a}{r_a} \delta^{pq} + \frac{G}{c^4} \sum_{a=1}^2 \frac{m_a}{r_a} [-(\vec{n}_a \cdot \vec{v}_a)^2 \delta^{pq} + 4v_a^p v_a^q], \quad (23)$$

where v_a^p denotes the velocity of the mass m_a and n_a^p is the unit vector defined by $n_a^p = r_a^p/r_a$.

Here, it is worthwhile to point out that the parts of the linear gravitational field in $h_{00}^{(1)}$ and $h_{pq}^{(1)}$ that contain the accelerations of the masses were introduced into the part of the gravitational field quadratic in G after substituting the accelerations by explicit functions of the coordinate positions of the masses by means of the Newtonian equations of motion.

B. The barycentric coordinate system

We use a harmonic coordinate system, the origin of which coincides with the 1PN-center of mass. Using the 1PN-accurate center of mass theorem of Ref. [11], we can express the individual center-of-mass frame positions of the two masses in terms of the relative position $\vec{r}_{12} \equiv \vec{x}_1 - \vec{x}_2$ and the relative velocity $\vec{v}_{12} \equiv \vec{v}_1 - \vec{v}_2$ as

$$\vec{x}_1 = \left[X_2 + \frac{1}{c^2} \epsilon_{1\text{PN}} \right] \vec{r}_{12}, \quad (24)$$

$$\vec{x}_2 = \left[-X_1 + \frac{1}{c^2} \epsilon_{1\text{PN}} \right] \vec{r}_{12}, \quad (25)$$

where X_1 , X_2 and $\epsilon_{1\text{PN}}$ are given by

$$X_1 \equiv \frac{m_1}{M}, \quad (26)$$

$$X_2 \equiv \frac{m_2}{M}, \quad (27)$$

$$\epsilon_{1\text{PN}} = \frac{\nu(m_1 - m_2)}{2M} \left[v_{12}^2 - \frac{GM}{r_{12}} \right]. \quad (28)$$

Here, we have introduced

$$M \equiv m_1 + m_2, \quad v_{12} = |\vec{v}_{12}| \quad (29)$$

and

$$\nu \equiv \frac{m_1 m_2}{M^2}. \quad (30)$$

It is important to remark that in our computation of the postlinear light deflection up to the order G^2/c^4 , we need only to consider the 1PN-corrections to the Newtonian center of mass, because, as we shall see in Sec. VIII, the 2PN-corrections to the Newtonian center of mass are related to postlinear light deflection terms of order higher than G^2/c^4 .

IV. LIGHT DEFLECTION IN THE LINEAR GRAVITATIONAL FIELD OF TWO BOUNDED POINTLIKE MASSES

In this section we compute the perturbation term $\delta l_{(1)}^i(\tau)$ and the corresponding angle of light deflection for an observer situated at infinity in an asymptotically flat space. To compute the perturbation term $\delta l_{(1)}^i(\tau)$ we have to introduce the linear metric coefficients given by Eqs. (21)–(23) into Eq. (15). After introducing the linear metric coefficients into Eq. (15), we find

$$\begin{aligned} \delta l_{(1)}^i(\tau) = & -2G \sum_{a=1}^2 m_a \int_{-\infty}^{\tau} d\sigma \frac{1}{r_a^3} [z^i - x_a^i(t)]|_{(\leftarrow)} + 4 \frac{G}{c} \sum_{a=1}^2 m_a \int_{-\infty}^{\tau} d\sigma \frac{1}{r_a^3} (\vec{e}_{(0)} \cdot \vec{v}_a(t)) [z^i - x_a^i(t)]|_{(\leftarrow)} \\ & - 2 \frac{G}{c^2} \sum_{a=1}^2 m_a \int_{-\infty}^{\tau} d\sigma \frac{1}{r_a^3} [v_a^2(t) + (\vec{e}_{(0)} \cdot \vec{v}_a(t))^2] [z^i - x_a^i(t)]|_{(\leftarrow)} \\ & + 3 \frac{G}{c^2} \sum_{a=1}^2 m_a \int_{-\infty}^{\tau} d\sigma \frac{1}{r_a^3} (\vec{r}_a \cdot \vec{v}_a(t))^2 [z^i - x_a^i(t)]|_{(\leftarrow)} \\ & - 4 \frac{G}{c^2} \sum_{a=1}^2 m_a \int_{-\infty}^{\tau} d\sigma \frac{1}{r_a^3} (\vec{e}_{(0)} \cdot \vec{v}_a(t)) [c\sigma - (\vec{e}_{(0)} \cdot \vec{x}_a(t))] l_{(0)}^i|_{(\leftarrow)} \\ & - 2 \frac{G}{c^2} \sum_{a=1}^2 m_a \int_{-\infty}^{\tau} d\sigma \frac{1}{r_a^3} \left\{ (\vec{e}_{(0)} \cdot \vec{v}_a(t)) c\sigma + [\vec{\xi} \cdot \vec{v}_a(t) - \vec{x}_a(t) \cdot \vec{v}_a(t)] v_a^i(t) \right\} |_{(\leftarrow)} + 4 \frac{G}{c^2} \sum_{a=1}^2 \frac{m_a}{r_a} v_a^i(t) \\ & - 4 \frac{G}{c^2} \sum_{a=1}^2 \frac{m_a}{r_a} l_{(0)}^i - 4 \frac{G}{c^3} \sum_{a=1}^2 \frac{m_a}{r_a} (\vec{e}_{(0)} \cdot \vec{v}_a(t)) v_a^i(t) + 2 \frac{G}{c^4} \sum_{a=1}^2 m_a \left\{ \frac{(\vec{r}_a \cdot \vec{v}_a(t))^2}{r_a^3} - 2 \frac{1}{r_a} v_a^2(t) \right\} l_{(0)}^i, \quad (31) \end{aligned}$$

where $\vec{r}_a = \vec{z} - \vec{x}_a(t)$ and $r_a = |\vec{r}_a|$.

Because the linear metric coefficients are functions of the positions and velocities of the masses $\vec{x}_a(t)$ and $\vec{v}_a(t)$, respectively, the expression for $\delta l_{(1)}^i(\tau)$ given in Eq. (31) is a function of these quantities. This means that we

have to take into account the motion of the masses when we are going to compute the integrals in Eq. (31). Considering that the influence of the gravitational field on the light propagation is strongest near the barycenter of the binary and that the velocities of the masses are small with respect to the velocity of light, we are allowed to make the following approximations:

- (1) We may assume that the linear gravitational field is determined by the positions and velocities of the masses taken at the time of closest approach ($t = t^*$) of the unperturbed light ray to the barycenter of the binary (i.e. to the origin of the asymptotically flat

harmonic coordinate system). The expression, resulting from Eq. (31) after setting $t = t^*$ for the positions and velocities and computing the integrals, is denoted by $\delta l_{(1)I}^i(\tau)$;

- (2) We treat the effect of the motion of the masses on light propagation as a correction to the expression of $\delta l_{(1)I}^i(\tau)$, which we denote by $\delta l_{(1)II}^i(\tau)$. We shall compute this correction in Sec. VI.

After fixing the values of the quantities $\vec{x}_a(t)$ and $\vec{v}_a(t)$ to $\vec{x}_a(t^*)$ and $\vec{v}_a(t^*)$ and evaluating the integrals in Eq. (31), we find

$$\begin{aligned}
 \delta l_{(1)I}^i(\tau) = & -2\frac{G}{c} \sum_{a=1}^2 m_a B_a [\xi^i - x_a^i(t^*)] - \frac{G}{c^2} \sum_{a=1}^2 m_a \left\{ 2A_a + \frac{4}{r_a} \right\} l_{(0)}^i + 4\frac{G}{c^2} \sum_{a=1}^2 m_a (\vec{e}_{(0)} \cdot \vec{v}_a(t^*)) B_a [\xi^i - x_a^i(t^*)] \\
 & + 4\frac{G}{c^2} \sum_{a=1}^2 \frac{m_a}{r_a} v_a^i(t^*) + 4\frac{G}{c^3} \sum_{a=1}^2 m_a (\vec{e}_{(0)} \cdot \vec{v}_a(t^*)) \left\{ A_a + \frac{1}{r_a} \right\} l_{(0)}^i \\
 & + \frac{G}{c^3} \sum_{a=1}^2 m_a (\vec{e}_{(0)} \cdot \vec{v}_a(t^*))^2 \{ 3F_{a2} - 2B_a \} [\xi^i - x_a^i(t^*)] \\
 & + 6\frac{G}{c^3} \sum_{a=1}^2 m_a (\vec{e}_{(0)} \cdot \vec{v}_a(t^*)) [\vec{\xi} \cdot \vec{v}_a(t^*) - \vec{x}_a(t^*) \cdot \vec{v}_a(t^*)] F_{a3} [\xi^i - x_a^i(t^*)] \\
 & - 2\frac{G}{c^3} \sum_{a=1}^2 m_a v_a^2(t^*) B_a [\xi^i - x_a^i(t^*)] + 3\frac{G}{c^3} \sum_{a=1}^2 m_a [\vec{\xi} \cdot \vec{v}_a(t^*) - \vec{x}_a(t^*) \cdot \vec{v}_a(t^*)]^2 F_{a4} [\xi^i - x_a^i(t^*)] \\
 & - \frac{G}{c^3} \sum_{a=1}^2 m_a \left\{ (\vec{e}_{(0)} \cdot \vec{v}_a(t^*)) \left[2A_a + \frac{4}{r_a} \right] + 2[\vec{\xi} \cdot \vec{v}_a(t^*) - \vec{x}_a(t^*) \cdot \vec{v}_a(t^*)] B_a \right\} v_a^i(t^*) \\
 & + 6\frac{G}{c^4} \sum_{a=1}^2 m_a (\vec{e}_{(0)} \cdot \vec{v}_a(t^*)) [\vec{\xi} \cdot \vec{v}_a(t^*) - \vec{x}_a(t^*) \cdot \vec{v}_a(t^*)] F_{a2} l_{(0)}^i + \frac{G}{c^4} \sum_{a=1}^2 m_a (\vec{e}_{(0)} \cdot \vec{v}_a(t^*))^2 \{ 3F_{a1} - 2A_a \} l_{(0)}^i \\
 & - \frac{G}{c^4} \sum_{a=1}^2 m_a v_a^2(t^*) \left\{ 2A_a + \frac{4}{r_a} \right\} l_{(0)}^i + 2\frac{G}{c^4} \sum_{a=1}^2 \frac{m_a}{r_a^3} (\vec{r} \cdot \vec{v}_a(t^*))^2 l_{(0)}^i \\
 & + 3\frac{G}{c^4} \sum_{a=1}^2 m_a [\vec{\xi} \cdot \vec{v}_a(t^*) - \vec{x}_a(t^*) \cdot \vec{v}_a(t^*)]^2 F_{a3} l_{(0)}^i, \tag{32}
 \end{aligned}$$

where the functions A_a , B_a , F_{a1} , F_{a2} , F_{a3} , and F_{a4} are given by

$$A_a = \frac{1}{r_a R_a} [-r_a^2(0, t^*) + (\vec{e}_{(0)} \cdot \vec{x}_a(t^*)) (c\tau + r_a)], \tag{33}$$

$$B_a = \frac{1}{r_a R_a} [-(\vec{e}_{(0)} \cdot \vec{x}_a(t^*)) + c\tau + r_a], \tag{34}$$

$$\begin{aligned}
 F_{a1} = & \frac{1}{3r_a^3 R_a^2} \{ -r_a^2(0, t^*) [2r_a^4(0, t^*) - (\vec{e}_{(0)} \cdot \vec{x}_a(t^*)) (3r_a^2(0, t^*) - (\vec{e}_{(0)} \cdot \vec{x}_a(t^*))^2) r_a] \\
 & + 2(\vec{e}_{(0)} \cdot \vec{x}_a(t^*)) [3r^4(0, t^*) - (\vec{e}_{(0)} \cdot \vec{x}_a(t^*)) (3r_a^2(0, t^*) - (\vec{e}_{(0)} \cdot \vec{x}_a(t^*))^2) r_a] c\tau \\
 & - [3r_a^4(0, t^*) + 3(\vec{e}_{(0)} \cdot \vec{x}_a(t^*))^2 r_a^2(0, t^*) - (\vec{e}_{(0)} \cdot \vec{x}_a(t^*)) (3r_a^2(0, t^*) - (\vec{e}_{(0)} \cdot \vec{x}_a(t^*))^2) r_a] c^2 \tau^2 \\
 & + (\vec{e}_{(0)} \cdot \vec{x}_a(t^*)) [3r_a^2(0, t^*) - (\vec{e}_{(0)} \cdot \vec{x}_a(t^*))^2] c^3 \tau^3 \}, \tag{35}
 \end{aligned}$$

$$\begin{aligned}
F_{a2} = & \frac{1}{3r_a^3 R_a^2} \{-2(\vec{e}_{(0)} \cdot \vec{x}_a(t^*))r_a^4(0, t^*) + [r_a^2(0, t^*) + (\vec{e}_{(0)} \cdot \vec{x}_a(t^*))^2]r_a^2(0, t^*)r_a \\
& - 2(\vec{e}_{(0)} \cdot \vec{x}_a(t^*))[(r_a^2(0, t^*) + (\vec{e}_{(0)} \cdot \vec{x}_a(t^*))^2)r_a - 3(\vec{e}_{(0)} \cdot \vec{x}_a(t^*))r_a^2(0, t^*)]c\tau \\
& + [r_a^2(0, t^*) + (\vec{e}_{(0)} \cdot \vec{x}_a(t^*))^2][r_a - 3(\vec{e}_{(0)} \cdot \vec{x}_a(t^*))]c^2\tau^2 + [r_a^2(0, t^*) + (\vec{e}_{(0)} \cdot \vec{x}_a(t^*))^2]c^3\tau^3\}, \quad (36)
\end{aligned}$$

$$\begin{aligned}
F_{a3} = & \frac{1}{3r_a^3 R_a^2} \{-r_a^2(0, t^*)[r_a^2(0, t^*) + (\vec{e}_{(0)} \cdot \vec{x}_a(t^*))^2 - 2(\vec{e}_{(0)} \cdot \vec{x}_a(t^*))r_a] \\
& + (\vec{e}_{(0)} \cdot \vec{x}_a(t^*))[3r_a^2(0, t^*) + 3(\vec{e}_{(0)} \cdot \vec{x}_a(t^*))^2 - 4(\vec{e}_{(0)} \cdot \vec{x}_a(t^*))r_a]c\tau + 2(\vec{e}_{(0)} \cdot \vec{x}_a(t^*))r_a - 3(\vec{e}_{(0)} \cdot \vec{x}_a(t^*))c^2\tau^2 \\
& + 2(\vec{e}_{(0)} \cdot \vec{x}_a(t^*))c^3\tau^3\}, \quad (37)
\end{aligned}$$

$$\begin{aligned}
F_{a4} = & \frac{1}{3r_a^3 R_a^2} \{(\vec{e}_{(0)} \cdot \vec{x}_a(t^*))[-3r_a^2(0, t^*) + (\vec{e}_{(0)} \cdot \vec{x}_a(t^*))^2] + 2r_a^2(0, t^*)r_a \\
& + [3r_a^2(0, t^*) + 3(\vec{e}_{(0)} \cdot \vec{x}_a(t^*))^2 - 4(\vec{e}_{(0)} \cdot \vec{x}_a(t^*))r_a]c\tau + 2[r_a - 3(\vec{e}_{(0)} \cdot \vec{x}_a(t^*))]c^2\tau^2 + 2c^3\tau^3\}. \quad (38)
\end{aligned}$$

Here, the suffix a labels the masses and r_a is the distance between the position of the photon along its unperturbed trajectory and the position of the mass m_a at the time t^* . Explicitly the distance r_a is given by

$$r_a = r_a(\tau, t^*) = [c^2\tau^2 + \xi^2 - 2c\tau\vec{e}_{(0)} \cdot \vec{x}_a(t^*) - 2\vec{\xi} \cdot \vec{x}_a(t^*) + x_a^2(t^*)]^{1/2}. \quad (39)$$

It follows from the expression for r_a that $r_a(0, t^*)$ is the distance between the point of closest approach of the unperturbed light ray to the origin of the coordinate system and the position of the mass m_a at the time t^* . The quantity R_a appearing in Eqs. (33)–(38) is given by

$$R_a = r_a^2(0, t^*) - (\vec{e}_{(0)} \cdot \vec{x}_a(t^*))^2. \quad (40)$$

To get the expression for the angle of light deflection we have to introduce $\delta l_{(1)I}^i(\tau)$ into Eq. (18) and compute the limit $\tau \rightarrow \infty$. We have to compute the limit $\tau \rightarrow \infty$, because we assume that the observer is located at infinity in an asymptotically flat space.

After introducing the perturbation $\delta l_{(1)I}^i(\tau)$ into Eq. (18) and computing the limit for $\tau \rightarrow \infty$, we find

$$\begin{aligned}
\alpha_{(1)I}^i = & \lim_{\tau \rightarrow \infty} \left[\frac{1}{c} P_q^i \delta l_{(1)I}^q(\tau) \right] \\
= & -4 \frac{G}{c^2} \sum_{a=1}^2 \frac{m_a}{R_a} [\xi^i - P_q^i x_a^q(t^*)] + 8 \frac{G}{c^3} \sum_{a=1}^2 \frac{m_a}{R_a} (\vec{e}_{(0)} \cdot \vec{v}_a(t^*)) [\xi^i - P_q^i x_a^q(t^*)] - 4 \frac{G}{c^4} \sum_{a=1}^2 \frac{m_a}{R_a} v_a^2(t^*) [\xi^i - P_q^i x_a^q(t^*)] \\
& - 4 \frac{G}{c^4} \sum_{a=1}^2 \frac{m_a}{R_a} (\vec{e}_{(0)} \cdot \vec{v}_a(t^*))^2 [\xi^i - P_q^i x_a^q(t^*)] - 4 \frac{G}{c^4} \sum_{a=1}^2 \frac{m_a}{R_a} \{(\vec{e}_{(0)} \cdot \vec{x}_a(t^*))(\vec{e}_{(0)} \cdot \vec{v}_a(t^*)) \\
& + \vec{\xi} \cdot \vec{v}_a(t^*) - \vec{x}_a(t^*) \cdot \vec{v}_a(t^*)\} P_q^i v_a^q(t^*) + 2 \frac{G}{c^4} \sum_{a=1}^2 \frac{m_a}{R_a} (\vec{e}_{(0)} \cdot \vec{v}_a(t^*))^2 [\xi^i - P_q^i x_a^q(t^*)] \\
& + 4 \frac{G}{c^4} \sum_{a=1}^2 \frac{m_a}{R_a^2} (\vec{e}_{(0)} \cdot \vec{v}_a(t^*))^2 (\vec{e}_{(0)} \cdot \vec{x}_a(t^*))^2 [\xi^i - P_q^i x_a^q(t^*)] \\
& + 8 \frac{G}{c^4} \sum_{a=1}^2 \frac{m_a}{R_a^2} (\vec{e}_{(0)} \cdot \vec{x}_a(t^*)) (\vec{e}_{(0)} \cdot \vec{v}_a(t^*)) [\vec{\xi} \cdot \vec{v}_a(t^*) - \vec{x}_a(t^*) \cdot \vec{v}_a(t^*)] [\xi^i - P_q^i x_a^q(t^*)] \\
& + 4 \frac{G}{c^4} \sum_{a=1}^2 \frac{m_a}{R_a^2} [(\vec{\xi} \cdot \vec{v}_a(t^*))^2 - 2(\vec{\xi} \cdot \vec{v}_a(t^*))(\vec{x}_a(t^*) \cdot \vec{v}_a(t^*)) + (\vec{x}_a(t^*) \cdot \vec{v}_a(t^*))^2] [\xi^i - P_q^i x_a^q(t^*)]. \quad (41)
\end{aligned}$$

V. THE POSTLINEAR LIGHT DEFLECTION IN THE POSTLINEAR GRAVITATIONAL FIELD OF TWO BOUNDED POINTLIKE MASSES

In this section we present our computations for light deflection in the postlinear gravitational field of two bounded pointlike masses. In our computations we assume that both the light source and the observer are at infinity in an asymptotically flat space so that the effects of $h_{\mu\nu}^{(1)}$ and $h_{\mu\nu}^{(2)}$ near the light source and near the observer are negligible. We take into account only the terms of the order G^2/c^4 . From Eqs. (16)–(18) we see that $\alpha_{(2)}^i$ is a function of the postlinear metric coefficients $h_{\mu\nu}^{(2)}$ and of the linear metric coefficients $h_{\mu\nu}^{(1)}$. To facilitate the computations we separate the light deflection terms that are functions of the postlinear metric coefficients from the terms that are functions of the linear metric coefficients and the perturbations of the first order in G of the vector tangent to the unperturbed light ray. First we compute the terms of $\alpha_{(2)}^i$ that are functions of the postlinear metric coefficients, which we denote by $\alpha_{(2)I}^i$.

A. The postlinear light deflection terms that depend on the metric coefficients quadratic in G

It follows from Eqs. (16) and (18) that a part of the postlinear light deflection is given by:

$$\alpha_{(2)I}^i = \frac{1}{c} P_q^i \left[\frac{1}{2} c^2 \int_{-\infty}^{\infty} d\tau h_{00,q}^{(2)} |_{(\leftarrow)} + \int_{-\infty}^{\infty} d\tau \left[\frac{1}{2} h_{mn,q}^{(2)} - h_{qm,n}^{(2)} \right] l_{(0)}^m l_{(0)}^n |_{(\leftarrow)} \right]. \quad (42)$$

Upon introducing the postlinear metric (19) and (20) into (42) we obtain integrals whose integrands are functions of the distances r_1 , r_2 , S and their inverses. Through the distances r_1 , r_2 and S , the resulting integrals from Eq. (42) are functions of the positions of the masses $\vec{x}_a(t)$.

For the same reason as in the case of the linear light deflection we are here allowed to fix the values of the

positions of the masses $\vec{x}_a(t)$ to their values at the time t^* before performing the integration. The resulting integrals are given explicitly in Appendix A.

