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Oscillatons formed by nonlinear gravity

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Oscillatons are solutions of the coupled Einstein-Klein-Gordon equations that are globally regular and asymptotically flat. By means of a Legendre transformation we are able to visualize the behavior of the corresponding objects in nonlinear gravity where the scalar field has been absorbed by means of the conformal mapping.

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I. INTRODUCTION

Nonlinear modifications of the Einstein-Hilbert action have a long history [1] (for a discussion on recent issues see [2,3], and references therein). They have been of interest for a variety of reasons. For instance, it has been claimed, that they could be good renormalizable models for quantum gravity [4–6]. Also some nonlinear Lagrangians can be chosen with the property that the field equations of the metric are second order, these are the so-called Lovelock actions [7–11], which arise from dimensional reduction of the Euler characteristic. Also string theory predicts an effective gravitational action containing the usual Einstein term plus higher-order perturbative corrections in the curvature [12,13]. When the Lagrangian is an exact function of the Ricci scalar, a mapping exists to the usual Hilbert-Einstein Lagrangian with a specific self-interacting real scalar field; the equations of motion of the latter being the so-called Einstein-Klein-Gordon (EKG) system. This idea has been used, in different manners to relate well-known scalar inflationary potentials with pure-gravity higher-order curvature scalar Lagrangians [14–19].

On the other hand, the EKG system has been studied in many situations. In particular, it has been shown that there are spherically symmetric solutions that are *globally regular* and asymptotically flat; these solutions are called *oscillatons* [20–27] .

Our aim in this paper is to investigate the mapping of oscillatons to nonlinear gravity (NLG) theories. We shall search for the corresponding objects in these theories, and whether we can get interesting information relating the results in the NLG theory and those in the EKG system.

We will limit our models to specific scalar potentials for which NLG theories can be constructed, and we will study the ‘‘mapped’’ objects that may arise in them. It is not argued that these gravity models would explain all the range of gravity experiments [29]. Probably, as in string theory [12,13,30], higher-order perturbative corrections to the Einstein-Hilbert action depending also on combinations of the Riemann and Ricci tensors should also be present in a more realistic theory, and the models considered here would be, possibly, a limit of these theories.

To set the stage for our analysis, we begin by recalling some properties of the NLG and EKG systems. For the former, we write

$$\mathcal{L} = \sqrt{-g} f(R), \tag{1}$$

where f is an arbitrary function of the scalar curvature R. The field equations derived from (1) are

$$f'(R)R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} f(R) + (g_{\mu\nu}\Box - \nabla_\mu \nabla_\nu) f'(R) = 0, \tag{2}$$

in which $f'(R) \equiv (df/dR) \neq 0$.

Following a well-known procedure [14,15,31], we now establish the connection between this NLG theory and the EKG formalism. We first define the new variables

$$e^{\sqrt{(2/3)\kappa_0}\Phi} = f'(R), \qquad \tilde{g}_{\mu\nu} = f'(R) g_{\mu\nu}, \tag{3}$$

where the conformal transformation between the metrics is invertible and then well defined. Inserting this into Eq. (2), we directly obtain the EKG equations

$$\tilde{G}_{\mu\nu} := \tilde{R}_{\mu\nu} - \frac{1}{2}\tilde{g}_{\mu\nu}\tilde{R} = \kappa_0 T_{\mu\nu}(\Phi), \tag{4a}$$

$$\tilde{\Box}\Phi - \frac{dV}{d\Phi} = 0, \tag{4b}$$

where

$$T_{\mu\nu}(\Phi) = \Phi_{,\mu}\Phi_{,\nu} - \frac{1}{2}\tilde{g}_{\mu\nu}[\Phi^{,\sigma}\Phi_{,\sigma} + 2V(\Phi)], \tag{5}$$

is the energy-momentum tensor of the scalar field Φ endowed with a scalar field potential $V(\Phi)$. The scalar field potential is related to f and R through

$$V(\Phi) = \frac{1}{2\kappa_0 f'^2}(Rf' - f). \tag{6}$$

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Also, $\kappa_0 = 8\pi G$ (we use units in which $\hbar = c = 1$, and then the Plank mass is $m_{\rm Pl} = G^{1/2}$) and $\tilde{\Box} = (1/\sqrt{-\tilde{g}})\partial_\mu(\sqrt{-\tilde{g}}\tilde{g}^{\mu\nu}\partial_\nu)$ is the covariant d'Alembertian operator. Notice that all metric quantities in the EKG system will be denoted with a tilde. The corresponding Lagrangian is

$$\tilde{\mathcal{L}} = \sqrt{-\tilde{g}}[\tilde{R} - \Phi^{,\sigma}\Phi_{,\sigma} - 2V(\Phi)], \tag{7}$$

where we see that the scalar field is minimally coupled to the metric tensor $\tilde{g}_{\mu\nu}$.