To evaluate the integrals that cannot be represented by elementary functions we resort as usual to a series expansion of the integrands. To perform the series expansion we consider the integrands as functions of the distances r_1 , r_2 and S . Then, we expand these functions in a Taylor series about the origin of the coordinate system $\vec{x}_1 = \vec{x}_2 = 0$ to the second order. We need only to perform the Taylor expansion up to second order, since with an expansion to this order we obtain a result which is sufficiently accurate for the applications that we shall consider in this paper.

The positions of the masses in the center-of-mass frame defined in subsection III B are given by Eqs. (24) and (25). Here, we do not need to take into account the 1PN-corrections in the positions of the masses, because if we introduce these into Eq. (42) we will obtain terms of higher order than G^2/c^4 .

In Sec. VIII we shall compute the postlinear light deflection terms resulting from the introduction of the 1PN-corrections in the positions of the masses into the equation for the linear light deflection.

Also, we do not need to consider here the correction terms arising from introducing the motions of the masses into Eq. (42), because these terms are of higher order than G^2/c^4 . The correction to the postlinear light deflection arising from introducing the motion of the masses into the expression for the linear perturbation is denoted by $\alpha_{(2)III}^i$ and we shall compute it in Sec. VI.

B. The postlinear light deflection terms that are dependent on the metric coefficients linear in G

The postlinear light deflection terms which are functions of the linear metric coefficients and the linear perturbations $\delta \vec{l}_{(1)}(\tau)$ we denote by $\alpha_{(2)II}^i$. It follows from Eqs. (15) and (18) that the resulting expression for the postlinear light deflection $\alpha_{(2)II}^i$ is given by

$$\alpha_{(2)II}^i = \frac{1}{c} P_q^i \left[-\frac{1}{2} c^2 \int_{-\infty}^{\infty} d\tau h^{(1)qm} h_{00,m}^{(1)} |_{(\leftarrow)} + \int_{-\infty}^{\infty} d\tau \left[h^{(1)qp} \left(h_{mp,n}^{(1)} - \frac{1}{2} h_{mn,p}^{(1)} \right) \right] l_{(0)}^m l_{(0)}^n |_{(\leftarrow)} + c \int_{-\infty}^{\infty} d\tau [h_{0m,q}^{(1)} - h_{0q,m}^{(1)}] \delta l_{(1)}^m(\tau) |_{(\leftarrow)} + \int_{-\infty}^{\infty} d\tau [h_{mn,q}^{(1)} \delta l_{(1)}^m(\tau) l_{(0)}^n - h_{mq,n}^{(1)} \delta l_{(1)}^m(\tau) l_{(0)}^m - h_{mq,n}^{(1)} l_{(0)}^m \delta l_{(1)}^n(\tau)] |_{(\leftarrow)} - \int_{-\infty}^{\infty} d\tau h_{00,k}^{(1)} l_{(0)}^k \delta l_{(1)}^q(\tau) |_{(\leftarrow)} - \frac{1}{c} \int_{-\infty}^{\infty} d\tau h_{0p,m}^{(1)} l_{(0)}^m l_{(0)}^p \delta l_{(1)}^q(\tau) |_{(\leftarrow)} \right]. \quad (43)$$

To compute $\alpha_{(2)II}^i$, we introduce the expressions for the perturbations $\delta l_{(1)}^i(\tau)$ given by Eq. (63) and the metric functions (21), (23), and (22) into the expression for $\alpha_{(2)II}^i$. Here, we may use the same approximations as before, i.e. we can fix the values of the positions and velocities of the masses to their values at the time t^* before performing the integrals. The resulting integrals are given in Appendix B. As explained in the preceding section, with the help of a Taylor expansion of the integrands we can evaluate the integrals, which cannot be represented by elementary functions.

VI. LIGHT DEFLECTION AND THE MOTION OF THE MASSES

In this section we compute the correction terms to the linear and postlinear light deflection arising from the effect of the motion of the masses on light propagation. The correction terms to the linear and postlinear light deflection can be found by means of Taylor expansions of the linear perturbation (31) in which the coefficients depend on the sources' coordinates x_a^i and their successive derivatives with regard to t , namely

$$\frac{dx_a^i}{dt} = v_a^i(t), \frac{d^2x_a^i}{dt^2} = \frac{dv_a^i}{dt} = a_a^i(t), \dots,$$

taken at the time t^* .

A. The linear light deflection and the motion of the masses

The correction terms to the linear perturbation arising from the Taylor expansion of Eq. (31) are given by

$$\begin{aligned} \delta l_{(1)\Pi}^i(\tau) = & G \sum_{a=1}^2 m_a \int_{-\infty}^{\tau} d\sigma \left\{ \left[-\frac{6}{r_a^5} (\vec{r}_a \cdot \vec{v}_a(t^*)) r_a^i + \frac{2}{r_a^3} v_a^i(t^*) \right] \sigma \right. \\ & + \left. \left[-\frac{15}{r_a^7} (\vec{r}_a \cdot \vec{v}_a(t^*))^2 r_a^i + \frac{6}{r_a^5} (\vec{r}_a \cdot \vec{v}_a(t^*)) v_a^i(t^*) + \frac{3}{r_a^5} v_a^2(t^*) r_a^i \right] \sigma^2 \right\} |_{(\leftarrow)} \\ & + \frac{G}{c} \sum_{a=1}^2 m_a (\vec{e}_{(0)} \cdot \vec{v}_a(t^*)) \int_{-\infty}^{\tau} d\sigma \left\{ \left[\frac{12}{r_a^5} (\vec{r}_a \cdot \vec{v}_a(t^*)) r_a^i - \frac{4}{r_a^3} v_a^i(t^*) \right] \sigma \right. \\ & + \left. \left[\frac{30}{r_a^7} (\vec{r}_a \cdot \vec{v}_a(t^*))^2 r_a^i - \frac{12}{r_a^5} (\vec{r}_a \cdot \vec{v}_a(t^*)) v_a^i(t^*) - \frac{6}{r_a^5} v_a^2(t^*) r_a^i \right] \sigma^2 \right\} |_{(\leftarrow)} \\ & + \frac{G}{c^2} \sum_{a=1}^2 m_a \int_{-\infty}^{\tau} d\sigma \left\{ [v^2(t^*) + (\vec{e}_{(0)} \cdot \vec{v}(t^*))^2] \left[\left(-\frac{6}{r_a^5} (\vec{r}_a \cdot \vec{v}_a(t^*)) r_a^i + \frac{2}{r_a^3} v_a^i(t^*) \right) \sigma \right. \right. \\ & + \left. \left. \left(-\frac{15}{r_a^7} (\vec{r}_a \cdot \vec{v}_a(t^*))^2 r_a^i + \frac{6}{r_a^5} (\vec{r}_a \cdot \vec{v}_a(t^*)) v_a^i(t^*) + \frac{3}{r_a^5} v_a^2(t^*) r_a^i \right) \sigma^2 \right] \right. \\ & + \left. \left[\left(\frac{15}{r_a^7} (\vec{r}_a \cdot \vec{v}_a(t^*))^3 r_a^i - \frac{3}{r_a^5} (\vec{r}_a \cdot \vec{v}_a(t^*))^2 v_a^i(t^*) - \frac{6}{r_a^5} (\vec{r}_a \cdot \vec{v}_a(t^*)) v_a^2(t^*) r_a^i \right) \sigma \right. \right. \\ & + \left. \left. \left(\frac{105}{2} \frac{(\vec{r}_a \cdot \vec{v}_a(t^*))^4}{r_a^9} r_a^i - \frac{15}{r_a^7} (\vec{r}_a \cdot \vec{v}_a(t^*))^3 v_a^i(t^*) - \frac{75}{2} \frac{(\vec{r}_a \cdot \vec{v}_a(t^*))^2}{r_a^7} v_a^2(t^*) r_a^i + \frac{6}{r_a^5} (\vec{r}_a \cdot \vec{v}_a(t^*)) v_a^2(t^*) v_a^i(t^*) \right. \right. \\ & + \left. \left. \frac{3}{r_a^5} v_a^4(t^*) r_a^i \right) \sigma^2 \right] - (\vec{e}_{(0)} \cdot \vec{v}_a(t^*)) [c\sigma - (\vec{e}_{(0)} \cdot \vec{x}_a(t^*))] \\ & \times \left[\frac{12}{r_a^5} (\vec{r}_a \cdot \vec{v}_a(t^*)) \sigma + \left(\frac{30}{r_a^7} (\vec{r}_a \cdot \vec{v}_a(t^*))^2 - \frac{6}{r_a^5} v_a^2(t^*) \right) \sigma^2 \right] l_{(0)}^i + (\vec{e}_{(0)} \cdot \vec{v}_a(t^*))^2 \left[\frac{4}{r_a^3} \sigma + \frac{12}{r_a^5} (\vec{r}_a \cdot \vec{v}_a(t^*)) \sigma^2 \right] l_{(0)}^i \\ & + [(\vec{e}_{(0)} \cdot \vec{v}_a(t^*)) c\sigma + (\vec{\xi} \cdot \vec{v}_a(t^*) - \vec{x}_a(t^*) \cdot \vec{v}_a(t^*))] \\ & \times \left[-\frac{6}{r_a^5} (\vec{r}_a \cdot \vec{v}_a(t^*)) \sigma + \left(-\frac{15}{r_a^7} (\vec{r}_a \cdot \vec{v}_a(t^*))^2 + \frac{3}{r_a^5} v_a^2(t^*) \right) \sigma^2 \right] v_a^i(t^*) \\ & + v_a^2(t^*) \left[\frac{2}{r_a^3} \sigma + \frac{6}{r_a^5} (\vec{r}_a \cdot \vec{v}_a(t^*)) \sigma^2 \right] v_a^i(t^*) \Big\} |_{(\leftarrow)} \\ & + \frac{G}{c^2} \sum_{a=1}^2 m_a \left\{ \frac{4}{r_a^3} (\vec{r}_a \cdot \vec{v}_a(t^*)) \tau + \left[\frac{6}{r_a^5} (\vec{r}_a \cdot \vec{v}_a(t^*))^2 - \frac{2}{r_a^3} v_a^2(t^*) \right] \tau^2 \right\} [v_a^i(t^*) - l_{(0)}^i] \\ & + \frac{G}{c^3} \sum_{a=1}^2 m_a (\vec{e}_{(0)} \cdot \vec{v}_a(t^*)) \left\{ -\frac{4}{r_a^3} (\vec{r}_a \cdot \vec{v}_a(t^*)) \tau + \left[-\frac{6}{r_a^5} (\vec{r}_a \cdot \vec{v}_a(t^*))^2 + \frac{2}{r_a^3} v_a^2(t^*) \right] \tau^2 \right\} v_a^i(t^*) \\ & + \frac{G}{c^4} \sum_{a=1}^2 m_a \left\{ \left[\frac{6}{r_a^5} (\vec{r}_a \cdot \vec{v}_a(t^*))^3 - \frac{4}{r_a^3} (\vec{r}_a \cdot \vec{v}_a(t^*)) v_a^2(t^*) \right] \tau + \left[\frac{15}{r_a^7} (\vec{r}_a \cdot \vec{v}_a(t^*))^4 - \frac{15}{r_a^5} (\vec{r}_a \cdot \vec{v}_a(t^*))^2 v_a^2(t^*) + \frac{2}{r_a^3} v_a^4(t^*) \right] \tau^2 \right. \\ & \left. - 4v_a^2(t^*) \left[\frac{(\vec{r}_a \cdot \vec{v}_a(t^*))}{r_a^3} \tau + \left(\frac{3}{2} \frac{(\vec{r}_a \cdot \vec{v}_a(t^*))^2}{r_a^5} - \frac{1}{2} \frac{v_a^2(t^*)}{r_a^3} \right) \tau^2 \right] \right\} l_{(0)}^i \end{aligned} \quad (44)$$

Considering that in the present work we compute the postlinear light deflection up to the order G^2/c^4 , we must retain the linear light deflection terms up to the order G/c^4 . Notice that the linear terms of the order G/c^4 are of the same order as the postlinear terms of the order G^2/c^4 since for a system of bounded pointlike masses, the virial theorem applies (i.e. $v_a^2 \sim G/d$) and, considering that the terms of the order G/c^4 are also terms in v_a^2 , it is easy to see that these terms are of the same order as the postlinear terms of the order G^2/c^4 . Linear light deflection terms of course we get from the perturbation $\delta l_{(1)\Pi}^i(\tau)$ too. To obtain the perturbation $\delta l_{(1)\Pi}^i(\tau)$ we have to evaluate the integrals in the expression above. Taking into account that we are only interested in the angle of light deflection to the order G/c^4 , we need only retain the terms of the order G/c^2 and G/c^3 , since the expression for the light deflection angle (Eq. (49)) contains a further factor $1/c$. After performing the integration of the expression above and retaining only terms of the order G/c^2 and G/c^3 , we finally find

$$\begin{aligned}
 \delta l_{(1)\Pi}^i(\tau) = & -6 \frac{G}{c^2} \sum_{a=1}^2 m_a [(\vec{e}_{(0)} \cdot \vec{v}_a(t^*)) F_{a2} + [\vec{\xi} \cdot \vec{v}_a(t^*) - \vec{x}_a(t^*) \cdot \vec{v}_a(t^*)] F_{a3}] [\xi^i - x_a^i(t^*)] \\
 & - 4 \frac{G}{c^2} \sum_{a=1}^2 \frac{m_a}{r_a^3} [(\vec{e}_{(0)} \cdot \vec{v}_a(t^*)) c\tau + [\vec{\xi} \cdot \vec{v}_a(t^*) - \vec{x}_a(t^*) \cdot \vec{v}_a(t^*)] \tau] l_{(0)}^i \\
 & + 2 \frac{G}{c^2} \sum_{a=1}^2 m_a \left\{ A_a + \frac{2}{r_a^3} [(\vec{e}_{(0)} \cdot \vec{v}_a(t^*)) c\tau + [\vec{\xi} \cdot \vec{v}_a(t^*) - \vec{x}_a(t^*) \cdot \vec{v}_a(t^*)] \tau] \right\} v_a^i(t^*) \\
 & + 4 \frac{G}{c^2} \sum_{a=1}^2 \frac{m_a}{r_a^3} [(\vec{e}_{(0)} \cdot \vec{v}_a(t^*)) c\tau + [\vec{\xi} \cdot \vec{v}_a(t^*) - \vec{x}_a(t^*) \cdot \vec{v}_a(t^*)] \tau] \tau v_a^i(t^*) \\
 & + 12 \frac{G}{c^3} \sum_{a=1}^2 m_a (\vec{e}_{(0)} \cdot \vec{v}_a(t^*)) [(\vec{e}_{(0)} \cdot \vec{v}_a(t^*)) F_{a2} + [\vec{\xi} \cdot \vec{v}_a(t^*) - \vec{x}_a(t^*) \cdot \vec{v}_a(t^*)] F_{a3}] [\xi^i - x_a^i(t^*)] \\
 & - \frac{G}{c^3} \sum_{a=1}^2 m_a [15(\vec{e}_{(0)} \cdot \vec{v}_a(t^*))^2 G_{a2} + 30(\vec{e}_{(0)} \cdot \vec{v}_a(t^*)) [\vec{\xi} \cdot \vec{v}_a(t^*) - \vec{x}_a(t^*) \cdot \vec{v}_a(t^*)] G_{a3} \\
 & + 15[\vec{\xi} \cdot \vec{v}_a(t^*) - \vec{x}_a(t^*) \cdot \vec{v}_a(t^*)]^2 G_{a4} - 3v_a^2(t^*) F_{a2}] [\xi^i - x_a^i(t^*)] \\
 & - 6 \frac{G}{c^3} \sum_{a=1}^2 m_a [(\vec{e}_{(0)} \cdot \vec{v}_a(t^*)) F_{a1} + [\vec{\xi} \cdot \vec{v}_a(t^*) - \vec{x}_a(t^*) \cdot \vec{v}_a(t^*)] F_{a2}] l_{(0)}^i \\
 & + 6 \frac{G}{c^3} \sum_{a=1}^2 m_a [(\vec{e}_{(0)} \cdot \vec{v}_a(t^*)) F_{a1} + [\vec{\xi} \cdot \vec{v}_a(t^*) - \vec{x}_a(t^*) \cdot \vec{v}_a(t^*)] F_{a2}] v_a^i(t^*) \\
 & - 4 \frac{G}{c^3} \sum_{a=1}^2 m_a (\vec{e}_{(0)} \cdot \vec{v}_a(t^*)) \left\{ A_a + \frac{1}{r_a^3} [(\vec{e}_{(0)} \cdot \vec{v}_a(t^*)) c\tau + [\vec{\xi} \cdot \vec{v}_a(t^*) - \vec{x}_a(t^*) \cdot \vec{v}_a(t^*)] \tau] \right\} v_a^i(t^*), \quad (45)
 \end{aligned}$$

where the functions $A_a, F_{a1}, F_{a2}, F_{a3}$ are given in Sec. IV by Eqs. (33)–(38). The functions G_{a2}, G_{a3} and G_{a4} are given by

$$\begin{aligned}
 G_{a2} = & \frac{1}{15r_a^5 R_a^3} \{ [3r_a^8(0, t^*) r_a - 8(\vec{e}_{(0)} \cdot \vec{x}_a(t^*)) r_a^8(0, t^*) + 6(\vec{e}_{(0)} \cdot \vec{x}_a(t^*))^2 r_a^6(0, t^*) r_a - (\vec{e}_{(0)} \cdot \vec{x}_a(t^*))^4 r_a^4(0, t^*) r_a] \\
 & - [12(\vec{e}_{(0)} \cdot \vec{x}_a(t^*)) r_a^6(0, t^*) r_a - 40(\vec{e}_{(0)} \cdot \vec{x}_a(t^*))^2 r_a^6(0, t^*) + 24(\vec{e}_{(0)} \cdot \vec{x}_a(t^*))^3 r_a^4(0, t^*) r_a \\
 & - 4(\vec{e}_{(0)} \cdot \vec{x}_a(t^*))^5 r_a^2(0, t^*) r_a] c\tau - [-6r_a^6(0, t^*) r_a + 20(\vec{e}_{(0)} \cdot \vec{x}_a(t^*)) r_a^6(0, t^*) - 24(\vec{e}_{(0)} \cdot \vec{x}_a(t^*))^2 r_a^4(0, t^*) r_a \\
 & + 60(\vec{e}_{(0)} \cdot \vec{x}_a(t^*))^3 r_a^4(0, t^*) - 22(\vec{e}_{(0)} \cdot \vec{x}_a(t^*))^4 r_a^2(0, t^*) r_a + 4(\vec{e}_{(0)} \cdot \vec{x}_a(t^*))^6 r_a] c^2 \tau^2 \\
 & - [12(\vec{e}_{(0)} \cdot \vec{x}_a(t^*)) r_a^4(0, t^*) r_a - 60(\vec{e}_{(0)} \cdot \vec{x}_a(t^*))^2 r_a^4(0, t^*) + 24(\vec{e}_{(0)} \cdot \vec{x}_a(t^*))^3 r_a^2(0, t^*) r_a - 20(\vec{e}_{(0)} \cdot \vec{x}_a(t^*))^4 r_a^2(0, t^*) \\
 & - 4(\vec{e}_{(0)} \cdot \vec{x}_a(t^*))^5 r_a] c^3 \tau^3 - [-3r_a^4(0, t^*) r_a + 15(\vec{e}_{(0)} \cdot \vec{x}_a(t^*)) r_a^4(0, t^*) - 6(\vec{e}_{(0)} \cdot \vec{x}_a(t^*))^2 r_a^2(0, t^*) r_a \\
 & + 30(\vec{e}_{(0)} \cdot \vec{x}_a(t^*))^3 r_a^2(0, t^*) + (\vec{e}_{(0)} \cdot \vec{x}_a(t^*))^4 r_a - 5(\vec{e}_{(0)} \cdot \vec{x}_a(t^*))^5] c^4 \tau^4 \\
 & - [-3r_a^4(0, t^*) - 6(\vec{e}_{(0)} \cdot \vec{x}_a(t^*))^2 r_a^2(0, t^*) + (\vec{e}_{(0)} \cdot \vec{x}_a(t^*))^4] c^5 \tau^5 \}, \quad (46)
 \end{aligned}$$

$$\begin{aligned}
G_{a3} = & \frac{1}{15r_a^5 R_a^3} \{-2r_a^4(0, t^*)[r_a^4(0, t^*) - (\vec{e}_{(0)} \cdot \vec{x}_a(t^*))^3 r_a + 3(\vec{e}_{(0)} \cdot \vec{x}_a(t^*))r_a^2(0, t^*)[-r_a + (\vec{e}_{(0)} \cdot \vec{x}_a(t^*))]] \\
& + 2(\vec{e}_{(0)} \cdot \vec{x}_a(t^*))r_a^2(0, t^*)[5r_a^4(0, t^*) - 4(\vec{e}_{(0)} \cdot \vec{x}_a(t^*))^3 r_a + 3(\vec{e}_{(0)} \cdot \vec{x}_a(t^*))r_a^2(0, t^*)[-4r_a + 5(\vec{e}_{(0)} \cdot \vec{x}_a(t^*))]]c\tau \\
& - [5r_a^6(0, t^*) - 8(\vec{e}_{(0)} \cdot \vec{x}_a(t^*))^5 r_a + 6(\vec{e}_{(0)} \cdot \vec{x}_a(t^*))r_a^4(0, t^*)[-2r_a + 5(\vec{e}_{(0)} \cdot \vec{x}_a(t^*))]] \\
& + (\vec{e}_{(0)} \cdot \vec{x}_a(t^*))^3 r_a^6(0, t^*)[-28r_a + 45(\vec{e}_{(0)} \cdot \vec{x}_a(t^*))]]c^2\tau^2 + (\vec{e}_{(0)} \cdot \vec{x}_a(t^*)) [3r_a^2(0, t^*) + (\vec{e}_{(0)} \cdot \vec{x}_a(t^*))^2] \\
& \times [5r_a^2(0, t^*) + (\vec{e}_{(0)} \cdot \vec{x}_a(t^*))[-8r_a + 15(\vec{e}_{(0)} \cdot \vec{x}_a(t^*))]]c^3\tau^3 - (\vec{e}_{(0)} \cdot \vec{x}_a(t^*))[-r_a + 5(\vec{e}_{(0)} \cdot \vec{x}_a(t^*))] \\
& \times [3r_a^2(0, t^*) + (\vec{e}_{(0)} \cdot \vec{x}_a(t^*))^2]c^4\tau^4 + 2(\vec{e}_{(0)} \cdot \vec{x}_a(t^*)) [3r_a^2(0, t^*) + (\vec{e}_{(0)} \cdot \vec{x}_a(t^*))^2]c^5\tau^5\}, \tag{47}
\end{aligned}$$

$$\begin{aligned}
G_{a4} = & \frac{1}{15r_a^5 R_a^3} \{2r_a^4(0, t^*)[r_a^2(0, t^*)[r_a - 3(\vec{e}_{(0)} \cdot \vec{x}_a(t^*))] + (\vec{e}_{(0)} \cdot \vec{x}_a(t^*))^2[3r_a - (\vec{e}_{(0)} \cdot \vec{x}_a(t^*))]] \\
& + 2(\vec{e}_{(0)} \cdot \vec{x}_a(t^*))r_a^2(0, t^*)[(\vec{e}_{(0)} \cdot \vec{x}_a(t^*))^2[-12r_a + 5(\vec{e}_{(0)} \cdot \vec{x}_a(t^*))] + r_a^2(0, t^*)[-4r_a + 15(\vec{e}_{(0)} \cdot \vec{x}_a(t^*))]]c\tau \\
& + [r_a^2(0, t^*) + 3(\vec{e}_{(0)} \cdot \vec{x}_a(t^*))^2][r_a^2(0, t^*)[4r_a - 15(\vec{e}_{(0)} \cdot \vec{x}_a(t^*))] + (\vec{e}_{(0)} \cdot \vec{x}_a(t^*))^2[8r_a - 5(\vec{e}_{(0)} \cdot \vec{x}_a(t^*))]]c^2\tau^2 \\
& + [r_a^2(0, t^*) + 3(\vec{e}_{(0)} \cdot \vec{x}_a(t^*))^2][5r_a^2(0, t^*) + (\vec{e}_{(0)} \cdot \vec{x}_a(t^*))[-8r_a + 15(\vec{e}_{(0)} \cdot \vec{x}_a(t^*))]]c^3\tau^3 \\
& + 2[r_a - 5(\vec{e}_{(0)} \cdot \vec{x}_a(t^*))][r_a^2(0, t^*) + 3(\vec{e}_{(0)} \cdot \vec{x}_a(t^*))^2]c^4\tau^4 + 2[r_a^2(0, t^*) + 3(\vec{e}_{(0)} \cdot \vec{x}_a(t^*))^2]c^5\tau^5\}. \tag{48}
\end{aligned}$$