As a final point in this section, we would like to mention the possible *vacuum* solutions of Eqs. (2) known so far. It has been shown in [32] that, provided $f(R) = R + a_2R^2 + \cdots$ [33], if $a_2 > 0$ then the only static spherically symmetric and asymptotically flat solution with a regular horizon is the Schwarzschild solution. As it can be seen, the statement above is not as strong as the Birkhoff theorem that arises in general relativity, that a spherically symmetric gravitational field in empty space must be static, with a metric given by the Schwarzschild one [34,35].

We should emphasize that, as we shall show below, the transformation (3) will allow us to map known EKG-oscillaton solutions onto a *full* NLG theory; that is, the transformation will not be performed on the perturbative expansion of a NLG theory.

Although we will exhibit the corresponding perturbative expansion of the resulting NLG Lagrangian, it is not the aim of this work to find solutions to any NLG perturbative expansion. Nevertheless, the search for oscillaton kind of solutions to a NLG theory could be of interest for the case of other Lagrangians that, in particular, are expressed through a (well defined) perturbative expansion.

Even though we have shown the *formal* equivalence between two conformally related frames, it should be reminded that this does not imply a *physical* equivalence too. We will make use of the former to find solutions to Eq. (2), and will comment on the latter equivalence in the last sections. However, the present manuscript should not be seen as a discussion on the trueness of one particular frame, but rather as another example that may help us to understand some of the (yet hidden) particularities of the NLG frame.

The sections are organized as follows. In Sec. II, we shall present the solutions to the EKG system and many of their interesting features. In Sec. III, we find the corresponding NLG theory and exhibit a way to compare the mass-radius relation of the objects formed in both systems. Section IV is devoted to final remarks.

II. OSCILLATONS: NUMERICAL SOLUTIONS OF THE EKG SYSTEM

Regular and asymptotically flat solutions of the EKG equations (4) are called oscillatons, and then we expect them to be related to the vacuum solutions of the field equations (2). As an example, we will focus our attention in the simplest oscillaton, which arises from a quadratic scalar potential of the form $V(\Phi) = (1/2)m_\Phi^2\Phi^2$, in the spherically symmetric case. Using the polar-areal slicing, we write the line element as

$$ds^2 = -\tilde{\alpha}^2(\tilde{t},\tilde{r})d\tilde{t}^2 + \tilde{a}^2(\tilde{t},\tilde{r})d\tilde{r}^2 + \tilde{r}^2 d\Omega, \tag{8}$$

where $\tilde{\alpha}(\tilde{t},\tilde{r})$ and $\tilde{a}(\tilde{t},\tilde{r})$ are the metric functions that appear on the left hand side of Eq. (4a).

It is not possible to find analytical solutions to the EKG system, but Eqs. (4) can be solved numerically instead. In order to find well-behaved solutions, the numerical solution should be fully time dependent. The most popular solution is due to Seidel and Suen [20,21], that used Fourier expansions of the functions involved in the EKG equations. The method has been refined in [22,23] to facilitate the numerical solution; and it is the latter which is briefly described next to construct the oscillaton solutions.

The Fourier series of the fields are of the form

$$\sqrt{8\pi G}\Phi(\tilde{t},\tilde{r}) = \sum_{j=1}^{\infty}\phi_j(\tilde{r})\cos(j\omega\tilde{t}), \tag{9a}$$

$$\tilde{\alpha}^2(\tilde{t},\tilde{r}) = \sum_{j=0}^{\infty}\tilde{\alpha}_j(\tilde{r})\cos(j\omega\tilde{t}), \tag{9b}$$

$$\tilde{a}^2(\tilde{t},\tilde{r}) = \sum_{j=0}^{\infty}\tilde{a}_j(\tilde{r})\cos(j\omega\tilde{t}), \tag{9c}$$

where ω is the fundamental angular frequency of the system.