After introducing the perturbation $\delta l_{(1)\text{II}}^i(\tau)$ into Eq. (18) and computing the limit for $\tau \rightarrow \infty$, we obtain

$$\begin{aligned}
\alpha_{(1)\text{II}}^i = & \lim_{\tau \rightarrow \infty} \left[\frac{1}{c} P_q^i \delta l_{(1)\text{II}}^q(\tau) \right] \\
= & -4 \frac{G}{c^3} \sum_{a=1}^2 \frac{m_a}{R_a} (\vec{e}_{(0)} \cdot \vec{v}_a(t^*)) [\xi^i - P_q^i x_a^q(t^*)] - 8 \frac{G}{c^3} \sum_{a=1}^2 \frac{m_a}{R_a^2} (\vec{e}_{(0)} \cdot \vec{x}_a(t^*)) [(\vec{e}_{(0)} \cdot \vec{x}_a(t^*)) (\vec{e}_{(0)} \cdot \vec{v}_a(t^*)) \\
& + \vec{\xi} \cdot \vec{v}_a(t^*) - \vec{x}_a(t^*) \cdot \vec{v}_a(t^*)] [\xi^i - P_q^i x_a^q(t^*)] + 4 \frac{G}{c^3} \sum_{a=1}^2 \frac{m_a}{R_a} (\vec{e}_{(0)} \cdot \vec{x}_a(t^*)) P_q^i v_a^q(t^*) \\
& + 2 \frac{G}{c^4} \sum_{a=1}^2 \frac{m_a}{R_a} v_a^2(t^*) [\xi^i - P_q^i x_a^q(t^*)] + 4 \frac{G}{c^4} \sum_{a=1}^2 \frac{m_a}{R_a^2} v_a^2(t^*) (\vec{e}_{(0)} \cdot \vec{x}_a(t^*))^2 [\xi^i - P_q^i x_a^q(t^*)] \\
& + 2 \frac{G}{c^4} \sum_{a=1}^2 \frac{m_a}{R_a} (\vec{e}_{(0)} \cdot \vec{v}_a(t^*))^2 [\xi^i - P_q^i x_a^q(t^*)] - 8 \frac{G}{c^4} \sum_{a=1}^2 \frac{m_a}{R_a} (\vec{e}_{(0)} \cdot \vec{x}_a(t^*))^2 (\vec{e}_{(0)} \cdot \vec{v}_a(t^*))^2 [\xi^i - P_q^i x_a^q(t^*)] \\
& - 16 \frac{G}{c^4} \sum_{a=1}^2 \frac{m_a}{R_a^3} (\vec{e}_{(0)} \cdot \vec{x}_a(t^*))^4 (\vec{e}_{(0)} \cdot \vec{v}_a(t^*))^2 [\xi^i - P_q^i x_a^q(t^*)] - 8 \frac{G}{c^4} \sum_{a=1}^2 \frac{m_a}{R_a^2} (\vec{e}_{(0)} \cdot \vec{x}_a(t^*)) (\vec{e}_{(0)} \cdot \vec{v}_a(t^*)) \\
& \times [\vec{\xi} \cdot \vec{v}_a(t^*) - \vec{x}_a(t^*) \cdot \vec{v}_a(t^*)] [\xi^i - P_q^i x_a^q(t^*)] - 32 \frac{G}{c^4} \sum_{a=1}^2 \frac{m_a}{R_a^3} (\vec{e}_{(0)} \cdot \vec{x}_a(t^*))^3 (\vec{e}_{(0)} \cdot \vec{v}_a(t^*)) \\
& \times [\vec{\xi} \cdot \vec{v}_a(t^*) - \vec{x}_a(t^*) \cdot \vec{v}_a(t^*)] [\xi^i - P_q^i x_a^q(t^*)] - 4 \frac{G}{c^4} \sum_{a=1}^2 \frac{m_a}{R_a^2} [\vec{\xi} \cdot \vec{v}_a(t^*) - \vec{x}_a(t^*) \cdot \vec{v}_a(t^*)]^2 [\xi^i - P_q^i x_a^q(t^*)] \\
& - 16 \frac{G}{c^4} \sum_{a=1}^2 \frac{m_a}{R_a^3} (\vec{e}_{(0)} \cdot \vec{x}_a(t^*))^2 [\vec{\xi} \cdot \vec{v}_a(t^*) - \vec{x}_a(t^*) \cdot \vec{v}_a(t^*)]^2 [\xi^i - P_q^i x_a^q(t^*)] \\
& + 4 \frac{G}{c^4} \sum_{a=1}^2 \frac{m_a}{R_a} (\vec{e}_{(0)} \cdot \vec{x}_a(t^*)) (\vec{e}_{(0)} \cdot \vec{v}_a(t^*)) P_q^i v_a^q(t^*) + 8 \frac{G}{c^4} \sum_{a=1}^2 \frac{m_a}{R_a^2} (\vec{e}_{(0)} \cdot \vec{x}_a(t^*))^2 \{ [\vec{\xi} \cdot \vec{v}_a(t^*) - \vec{x}_a(t^*) \cdot \vec{v}_a(t^*)] \\
& + (\vec{e}_{(0)} \cdot \vec{x}_a(t^*)) (\vec{e}_{(0)} \cdot \vec{v}_a(t^*)) \} P_q^i v_a^q(t^*) + 4 \frac{G}{c^4} \sum_{a=1}^2 \frac{m_a}{R_a} [\vec{\xi} \cdot \vec{v}_a(t^*) - \vec{x}_a(t^*) \cdot \vec{v}_a(t^*)] P_q^i v_a^q(t^*), \tag{49}
\end{aligned}$$

where the quantity R_a is given by Eq. (40).

B. The postlinear light deflection and the motion of the masses

The correction terms to the postlinear light deflection to the order G^2/c^4 resulting from the Taylor expansion of Eq. (31) are given by

$$\begin{aligned} \alpha_{(2)\text{III}}^i = & -3 \frac{G}{c} \sum_{a=1}^2 m_a \int_{-\infty}^{\infty} d\tau \frac{\tau^2}{r_a^5} [c\tau(\vec{e}_{(0)} \cdot \vec{a}_a(t^*)) + \vec{\xi} \cdot \vec{a}_a(t^*) - \vec{x}_a(t^*) \cdot \vec{a}_a(t^*)][\xi^i - P_q^i x_a^q(t^*)] \\ & + \frac{G}{c} \sum_{a=1}^2 m_a \int_{-\infty}^{\infty} d\tau \frac{\tau^2}{r_a^3} P_q^i a_a^q(t^*) + 4 \frac{G}{c^2} \sum_{a=1}^2 m_a \int_{-\infty}^{\infty} d\tau \frac{\tau}{r_a^3} (\vec{e}_{(0)} \cdot \vec{a}_a(t^*)) [\xi^i - P_q^i x_a^q(t^*)]. \end{aligned} \quad (50)$$

In the preceding equation the second integral diverges. Therefore, we resort to a Taylor expansion of its integrand about the origin of the coordinate system $\vec{x}_a = 0$, up to second order. Then only the first term of the Taylor expansion is a divergent integral and it is given by

$$\frac{G}{c} \sum_{a=1}^2 m_a \int_{-\infty}^{\infty} d\tau \tau^2 \frac{1}{[c^2 \tau^2 + \xi^2]^{3/2}} P_q^i a_a^q(t^*).$$

Because in this case we do not need to take into account the 1PN-corrections in the positions of the masses, we can assume that the origin of the coordinate system which is located at the 1PN-center of mass coincides with the position of the Newtonian center of mass. Taking into account the consequence of the Newtonian center-of-mass theorem

$$\sum_{a=1}^2 m_a \vec{a}_a = 0,$$

it is easy to see that the divergent integral vanishes. After performing the integration with respect to the parameter τ we find:

$$\begin{aligned} \alpha_{(2)\text{III}}^i = & 2 \frac{G}{c^4} \sum_{a=1}^2 \frac{m_a}{R_a} (\vec{e}_{(0)} \cdot \vec{x}_a(t^*)) (\vec{e}_{(0)} \cdot \vec{a}_a(t^*)) [\xi^i - P_q^i x_a^q(t^*)] - 4 \frac{G}{c^4} \sum_{a=1}^2 \frac{m_a}{R_a^2} (\vec{e}_{(0)} \cdot \vec{x}_a(t^*))^3 (\vec{e}_{(0)} \cdot \vec{a}_a(t^*)) [\xi^i - P_q^i x_a^q(t^*)] \\ & - 2 \frac{G}{c^4} \sum_{a=1}^2 \frac{m_a}{R_a} [\vec{\xi} \cdot \vec{a}_a(t^*) - \vec{x}_a(t^*) \cdot \vec{a}_a(t^*)][\xi^i - P_q^i x_a^q(t^*)] \\ & - 4 \frac{G}{c^4} \sum_{a=1}^2 \frac{m_a}{R_a^2} (\vec{e}_{(0)} \cdot \vec{x}_a(t^*))^2 [\vec{\xi} \cdot \vec{a}_a(t^*) - \vec{x}_a(t^*) \cdot \vec{a}_a(t^*)][\xi^i - P_q^i x_a^q(t^*)] \\ & + \frac{G}{c^4} \sum_{a=1}^2 m_a \left\{ 2 \frac{\vec{\xi} \cdot \vec{x}_a(t^*)}{\xi^2} - \frac{x_a^2(t^*)}{\xi^2} + 3 \frac{(\vec{e}_{(0)} \cdot \vec{x}_a(t^*))^2}{\xi^2} + 2 \frac{(\vec{\xi} \cdot \vec{x}_a(t^*))^2}{\xi^4} \right\} P_q^i a_a^q(t^*), \end{aligned} \quad (51)$$

where the quantity R_a is given by Eq. (40). Note that in Eq. (50) two of the three integrals were exactly integrated, so that the resulting expression given by Eq. (51) is a combination of exact terms with a term which is represented as an expansion in powers of $(x_a(t^*)/\xi)$. In view of further applications and for the sake of uniformity we perform the Taylor expansion of the exact terms about the origin of the coordinate system $\vec{x}_a = 0$ up to second order. After performing the Taylor expansion of the exact terms in Eq. (51) about the origin of the coordinate system $\vec{x}_a = 0$, up to second order, we find

$$\begin{aligned}
\alpha^i_{(2)\text{III}} = & 2 \frac{G}{c^4} \sum_{a=1}^2 m_a (\vec{e}_{(0)} \cdot \vec{x}_a(t^*)) (\vec{e}_{(0)} \cdot \vec{a}_a(t^*)) \left\{ \frac{1}{\xi^2} + 2 \frac{(\vec{\xi} \cdot \vec{x}_a(t^*))}{\xi^4} - \frac{x_a^2(t^*)}{\xi^4} + \frac{(\vec{e}_{(0)} \cdot \vec{x}_a(t^*))^2}{\xi^4} + 4 \frac{(\vec{\xi} \cdot \vec{x}_a(t^*))^2}{\xi^6} \right\} \\
& \times [\xi^i - P_q^i x_a^q(t^*)] - 4 \frac{G}{c^4} \sum_{a=1}^2 m_a (\vec{e}_{(0)} \cdot \vec{x}_a(t^*))^3 (\vec{e}_{(0)} \cdot \vec{a}_a(t^*)) \left\{ \frac{1}{\xi^4} + 4 \frac{\vec{\xi} \cdot \vec{x}_a(t^*)}{\xi^6} - 2 \frac{x_a^2(t^*)}{\xi^6} + 2 \frac{(\vec{e}_{(0)} \cdot \vec{x}_a(t^*))^2}{\xi^6} \right\} \\
& + 12 \frac{(\vec{\xi} \cdot \vec{x}_a(t^*))^2}{\xi^8} \left\{ \xi^i - P_q^i x_a^q(t^*) \right\} - 2 \frac{G}{c^4} \sum_{a=1}^2 m_a [\vec{\xi} \cdot \vec{a}_a(t^*) - \vec{x}_a(t^*) \cdot \vec{a}_a(t^*)] \left\{ \frac{1}{\xi^2} + 2 \frac{(\vec{\xi} \cdot \vec{x}_a(t^*))}{\xi^4} - \frac{x_a^2(t^*)}{\xi^4} \right\} \\
& + \frac{(\vec{e}_{(0)} \cdot \vec{x}_a(t^*))^2}{\xi^4} + 4 \frac{(\vec{\xi} \cdot \vec{x}_a(t^*))^2}{\xi^6} \left\{ \xi^i - P_q^i x_a^q(t^*) \right\} - 4 \frac{G}{c^4} \sum_{a=1}^2 m_a (\vec{e}_{(0)} \cdot \vec{x}_a(t^*))^2 [\vec{\xi} \cdot \vec{a}_a(t^*) - \vec{x}_a(t^*) \cdot \vec{a}_a(t^*)] \\
& \times \left\{ \frac{1}{\xi^4} + 4 \frac{\vec{\xi} \cdot \vec{x}_a(t^*)}{\xi^6} - 2 \frac{x_a^2(t^*)}{\xi^6} + 2 \frac{(\vec{e}_{(0)} \cdot \vec{x}_a(t^*))^2}{\xi^6} + 12 \frac{(\vec{\xi} \cdot \vec{x}_a(t^*))^2}{\xi^8} \right\} [\xi^i - P_q^i x_a^q(t^*)] \\
& + \frac{G}{c^4} \sum_{a=1}^2 m_a \left\{ 2 \frac{\vec{\xi} \cdot \vec{x}_a(t^*)}{\xi^2} - \frac{x_a^2(t^*)}{\xi^2} + 3 \frac{(\vec{e}_{(0)} \cdot \vec{x}_a(t^*))^2}{\xi^2} + 2 \frac{(\vec{\xi} \cdot \vec{x}_a(t^*))^2}{\xi^4} \right\} P_q^i a_a^q(t^*). \tag{52}
\end{aligned}$$

To replace the accelerations in the preceding equation by functions of the positions we shall use the Newtonian equations of motion. After replacing the accelerations in Eq. (52) and expressing the positions of the masses by their center-of-mass frame coordinates without considering the 1PN-corrections we find,

$$\begin{aligned}
\alpha^i_{(2)\text{III}} = & \frac{G^2 m_1 m_2}{c^4 r_{12}^2} \left\{ 2(\vec{e}_\xi \cdot \vec{n}_{12}) - 2X_2[1 - 2(\vec{e}_\xi \cdot \vec{n}_{12})^2 - 3(\vec{e}_{(0)} \cdot \vec{n}_{12})^2] \left(\frac{r_{12}}{\xi} \right) \right. \\
& - 6X_2^2(\vec{e}_\xi \cdot \vec{n}_{12}) \left[1 - \frac{4}{3}(\vec{e}_\xi \cdot \vec{n}_{12})^2 - 3(\vec{e}_{(0)} \cdot \vec{n}_{12})^2 \right] \left(\frac{r_{12}}{\xi} \right)^2 + 2X_2^3[1 - 4(\vec{e}_\xi \cdot \vec{n}_{12})^2 - 3(\vec{e}_{(0)} \cdot \vec{n}_{12})^2] \left(\frac{r_{12}}{\xi} \right)^3 \\
& + \mathcal{O} \left[\left(\frac{r_{12}}{\xi} \right)^4 \right] \left. \right\} e_\xi^i - \frac{G^2 m_1 m_2}{c^4 r_{12}^2} \left\{ 2X_2(\vec{e}_\xi \cdot \vec{n}_{12}) \left(\frac{r_{12}}{\xi} \right) - 2X_2^2[1 - 2(\vec{e}_\xi \cdot \vec{n}_{12})^2 - 3(\vec{e}_{(0)} \cdot \vec{n}_{12})^2] \left(\frac{r_{12}}{\xi} \right)^2 \right. \\
& - 6X_2^2(\vec{e}_\xi \cdot \vec{n}_{12}) \left[1 - \frac{4}{3}(\vec{e}_\xi \cdot \vec{n}_{12})^2 - 3(\vec{e}_{(0)} \cdot \vec{n}_{12})^2 \right] \left(\frac{r_{12}}{\xi} \right)^3 + 2X_2^3[1 - 4(\vec{e}_\xi \cdot \vec{n}_{12})^2 - 3(\vec{e}_{(0)} \cdot \vec{n}_{12})^2] \left(\frac{r_{12}}{\xi} \right)^4 \\
& + \mathcal{O} \left[\left(\frac{r_{12}}{\xi} \right)^5 \right] \left. \right\} P_q^i n_{12}^q + \frac{G^2 m_1 m_2}{c^4 r_{12}^2} \left\{ -2X_2(\vec{e}_\xi \cdot \vec{n}_{12}) \left(\frac{r_{12}}{\xi} \right) + X_2^2[1 - 2(\vec{e}_\xi \cdot \vec{n}_{12})^2 - 3(\vec{e}_{(0)} \cdot \vec{n}_{12})^2] \left(\frac{r_{12}}{\xi} \right)^2 \right. \\
& + 2X_2^2(\vec{e}_\xi \cdot \vec{n}_{12}) \left[1 - \frac{4}{3}(\vec{e}_\xi \cdot \vec{n}_{12})^2 - 3(\vec{e}_{(0)} \cdot \vec{n}_{12})^2 \right] \left(\frac{r_{12}}{\xi} \right)^3 + \mathcal{O} \left[\left(\frac{r_{12}}{\xi} \right)^4 \right] \left. \right\} P_q^i n_{12}^q \\
& - 8 \frac{G^2 m_1 m_2}{c^4 r_{12}^2} X_2 (\vec{e}_{(0)} \cdot \vec{n}_{12})^2 \left\{ \left(\frac{r_{12}}{\xi} \right) + 2X_2(\vec{e}_\xi \cdot \vec{n}_{12}) \left(\frac{r_{12}}{\xi} \right)^2 + \mathcal{O} \left[\left(\frac{r_{12}}{\xi} \right)^3 \right] \right\} e_\xi^i \\
& + 8 \frac{G^2 m_1 m_2}{c^4 r_{12}^2} X_2^2 (\vec{e}_{(0)} \cdot \vec{n}_{12})^2 \left\{ \left(\frac{r_{12}}{\xi} \right)^2 + 2X_2(\vec{e}_\xi \cdot \vec{n}_{12}) \left(\frac{r_{12}}{\xi} \right)^3 + \mathcal{O} \left[\left(\frac{r_{12}}{\xi} \right)^4 \right] \right\} P_q^i n_{12}^q + (1 \leftrightarrow 2), \tag{53}
\end{aligned}$$

where the quantities \vec{n}_{12} , \vec{v}_{12} and r_{12} are taken at the time t^* .