Imposing boundary conditions of regularity at the origin and of asymptotic flatness, the EKG system becomes an eigenvalue problem. For each value of, say, $\phi_1(0)$, there is a set of eigenvalues of ω and $\tilde{\alpha}_j(0)$ for which the boundary conditions are fulfilled (for more details see [23]). The Fourier series have to be truncated by hand, and just a few of the Fourier coefficients are taken into account. However, all the solutions show convergence: the higher the Fourier mode, the less it contributes to the series.

Typical profiles of the Fourier modes of the radial metric field $\tilde{g}_{rr} \equiv \tilde{a}^2$ are shown in Fig. 1, up to $j = 12$, which corresponds to an oscillaton with total mass $M_{\rm osc} = 0.571(m_{\rm Pl}^2/m_\Phi)$ (here $\phi_1(0) = 2\sqrt{2}$). It should be noticed that the time-dependent Fourier terms are confined, and that outside a typical radius, say $\tilde{r}_{95}$ (the radius within which 95% of the total mass is contained in), the metric matches the Schwarzschild solution,

$$\tilde{\alpha}^2(\tilde{t},\tilde{r} > \tilde{r}_{95}) = \tilde{a}^{-2}(\tilde{t},\tilde{r} > \tilde{r}_{95}) = 1 - \frac{2GM_{\rm osc}}{\tilde{r}}. \tag{10}$$

We should add here that the mass of an oscillaton is finite due to the fact that the scalar field vanishes exponentially for $\tilde{r} > \tilde{r}_{95}$.

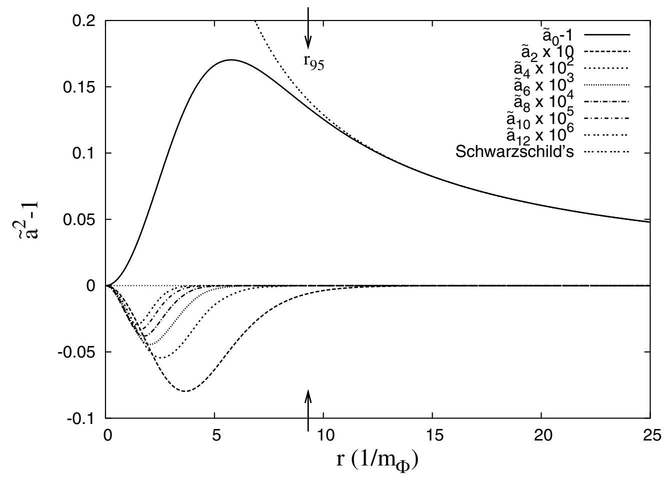

FIG. 1. The Fourier coefficients corresponding to the radial metric function $\tilde{g}_{rr} = \tilde{a}^2$ according to expansion (9c), for an oscillaton with total mass $M = 0.571(m_{\rm Pl}^2/m_\Phi)$ and for which $\phi_1(0) = 2\sqrt{2}$; also shown is its 95% radius $r_{95} = 9.34 m_\Phi^{-1}$ (For oscillatons, m_Φ becomes the natural unit of distance). The Fourier series was truncated at $j = 12$, and the profiles were appropriately scaled to show them all in the same plot. The convergence of the solution is manifest.

A curious point is that, by construction of the solution, Φ has only odd multiples of the fundamental angular frequency ω, and the metric functions $\tilde{\alpha}$ and $\tilde{a}$ have the even ones. This seems to indicate that the oscillaton in Fig. 1 and alike are the simplest configurations (the less massive) one can construct for the EKG system: the inclusion of all cosine coefficients or of sine terms would result in more massive oscillatons [36].

An important issue of oscillatons that has not been fully tackled is that of their stability. The numerical experience so far points out that there exist intrinsically *stable* (S-branch) and *unstable* (U-branch) oscillatons [24]; cf. the more detailed description in catastrophe language in [37,38]. The U-branch solutions migrate away from the equilibrium solution under small perturbations. Stable oscillatons, on the other hand, do not migrate if perturbed a little. Moreover, it seems that they play the role of *final* states an arbitrary scalar field configuration evolves towards to. However, there is not a definite proof for the stability of oscillatons, but it seems that, if oscillatons are not fully stable, they are long lived at least [20,24,26].

III. THE MASS-RADIUS RELATION IN THE NLG AND EKG SYSTEMS

Now, we are to write explicitly the connection between the NLG and the EKG systems for the particular example of a quadratic oscillaton. Equation (6) is usually taken as a differential equation for $f(R)$ to be solved in terms of the curvature R. As can be seen from Eq. (3), one obtains a highly nonlinear differential equation, which is very difficult to solve.

This can be ameliorated if we derive Eq. (6) once again with respect to R, which yields

$$2\kappa_0 f'' \left[2f'V + \frac{1}{\sqrt{(2/3)\kappa_0}} f' \frac{dV}{d\Phi} - \frac{R}{2\kappa_0} \right] = 0. \qquad (11)$$

The obvious and trivial solution, which appears for all cases, is $f''(R) = 0$, which means $f(R) = AR + B$, with A and B arbitrary constants, and also that $\Phi = $ const. [39].

It turns out that, in the general case, f and R can be given in parametric form in terms of Φ by means of Eqs. (3), (6), and (11). To simplify the calculations, we define a new dimensionless variable by $\ln(x) = \sqrt{(2/3)\kappa_0}\Phi$. Thus,

$$f = 2\kappa_0 x^2 \left[V(x) + x \frac{dV(x)}{dx} \right], \qquad (12a)$$

$$R = 4\kappa_0 x \left[V(x) + \frac{x}{2} \frac{dV(x)}{dx} \right]. \qquad (12b)$$

In this form, it is easily seen that the Liouville theory, for which $V(\Phi) = V_0 e^{\lambda\Phi}$, where V_0 and λ are constants, is one of the scalar-tensor cases that can be solved exactly [14,15,31]. However, it seems that the Liouville theory does not provide regular nor asymptotically flat solutions of the EKG system; see for instance [40,41].

On the other hand, for the quadratic case $V(\Phi) = (1/2)m_\Phi^2\Phi^2$, Eqs. (12) are rewritten as [31]

$$f = \frac{3}{2} m_\Phi^2 x^2 \ln(x)[\ln(x) + 2], \qquad (13a)$$

$$R = 3m_\Phi^2 x \ln(x)[\ln(x) + 1]. \qquad (13b)$$

A Taylor expansion of Eqs. (13) around $f(0) = 0$ ($x = 1$) gives

$$f(R) = R + \frac{1}{6m_\Phi^2} R^2 - \frac{1}{18 m_\Phi^4} R^3 + \mathcal{O}(R^4). \qquad (14)$$

Hence, a quadratic oscillaton corresponds to an R-regular NLG theory.

It has been argued before that Lagrangians of the form (14) accomplish some desirable features [43]. For instance, the presence of an R term ensures that the EKG exists near Minkowski spacetime, and the existence of an $a_2 R^2$ term ensures regularity of the conformal transformation to flat space; moreover, if $a_2 > 0$ then the Minkowski space is a stable ground state solution. It also seems that curvature corrections at all orders are essential in order to regulate gravity [30].

It should be remarked that NLG theories with characteristics as mentioned in the above paragraph correspond to a very particular class of scalar potentials and probably are a limit of a more general theory [12]. From Eqs. (13), it can be seen that Lagrangians of the form (14) correspond to the case in which $V(0) = (dV/d\Phi)(0) = 0$, i.e., the scalar potential $V(\Phi)$ has a critical point at $\Phi = 0$. Explicitly, the first Taylor coefficients for this type of potentials, calculated from Eqs. (12), are

$$a_0 = 0, \qquad a_1 = x|_{x=1} = 1,$$
$$a_2 = \frac{1}{2}\frac{1}{\dot{R}}\bigg|_{x=1} = \frac{1}{6V''(0)},$$
$$a_3 = -\frac{1}{6}\frac{\ddot{R}}{\dot{R}^3}\bigg|_{x=1} = -\frac{1}{18[V''(0)]^2}\left(1 + \frac{1}{\sqrt{6\kappa_0}}\frac{V'''(0)}{V''(0)}\right),$$
$$\ldots$$

where $\dot{} \equiv \frac{d}{dx}$ and $' \equiv \frac{d}{d\Phi}$. Higher Fourier terms depend on higher derivatives of the scalar potential in a complicated way, so that we do not show them here. However, it is easily seen that $a_2 > 0$ *only if the scalar field is also massive, i.e.,* $m_\Phi^2 \equiv V''(0) > 0$.