VII. THE POSTLINEAR LIGHT DEFLECTION AND THE PERTURBED LIGHT RAY TRAJECTORY

If we introduce into the equations for the linear perturbations (15) and (44) the expression for the perturbed light ray trajectory, we get additional postlinear light deflection terms. To compute these terms we have first to find the expression for the perturbation of the photon's trajectory that is linear in G . We obtain this perturbation by integrat-

ing the expression for the total linear perturbation given by Eq. (63) with respect to the parameter τ . The expression for the total linear perturbation is obtained by summing up the expressions for the linear perturbations $\delta l_{(1)\text{I}}^i(\tau)$ and $\delta l_{(1)\text{II}}^i(\tau)$. Considering that in this paper we compute the postlinear light deflection to the order G^2/c^4 we do not need to retain in the expression resulting from the integration of Eq. (63) the terms of the order G/c^4 , since these terms are related to the postlinear light deflection terms of higher order than G^2/c^4 . After performing the integration of Eq. (63) with regard to τ and retaining only terms of the order $\mathcal{O}(G/c^2)$ and $\mathcal{O}(G/c^3)$ we obtain

$$\begin{aligned}
 \delta \vec{z}_{(1)}^i(\tau) = & -2 \frac{G}{c^2} \sum_{a=1}^2 m_a \mathcal{B}_a [\xi^i - x_a^i(t^*)] - 2 \frac{G}{c^3} \sum_{a=1}^2 m_a (\vec{e}_{(0)} \cdot \vec{x}_a(t^*)) \mathcal{B}_a l_{(0)}^i - 2 \frac{G}{c^3} \sum_{a=1}^2 m_a \ln \left[\frac{c\tau - \vec{e}_{(0)} \cdot \vec{x}_a(t^*) + r_a}{r_a(0, t^*) - \vec{e}_{(0)} \cdot \vec{x}_a(t^*)} \right] l_{(0)}^i \\
 & + 4 \frac{G}{c^3} \sum_{a=1}^2 m_a (\vec{e}_{(0)} \cdot \vec{v}_a(t^*)) \mathcal{B}_a [\xi^i - x_a^i(t^*)] + 2 \frac{G}{c^3} \sum_{a=1}^2 m_a (\vec{e}_{(0)} \cdot \vec{x}_a(t^*)) \mathcal{B}_a v_a^i(t^*) \\
 & + 2 \frac{G}{c^3} \sum_{a=1}^2 m_a \ln \left[\frac{c\tau - \vec{e}_{(0)} \cdot \vec{x}_a(t^*) + r_a}{r_a(0, t^*) - \vec{e}_{(0)} \cdot \vec{x}_a(t^*)} \right] v_a^i(t^*) - 6 \frac{G}{c^3} \sum_{a=1}^2 m_a [(\vec{e}_{(0)} \cdot \vec{v}_a(t^*)) \mathcal{F}_{a2} + (\vec{r}_a(0, t^*) \cdot \vec{v}_a(t^*)) \mathcal{F}_{a3}] \\
 & \times [\xi^i - x_a^i(t^*)] + \mathcal{O}\left(\frac{G}{c^4}\right), \tag{54}
 \end{aligned}$$

where the functions \mathcal{B}_a , \mathcal{F}_{a2} and \mathcal{F}_{a3} are given by

$$\begin{aligned}
 \mathcal{B}_a &= \frac{1}{R_a} [c\tau + r_a], \\
 \mathcal{F}_{a2} &= \frac{1}{3r_a R_a^3} \{ (\vec{e}_{(0)} \cdot \vec{x}_a(t^*))^2 [3r_a^4(0, t^*) + r_a^2 R_a + (\vec{e}_{(0)} \cdot \vec{x}_a(t^*))^2 r_a^2(0, t^*)] \\
 & \quad + r_a^2(0, t^*) [r_a^4(0, t^*) + r_a^2 R_a - 5(\vec{e}_{(0)} \cdot \vec{x}_a(t^*))^2 r_a^2(0, t^*)] + [-2(\vec{e}_{(0)} \cdot \vec{x}_a(t^*)) r_a^4(0, t^*) + (r_a^2(0, t^*) \\
 & \quad + (\vec{e}_{(0)} \cdot \vec{x}_a(t^*))^2) r_a R_a + 2(\vec{e}_{(0)} \cdot \vec{x}_a(t^*))^3 (2r_a^2(0, t^*) - (\vec{e}_{(0)} \cdot \vec{x}_a(t^*))^2)] c\tau \}, \\
 \mathcal{F}_{a3} &= \frac{1}{3r_a R_a^2} \{ 2(\vec{e}_{(0)} \cdot \vec{x}_a(t^*)) r_a^2(0, t^*) - [R_a - 2(\vec{e}_{(0)} \cdot \vec{x}_a(t^*)) r_a + 4(\vec{e}_{(0)} \cdot \vec{x}_a(t^*))^2] c\tau + 2(\vec{e}_{(0)} \cdot \vec{x}_a(t^*)) c^2 \tau^2 \}. \tag{55}
 \end{aligned}$$

As our integration constant we have chosen in Eq. (54)

$$K^i = 2 \frac{G}{c^3} \sum_{a=1}^2 m_a \ln [2(r_a(0, t^*) - \vec{e}_{(0)} \cdot \vec{x}_a(t^*))] l_{(0)}^i - 2 \frac{G}{c^3} \sum_{a=1}^2 m_a \ln [2(r_a(0, t^*) - \vec{e}_{(0)} \cdot \vec{x}_a(t^*))] v_a^i(t^*) + \mathcal{O}\left(\frac{G}{c^4}\right),$$

because with this integration constant we recover from our expression for the postlinear light deflection the correct expression for the postlinear light deflection in the event that the value of one of the masses is equal to zero (i.e. the Epstein-Shapiro postlinear light deflection).

It follows from Eq. (18) that the expression for the linear light deflection for an observer located at infinity is given by

$$\begin{aligned}
 \alpha_{(1)}^i &= \lim_{\tau \rightarrow \infty} \left\{ \frac{1}{c} P_q^i \delta l_{(1)}^q(\tau) \right\}, \\
 &= \lim_{\tau \rightarrow \infty} \left\{ \frac{1}{c} P_q^i [\delta l_{(1)\text{I}}^q(\tau) + \delta l_{(1)\text{II}}^q(\tau)] \right\}, \tag{56}
 \end{aligned}$$

where the perturbations $\delta l_{(1)\text{I}}^q(\tau)$ and $\delta l_{(1)\text{II}}^q(\tau)$ are given by Eqs. (15) and (44). After introducing the perturbation $\delta \vec{z}_{(1)}$ into the equations for $\delta l_{(1)\text{I}}^q(\tau)$ and $\delta l_{(1)\text{II}}^q(\tau)$ we get a perturbed linear light deflection. Because the perturbation $\delta \vec{z}_{(1)}$ is a small quantity compared to $\vec{z}(\tau)_{\text{unpert.}}$, we can resort to a Taylor expansion of the perturbed linear light deflection about $\delta \vec{z}_{(1)} = 0$ in order to get the terms of the perturbed linear light deflection that are quadratic in G . We denote these terms by $\alpha_{(2)\text{IV}}^i$ and they are given by

$$\begin{aligned}
\alpha_{(2)IV}^i &= \lim_{\tau \rightarrow \infty} \left\{ \frac{1}{c} P_q^i \left[\left(\frac{\partial \delta l_{(1)I}^q(\text{Pert.})}{\partial \delta z_{(1)}^m} \right)_{\delta \vec{z}_{(1)}=0} \delta z_{(1)}^m + \left(\frac{\partial \delta l_{(1)II}^q(\text{Pert.})}{\partial \delta z_{(1)}^m} \right)_{\delta \vec{z}_{(1)}=0} \delta z_{(1)}^m \right] \right\} \\
&= \frac{G}{c} \sum_{a=1}^2 m_a \int_{-\infty}^{\infty} d\tau \left\{ \left[\frac{6}{r_a^5} (\vec{r}_a \cdot \delta \vec{z}_{(1)}) - 6 \left[\frac{1}{r_a^5} (\vec{v}_a(t^*) \cdot \delta \vec{z}_{(1)}) - \frac{5}{r_a^7} (\vec{r}_a \cdot \vec{v}_a(t^*)) (\vec{r}_a \cdot \delta \vec{z}_{(1)}) \right] \tau \right. \right. \\
&\quad - 15 \left[\frac{2}{r_a^7} (\vec{r}_a \cdot \vec{v}_a(t^*)) (\vec{v}_a(t^*) \cdot \delta \vec{z}_{(1)}) + \frac{1}{r_a^7} v_a^2(t^*) (\vec{r}_a \cdot \delta \vec{z}_{(1)}) - \frac{7}{r_a^9} (\vec{r}_a \cdot \vec{v}_a(t^*))^2 (\vec{r}_a \cdot \delta \vec{z}_{(1)}) \right] \tau^2 \left. \right] P_q^i r_a^q \\
&\quad - \left[\frac{2}{r_a^3} + \frac{6}{r_a^5} (\vec{r}_a \cdot \vec{v}_a(t^*)) \tau - 3 \left[\frac{1}{r_a^5} v_a^2(t^*) - \frac{5}{r_a^7} (\vec{r}_a \cdot \vec{v}_a(t^*))^2 \right] \tau^2 \right] P_q^i \delta z_{(1)}^q \\
&\quad - \left. \left[\frac{6}{r_a^5} (\vec{r}_a \cdot \delta \vec{z}_{(1)}) \tau - 3 \left[\frac{2}{r_a^5} (\vec{v}_a \cdot \delta \vec{z}_{(1)}) - \frac{10}{r_a^7} (\vec{r}_a \cdot \vec{v}_a(t^*)) (\vec{r}_a \cdot \delta \vec{z}_{(1)}) \right] \tau^2 \right] P_q^i v_a^q(t^*) \right\}_{|(\leftarrow)} \\
&\quad + \frac{G}{c^2} \sum_{a=1}^2 m_a (\vec{e}_{(0)} \cdot \vec{v}_a(t^*)) \int_{-\infty}^{\infty} d\tau \left\{ -6 \left[\frac{2}{r_a^5} (\vec{r}_a \cdot \delta \vec{z}_{(1)}) - 2 \left[\frac{1}{r_a^5} (\vec{v}_a(t^*) \cdot \delta \vec{z}_{(1)}) - \frac{5}{r_a^7} (\vec{r}_a \cdot \vec{v}_a(t^*)) (\vec{r}_a \cdot \delta \vec{z}_{(1)}) \right] \tau \right. \right. \\
&\quad - 5 \left[\frac{1}{r_a^7} v_a^2(t^*) (\vec{r}_a \cdot \delta \vec{z}_{(1)}) + \frac{2}{r_a^7} (\vec{r}_a \cdot \vec{v}_a(t^*)) (\vec{v}_a(t^*) \cdot \delta \vec{z}_{(1)}) - \frac{7}{r_a^9} (\vec{r}_a \cdot \vec{v}_a(t^*))^2 (\vec{r}_a \cdot \delta \vec{z}_{(1)}) \right] \tau^2 \left. \right] P_q^i r_a^q \\
&\quad + 2 \left[\frac{2}{r_a^3} + \frac{6}{r_a^5} (\vec{r}_a \cdot \vec{v}_a(t^*)) \tau - 3 \left[\frac{1}{r_a^5} v_a^2(t^*) - \frac{5}{r_a^7} (\vec{r}_a \cdot \vec{v}_a(t^*))^2 \right] \tau^2 \right] P_q^i \delta z_{(1)}^q \\
&\quad + \left. 12 \left[\frac{1}{r_a^5} (\vec{r}_a \cdot \delta \vec{z}_{(1)}) \tau - \left[\frac{1}{r_a^5} (\vec{v}_a(t^*) \cdot \delta \vec{z}_{(1)}) - \frac{5}{r_a^7} (\vec{r}_a \cdot \vec{v}_a(t^*)) (\vec{r}_a \cdot \delta \vec{z}_{(1)}) \right] \tau^2 \right] P_q^i v_a^q(t^*) \right\}_{|(\leftarrow)} + \frac{G}{c^3} \{ \cdot \cdot \cdot \}. \quad (57)
\end{aligned}$$

In the equation above we need not write out the expression $G/c^3 \{ \cdot \cdot \cdot \}$ explicitly since it contributes only terms of order greater than G^2/c^4 . After substituting the perturbation (54) into the preceding equation we obtain the integrals for the postlinear light deflection $\alpha_{(2)IV}^i$. Here, we take into account only the integrals of the order G^2/c^4 . These integrals are given in an explicit form in Appendix C.

VIII. LIGHT DEFLECTION AND THE CENTER OF MASS

In this section we compute the corrections to the linear and the postlinear light deflection resulting from the introduction of the 1PN-corrections to the positions of the masses in the equations for the linear perturbations $\delta l_{(1)I}^q(\tau)$ and $\delta l_{(1)II}^q(\tau)$ given by Eqs. (15) and (44). It follows from Eqs. (24) and (25) that the 1PN-corrections in the positions are given by

$$\delta \vec{x}_1 = \delta \vec{x}_2 = \frac{1}{c^2} \left[\frac{\nu(m_1 - m_2)}{2M} \left[v_{12}^2 - \frac{GM}{r_{12}} \right] \right] \vec{r}_{12}. \quad (58)$$

From expression (58), it is easy to see that the corrections vanish when $m_1 = m_2$. The corrections also vanish for the case of circular orbits. After introducing the 1PN-corrections into the equations for the linear perturbations $\delta l_{(1)I}^q(\tau)$ and $\delta l_{(1)II}^q(\tau)$ we obtain the expression for the perturbed linear light deflection. Because the corrections $\delta \vec{x}_a$ are small quantities compared to \vec{x}_a , we can resort to a Taylor expansion of the perturbed linear light deflection about $\delta \vec{x}_a = 0$ in order to find the correction terms for the linear and postlinear light deflection. We denote these terms by $\tilde{\alpha}_{(1)(2)}^i$ and they are given by

$$\begin{aligned}
 \tilde{\alpha}_{(1)(2)}^i &= \lim_{\tau \rightarrow \infty} \left\{ \frac{1}{c} P_q^i \left[\left(\frac{\partial \delta l_{(1)I}^{q(\text{Pert.})}}{\partial \delta x_a^m} \right)_{\delta \tilde{x}_a=0} \delta x_a^m + \left(\frac{\partial \delta l_{(1)II}^{q(\text{Pert.})}}{\partial \delta x_a^m} \right)_{\delta \tilde{x}_a=0} \delta x_a^m \right] \right\} \\
 &= \frac{G}{c} \sum_{a=1}^2 m_a \int_{-\infty}^{\infty} d\tau \left\{ \left[-\frac{6}{r_a^5} (\tilde{r}_a \cdot \delta \tilde{x}_a) + 6 \left[\frac{1}{r_a^5} (\tilde{v}_a(t^*) \cdot \delta \tilde{x}_a) - \frac{5}{r_a^7} (\tilde{r}_a \cdot \tilde{v}_a(t^*)) (\tilde{r}_a \cdot \delta \tilde{x}_a) \right] \tau \right. \right. \\
 &\quad + 15 \left[\frac{2}{r_a^7} (\tilde{r}_a \cdot \tilde{v}_a(t^*)) (\tilde{v}_a(t^*) \cdot \delta \tilde{x}_a) + \frac{1}{r_a^7} v_a^2(t^*) (\tilde{r}_a \cdot \delta \tilde{x}_a) - \frac{7}{r_a^9} (\tilde{r}_a \cdot \tilde{v}_a(t^*))^2 (\tilde{r}_a \cdot \delta \tilde{x}_a) \right] \tau^2 \left. \right] P_q^i r_a^q \\
 &\quad + \left[\frac{2}{r_a^3} + \frac{6}{r_a^5} (\tilde{r}_a \cdot \tilde{v}_a(t^*)) \tau - 3 \left[\frac{1}{r_a^5} v_a^2(t^*) - \frac{5}{r_a^7} (\tilde{r}_a \cdot \tilde{v}_a(t^*))^2 \right] \tau^2 \right] P_q^i \delta x_a^q \\
 &\quad + \left. \left[\frac{6}{r_a^5} (\tilde{r}_a \cdot \delta \tilde{x}_a) \tau - 3 \left[\frac{2}{r_a^5} (\tilde{v}_a \cdot \delta \tilde{x}_a) - \frac{10}{r_a^7} (\tilde{r}_a \cdot \tilde{v}_a(t^*)) (\tilde{r}_a \cdot \delta \tilde{x}_a) \right] \tau^2 \right] P_q^i v_a^q(t^*) \right\} \Big|_{l(\leftarrow)} \\
 &\quad + \frac{G}{c^2} \sum_{a=1}^2 m_a (\tilde{e}_{(0)} \cdot \tilde{v}_a(t^*)) \int_{-\infty}^{\infty} d\tau \left\{ 6 \left[\frac{2}{r_a^5} (\tilde{r}_a \cdot \delta \tilde{x}_a) - 2 \left[\frac{1}{r_a^5} (\tilde{v}_a(t^*) \cdot \delta \tilde{x}_a) - \frac{5}{r_a^7} (\tilde{r}_a \cdot \tilde{v}_a(t^*)) (\tilde{r}_a \cdot \delta \tilde{x}_a) \right] \tau \right. \right. \\
 &\quad - 5 \left[\frac{1}{r_a^7} v_a^2(t^*) (\tilde{r}_a \cdot \delta \tilde{x}_a) + \frac{2}{r_a^7} (\tilde{r}_a \cdot \tilde{v}_a(t^*)) (\tilde{v}_a(t^*) \cdot \delta \tilde{x}_a) - \frac{7}{r_a^9} (\tilde{r}_a \cdot \tilde{v}_a(t^*))^2 (\tilde{r}_a \cdot \delta \tilde{x}_a) \right] \tau^2 \left. \right] P_q^i r_a^q \\
 &\quad - 2 \left[\frac{2}{r_a^3} + \frac{6}{r_a^5} (\tilde{r}_a \cdot \tilde{v}_a(t^*)) \tau - 3 \left[\frac{1}{r_a^5} v_a^2(t^*) - \frac{5}{r_a^7} (\tilde{r}_a \cdot \tilde{v}_a(t^*))^2 \right] \tau^2 \right] P_q^i \delta x_a^q \\
 &\quad - \left. 12 \left[\frac{1}{r_a^5} (\tilde{r}_a \cdot \delta \tilde{x}_a) \tau - \left[\frac{1}{r_a^5} (\tilde{v}_a(t^*) \cdot \delta \tilde{x}_a) - \frac{5}{r_a^7} (\tilde{r}_a \cdot \tilde{v}_a(t^*)) (\tilde{r}_a \cdot \delta \tilde{x}_a) \right] \tau^2 \right] P_q^i v_a^q(t^*) \right\} \Big|_{l(\leftarrow)} + \frac{G}{c^3} \{ \dots \}. \quad (59)
 \end{aligned}$$

For the same reason as in Eq. (57), we do not need to write out the expression $G/c^3\{\dots\}$ explicitly in Eq. (59). After substituting the 1PN-corrections given by Eq. (58) into the equation above and taking into account only the terms of the orders G/c^4 and G^2/c^4 we find,

$$\begin{aligned}
 \tilde{\alpha}_{(1)(2)}^i &= -6 \frac{G}{c^3} \sum_{a=2}^2 m_a \left[\frac{\nu(m_1 - m_2)}{2M} \left[v_{12}^2(t^*) - \frac{GM}{r_{12}(t^*)} \right] \right] \int_{-\infty}^{\infty} d\tau \frac{1}{r_a^5} [c\tau \tilde{e}_{(0)} \cdot \tilde{r}_{12}(t^*) - \tilde{x}_a \cdot \tilde{r}_{12}(t^*)] [\xi^i - P_q^i x_a^q(t^*)] \\
 &\quad + 2 \frac{G}{c^3} \sum_{a=2}^2 m_a \left[\frac{\nu(m_1 - m_2)}{2M} \left[v_{12}^2(t^*) - \frac{GM}{r_{12}(t^*)} \right] \right] \int_{-\infty}^{\infty} d\tau \frac{1}{r_a^3} P_q^i r_{12}^q(t^*) + (1 \leftrightarrow 2). \quad (60)
 \end{aligned}$$

Here, we have already replaced the photon trajectory by its unperturbed approximation $\tilde{z}(\tau)_{\text{unpert.}} = \tau \tilde{l}_{(0)} + \tilde{\xi}$.

After performing the integration we obtain,

$$\begin{aligned}
 \tilde{\alpha}_{(1)(2)}^i &= 8 \frac{G}{c^4} \left[\frac{\nu(m_1 - m_2)}{2M} \left[v_{12}^2(t^*) - \frac{GM}{r_{12}(t^*)} \right] \right] \sum_{a=1}^2 \frac{m_a}{R_a^2} [(\tilde{x}_a(t^*) \cdot \tilde{r}_{12}(t^*)) - (\tilde{e}_{(0)} \cdot \tilde{x}_a(t^*)) (\tilde{e}_{(0)} \cdot \tilde{r}_{12}(t^*))] \\
 &\quad \times [\xi^i - P_q^i x_a^q(t^*)] + 4 \frac{G}{c^4} \left[\frac{\nu(m_1 - m_2)}{2M} \left[v_{12}^2(t^*) - \frac{GM}{r_{12}(t^*)} \right] \right] \sum_{a=1}^2 \frac{m_a}{R_a} P_q^i r_{12}^q(t^*), \quad (61)
 \end{aligned}$$

where the quantity R_a is given by Eq. (40). Finally, considering further applications we express the positions of the masses by their center-of-mass coordinates and expand the preceding expression about the origin of the coordinate system to the second order in (r_{12}/ξ) to obtain,

$$\begin{aligned}
 \tilde{\alpha}_{(1)(2)}^i &= 8X_2 \frac{G}{c^4} m_1 \left[\frac{\nu(m_1 - m_2)}{2M} \left[v_{12}^2 - \frac{GM}{r_{12}} \right] \right] \left\{ \left[1 - (\tilde{e}_{(0)} \cdot \tilde{n}_{12})^2 \right] \left(\frac{r_{12}}{\xi} \right)^2 + \mathcal{O} \left[\left(\frac{r_{12}}{\xi} \right)^3 \right] \right\} \frac{e^i}{\xi} \\
 &\quad + 4 \frac{G}{c^4} m_1 \left[\frac{\nu(m_1 - m_2)}{2M} \left[v_{12}^2 - \frac{GM}{r_{12}} \right] \right] \left\{ \left(\frac{r_{12}}{\xi} \right) + 2X_2 (\tilde{e}_{\xi} \cdot \tilde{n}_{12}) \left(\frac{r_{12}}{\xi} \right)^2 + \mathcal{O} \left[\left(\frac{r_{12}}{\xi} \right)^3 \right] \right\} \frac{1}{\xi} P_q^i n_{12}^q + (1 \leftrightarrow 2). \quad (62)
 \end{aligned}$$

As in Sec. VI, the quantities \tilde{n}_{12} , \tilde{v}_{12} and r_{12} are taken at the time t^* .