What else can we say of the NLG theory in (13)? First, the line element in the NLG theory is spherically symmetric and fully time dependent, properties preserved by the conformal transformation (3). The metric fields have Fourier expansions of the form

$$\alpha^2 = e^{-\sqrt{(2/3)\kappa_0}\Phi}\tilde{\alpha}^2 = \sum_{j=0}^{\infty} \alpha_j(\tilde{r})\cos(j\omega\tilde{t}), \qquad (15a)$$

$$a^2 = e^{-\sqrt{(2/3)\kappa_0}\Phi}\tilde{a}^2 = \sum_{j=0}^{\infty} a_j(\tilde{r})\cos(j\omega\tilde{t}), \qquad (15b)$$

where, in contrast to Eqs. (9) of an EKG oscillaton, all the Fourier coefficients, odd and even, are nontrivial. Here, it is the scalar field Φ which provides the odd coefficients.

Second, the *exterior* form for both metrics g and $\tilde{g}$ is the Schwarzschild solution (10). In consequence, the ADM mass is the same in both the NLG and the EKG.

Therefore, this theory permits the existence of regular objects made of *pure* gravity, which are, in addition, *vacuum* solutions. Hence, there may be in some cases a complementary second part of the theorem mentioned before [32]. We have shown here that there is a nonlinear Lagrangian of the form $f(R) = R + a_2R^2 + \cdots$, with $a_2 > 0$, that has *two type of asymptotically flat vacuum solutions*: Schwarzschild (static), and that corresponding to oscillatons (time dependent). It would be interesting to investigate whether the above statement is true for any massive real scalar field, since it has not been proved completely that the EKG equations have regular stable solutions only in the case $m_\Phi^2 > 0$; though this seems to be the case, see for instance [44].

We observe that the form of *all* of the coefficients in the Taylor expansion of the exact result (13) is determinant for the existence of oscillaton solutions; had we started with the approximate expansion (14), the results could have in general differed from the results obtained in here, even approximately.

On the other hand, it should be noticed that the simplest solution to Eqs. (2) is the metric $g_{\mu\nu}$ whose coefficients are expanded in a Fourier series of the form (15). As we said before, this metric is spherically symmetric but is not written in its so-called standard form. An observer in the NLG frame would, however, notice that the coordinates $\tilde{t}, \tilde{r}$ are its usual Schwarzschild coordinates far from $\tilde{r} = 0$. This would indicate her/him that there is an *exterior* (static) vacuum solution and an *interior* (time dependent) one.

This observer would also find more handy to continue using $(\tilde{t}, \tilde{r})$ as her/his metric coordinates, and then to preserve $g_{\mu\nu}$ in its simplest form. Thinking of this possibility, we show a comparison between the radial metric functions in the EKG and NLG cases in Fig. 2. We also plot the corresponding M vs $\tilde{r}_{\max}$ graphs for both cases, where $\tilde{r}_{\max}$ is the position of the maximum of the radial metric function (15b) at $\tilde{t} = 0$. There is a maximum mass for the NLG