IX. THE TOTAL LINEAR PERTURBATION AND THE LINEAR LIGHT DEFLECTION IN THE GRAVITATIONAL FIELD OF TWO BOUNDED MASSES

To obtain the total linear perturbation $\delta l_{(1)}^i(\tau)$ we have to sum up the expressions for the linear perturbations $\delta l_{(1)I}^i(\tau)$ and $\delta l_{(1)II}^i(\tau)$ given by Eqs. (32) and (45). The resulting expression is given by

$$\begin{aligned}
\delta l_{(1)}^i(\tau) = & -2\frac{G}{c} \sum_{a=1}^2 m_a B_a [\xi^i - x_a^i(t^*)] - \frac{G}{c^2} \sum_{a=1}^2 m_a \left\{ 2A_a + \frac{4}{r_a} + \frac{4}{r_a^3} [(\vec{e}_{(0)} \cdot \vec{v}_a(t^*))c\tau^2 + [\vec{\xi} \cdot \vec{v}_a(t^*) - \vec{x}_a(t^*) \cdot \vec{v}_a(t^*)]\tau] \right\} l_{(0)}^i \\
& + \frac{G}{c^2} \sum_{a=1}^2 m_a \{ 4(\vec{e}_{(0)} \cdot \vec{v}_a(t^*))B_a - 6(\vec{e}_{(0)} \cdot \vec{v}_a(t^*))F_{a2} - 6[\vec{\xi} \cdot \vec{v}_a(t^*) - \vec{x}_a(t^*) \cdot \vec{v}_a(t^*)]F_{a3} \} [\xi^i - x_a^i(t^*)] \\
& + 2\frac{G}{c^2} \sum_{a=1}^2 m_a \left\{ A_a + \frac{2}{r_a} + \frac{2}{r_a^3} [(\vec{e}_{(0)} \cdot \vec{v}_a(t^*))c\tau^2 + [\vec{\xi} \cdot \vec{v}_a(t^*) - \vec{x}_a(t^*) \cdot \vec{v}_a(t^*)]\tau] \right\} v_a^i(t^*) \\
& + \frac{G}{c^3} \sum_{a=1}^2 m_a \left\{ 4(\vec{e}_{(0)} \cdot \vec{v}_a(t^*)) \left[A_a + \frac{1}{r_a} \right] - 6[(\vec{e}_{(0)} \cdot \vec{v}_a(t^*))F_{a1} + [\vec{\xi} \cdot \vec{v}_a(t^*) - \vec{x}_a(t^*) \cdot \vec{v}_a(t^*)]F_{a2}] \right\} l_{(0)}^i \\
& + \frac{G}{c^3} \sum_{a=1}^2 m_a \{ (\vec{e}_{(0)} \cdot \vec{v}_a(t^*))^2 [15F_{a2} - 2B_a - 15G_{a2}] + (\vec{e}_{(0)} \cdot \vec{v}_a(t^*)) [\vec{\xi} \cdot \vec{v}_a(t^*) - \vec{x}_a(t^*) \cdot \vec{v}_a(t^*)] \\
& \times [18F_{a3} - 30G_{a3}] + [\vec{\xi} \cdot \vec{v}_a(t^*) - \vec{x}_a(t^*) \cdot \vec{v}_a(t^*)]^2 [3F_{a4} - 15G_{a4}] + v_a^2(t^*) [3F_{a2} - 2B_a] \} [\xi^i - x_a^i(t^*)] \\
& + \frac{G}{c^3} \sum_{a=1}^2 m_a \left\{ (\vec{e}_{(0)} \cdot \vec{v}_a(t^*)) \left[6F_{a1} - 6A_a - \frac{4}{r_a} - \frac{4}{r_a^3} (\vec{e}_{(0)} \cdot \vec{v}_a(t^*))c\tau^2 - \frac{4}{r_a^3} [\vec{\xi} \cdot \vec{v}_a(t^*) - \vec{x}_a(t^*) \cdot \vec{v}_a(t^*)]\tau \right] \right. \\
& \left. + [\vec{\xi} \cdot \vec{v}_a(t^*) - \vec{x}_a(t^*) \cdot \vec{v}_a(t^*)] [6F_{a2} - 2B_a] \right\} v_a^i(t^*) + \mathcal{O}\left(\frac{G}{c^4}\right). \tag{63}
\end{aligned}$$

In the expression above we need only retain the terms of the order G/c^2 and G/c^3 , since the terms of the order G/c^4 are related to linear and postlinear light deflection terms of order higher than the terms which we compute in this paper. After introducing the perturbation $\delta l_{(1)}^i(\tau)$ into Eq. (18) and computing the limit for $\tau \rightarrow \infty$, we find

$$\begin{aligned}
\alpha_{(1)}^i = & \lim_{\tau \rightarrow \infty} \left[\frac{1}{c} P_q^i \delta l_{(1)}^q(\tau) \right] \\
= & -4\frac{G}{c^2} \sum_a \frac{m_a}{R_a} [\xi^i - P_q^i x_a^q(t^*)] + 4\frac{G}{c^3} \sum_{a=1}^2 \frac{m_a}{R_a} (\vec{e}_{(0)} \cdot \vec{v}_a(t^*)) [\xi^i - P_q^i x_a^q(t^*)] \\
& + 4\frac{G}{c^3} \sum_{a=1}^2 \frac{m_a}{R_a} (\vec{e}_{(0)} \cdot \vec{x}_a(t^*)) P_q^i v_a^q(t^*) - 8\frac{G}{c^3} \sum_{a=1}^2 \frac{m_a}{R_a^2} (\vec{e}_{(0)} \cdot \vec{x}_a(t^*)) [\vec{\xi} \cdot \vec{v}_a(t^*) - \vec{x}_a(t^*) \cdot \vec{v}_a(t^*)] \\
& + (\vec{e}_{(0)} \cdot \vec{x}_a(t^*)) (\vec{e}_{(0)} \cdot \vec{v}_a(t^*)) [\xi^i - P_q^i x_a^q(t^*)] - 2\frac{G}{c^4} \sum_{a=1}^2 \frac{m_a}{R_a} v_a^2(t^*) [\xi^i - P_q^i x_a^q(t^*)] \\
& + 4\frac{G}{c^4} \sum_{a=1}^2 \frac{m_a}{R_a^2} v_a^2(t^*) (\vec{e}_{(0)} \cdot \vec{x}_a(t^*))^2 [\xi^i - P_q^i x_a^q(t^*)] - 16\frac{G}{c^4} \sum_{a=1}^2 \frac{m_a}{R_a^3} (\vec{e}_{(0)} \cdot \vec{x}_a(t^*))^2 [\vec{\xi} \cdot \vec{v}_a(t^*) - \vec{x}_a(t^*) \cdot \vec{v}_a(t^*)]^2 \\
& \times [\xi^i - P_q^i x_a^q(t^*)] - 32\frac{G}{c^4} \sum_{a=1}^2 \frac{m_a}{R_a^3} (\vec{e}_{(0)} \cdot \vec{x}_a(t^*))^3 (\vec{e}_{(0)} \cdot \vec{v}_a(t^*)) [\vec{\xi} \cdot \vec{v}_a(t^*) - \vec{x}_a(t^*) \cdot \vec{v}_a(t^*)] [\xi^i - P_q^i x_a^q(t^*)] \\
& - 16\frac{G}{c^4} \sum_{a=1}^2 \frac{m_a}{R_a^3} (\vec{e}_{(0)} \cdot \vec{x}_a(t^*))^4 (\vec{e}_{(0)} \cdot \vec{v}_a(t^*))^2 [\xi^i - P_q^i x_a^q(t^*)] + 8\frac{G}{c^4} \sum_{a=1}^2 \frac{m_a}{R_a^2} (\vec{e}_{(0)} \cdot \vec{x}_a(t^*))^2 [\vec{\xi} \cdot \vec{v}_a(t^*) - \vec{x}_a(t^*) \cdot \vec{v}_a(t^*)] \cdot \vec{v}_a(t^*) \\
& + (\vec{e}_{(0)} \cdot \vec{x}_a(t^*)) (\vec{e}_{(0)} \cdot \vec{v}_a(t^*)) P_q^i v_a^q(t^*) - 4\frac{G}{c^4} \sum_{a=1}^2 \frac{m_a}{R_a^2} (\vec{e}_{(0)} \cdot \vec{x}_a(t^*))^2 (\vec{e}_{(0)} \cdot \vec{v}_a(t^*))^2 [\xi^i - P_q^i x_a^q(t^*)], \tag{64}
\end{aligned}$$

where R_a is given by Eq. (40).

Considering that the expression for the postlinear light deflection computed in this paper is given in terms of the center-of-mass-frame coordinates and as an expansion in powers of (r_{12}/ξ) , we must perform the expansion of the expression above about the origin of the coordinate system $\vec{x}_a = 0$ and express the positions of the masses in terms of their center of mass coordinates. After expressing the positions of the masses by their center-of-mass frame coordinates without considering the 1PN-corrections and expanding the expression given by Eq. (64) about the origin of the coordinate system to the third order in (r_{12}/ξ) we finally obtain,

$$\begin{aligned}
 \alpha_{(1)}^i = & \frac{Gm_1}{c^2\xi} \left\{ \left[-4 + 4X_2 \frac{1}{c} (\vec{e}_{(0)} \cdot \vec{v}_{12}) - 2X_2^2 \frac{v_{12}^2}{c^2} \right] + \left[-8X_2 (\vec{e}_\xi \cdot \vec{n}_{12}) + 8X_2^2 (\vec{e}_\xi \cdot \vec{n}_{12}) \frac{1}{c} (\vec{e}_{(0)} \cdot \vec{v}_{12}) \right. \right. \\
 & - 8X_2^2 \frac{1}{c} (\vec{e}_\xi \cdot \vec{v}_{12}) (\vec{e}_{(0)} \cdot \vec{n}_{12}) - 4X_2^3 (\vec{e}_\xi \cdot \vec{n}_{12}) \frac{v_{12}^2}{c^2} \left. \right] \left(\frac{r_{12}}{\xi} \right) + \left[4X_2^2 - 16X_2^2 (\vec{e}_\xi \cdot \vec{n}_{12})^2 - 4X_2^2 (\vec{e}_{(0)} \cdot \vec{n}_{12})^2 \right. \\
 & - 4X_2^3 \frac{1}{c} (\vec{e}_{(0)} \cdot \vec{v}_{12}) + 16X_2^3 (\vec{e}_\xi \cdot \vec{n}_{12})^2 \frac{1}{c} (\vec{e}_{(0)} \cdot \vec{v}_{12}) - 32X_2^3 \frac{1}{c} (\vec{e}_\xi \cdot \vec{v}_{12}) (\vec{e}_\xi \cdot \vec{n}_{12}) (\vec{e}_{(0)} \cdot \vec{n}_{12}) \\
 & - 4X_2^3 \frac{1}{c} (\vec{e}_{(0)} \cdot \vec{v}_{12}(t^*)) (\vec{e}_{(0)} \cdot \vec{n}_{12})^2 + 8X_2^3 (\vec{e}_{(0)} \cdot \vec{n}_{12}) \frac{1}{c} (\vec{n}_{12} \cdot \vec{v}_{12}) + 2X_2^4 \frac{v_{12}^2}{c^2} - 8X_2^4 (\vec{e}_\xi \cdot \vec{n}_{12})^2 \frac{v_{12}^2}{c^2} \\
 & + 2X_2^4 (\vec{e}_{(0)} \cdot \vec{n}_{12})^2 \frac{v_{12}^2}{c^2} - 16X_2^4 \frac{1}{c^2} (\vec{e}_\xi \cdot \vec{v}_{12})^2 (\vec{e}_{(0)} \cdot \vec{n}_{12})^2 - 4X_2^4 \frac{1}{c^2} (\vec{e}_{(0)} \cdot \vec{v}_{12})^2 (\vec{e}_{(0)} \cdot \vec{n}_{12})^2 \left. \right] \left(\frac{r_{12}}{\xi} \right)^2 \\
 & + \left[16X_2^4 \frac{1}{c} (\vec{e}_\xi \cdot \vec{v}_{12}) (\vec{e}_{(0)} \cdot \vec{n}_{12}) - 96X_2^4 \frac{1}{c} (\vec{e}_\xi \cdot \vec{v}_{12}) (\vec{e}_\xi \cdot \vec{n}_{12})^2 (\vec{e}_{(0)} \cdot \vec{n}_{12}) - 32X_2^4 (\vec{e}_\xi \cdot \vec{n}_{12}) \frac{1}{c} (\vec{e}_{(0)} \cdot \vec{v}_{12}) (\vec{e}_{(0)} \cdot \vec{n}_{12})^2 \right. \\
 & - 16X_2^4 \frac{1}{c} (\vec{e}_\xi \cdot \vec{v}_{12}) (\vec{e}_{(0)} \cdot \vec{n}_{12})^3 + 32X_2^4 (\vec{e}_\xi \cdot \vec{n}_{12}) (\vec{e}_{(0)} \cdot \vec{n}_{12}) \frac{1}{c} (\vec{n}_{12} \cdot \vec{v}_{12}) + 16X_2^5 (\vec{e}_\xi \cdot \vec{n}_{12}) (\vec{e}_{(0)} \cdot \vec{n}_{12})^2 \frac{v_{12}^2}{c^2} \\
 & - 96X_2^5 \frac{1}{c^2} (\vec{e}_\xi \cdot \vec{v}_{12})^2 (\vec{e}_\xi \cdot \vec{n}_{12}) (\vec{e}_{(0)} \cdot \vec{n}_{12})^2 - 16X_2^5 (\vec{e}_\xi \cdot \vec{n}_{12}) \frac{1}{c^2} (\vec{e}_{(0)} \cdot \vec{v}_{12})^2 (\vec{e}_{(0)} \cdot \vec{n}_{12})^2 \\
 & \left. - 32X_2^5 \frac{1}{c^2} (\vec{e}_\xi \cdot \vec{v}_{12}) (\vec{e}_{(0)} \cdot \vec{v}_{12}) (\vec{e}_{(0)} \cdot \vec{n}_{12})^3 + 32X_2^5 \frac{1}{c^2} (\vec{e}_\xi \cdot \vec{v}_{12}) (\vec{e}_{(0)} \cdot \vec{n}_{12})^2 (\vec{n}_{12} \cdot \vec{v}_{12}) \right] \left(\frac{r_{12}}{\xi} \right)^3 \\
 & + \mathcal{O} \left[\left(\frac{r_{12}}{\xi} \right)^4 \right] \left\} e_\xi^i + \frac{Gm_1}{c^2\xi} \left\{ \left[4X_2 - 4X_2^2 \frac{1}{c} (\vec{e}_{(0)} \cdot \vec{v}_{12}) + 2X_2^3 \frac{v_{12}^2}{c^2} \right] \left(\frac{r_{12}}{\xi} \right) \right. \\
 & + \left[8X_2^2 (\vec{e}_\xi \cdot \vec{n}_{12}) - 8X_2^3 (\vec{e}_\xi \cdot \vec{n}_{12}) \frac{1}{c} (\vec{e}_{(0)} \cdot \vec{v}_{12}) + 8X_2^3 \frac{1}{c} (\vec{e}_\xi \cdot \vec{v}_{12}) (\vec{e}_{(0)} \cdot \vec{n}_{12}) + 4X_2^4 (\vec{e}_\xi \cdot \vec{n}_{12}) \frac{v_{12}^2}{c^2} \right] \left(\frac{r_{12}}{\xi} \right)^2 \\
 & + \left[-4X_2^3 + 16X_2^3 (\vec{e}_\xi \cdot \vec{n}_{12})^2 + 4X_2^3 (\vec{e}_{(0)} \cdot \vec{n}_{12})^2 + 4X_2^4 \frac{1}{c} (\vec{e}_{(0)} \cdot \vec{v}_{12}) - 16X_2^4 (\vec{e}_\xi \cdot \vec{n}_{12})^2 \frac{1}{c} (\vec{e}_{(0)} \cdot \vec{v}_{12}) \right. \\
 & + 32X_2^4 \frac{1}{c} (\vec{e}_\xi \cdot \vec{v}_{12}) (\vec{e}_\xi \cdot \vec{n}_{12}) (\vec{e}_{(0)} \cdot \vec{n}_{12}) + 4X_2^4 \frac{1}{c} (\vec{e}_{(0)} \cdot \vec{v}_{12}) (\vec{e}_{(0)} \cdot \vec{n}_{12})^2 - 8X_2^4 (\vec{e}_{(0)} \cdot \vec{n}_{12}) \frac{1}{c} (\vec{n}_{12} \cdot \vec{v}_{12}) - 2X_2^5 \frac{v_{12}^2}{c^2} \\
 & \left. + 8X_2^5 (\vec{e}_\xi \cdot \vec{n}_{12})^2 \frac{v_{12}^2}{c^2} - 2X_2^5 (\vec{e}_{(0)} \cdot \vec{n}_{12})^2 \frac{v_{12}^2}{c^2} + 16X_2^5 \frac{1}{c^2} (\vec{e}_\xi \cdot \vec{v}_{12})^2 (\vec{e}_{(0)} \cdot \vec{n}_{12})^2 + 4X_2^5 \frac{1}{c^2} (\vec{e}_{(0)} \cdot \vec{v}_{12})^2 (\vec{e}_{(0)} \cdot \vec{n}_{12})^2 \right] \left(\frac{r_{12}}{\xi} \right)^3 \\
 & + \mathcal{O} \left[\left(\frac{r_{12}}{\xi} \right)^4 \right] \left\} P_q^i n_{12}^q + \frac{Gm_1}{c^2\xi} \left\{ 4X_2^2 (\vec{e}_{(0)} \cdot \vec{n}_{12}) \left(\frac{r_{12}}{\xi} \right) + \left[8X_2^3 (\vec{e}_\xi \cdot \vec{n}_{12}) (\vec{e}_{(0)} \cdot \vec{n}_{12}) + 8X_2^4 \frac{1}{c} (\vec{e}_\xi \cdot \vec{v}_{12}) (\vec{e}_{(0)} \cdot \vec{n}_{12})^2 \right] \left(\frac{r_{12}}{\xi} \right)^2 \right. \\
 & + \left[-4X_2^4 (\vec{e}_{(0)} \cdot \vec{n}_{12}) + 16X_2^4 (\vec{e}_\xi \cdot \vec{n}_{12})^2 (\vec{e}_{(0)} \cdot \vec{n}_{12}) + 4X_2^4 (\vec{e}_{(0)} \cdot \vec{n}_{12})^3 + 32X_2^5 \frac{1}{c} (\vec{e}_\xi \cdot \vec{v}_{12}) (\vec{e}_\xi \cdot \vec{n}_{12}) (\vec{e}_{(0)} \cdot \vec{n}_{12})^2 \right. \\
 & \left. + 8X_2^5 \frac{1}{c} (\vec{e}_{(0)} \cdot \vec{v}_{12}) (\vec{e}_{(0)} \cdot \vec{n}_{12})^3 - 8X_2^5 (\vec{e}_{(0)} \cdot \vec{n}_{12})^2 \frac{1}{c} (\vec{n}_{12} \cdot \vec{v}_{12}) \right] \left(\frac{r_{12}}{\xi} \right)^3 + \mathcal{O} \left[\left(\frac{r_{12}}{\xi} \right)^4 \right] \left\} \left(\frac{v_{12}}{c} \right) P_q^i e_{v_{12}}^q + (1 \leftrightarrow 2), \quad (65)
 \end{aligned}$$

where the quantities \vec{n}_{12} , \vec{v}_{12} and r_{12} are taken at the time t^* . Notice that in Eq. (65) we chose the order of the expansion in an arbitrary manner in order to show the structure of the terms belonging to the linear light deflection. In concrete applications we have to choose the order

of the expansion of the linear light deflection in accordance with the accuracy reached by the postlinear light deflection.

To obtain the total expression for the angle of light deflection linear in G we have to add to the preceding

equation the correction terms arising from the part of $\tilde{\alpha}_{(1)(2)}^i$, which is linear in G . The expression for $\tilde{\alpha}_{(1)(2)}^i$ is given by Eq. (62).

X. THE POSTLINEAR LIGHT DEFLECTION

The final expression for the postlinear light deflection in the gravitational field of two bounded masses is obtained

$$\begin{aligned}
\alpha_{(2)}^i = & \left\{ \frac{G^2 m_1^2}{c^4 \xi^2} \left[-\frac{15}{4} \pi - X_2 (8(\tilde{e}_{(0)} \cdot \tilde{n}_{12}) + \frac{45}{4} \pi (\tilde{e}_\xi \cdot \tilde{n}_{12})) \left(\frac{r_{12}}{\xi} \right) + \mathcal{O} \left[\left(\frac{r_{12}}{\xi} \right)^2 \right] \right] \right. \\
& + \frac{G^2 m_2^2}{c^4 \xi^2} \left[-\frac{15}{4} \pi + X_1 (8(\tilde{e}_{(0)} \cdot \tilde{n}_{12}) + \frac{45}{4} \pi (\tilde{e}_\xi \cdot \tilde{n}_{12})) \left(\frac{r_{12}}{\xi} \right) + \mathcal{O} \left[\left(\frac{r_{12}}{\xi} \right)^2 \right] \right] \left. \right\} e_\xi^i + 4 \frac{G^2 m_1 m_2}{c^4 \xi r_{12}} e_\xi^i \\
& + \frac{G^2 m_1 m_2}{c^4 \xi^2} \left\{ -\frac{15}{2} \pi - 4(X_1 - X_2)(\tilde{e}_\xi \cdot \tilde{n}_{12}) + \left[-\frac{2}{3} - (X_1 - X_2)^2 - 3(X_1^2 + X_2^2) + 2(X_1^3 + X_2^3) \right. \right. \\
& + 8(X_1 - X_2)(\tilde{e}_{(0)} \cdot \tilde{n}_{12}) + \frac{45}{4} \pi (X_1 - X_2)(\tilde{e}_\xi \cdot \tilde{n}_{12}) + \left[\frac{2}{3} + \frac{5}{3}(X_1^2 + X_2^2) - 6(X_1^3 + X_2^3) + 3(X_1^4 + X_2^4) \right. \\
& + \left. \frac{4}{3} X_1 X_2 - 2X_1^2 X_2^2 + 2X_1 X_2 (X_1^2 + X_2^2) \right] (\tilde{e}_{(0)} \cdot \tilde{n}_{12})^2 + \left[\frac{44}{3} - 8X_1 X_2 (3 + X_1 X_2) + 4(X_1^4 + X_2^4) \right] (\tilde{e}_\xi \cdot \tilde{n}_{12})^2 \\
& + \frac{16}{3} [X_1^2 + X_2^2 - X_1 X_2 - 3(X_1^3 + X_2^3)] (\tilde{e}_{(0)} \cdot \tilde{n}_{12})^2 (\tilde{e}_\xi \cdot \tilde{n}_{12})^2 - 16(X_1^3 + X_2^3) (\tilde{e}_\xi \cdot \tilde{n}_{12})^4 \\
& + \left. \frac{2}{3} [2(X_1^2 + X_2^2) - 2X_1 X_2 - 3(X_1^3 + X_2^3)] (\tilde{e}_{(0)} \cdot \tilde{n}_{12})^4 \right] \left(\frac{r_{12}}{\xi} \right) + \left[\mathcal{O} \left(\frac{r_{12}}{\xi} \right)^2 \right] \left. \right\} e_\xi^i \\
& + \frac{G^2 m_1^2}{c^4 \xi^2} \left[\frac{15}{4} \pi X_2 \left(\frac{r_{12}}{\xi} \right) + X_2^2 (8(\tilde{e}_{(0)} \cdot \tilde{n}_{12}) + \frac{45}{4} \pi (\tilde{e}_\xi \cdot \tilde{n}_{12})) \left(\frac{r_{12}}{\xi} \right)^2 + \mathcal{O} \left[\left(\frac{r_{12}}{\xi} \right)^3 \right] \right] P_q^i n_{12}^q \\
& + \frac{G^2 m_2^2}{c^4 \xi^2} \left[-\frac{15}{4} \pi X_1 \left(\frac{r_{12}}{\xi} \right) + X_1^2 (8(\tilde{e}_{(0)} \cdot \tilde{n}_{12}) + \frac{45}{4} \pi (\tilde{e}_\xi \cdot \tilde{n}_{12})) \left(\frac{r_{12}}{\xi} \right)^2 + \mathcal{O} \left[\left(\frac{r_{12}}{\xi} \right)^3 \right] \right] P_q^i n_{12}^q \\
& + \frac{G^2 m_1 m_2}{c^4 \xi^2} \left\{ 2(X_1 - X_2) + \left[-\frac{15}{4} \pi (X_1 - X_2) + \left[-\frac{22}{3} + 4X_1 X_2 (3 + X_1 X_2) + 4(X_1^3 + X_2^3) - 2(X_1^4 + X_2^4) \right] \right. \right. \\
& \times (\tilde{e}_\xi \cdot \tilde{n}_{12}) + \left. \left. -\frac{8}{3} [1 - 3X_1 X_2] (\tilde{e}_\xi \cdot \tilde{n}_{12}) (\tilde{e}_{(0)} \cdot \tilde{n}_{12})^2 \right] \left(\frac{r_{12}}{\xi} \right) + \mathcal{O} \left[\left(\frac{r_{12}}{\xi} \right)^2 \right] \right\} P_q^i n_{12}^q + \alpha_{(2)V}^i, \tag{66}
\end{aligned}$$

where $\alpha_{(2)V}^i$ is given by the part of $\tilde{\alpha}_{(1)(2)}^i$ [i.e. of Eq. (62)], which is quadratic in G . Here, it should be pointed out that some terms which belong to the postlinear light deflection given by Eq. (66) (e.g. the term $4 \frac{G^2 m_1 m_2}{c^4 \xi r_{12}} e_\xi^i$) are related to terms in v_{12}^2 of the linear light deflection given by Eq. (65) through the virial theorem. In the next section we shall group these terms together before computing the limit $r_{12} \rightarrow 0$ in Eqs. (65) and (66) in order to remove the formal divergences. As in the case of the linear light deflection the order of the expansion in Eq. (66) was chosen in an arbitrary manner in order to show the structure of the terms belonging to the postlinear light deflection. In concrete applications the order of the expansion of the postlinear light deflection is to be chosen in accordance with the accuracy reached by the linear light deflection.

by summing up the parts of the light deflection, which are given in the preceding sections and in Appendices A, B, and C. The final expression for the angle of light deflection quadratic in G to the first order in (r_{12}/ξ) , in which the positions of the masses are expressed in the center-of-mass frame coordinates, is given by

XI. RESULTS

The resulting expressions for the angle of light deflection linear and quadratic in G are given in Secs. IX and X, respectively. To study the important features of the derived results, let us consider special cases. The results take a particularly simple form for the following cases.

A. The value of one of the two masses is equal to zero

If in Eqs. (65) and (66) we put the value of one of the masses equal to zero (e.g. $m_1 = M$, $m_2 = 0$) we obtain expressions for the light deflection angle for a static point-like mass,

$$\alpha_{(1)(E)}^i = -4 \frac{GM_{\text{ADM}}}{c^2 \xi} e_\xi^i, \tag{67}$$

$$\alpha_{(2)(E-S)}^i = -\frac{15}{4}\pi \frac{G^2 M_{\text{ADM}}^2}{c^4 \xi^2} e_\xi^i, \quad (68)$$

where in this case the ADM mass M_{ADM} is equal to the mass M . The deflection angle linear in G is the well known ‘‘Einstein angle’’ (see [1]). The light deflection angle given by Eq. (68) is the post-post-Newtonian light deflection for a pointlike mass, which was obtained for the first time by Epstein and Shapiro and by other authors in 1980 (see [12,13]).

B. The light deflection when $r_{12} \rightarrow 0$

In this subsection we are going to compute the limit of the expression for the linear and postlinear angle of light deflection [Eqs. (65) and (66)] in the event that the distance r_{12} between the components of the binary goes towards zero (i.e. $r_{12} \rightarrow 0$). As we explained at the end of the preceding section, we have to group together the terms of Eqs. (62), (65), and (66) in an appropriate manner in order to remove the formal divergences. By inspection, it is clear to see that the remaining terms in Eqs. (62), (65), and (66) are given by

$$\begin{aligned} \alpha_{(1)}^i &= \frac{Gm_1}{c^2 \xi} \left\{ \left[-4 + 4X_2 \frac{1}{c} (\vec{e}_{(0)} \cdot \vec{v}_{12}) - 2X_2^2 \frac{v_{12}^2}{c^2} \right] \right. \\ &\quad \left. - 4X_2^3 (\vec{e}_\xi \cdot \vec{n}_{12}) \frac{v_{12}^2}{c^2} \left(\frac{r_{12}}{\xi} \right) \right\} e_\xi^i \\ &\quad + \frac{Gm_2}{c^2 \xi} \left\{ \left[-4 - 4X_1 \frac{1}{c} (\vec{e}_{(0)} \cdot \vec{v}_{12}) - 2X_1^2 \frac{v_{12}^2}{c^2} \right] \right. \\ &\quad \left. + 4X_1^3 (\vec{e}_\xi \cdot \vec{n}_{12}) \frac{v_{12}^2}{c^2} \left(\frac{r_{12}}{\xi} \right) \right\} e_\xi^i \\ &\quad + \left\{ 2 \frac{Gm_1}{c^2 \xi} X_2^3 - 2 \frac{Gm_2}{c^2 \xi} X_1^3 \right\} \frac{v_{12}^2}{c^2} \left(\frac{r_{12}}{\xi} \right) P_{q,n}^i, \quad (69) \end{aligned}$$

$$\begin{aligned} \tilde{\alpha}_{(1)(2)}^i &= \left[\frac{\nu(m_1 - m_2)}{2M} \left[v_{12}^2 - \frac{GM}{r_{12}} \right] \right] \\ &\quad \times \left\{ 4 \frac{Gm_1}{c^4 \xi} + 4 \frac{Gm_2}{c^4 \xi} \right\} \left(\frac{r_{12}}{\xi} \right) P_{q,n}^i, \quad (70) \end{aligned}$$

$$\begin{aligned} \alpha_{(2)}^i &= \left\{ -\frac{15}{4}\pi \frac{G^2 m_1^2}{c^4 \xi^2} - \frac{15}{4}\pi \frac{G^2 m_2^2}{c^4 \xi^2} + 4 \frac{G^2 m_1 m_2}{c^4 \xi r_{12}} \right. \\ &\quad \left. + \frac{G^2 m_1 m_2}{c^4 \xi^2} \left[-\frac{15}{2}\pi - 4(X_1 - X_2) (\vec{e}_\xi \cdot \vec{n}_{12}) \right] \right\} e_\xi^i \\ &\quad + 2 \frac{G^2 m_1 m_2}{c^4 \xi^2} (X_1 - X_2) P_{q,n}^i. \quad (71) \end{aligned}$$

After grouping together the terms that are related through the virial theorem in Eqs. (69)–(71) we get

$$\begin{aligned} \alpha_{(1)}^i &= \left\{ \left[-4 \frac{G}{c^2 \xi} (m_1 + m_2) - 2 \frac{G}{c^2 \xi} [m_1 X_2^2 + m_2 X_1^2] \frac{v_{12}^2}{c^2} \right. \right. \\ &\quad \left. \left. + 4 \frac{G^2 m_1 m_2}{c^4 \xi r_{12}} \right] + \left[-4 \frac{Gm_1}{c^2 \xi} X_2^3 \frac{v_{12}^2}{c^2} + 4 \frac{Gm_2}{c^2 \xi} X_1^3 \frac{v_{12}^2}{c^2} \right. \right. \\ &\quad \left. \left. - 4 \frac{G^2 m_1 m_2}{c^4 \xi r_{12}} (X_1 - X_2) \right] (\vec{e}_\xi \cdot \vec{n}_{12}) \left(\frac{r_{12}}{\xi} \right) \right\} e_\xi^i \\ &\quad + 2 \frac{G}{c^2 \xi} \left\{ [m_1 X_2^3 - m_2 X_1^3] \right. \\ &\quad \left. + \frac{\nu(m_1 - m_2)}{M} (m_1 + m_2) \right\} \frac{v_{12}^2}{c^2} \left(\frac{r_{12}}{\xi} \right) P_{q,n}^i, \quad (72) \end{aligned}$$

$$\begin{aligned} \alpha_{(2)}^i &= -\frac{15}{4}\pi \frac{G^2}{c^4 \xi^2} \{m_1^2 + 2m_1 m_2 + m_2^2\} e_\xi^i \\ &\quad + 2 \left\{ \frac{G^2 m_1 m_2}{c^4 \xi r_{12}} (X_1 - X_2) - \frac{\nu(m_1 - m_2)}{M} \frac{GM}{r_{12}} \right. \\ &\quad \left. \times \left[\frac{Gm_1}{c^4 \xi} + \frac{Gm_2}{c^4 \xi} \right] \right\} \left(\frac{r_{12}}{\xi} \right) P_{q,n}^i. \quad (73) \end{aligned}$$

After simplifying Eqs. (72) and (73) we finally obtain

$$\begin{aligned} \alpha_{(1)}^i &= -4 \frac{GM_{\text{ADM}}}{c^2 \xi} e_\xi^i + 4 \frac{Gm_1 m_2}{c^4 \xi M^2} (m_1 - m_2) \\ &\quad \times \left[v_{12}^2 - \frac{GM}{r_{12}} \right] (\vec{e}_\xi \cdot \vec{n}_{12}) \left(\frac{r_{12}}{\xi} \right) e_\xi^i, \quad (74) \end{aligned}$$

$$\alpha_{(2)}^i = -\frac{15}{4}\pi \frac{G^2 M^2}{c^4 \xi^2} e_\xi^i, \quad (75)$$

where

$$M_{\text{ADM}} = M + \frac{1}{2}\mu \frac{v_{12}^2}{c^2} - \frac{Gm_1 m_2}{c^2 r_{12}} \quad (76)$$

is the ADM mass of the system with $\mu = m_1 m_2 / M$ and $M = m_1 + m_2$. In Eqs. (69)–(74), the quantities \vec{n}_{12} , v_{12} and r_{12} are taken at the time t^* . Note that the second term in Eq. (74) goes to zero when $r_{12} \rightarrow 0$ since the expression in brackets remains finite. At the end, we recover the Einstein angle [i.e. Eq. (67)] with the ADM mass as given by Eq. (76) as well as the Epstein-Shapiro angle [i.e. Eq. (68)].

C. The values of the two masses are equal and the light ray is originally orthogonal to the orbital plane of the binary

In this case we choose $M/2$ for the value of the masses in Eqs. (65) and (66) and assume that the light ray is originally propagating orthogonal to the orbital plane of the two bounded masses (i.e. $\vec{e}_{(0)} \cdot \vec{n}_{12} = 0$, $\vec{e}_{(0)} \cdot \vec{v}_{12} = 0$). After introducing the ADM mass in the resulting expression for the angle of light deflection linear and quadratic in G and rearranging the terms, we finally find

$$\begin{aligned}
\alpha_{(1)\perp}^i = & \frac{GM_{\text{ADM}}}{c^2\xi} \left\{ -4 + [1 - 4(\vec{e}_\xi \cdot \vec{n}_{12})^2] \left(\frac{r_{12}}{\xi}\right)^2 + \left[-\frac{1}{4} + 3(\vec{e}_\xi \cdot \vec{n}_{12})^2 - 4(\vec{e}_\xi \cdot \vec{n}_{12})^4 \right] \left(\frac{r_{12}}{\xi}\right)^4 \right. \\
& + \left[\frac{1}{16} - \frac{3}{2}(\vec{e}_\xi \cdot \vec{n}_{12})^2 + 5(\vec{e}_\xi \cdot \vec{n}_{12})^4 - 4(\vec{e}_\xi \cdot \vec{n}_{12})^6 \right] \left(\frac{r_{12}}{\xi}\right)^6 + \left[-\frac{1}{64} + \frac{5}{8}(\vec{e}_\xi \cdot \vec{n}_{12})^2 - \frac{15}{4}(\vec{e}_\xi \cdot \vec{n}_{12})^4 \right. \\
& + 7(\vec{e}_\xi \cdot \vec{n}_{12})^6 - 4(\vec{e}_\xi \cdot \vec{n}_{12})^8 \left. \right] \left(\frac{r_{12}}{\xi}\right)^8 + \left[\frac{1}{256} - \frac{15}{64}(\vec{e}_\xi \cdot \vec{n}_{12})^2 + \frac{35}{16}(\vec{e}_\xi \cdot \vec{n}_{12})^4 - 7(\vec{e}_\xi \cdot \vec{n}_{12})^6 + 9(\vec{e}_\xi \cdot \vec{n}_{12})^8 \right. \\
& - 4(\vec{e}_\xi \cdot \vec{n}_{12})^{10} \left. \right] \left(\frac{r_{12}}{\xi}\right)^{10} + \left[-\frac{1}{1024} + \frac{21}{256}(\vec{e}_\xi \cdot \vec{n}_{12})^2 - \frac{35}{32}(\vec{e}_\xi \cdot \vec{n}_{12})^4 + \frac{21}{4}(\vec{e}_\xi \cdot \vec{n}_{12})^6 - \frac{45}{4}(\vec{e}_\xi \cdot \vec{n}_{12})^8 \right. \\
& + 11(\vec{e}_\xi \cdot \vec{n}_{12})^{10} - 4(\vec{e}_\xi \cdot \vec{n}_{12})^{12} \left. \right] \left(\frac{r_{12}}{\xi}\right)^{12} + \mathcal{O}\left[\left(\frac{r_{12}}{\xi}\right)^{14}\right] \left. \right\} e_\xi^i + \frac{GM_{\text{ADM}}}{c^2\xi} (\vec{e}_\xi \cdot \vec{n}_{12}) \left[2\left(\frac{r_{12}}{\xi}\right)^2 \right. \\
& + [-1 + 2(\vec{e}_\xi \cdot \vec{n}_{12})^2] \left(\frac{r_{12}}{\xi}\right)^4 + \left[\frac{3}{8} - 2(\vec{e}_\xi \cdot \vec{n}_{12})^2 + 2(\vec{e}_\xi \cdot \vec{n}_{12})^4 \right] \left(\frac{r_{12}}{\xi}\right)^6 \\
& + \left[-\frac{1}{8} + \frac{5}{4}(\vec{e}_\xi \cdot \vec{n}_{12})^2 - 3(\vec{e}_\xi \cdot \vec{n}_{12})^4 + 2(\vec{e}_\xi \cdot \vec{n}_{12})^6 \right] \left(\frac{r_{12}}{\xi}\right)^8 + \left[\frac{5}{128} - \frac{5}{8}(\vec{e}_\xi \cdot \vec{n}_{12})^2 + \frac{21}{8}(\vec{e}_\xi \cdot \vec{n}_{12})^4 \right. \\
& - 4(\vec{e}_\xi \cdot \vec{n}_{12})^6 + 2(\vec{e}_\xi \cdot \vec{n}_{12})^8 \left. \right] \left(\frac{r_{12}}{\xi}\right)^{10} + \left[-\frac{3}{256} + \frac{35}{128}(\vec{e}_\xi \cdot \vec{n}_{12})^2 - \frac{7}{4}(\vec{e}_\xi \cdot \vec{n}_{12})^4 + \frac{9}{2}(\vec{e}_\xi \cdot \vec{n}_{12})^6 \right. \\
& \left. - 5(\vec{e}_\xi \cdot \vec{n}_{12})^8 + 2(\vec{e}_\xi \cdot \vec{n}_{12})^{10} \right] \left(\frac{r_{12}}{\xi}\right)^{12} + \mathcal{O}\left[\left(\frac{r_{12}}{\xi}\right)^{14}\right] \left. \right\} P_q^i n_{12}^q \quad (77)
\end{aligned}$$

and

$$\begin{aligned}
\alpha_{(2)\perp}^i = & \frac{G^2 M_{\text{ADM}}^2}{c^4 \xi^2} \left\{ -\frac{15}{4} \pi - \frac{1}{6} [1 - 7(\vec{e}_\xi \cdot \vec{n}_{12})^2 + 6(\vec{e}_\xi \cdot \vec{n}_{12})^4] \left(\frac{r_{12}}{\xi}\right) + \frac{75}{256} \pi [10 - 31(\vec{e}_\xi \cdot \vec{n}_{12})^2] \left(\frac{r_{12}}{\xi}\right)^2 \right. \\
& \left. + \mathcal{O}\left[\left(\frac{r_{12}}{\xi}\right)^3\right] \right\} e_\xi^i + \frac{G^2 M_{\text{ADM}}^2}{c^4 \xi^2} (\vec{e}_\xi \cdot \vec{n}_{12}) \left\{ -\frac{1}{3} \left(\frac{r_{12}}{\xi}\right) + \frac{465}{128} \pi \left(\frac{r_{12}}{\xi}\right)^2 + \mathcal{O}\left[\left(\frac{r_{12}}{\xi}\right)^3\right] \right\} P_q^i n_{12}^q \quad (78)
\end{aligned}$$

where in this case the ADM mass is given by

$$M_{\text{ADM}} = M \left[1 + \frac{1}{4} \left(\frac{v_{12}^2}{2c^2} - \frac{GM}{c^2 r_{12}} \right) \right]. \quad (79)$$

The expression for the linear light deflection given by Eq. (77) was expanded to the order $(r_{12}/\xi)^{12}$ in order to reach the accuracy of the postlinear light deflection given in Eq. (78). Note that an expansion to the order $(r_{12}/\xi)^{12}$ in the linear part of the light deflection, which is of the same accuracy as an expansion to the second order in (r_{12}/ξ) in the postlinear part, implies that $(r_{12}/\xi) \sim (GM_{\text{ADM}}/c^2\xi)^{1/10}$. In Eqs. (77)–(79), the quantities \vec{n}_{12} , v_{12} and r_{12} are taken at the time t^* . Note, that in this case the correction arising from the shift of the IPN–center of mass with respect to the Newtonian center of mass [see Eq. (62)] vanishes.