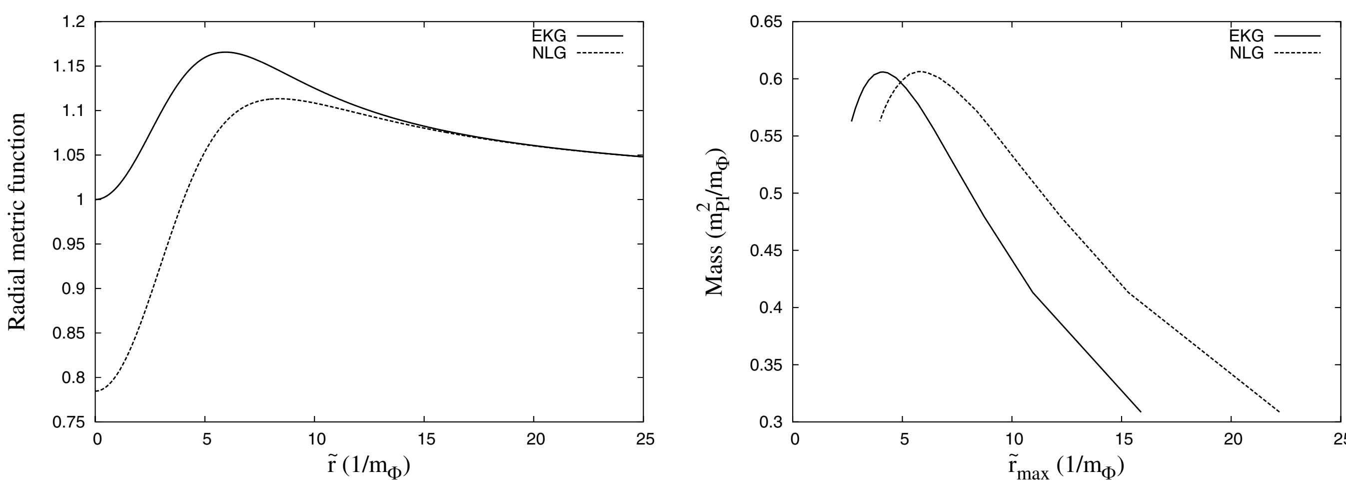

FIG. 2. (Left) Comparison of the radial metric function obtained from the EKG and the NLG cases for the same case shown in Fig. 1. It can be seen that the NLG solution is asymptotically flat and coincides with the EKG solution for $\tilde{r} \to \infty$; but it is conformally flat at $\tilde{r} = 0$. (Right) M vs $\tilde{r}_{\max}$ plots for the EKG and NLG solutions, the configuration corresponding to the leftmost (rightmost) point of the plot has $\sqrt{\kappa_0}\Phi(0, 0) = 0.8$ (0.01). The NLG oscillatons are less compact that their EKG-counterparts, so that they are distinguishable in principle, see text for details.

case, which could be interpreted as an indication of the existence of Stable and Unstable branches, as in the case of the EKG theory.

IV. FINAL REMARKS

If the NLG theory in (13) were the fundamental gravity theory, there would be many *dark pure-gravity* objects, corresponding to oscillatons, in the Universe. For a given mass, the object in the NLG theory is bigger than its EKG counter part (Fig. 2), so that one could, in principle, discriminate between them. Ultimately, by these means we could determine which system, the NLG or the EKG, is being measured.

On the other hand, we could ask at this point: how can these objects be formed from the NLG point of view? As we mentioned before, in the EKG, a self-gravitating scalar field evolves and settle-down onto an S oscillaton. Because of the spherical symmetry of the system, the only mechanism of relaxation permitted is the emission of *scalar radiation* (scalar field matter). This process has been dubbed *gravitational cooling* [21,24,25,45], which has been shown to be a very efficient mechanism for the relaxation of self-gravitating scalar fields.

On the other hand, any initial scalar field configuration in the EKG frame can be conformally transformed into a pure-gravity configuration in the NLG. Moreover, also the complete evolution of the EKG system can be conformally transformed and followed in the NLG. As the system is spherically symmetric also in the NLG, gravitational radiation is forbidden, and so there should exist a mechanism for the relaxation which would be the "conformal" partner of the gravitational cooling.

The only explanation we foresee is that, in the higher derivative NLG frame, gravity must be allowed to have a spin-0 component, which would provide the channel for gravitational cooling. In fact, the existence of *scalar gravitational waves* has been studied before, together with the idea that actual interferometers built for the detection of gravitational waves can also detect a scalar component of gravitational radiation [46,47].

It would be interesting to determine the scalar gravitational waves emitted for the system discussed here, for which one would expect that the nonlinear terms would, in a certain manner, take the role of the scalar field. The procedure would be the full numerical evolution of Eqs. (2). For this, it would also help the formal equivalence between the NLG and the EKG, as the numerical evolution of the metric fields α and a can be mapped directly from that of $\tilde{\alpha}$, $\tilde{a}$ and Φ. These calculations are in progress and will be reported elsewhere.

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