D. The values of the two masses are equal and the light ray is originally parallel to the orbital plane of the binary

In this case we choose the value of the masses equal to $M/2$ in Eqs. (65) and (66) and assume that the light ray is originally propagating parallel to the orbital plane of the binary (i.e. $\vec{e}_\xi \cdot \vec{n}_{12} = 0$). After introducing the ADM mass as given by Eq. (79) and rearranging the terms, we finally find:

$$\begin{aligned}
 \alpha_{(1)\parallel}^i = & \frac{GM_{\text{ADM}}}{c^2 \xi} \left\{ -4 + [1 - (\vec{e}_{(0)} \cdot \vec{n}_{12})^2] \left(\frac{r_{12}}{\xi} \right)^2 + \left[-\frac{1}{4} + \frac{1}{2} (\vec{e}_{(0)} \cdot \vec{n}_{12})^2 - \frac{1}{4} (\vec{e}_{(0)} \cdot \vec{n}_{12})^4 \right] \left(\frac{r_{12}}{\xi} \right)^4 \right. \\
 & + \left[\frac{1}{16} - \frac{3}{16} (\vec{e}_{(0)} \cdot \vec{n}_{12})^2 + \frac{3}{16} (\vec{e}_{(0)} \cdot \vec{n}_{12})^4 - \frac{1}{16} (\vec{e}_{(0)} \cdot \vec{n}_{12})^6 \right] \left(\frac{r_{12}}{\xi} \right)^6 \\
 & + \left[-\frac{1}{64} + \frac{1}{16} (\vec{e}_{(0)} \cdot \vec{n}_{12})^2 - \frac{3}{32} (\vec{e}_{(0)} \cdot \vec{n}_{12})^4 + \frac{1}{16} (\vec{e}_{(0)} \cdot \vec{n}_{12})^6 - \frac{1}{64} (\vec{e}_{(0)} \cdot \vec{n}_{12})^8 \right] \left(\frac{r_{12}}{\xi} \right)^8 \\
 & + \left[\frac{1}{256} - \frac{5}{256} (\vec{e}_{(0)} \cdot \vec{n}_{12})^2 + \frac{5}{128} (\vec{e}_{(0)} \cdot \vec{n}_{12})^4 - \frac{5}{128} (\vec{e}_{(0)} \cdot \vec{n}_{12})^6 + \frac{5}{256} (\vec{e}_{(0)} \cdot \vec{n}_{12})^8 - \frac{1}{256} (\vec{e}_{(0)} \cdot \vec{n}_{12})^{10} \right] \left(\frac{r_{12}}{\xi} \right)^{10} \\
 & + \left[-\frac{1}{1024} + \frac{3}{512} (\vec{e}_{(0)} \cdot \vec{n}_{12})^2 - \frac{15}{1024} (\vec{e}_{(0)} \cdot \vec{n}_{12})^4 + \frac{5}{256} (\vec{e}_{(0)} \cdot \vec{n}_{12})^6 - \frac{15}{1024} (\vec{e}_{(0)} \cdot \vec{n}_{12})^8 + \frac{3}{512} (\vec{e}_{(0)} \cdot \vec{n}_{12})^{10} \right. \\
 & \left. - \frac{1}{1024} (\vec{e}_{(0)} \cdot \vec{n}_{12})^{12} \right] \left(\frac{r_{12}}{\xi} \right)^{12} + \mathcal{O} \left[\left(\frac{r_{12}}{\xi} \right)^{14} \right] \left\} e_{\xi}^i + \frac{GM_{\text{ADM}}}{c^3 \xi} \left\{ -(\vec{e}_{(0)} \cdot \vec{v}_{12}) \left(\frac{r_{12}}{\xi} \right) + \left[\frac{1}{4} (\vec{e}_{(0)} \cdot \vec{v}_{12}) [1 + (\vec{e}_{(0)} \cdot \vec{n}_{12})^2] \right. \right. \\
 & \left. \left. - \frac{1}{2} (\vec{n}_{12} \cdot \vec{v}_{12}) (\vec{e}_{(0)} \cdot \vec{n}_{12}) \right] \left(\frac{r_{12}}{\xi} \right)^3 + \left[-\frac{1}{16} (\vec{e}_{(0)} \cdot \vec{v}_{12}) [1 + 2(\vec{e}_{(0)} \cdot \vec{n}_{12})^2 - 3(\vec{e}_{(0)} \cdot \vec{n}_{12})^4] \right. \right. \\
 & \left. \left. + \frac{1}{4} (\vec{n}_{12} \cdot \vec{v}_{12}) (\vec{e}_{(0)} \cdot \vec{n}_{12}) [1 - (\vec{e}_{(0)} \cdot \vec{n}_{12})^2] \right] \left(\frac{r_{12}}{\xi} \right)^5 + \left[\frac{1}{64} (\vec{e}_{(0)} \cdot \vec{v}_{12}) [1 + 3(\vec{e}_{(0)} \cdot \vec{n}_{12})^2 - 9(\vec{e}_{(0)} \cdot \vec{n}_{12})^4 \right. \right. \\
 & \left. \left. + 5(\vec{e}_{(0)} \cdot \vec{n}_{12})^6] - \frac{3}{32} (\vec{n}_{12} \cdot \vec{v}_{12}) (\vec{e}_{(0)} \cdot \vec{n}_{12}) [1 - 2(\vec{e}_{(0)} \cdot \vec{n}_{12})^2 + (\vec{e}_{(0)} \cdot \vec{n}_{12})^4] \right] \left(\frac{r_{12}}{\xi} \right)^7 + \mathcal{O} \left[\left(\frac{r_{12}}{\xi} \right)^9 \right] \right\} P_q^i n_{12}^q \\
 & + \frac{GM_{\text{ADM}}}{c^3 \xi} (\vec{e}_{(0)} \cdot \vec{n}_{12}) \left[\frac{1}{2} \left(\frac{r_{12}}{\xi} \right) - \frac{1}{8} [1 - (\vec{e}_{(0)} \cdot \vec{n}_{12})^2] \left(\frac{r_{12}}{\xi} \right)^3 + \frac{1}{32} [1 - 2(\vec{e}_{(0)} \cdot \vec{n}_{12})^2 + (\vec{e}_{(0)} \cdot \vec{n}_{12})^4] \left(\frac{r_{12}}{\xi} \right)^5 \right. \\
 & \left. - \frac{1}{128} [1 - 3(\vec{e}_{(0)} \cdot \vec{n}_{12})^2 + 3(\vec{e}_{(0)} \cdot \vec{n}_{12})^4 - (\vec{e}_{(0)} \cdot \vec{n}_{12})^6] \left(\frac{r_{12}}{\xi} \right)^7 + \mathcal{O} \left[\left(\frac{r_{12}}{\xi} \right)^9 \right] \right\} P_q^i v_{12}^q \\
 & + \frac{GM_{\text{ADM}}}{c^4 \xi} (\vec{e}_{(0)} \cdot \vec{n}_{12})^2 \left\{ \frac{1}{4} [v_{12}^2 - (\vec{e}_{(0)} \cdot \vec{v}_{12})^2] \left(\frac{r_{12}}{\xi} \right)^2 + \mathcal{O} \left[\left(\frac{r_{12}}{\xi} \right)^4 \right] \right\} e_{\xi}^i
 \end{aligned} \tag{80}$$

and

$$\begin{aligned}
 \alpha_{(2)\parallel}^i = & \frac{G^2 M_{\text{ADM}}^2}{c^4 \xi^2} \left\{ -\frac{15}{4} \pi - \frac{1}{24} [4 + (\vec{e}_{(0)} \cdot \vec{n}_{12})^2 + (\vec{e}_{(0)} \cdot \vec{n}_{12})^4] \left(\frac{r_{12}}{\xi} \right) + \frac{3}{256} \pi [250 - 797(\vec{e}_{(0)} \cdot \vec{n}_{12})^2] \left(\frac{r_{12}}{\xi} \right)^2 \right. \\
 & \left. + \mathcal{O} \left[\left(\frac{r_{12}}{\xi} \right)^3 \right] \right\} e_{\xi}^i + \frac{G^2 M_{\text{ADM}}^2}{c^4 \xi^2} \left\{ 4(\vec{e}_{(0)} \cdot \vec{n}_{12}) \left(\frac{r_{12}}{\xi} \right)^2 + \mathcal{O} \left[\left(\frac{r_{12}}{\xi} \right)^3 \right] \right\} P_q^i n_{12}^q
 \end{aligned} \tag{81}$$

where, the quantities \vec{n}_{12} , \vec{v}_{12} and r_{12} are taken at the time t^* .

In Eq. (80) the components e_{ξ}^i , $P_q^i n_{12}^q$ and $P_q^i v_{12}^q$ of the linear light deflection were expanded to the order $(r_{12}/\xi)^{12}$, $(r_{12}/\xi)^7$ and $(r_{12}/\xi)^7$, respectively, in order to reach the accuracy of the postlinear light deflection given in Eq. (81). As in the preceding subsection here the correction arising from the shift of the 1PN-center of mass with respect to the Newtonian center of mass [see Eq. (62)] vanishes.

Finally, we apply the formulas for the angle of light deflection (67)–(81) to the double pulsar PSR J0737-3039. The parameters of the pulsar PSR J0737-3039 (e.g. see [14]) are given in Table I.

We compute the angle of light deflection for the cases when the distance between the two stars r_{12} is maximal and minimal. In our computations we assume that the masses of the binary's components are equal, i.e. that the mass ratio R is equal to 1. For the impact parameter, we choose $\xi = 5r_{12}$. In order to compute the angle of light deflection

TABLE I. The parameters of PSR J0737-3039.

Orbital period P_b (day)	0.102 251 563 (1)
Eccentricity e	0.087 779 (5)
Total mass $m_A + m_B$ (M_{\odot})	2.588(3)
Mass ratio $R \equiv m_A/m_B$	1.069(6)

we have first to calculate r_{12} and v_{12} . Note that we only use Newtonian relations, since the uncertainties in observational data, although small, are nonetheless greater than the corrections that post-Newtonian corrections would yield.

To compute r_{12} we have to calculate the semimajor axis of the elliptical orbit by means of the following equation (because of the low accuracy of the observational data, only Newtonian relations can be used),

$$a = \sqrt[3]{\frac{G(m_1 + m_2)T^2}{4\pi^2}}, \quad (82)$$

where a denotes the semimajor axis and T the orbital period. The preceding equation follows from Kepler's third law (e.g. see [15]). The relationships between the distances $r_{12\max}$, $r_{12\min}$, and the semimajor axis a are given by,

$$r_{12\max} = a(1 + e), \quad (83)$$

$$r_{12\min} = a(1 - e). \quad (84)$$

We obtain the corresponding velocities to $r_{12\max}$ and $r_{12\min}$ from the equations given by

$$v_{12\min} = 8\pi^3 \sqrt{\frac{G(m_1 + m_2)}{a} \frac{(1 - e)}{(1 + e)}}, \quad (85)$$

and

$$v_{12\max} = 8\pi^3 \sqrt{\frac{G(m_1 + m_2)}{a} \frac{(1 + e)}{(1 - e)}}, \quad (86)$$

respectively.

After introducing the parameters of PSR J0737-3039 into the preceding equations and into the formulas for the light deflection given in this section we find the results presented in Table II.

Table II shows the angles of light deflection computed by means of Eqs. (78) and (81) corresponding to the Epstein-Shapiro angles of $\alpha_{(E-S)}^i = -1.55 \times 10^{-6} e_\xi^i$ arcsec and $\alpha_{(E-S)}^i = -2.20 \times 10^{-6} e_\xi^i$ arcsec. If we define the corrections to the Epstein-Shapiro angle that we calculated in this paper by $\delta\alpha_{(2)\perp,\parallel}^i = \alpha_{(2)\perp,\parallel}^i - \alpha_{(E-S)}^i$, we find:

- (1) light ray originally orthogonal to the orbital plane
 $\delta\alpha_{(2)\perp}^i = (-1.0 \times 10^{-7} e_\xi^i + 5 \times 10^{-8} P_q^i n_{12}^q)$ arcsec
for $\xi = 5r_{12\max}$, $\delta\alpha_{(2)\perp}^i = (-1.5 \times 10^{-7} e_\xi^i + 7 \times 10^{-8} P_q^i n_{12}^q)$ arcsec for $\xi = 5r_{12\min}$.
- (2) light ray originally parallel to the orbital plane
 $\delta\alpha_{(2)\parallel}^i = (-1.1 \times 10^{-7} e_\xi^i + 2 \times 10^{-8} P_q^i n_{12}^q)$ arcsec
for $\xi = 5r_{12\max}$, $\delta\alpha_{(2)\parallel}^i = (-1.6 \times 10^{-7} e_\xi^i + 3 \times 10^{-8} P_q^i n_{12}^q)$ arcsec for $\xi = 5r_{12\min}$.

From the values of $\delta\alpha_{(2)\perp}^i$ and $\delta\alpha_{(2)\parallel}^i$ we see that the corrections to the Epstein-Shapiro light deflection angle are slightly smaller for the case when the light ray is originally orthogonal to the orbital plane. Details related to the measurement of the corrections to the Epstein-Shapiro angle will be discussed in the next section.

XII. DISCUSSION AND CONCLUSIONS

The angle of light deflection in the postlinear gravitational field of two bounded pointlike masses has been computed to the second order in G/c^2 . Both the light source and the observer were assumed to be located at infinity in an asymptotically flat space. The light deflection linear in G has been exactly computed. It was shown that the expression obtained for the linear light deflection is fully equivalent to the expression given by Kopeikin

TABLE II. The angles of light deflection linear and quadratic in G are given in arcsec. We denote by $\alpha_{(1)(E)}^i$ and $\alpha_{(2)(E-S)}^i$ the Einstein angle and the Epstein-Shapiro angle. For the light ray originally orthogonal to the orbital plane we assume that $\vec{e}_\xi \cdot \vec{n}_{12} = 1$. For the light ray originally parallel to the orbital plane we assume that $\vec{e}_{(0)} \cdot \vec{n}_{12} = 1$.

	$r_{12\max} = 9.56 \times 10^{10}$ cm $v_{12\min} = 5.72 \times 10^7$ cm/s $\xi = 5r_{12\max}$	$r_{12\min} = 8.02 \times 10^{10}$ cm $v_{12\max} = 6.83 \times 10^7$ cm/s $\xi = 5r_{12\min}$
$\alpha_{(1)(E)}^i$	$-0.659 e_\xi^i$	$-0.785 e_\xi^i$
$\alpha_{(2)(E-S)}^i$	$-1.55 \times 10^{-6} e_\xi^i$	$-2.20 \times 10^{-6} e_\xi^i$
$\alpha_{(1)\perp}^i$	$-0.679 e_\xi^i + 0.013 P_q^i n_{12}^q$	$-0.809 e_\xi^i + 0.016 P_q^i n_{12}^q$
$\alpha_{(2)\perp}^i$	$-1.65 \times 10^{-6} e_\xi^i + 5 \times 10^{-8} P_q^i n_{12}^q$	$-2.35 \times 10^{-6} e_\xi^i + 7 \times 10^{-8} P_q^i n_{12}^q$
$\alpha_{(1)\parallel}^i$	$-0.659 e_\xi^i$	$-0.785 e_\xi^i$
$\alpha_{(2)\parallel}^i$	$-1.66 \times 10^{-6} e_\xi^i + 2 \times 10^{-8} P_q^i n_{12}^q$	$-2.36 \times 10^{-6} e_\xi^i + 3 \times 10^{-8} P_q^i n_{12}^q$

and Schäfer in [5] in the event that the velocities of the masses are small with respect to the velocity of light and the retarded times in the expression of Kopeikin and Schäfer are close to the time of closest approach of the unperturbed light ray to the origin of the coordinate system. To evaluate the integrals related to the light deflection quadratic in G , which could not be integrated by means of elementary functions, we resorted to a series expansion of the integrands. For this reason the resulting expressions for the angle of light deflection quadratic in G are only valid for the case when the distance between the two masses r_{12} is smaller than the impact parameter ξ (i.e. $r_{12}/\xi < 1$). The final result is given as a power series in r_{12}/ξ . The expression for the angle of light deflection in terms of the ADM mass to the order G^2/c^4 including a power expansion to the second order in (r_{12}/ξ) , in which $(r_{12}/\xi) \sim (GM_{\text{ADM}}/c^2\xi)^{1/10}$ is being assumed, is given in an explicit form for a binary with equal masses in the event that the light ray is originally orthogonal to the orbital plane of the binary. The expression for the angle of light deflection in terms of the ADM mass to the same order is also given for a binary with equal masses in the event that the light ray is originally parallel to the orbital plane of the binary. For a light ray originally propagating orthogonal to the orbital plane of a binary with equal masses the deflection angle takes a particularly simple form. In the case when one of the masses is equal to zero, we obtain the ‘‘Einstein angle’’ and the ‘‘Shapiro-Epstein light deflection angle’’, as we do when $r_{12} \rightarrow 0$. Application of the derived formulas for the deflection angle to the double pulsar PSR J0737-3039 has shown that the corrections to the ‘‘Einstein angle’’ are of the order 10^{-2} arcsec for the case when $r_{12}/\xi = 0.2$, see Table II. The corrections to the ‘‘Epstein-Shapiro light deflection angle’’ lie between 10^{-7} and 10^{-8} arcsec, see Table II. We conclude that the corrections to the ‘‘Epstein-Shapiro light deflection angle’’ are beyond the sensitivity of the current astronomical interferometers. Nevertheless, taking into account that the interferometer for the planned mission LATOR [16] will be able to measure light deflection angles of the order 10^{-8} arcsec, we believe that the corrections to the ‘‘Epstein-Shapiro light deflection angle’’ computed in the present work might be measured by space-borne interferometers in the foreseeable future.

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APPENDIX A: THE POSTLINEAR LIGHT DEFLECTION $\alpha_{(2)1}^i$

In the integrals that are given in this appendix as well as in Appendices B and C we already replaced the photon trajectory by its unperturbed approximation $\vec{z}(\tau) = \tau\vec{l}_{(0)} + \vec{\xi}$, where $\vec{l}_{(0)}$ is given by $\vec{l}_{(0)} = c\vec{e}_{(0)}$. The distances r_1 and r_2 are given by

$$r_a = [c^2\tau^2 + \xi^2 - 2c\tau\vec{e}_{(0)} \cdot \vec{x}_a(t^*) - 2\vec{\xi} \cdot \vec{x}_a(t^*) + x_a^2(t^*)]^{1/2},$$

with $a = 1$ and $a = 2$ for the distances r_1 and r_2 . The distance S is defined by

$$S = r_1 + r_2 + r_{12}.$$

Here, $r_{12} = |\vec{x}_1(t^*) - \vec{x}_2(t^*)|$ is the distance between the two masses m_1 and m_2 at the time t^* and the unit vector \vec{n}_{12} is given by

$$\vec{n}_{12} = \frac{1}{r_{12}}[\vec{x}_1(t^*) - \vec{x}_2(t^*)].$$

The positions of the masses in the center-of-mass frame without considering the 1PN-corrections are given by

$$\vec{x}_1 = X_2\vec{r}_{12}(t^*)$$

and

$$\vec{x}_2 = -X_1\vec{r}_{12}(t^*).$$

The integrals resulting from the introduction of the post-linear metric coefficients (19) and (20) into the expression for $\alpha_{(2)1}^i$ given by Eq. (42) are:

$$\begin{aligned}
\alpha_{(2)I}^i(\vec{\xi}) = & 2 \frac{G^2 m_1^2}{c^3} \int_{-\infty}^{\infty} d\tau \frac{1}{r_1^6} [c\tau - X_2 \vec{e}_{(0)} \cdot \vec{r}_{12}(t^*)]^2 [\xi^i - X_2 P_q^i r_{12}^q(t^*)] - 4 \frac{G^2 m_1 m_2}{c^3} \int_{-\infty}^{\infty} d\tau \frac{1}{r_{12} r_1 S^2} [\xi^i - X_2 P_q^i r_{12}^q(t^*)] \\
& + \frac{G^2 m_1 m_2}{c^3} \int_{-\infty}^{\infty} d\tau \frac{1}{r_{12}^3} \left[\frac{1}{r_2} - \frac{1}{r_1} \right] [\xi^i - X_2 P_q^i r_{12}^q(t^*)] + \frac{1}{2} \frac{G^2 m_1 m_2}{c^3} \int_{-\infty}^{\infty} d\tau \frac{1}{r_{12}^3} \left[\frac{1}{r_1} - \frac{r_2^2}{r_1^3} \right] [\xi^i - X_2 P_q^i r_{12}^q(t^*)] \\
& + 16 \frac{G^2 m_1 m_2}{c^3} \int_{-\infty}^{\infty} d\tau \frac{1}{r_1 r_2 S^3} (\vec{e}_{(0)} \cdot \vec{n}_{12}) [c\tau + X_1 \vec{e}_{(0)} \cdot \vec{r}_{12}(t^*)] [\xi^i - X_2 P_q^i r_{12}^q(t^*)] \\
& + 8 \frac{G^2 m_1 m_2}{c^3} \int_{-\infty}^{\infty} d\tau \frac{1}{r_1 S^3} (\vec{e}_{(0)} \cdot \vec{n}_{12})^2 [\xi^i - X_2 P_q^i r_{12}^q(t^*)] + 4 \frac{G^2 m_1 m_2}{c^3} \int_{-\infty}^{\infty} d\tau \frac{1}{r_{12} r_1 S^2} (\vec{e}_{(0)} \cdot \vec{n}_{12})^2 \\
& \times [\xi^i - X_2 P_q^i r_{12}^q(t^*)] + \frac{5}{2} \frac{G^2 m_1 m_2}{c^3} \int_{-\infty}^{\infty} d\tau \frac{1}{r_{12} r_1^3} [\xi^i - X_2 P_q^i r_{12}^q(t^*)] \\
& - 16 \frac{G^2 m_1 m_2}{c^3} \int_{-\infty}^{\infty} d\tau \frac{1}{r_{12} r_1 S^3} (\vec{e}_{(0)} \cdot \vec{n}_{12}) [c\tau - X_2 \vec{e}_{(0)} \cdot \vec{r}_{12}(t^*)] P_q^i r_{12}^q(t^*) \\
& - 8 \frac{G^2 m_1 m_2}{c^3} \int_{-\infty}^{\infty} d\tau \frac{1}{r_{12}^2 r_1 S^2} (\vec{e}_{(0)} \cdot \vec{n}_{12}) [c\tau - X_2 \vec{e}_{(0)} \cdot \vec{r}_{12}(t^*)] P_q^i r_{12}^q(t^*) \\
& - 4 \frac{G^2 m_1 m_2}{c^3} \int_{-\infty}^{\infty} d\tau \frac{1}{r_1 r_2 S^2} [\xi^i - X_2 P_q^i r_{12}^q(t^*)] + 4 \frac{G^2 m_1 m_2}{c^3} \int_{-\infty}^{\infty} d\tau \frac{1}{r_1 r_2^3 S^2} [c\tau + X_1 \vec{e}_{(0)} \cdot \vec{r}_{12}(t^*)]^2 \\
& \times [\xi^i - X_2 P_q^i r_{12}^q(t^*)] + 8 \frac{G^2 m_1 m_2}{c^3} \int_{-\infty}^{\infty} d\tau \frac{1}{r_1 r_2^2 S^3} [c\tau + X_1 \vec{e}_{(0)} \cdot \vec{r}_{12}(t^*)]^2 [\xi^i - X_2 P_q^i r_{12}^q(t^*)] \\
& - 4 \frac{G^2 m_1 m_2}{c^3} \int_{-\infty}^{\infty} d\tau \left[\frac{1}{r_{12} r_1 S^2} - \frac{1}{r_{12} r_1^3 S^2} [c\tau - X_2 \vec{e}_{(0)} \cdot \vec{r}_{12}(t^*)]^2 \right] P_q^i r_{12}^q(t^*) \\
& + 8 \frac{G^2 m_1 m_2}{c^3} \int_{-\infty}^{\infty} d\tau \frac{1}{r_{12} r_1^2 S^3} [c\tau - X_2 \vec{e}_{(0)} \cdot \vec{r}_{12}(t^*)]^2 P_q^i r_{12}^q(t^*) \\
& + 8 \frac{G^2 m_1 m_2}{c^3} \int_{-\infty}^{\infty} d\tau \frac{1}{r_{12} r_1 r_2 S^3} [c\tau - X_2 \vec{e}_{(0)} \cdot \vec{r}_{12}(t^*)] [c\tau + X_1 \vec{e}_{(0)} \cdot \vec{r}_{12}(t^*)] P_q^i r_{12}^q(t^*) + (1 \leftrightarrow 2). \tag{A1}
\end{aligned}$$

APPENDIX B: THE POSTLINEAR LIGHT DEFLECTION $\alpha_{(2)II}^i$

After introducing the expressions for the perturbations $\delta l_{(1)}^i(\tau)$ given by Eq. (63) and the metric coefficients (21)–(23) into the expression for $\alpha_{(2)II}^i$ given by Eq. (43) we find,

$$\begin{aligned}
\alpha_{(2)II}^i(\vec{\xi}) = & 12 \frac{G^2 m_1^2}{c^3} \int_{-\infty}^{\infty} d\tau \frac{1}{r_1^4} [\xi^i - X_2 P_q^i r_{12}^q(t^*)] + 12 \frac{G^2 m_1 m_2}{c^3} \int_{-\infty}^{\infty} d\tau \frac{1}{r_1^3 r_2} [\xi^i - X_2 P_q^i r_{12}^q(t^*)] \\
& + 4 \frac{G^2 m_1^2}{c^3} \int_{-\infty}^{\infty} d\tau \frac{1}{r_1^3} A_1 [\xi^i - X_2 P_q^i r_{12}^q(t^*)] + 4 \frac{G^2 m_1 m_2}{c^3} \int_{-\infty}^{\infty} d\tau \frac{1}{r_1^3} A_2 [\xi^i - X_2 P_q^i r_{12}^q(t^*)] \\
& - 4 \frac{G^2 m_1^2}{c^3} \int_{-\infty}^{\infty} d\tau \frac{1}{r_1^3} B_1 [\vec{e}_{(0)} \cdot (X_2 \vec{r}_{12}(t^*))] [\xi^i - X_2 P_q^i r_{12}^q(t^*)] \\
& + 4 \frac{G^2 m_1 m_2}{c^3} \int_{-\infty}^{\infty} d\tau \frac{1}{r_1^3} B_2 [\vec{e}_{(0)} \cdot (X_1 \vec{r}_{12}(t^*))] [\xi^i - X_2 P_q^i r_{12}^q(t^*)] \\
& - 8 \frac{G^2 m_1^2}{c^3} \int_{-\infty}^{\infty} d\tau \frac{1}{r_1^3} B_1 [c\tau - X_2 \vec{e}_{(0)} \cdot \vec{r}_{12}(t^*)] [\xi^i - X_2 P_q^i r_{12}^q(t^*)] \\
& - 8 \frac{G^2 m_1 m_2}{c^3} \int_{-\infty}^{\infty} d\tau \frac{1}{r_2^3} B_1 [c\tau + X_1 \vec{e}_{(0)} \cdot \vec{r}_{12}(t^*)] [\xi^i - X_2 P_q^i r_{12}^q(t^*)] + (1 \leftrightarrow 2). \tag{B1}
\end{aligned}$$

The functions A_1 , A_2 , B_1 and B_2 are given in Sec. IV.

APPENDIX C: THE POST-LINEAR LIGHT DEFLECTION $\alpha_{(2)IV}^i$

In this appendix we give the integrals of the order G^2/c^4 resulting from the introduction of the perturbation $\delta z_{(1)}^i(\tau)$ given by Eq. (54) into the expression for $\alpha_{(2)III}^i$ given by Eq. (57).

$$\begin{aligned}
 \alpha_{(2)IV}^i(\vec{\xi}) = & -12 \frac{G^2 m_1^2}{c^3} r_1^2(0, t^*) \int_{-\infty}^{\infty} d\tau \frac{1}{r_1^5} \mathcal{B}_1[\xi^i - X_2 P_q^i r_{12}^q(t^*)] - 12 \frac{G^2 m_1 m_2}{c^3} (\vec{r}_1(0, t^*) \cdot \vec{r}_2(0, t^*)) \\
 & \times \int_{-\infty}^{\infty} d\tau \frac{1}{r_1^5} \mathcal{B}_2[\xi^i - X_2 P_q^i r_{12}^q(t^*)] + 12 \frac{G^2 m_1^2}{c^3} (X_2 \vec{e}_{(0)} \cdot \vec{r}_{12}(t^*))^2 \int_{-\infty}^{\infty} d\tau \frac{1}{r_1^5} \mathcal{B}_1[\xi^i - X_2 P_q^i r_{12}^q(t^*)] \\
 & - 12 \frac{G^2 m_1 m_2}{c^3} (X_1 X_2) (\vec{e}_{(0)} \cdot \vec{r}_{12}(t^*))^2 \int_{-\infty}^{\infty} d\tau \frac{1}{r_1^5} \mathcal{B}_2[\xi^i - X_2 P_q^i r_{12}^q(t^*)] \\
 & - 12 \frac{G^2 m_1^2}{c^3} \int_{-\infty}^{\infty} d\tau \frac{1}{r_1^5} \ln \left[\frac{c\tau - X_2 \vec{e}_{(0)} \cdot \vec{r}_{12}(t^*) + r_1(\tau, t^*)}{r_1(0, t^*) - X_2 \vec{e}_{(0)} \cdot \vec{r}_{12}(t^*)} \right] [c\tau - X_2 \vec{e}_{(0)} \cdot \vec{r}_{12}(t^*)] [\xi^i - X_2 P_q^i r_{12}^q(t^*)] \\
 & - 12 \frac{G^2 m_1 m_2}{c^3} \int_{-\infty}^{\infty} d\tau \frac{1}{r_1^5} \ln \left[\frac{c\tau + X_1 \vec{e}_{(0)} \cdot \vec{r}_{12}(t^*) + r_2(\tau, t^*)}{r_2(0, t^*) + X_1 \vec{e}_{(0)} \cdot \vec{r}_{12}(t^*)} \right] [c\tau - X_2 \vec{e}_{(0)} \cdot \vec{r}_{12}(t^*)] [\xi^i - X_2 P_q^i r_{12}^q(t^*)] \\
 & + 4 \frac{G^2 m_1^2}{c^3} \int_{-\infty}^{\infty} d\tau \frac{1}{r_1^3} \mathcal{B}_1[\xi^i - X_2 P_q^i r_{12}^q(t^*)] + 4 \frac{G^2 m_1 m_2}{c^3} \int_{-\infty}^{\infty} d\tau \frac{1}{r_2^3} \mathcal{B}_1[\xi^i - X_2 P_q^i r_{12}^q(t^*)] + (1 \leftrightarrow 2), \quad (C1)
 \end{aligned}$$

where the functions \mathcal{B}_1 and \mathcal{B}_2 are given in Sec. VII.

APPENDIX D: THE LINEAR LIGHT DEFLECTION TERMS ARISING FROM THE TERMS OF $h_{00}^{(1)}$ AND $h_{pq}^{(1)}$ WHICH CONTAIN THE ACCELERATIONS OF THE MASSES

As we mentioned in Sec. III, the terms of the metric coefficients $h_{00}^{(1)}$ and $h_{pq}^{(1)}$ which contain the accelerations of the masses were introduced into the metric quadratic in G after substituting the accelerations by explicit functions of coordinate positions of the masses by means of the Newtonian equations of motion. To get the light deflection terms arising from these terms in a form suitable for the comparison of our computations with the linear light deflection computed by Kopeikin and Schäfer [5], we compute here the light deflection resulting from these terms before performing the substitution of the accelerations. The terms of $h_{00}^{(1)}$ and $h_{pq}^{(1)}$ which contain the accelerations are given by

$$\begin{aligned}
 \tilde{h}_{00}^{(1)} &= -\frac{G}{c^4} \sum_{a=1}^2 m_a (\vec{n}_a \cdot \vec{a}_a) = -\frac{G}{c^4} \sum_{a=1}^2 m_a \frac{(\vec{r}_a \cdot \vec{a}_a)}{r_a}, \\
 \tilde{h}_{pq}^{(1)} &= \tilde{h}_{00}^{(1)} \delta_{pq}. \quad (D1)
 \end{aligned}$$

From Eq. (18) it follows that the linear light deflection is given by

$$\alpha_{(1)}^i = \lim_{\tau \rightarrow \infty} \left\{ \frac{1}{c} P_q^i \delta l_{(1)}^q(\tau) \right\}, \quad (D2)$$

where $\delta l_{(1)}^q(\tau)$ is given by Eq. (15). After introducing the metric coefficients (D1) into the equation above we obtain

$$\begin{aligned}
 \tilde{\alpha}_{(1)}^i &= \frac{G}{c^3} \sum_{a=1}^2 m_a \int_{-\infty}^{\infty} d\tau \frac{1}{r_a^3} [c\tau (\vec{e}_{(0)} \cdot \vec{a}_a(t^*)) \\
 & + \vec{\xi} \cdot \vec{a}_a(t^*) - \vec{x}_a(t^*) \cdot \vec{a}_a(t^*)] [\xi^i - P_q^i x_a^q(t^*)] \\
 & - \frac{G}{c^3} \sum_{a=1}^2 m_a \int_{-\infty}^{\infty} d\tau \frac{1}{r_a} P_q^i a_a^q(t^*). \quad (D3)
 \end{aligned}$$

As in Eq. (50) the second integral in the preceding equation diverges. After performing the Taylor expansion of the second integrand about the origin of the coordinate system $\vec{x}_a = 0$ up to the second order and taking into account the Newtonian center-of-mass theorem we perform the integration of Eq. (D3). As result we find

$$\begin{aligned}
 \tilde{\alpha}_{(1)}^i &= 2 \frac{G}{c^4} \sum_{a=1}^2 \frac{m_a}{R_a} (\vec{e}_{(0)} \cdot \vec{x}_a(t^*)) (\vec{e}_{(0)} \cdot \vec{a}_a(t^*)) \\
 & \times [\xi^i - P_q^i x_a^q(t^*)] \\
 & + 2 \frac{G}{c^4} \sum_{a=1}^2 \frac{m_a}{R_a} [\vec{\xi} \cdot \vec{a}_a(t^*) - \vec{x}_a(t^*) \cdot \vec{a}_a(t^*)] \\
 & \times [\xi^i - P_q^i x_a^q(t^*)] + \frac{G}{c^4} \sum_{a=1}^2 m_a \left\{ -2 \frac{(\vec{\xi} \cdot \vec{x}_a(t^*))}{\xi^2} \right. \\
 & \left. - \frac{(\vec{e}_{(0)} \cdot \vec{x}_a(t^*))^2}{\xi^2} + \frac{x_a^2(t^*)}{\xi^2} - 2 \frac{(\vec{\xi} \cdot \vec{x}_a(t^*))^2}{\xi^4} \right\} P_q^i a_a^q(t^*). \quad (D4)
 \end{aligned}$$

APPENDIX E: THE ANGLE OF LIGHT DEFLECTION IN THE LINEAR GRAVITATIONAL FIELD OF TWO ARBITRARILY MOVING POINTLIKE MASSES

In this appendix we show that the expression for the light deflection angle in the linear gravitational field of two arbitrarily moving pointlike masses computed by Kopeikin and Schäfer [5] is fully equivalent to the linear

$$\begin{aligned} \alpha_{(1)}^i(\tau) = & -2 \frac{G}{c^2} \sum_{a=1}^2 \frac{m_a [1 - \frac{\vec{e}_{(0)} \cdot \vec{v}_a(s_a)}{c}]^2 [r_a(\tau, s_a) + (\vec{e}_{(0)} \cdot \vec{r}_a(\tau, s_a))] P_q^i r_a^q(\tau, s_a)}{\sqrt{1 - \frac{v^2(s_a)}{c^2}} [r_a^2(\tau, s_a) - (\vec{e}_{(0)} \cdot \vec{r}_a(\tau, s_a))^2] [r_a(\tau, s_a) - \frac{\vec{v}_a(s_a) \cdot \vec{r}_a(\tau, s_a)}{c}]} \\ & - 4 \frac{G}{c^3} \sum_{a=1}^2 \frac{m_a [1 - \frac{\vec{e}_{(0)} \cdot \vec{v}_a(s_a)}{c}]}{\sqrt{1 - \frac{v^2(s_a)}{c^2}} [r_a(\tau, s_a) - \frac{\vec{v}_a(s_a) \cdot \vec{r}_a(\tau, s_a)}{c}]} P_q^i v_a^q(s_a), \end{aligned} \quad (\text{E1})$$

where $\vec{v}_a(s_a)$ is the velocity of the a th mass, $\vec{e}_{(0)}$ is the unit vector tangent to the unperturbed light ray, $\vec{r}_a(\tau, s_a)$ is given by $\vec{r}_a(\tau, s_a) = \vec{z}(\tau) - \vec{x}_a(s_a)$ and $r_a(\tau, s_a)$ is the Euclidean norm of $\vec{r}_a(\tau, s_a)$. Here, s_a is the retarded time for the a th mass defined by the light-cone equation for s_a . The light-cone equation for s_a is given by

$$s_a + r_a(\tau, s_a) = \tau + t^*. \quad (\text{E2})$$

From Eq. (E1) it follows that for an observer located at infinity, the angle of light deflection is given by

$$\begin{aligned} \alpha_{(1)}^i &= \lim_{\tau \rightarrow \infty} \alpha_{(1)}^i(\tau) \\ &= -4 \frac{G}{c^2} \sum_{a=1}^2 \frac{m_a [1 - \frac{\vec{e}_{(0)} \cdot \vec{v}_a(s_a)}{c}]}{\sqrt{1 - \frac{v^2(s_a)}{c^2}} R_a(s_a)} [\xi^i - P_q^i x_a^q(s_a)], \end{aligned} \quad (\text{E3})$$

where the quantity $R_a(s_a)$ is given by

$$R_a(s_a) = r_a^2(0, s_a) - (\vec{e}_{(0)} \cdot \vec{x}_a(s_a))^2. \quad (\text{E4})$$

Here, it is worthwhile to note that the preceding expression for the light deflection angle is equivalent to the expression given by Eq. (139) in [5].

With the help of the light-cone equation (E2) and the relationship for the time of closest approach from [5],

$$\begin{aligned} t^* &= t - \frac{1}{c} (\vec{e}_{(0)} \cdot \vec{z}(t)) \\ &\simeq t - \frac{1}{c} \vec{e}_{(0)} \cdot \vec{r}_a(t, s_a) - \frac{1}{c} \vec{e}_{(0)} \cdot \vec{x}_a(s_a), \end{aligned} \quad (\text{E5})$$

light deflection computed in this paper in the event that the velocities of the masses are small with respect to the velocity of light and that the retarded times in the Kopeikin and Schäfer expression are close to the time of closest approach t^* . The light deflection angle given in [5] in the case that the source is located at infinity and the observer at $\vec{z}(\tau)$ is given by

it was shown that

$$s_a - t^* = \frac{1}{c} \vec{e}_{(0)} \cdot \vec{x}_a(s_a) - \frac{d_a^2}{2r_a} \simeq \frac{1}{c} \vec{e}_{(0)} \cdot \vec{x}_a(s_a), \quad (\text{E6})$$

where d_a is given by $d_a = |\vec{z}_a(s_a) - \vec{x}_a(s_a)|$.

If the velocities of the masses are small with respect to the velocity of light and the retarded times do not differ significantly from the time of closest approach t^* , we are allowed to use the Taylor expansion of the quantity

$$x_a^i(s_a) \simeq x_a^i(t^*) + v_a^i(t^*)(s_a - t^*) + \frac{1}{2} a_a^i(t^*)(s_a - t^*)^2 \quad (\text{E7})$$

After substituting into Eq. (E6), we find

$$\begin{aligned} s_a - t^* &\simeq \frac{1}{c} (\vec{e}_{(0)} \cdot \vec{x}_a(t^*)) + \frac{1}{c} (\vec{e}_{(0)} \cdot \vec{v}_a(t^*)) (s_a - t^*) \\ &\quad + \frac{1}{2c} (\vec{e}_{(0)} \cdot \vec{a}_a(t^*)) (s_a - t^*)^2. \end{aligned} \quad (\text{E8})$$

Now we solve Eq. (E7) iteratively with respect to $(s_a - t^*)$ to obtain

$$\begin{aligned} s_a - t^* &\simeq \frac{1}{c} \vec{e}_{(0)} \cdot \vec{x}_a(t^*) + \frac{1}{c^2} (\vec{e}_{(0)} \cdot \vec{x}_a(t^*)) (\vec{e}_{(0)} \cdot \vec{v}_a(t^*)) \\ &\quad + \mathcal{O}\left(\frac{1}{c^3}\right). \end{aligned} \quad (\text{E9})$$

After performing the Taylor expansion of the expression for the light deflection angle given by Eq. (E3) and taking into account Eq. (E9), we finally obtain

$$\begin{aligned}
 \alpha_{(1)}^i = & -4 \frac{G}{c^2} \sum_{a=1}^2 \frac{m_a}{R_a} [\xi^i - P_q^i x_a^q(t^*)] + 4 \frac{G}{c^3} \sum_{a=1}^2 \frac{m_a}{R_a} (\vec{e}_{(0)} \cdot \vec{v}_a(t^*)) [\xi^i - P_q^i x_a^q(t^*)] + 4 \frac{G}{c^3} \sum_{a=1}^2 \frac{m_a}{R_a} (\vec{e}_{(0)} \cdot \vec{x}_a(t^*)) P_q^i v_a^q(t^*) \\
 & - 8 \frac{G}{c^3} \sum_{a=1}^2 \frac{m_a}{R_a^2} (\vec{e}_{(0)} \cdot \vec{x}_a(t^*)) [\vec{\xi} \cdot \vec{v}_a(t^*) - \vec{x}_a(t^*) \cdot \vec{v}_a(t^*) + (\vec{e}_{(0)} \cdot \vec{x}_a(t^*)) (\vec{e}_{(0)} \cdot \vec{v}_a(t^*))] [\xi^i - P_q^i x_a^q(t^*)] \\
 & - 2 \frac{G}{c^4} \sum_{a=1}^2 \frac{m_a}{R_a} v_a^2(t^*) [\xi^i - P_q^i x_a^q(t^*)] + 4 \frac{G}{c^4} \sum_{a=1}^2 \frac{m_a}{R_a^2} v_a^2(t^*) (\vec{e}_{(0)} \cdot \vec{x}_a(t^*))^2 [\xi^i - P_q^i x_a^q(t^*)] \\
 & - 16 \frac{G}{c^4} \sum_{a=1}^2 \frac{m_a}{R_a^3} (\vec{e}_{(0)} \cdot \vec{x}_a(t^*))^2 [\vec{\xi} \cdot \vec{v}_a(t^*) - \vec{x}_a(t^*) \cdot \vec{v}_a(t^*)]^2 [\xi^i - P_q^i x_a^q(t^*)] \\
 & - 32 \frac{G}{c^4} \sum_{a=1}^2 \frac{m_a}{R_a^3} (\vec{e}_{(0)} \cdot \vec{x}_a(t^*))^3 (\vec{e}_{(0)} \cdot \vec{v}_a(t^*)) [\vec{\xi} \cdot \vec{v}_a(t^*) - \vec{x}_a(t^*) \cdot \vec{v}_a(t^*)] [\xi^i - P_q^i x_a^q(t^*)] \\
 & - 16 \frac{G}{c^4} \sum_{a=1}^2 \frac{m_a}{R_a^3} (\vec{e}_{(0)} \cdot \vec{x}_a(t^*))^4 (\vec{e}_{(0)} \cdot \vec{v}_a(t^*))^2 [\xi^i - P_q^i x_a^q(t^*)] \\
 & + 8 \frac{G}{c^4} \sum_{a=1}^2 \frac{m_a}{R_a^2} (\vec{e}_{(0)} \cdot \vec{x}_a(t^*))^2 [\vec{\xi} \cdot \vec{v}_a(t^*) - \vec{x}_a(t^*) \cdot \vec{v}_a(t^*) + (\vec{e}_{(0)} \cdot \vec{x}_a(t^*)) (\vec{e}_{(0)} \cdot \vec{v}_a(t^*))] P_q^i v_a^q(t^*) \\
 & - 4 \frac{G}{c^4} \sum_{a=1}^2 \frac{m_a}{R_a^2} (\vec{e}_{(0)} \cdot \vec{x}_a(t^*))^2 (\vec{e}_{(0)} \cdot \vec{v}_a(t^*))^2 [\xi^i - P_q^i x_a^q(t^*)] \\
 & - 4 \frac{G}{c^4} \sum_{a=1}^2 \frac{m_a}{R_a^2} (\vec{e}_{(0)} \cdot \vec{x}_a(t^*))^2 [\vec{\xi} \cdot \vec{a}_a(t^*) - \vec{x}_a(t^*) \cdot \vec{a}_a(t^*) + (\vec{e}_{(0)} \cdot \vec{x}_a(t^*)) (\vec{e}_{(0)} \cdot \vec{a}_a(t^*))] [\xi^i - P_q^i x_a^q(t^*)] \\
 & + 2 \frac{G}{c^4} \sum_{a=1}^2 \frac{m_a}{R_a} (\vec{e}_{(0)} \cdot \vec{x}_a(t^*))^2 P_q^i a_a^q(t^*) + 4 \frac{G}{c^4} \sum_{a=1}^2 \frac{m_a}{R_a} (\vec{e}_{(0)} \cdot \vec{x}_a(t^*)) (\vec{e}_{(0)} \cdot \vec{a}_a(t^*)) [\xi^i - P_q^i x_a^q(t^*)]. \tag{E10}
 \end{aligned}$$

It is easy to see that the expression above is equal to the sum of Eqs. (51), (64), and (D4). Note, that to obtain the term in $P_q^i a_a^q(t^*)$ of Eq. (E10), it is better to sum up the corresponding terms in Eqs. (50) and (D3) before performing the integration in order to remove the formal divergences. After summing up and performing the integration of these terms we get:

$$\begin{aligned}
 & \frac{G}{c} \sum_{a=1}^2 m_a \int_{-\infty}^{\infty} d\tau \frac{\tau^2}{r_a^3} P_q^i a_a^q(t^*) - \frac{G}{c^3} \sum_{a=1}^2 m_a \int_{-\infty}^{\infty} d\tau \frac{1}{r_a} P_q^i a_a^q(t^*) = \frac{G}{c^3} \sum_{a=1}^2 m_a \left\{ \int_{-\infty}^{\infty} d\tau \frac{1}{r_a^3} [c^2 \tau^2 - r_a^2] P_q^i a_a^q(t^*) \right\} \\
 & = -2 \frac{G}{c^4} \sum_{a=1}^2 m_a P_q^i a_a^q(t^*) + 2 \frac{G}{c^4} \sum_{a=1}^2 \frac{m_a}{R_a} (\vec{e}_{(0)} \cdot \vec{x}_a(t^*))^2 P_q^i a_a^q(t^*), \tag{E11}
 \end{aligned}$$

where the first term on the last line vanishes as consequence of the Newtonian center-of-mass theorem.

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