PHYSICAL REVIEW D **72,** 023517 (2005)

Do large-scale inhomogeneities explain away dark energy?

Ghazal Geshnizjani* and Daniel J. H. Chung†

Department of Physics, University of Wisconsin, Madison, Wisconsin 53706, USA

Niayesh Afshordi‡

Institute for Theory and Computation, Harvard-Smithsonian Center for Astrophysics, MS-51, 60 Garden Street, Cambridge, Massachusetts 02138, USA

(Received 30 March 2005; published 26 July 2005)

Recently, new arguments [E. Barausse, S. Matarrese, and A. Riotto, Phys. Rev. D **71**, 063537 (2005).][E. W. Kolb, S. Matarrese, A. Notari, and A. Riotto, hep-th/0503117 [Phys. Rev. Lett. (to be published)].] for how corrections from super-Hubble modes can explain the present-day acceleration of the universe have appeared in the literature. However, in this paper, we argue that, to second order in spatial gradients, these corrections only amount to a renormalization of local spatial curvature, and thus cannot account for the negative deceleration. Moreover, cosmological observations already put severe bounds on such corrections, at the level of a few percent, while in the context of inflationary models, these corrections are typically limited to $\sim 10^{-5}$. Currently there is no general constraint on the possible correction from higher order gradient terms, but we argue that such corrections are even more constrained in the context of inflationary models.

DOI: 10.1103/PhysRevD.72.023517 PACS numbers: 98.80.Cq

I. INTRODUCTION

The potential impact of large-scale perturbations on local cosmology has been a subject of interest in different contexts during the past decade [1]. In the standard theory of cosmological perturbations, it is always assumed that perturbations do not have any impact on the evolution of background cosmology. However, there is no *a priori* reason for neglecting the back-reaction from perturbations, particularly since Einstein's equations are highly nonlinear. Although it is easy to see that corrections do exist, an important question is whether these corrections, which are of second order or higher in perturbations, will ever become significant and if they do, what is the right way to distinguish the real physical effects from the gauge ambiguities in the calculations. The key difficulty is that if the perturbative effects are misinterpreted, one may overlook physical bounds already existing on such effects because the physical bounds are written in terms of variables not manifestly connected with the perturbations.

Recently, [12,13] argued that corrections due to the interplay between IR modes and UV modes may lead to an apparent late time acceleration of the universe, with no need for dark energy or a cosmological constant. The correction term that is claimed to determine the apparent acceleration is of the form $\varphi\nabla^2\varphi$, where φ is the gravitational potential. It is argued that this correction can have a large variance and its statistical nature may cause a negative value for the observed deceleration parameter.

Even without computation, one might guess that there is a problem with this correction becoming significant from a phenomenological point of view. For scale-invariant fluctuations of φ, the variance in $\varphi\nabla^2\varphi$ scales as λ^{-2}, where λ is the physical length scale. Therefore, if this correction is indeed ~ 1 on the present-day Hubble scale to explain away dark energy, it will be $\gg 1$ on smaller scales, which undermines the incredible success of linear structure formation theory in the low-redshift universe (see e.g., [14]).

In this paper, we investigate a related problem. In the following sections we will demonstrate that the perturbative corrections of the form $\varphi\nabla^2\varphi$ cannot lead to a negative deceleration parameter (at least not in the manner suggested in [12,13]), because this effect stems from a renormalization of the local spatial curvature [15]. We then argue how current cosmological observations put severe constraints on the magnitude of these corrections. Finally, we discuss the loopholes (e.g., our neglect of higher than second order gradients) in our argument before concluding.

II. CORRECTIONS TO DECELERATION PARAMETER DUE TO SPATIAL CURVATURE

The metric of a homogeneous and isotropic universe (Friedmann-Robertson-Walker; FRW metric) can be generally described as

$$ds^2 = -dt^2 + a^2(t)\left(1 + \frac{1}{4}\mathcal{K}r^2\right)^{-2}\delta_{ij}dx^i dx^j, \quad (1)$$

where a is the scale factor and $\mathcal{K}$ is the spatial curvature. It is customary to normalize $\mathcal{K}$ such that it takes the values of 0, $+1$, and -1 corresponding, respectively, to flat, closed, and open universes [16], but in general it could take any value. Einstein's equations for the above metric reduce to the Friedmann equations

*Electronic address: ghazal@physics.wisc.edu
†Electronic address: Danielchung@wisc.edu
‡Electronic address: nafshordi@cfa.harvard.edu

$$H^2 + \frac{\mathcal{K}}{a^2} = \frac{8\pi G}{3}\rho, \tag{2}$$

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}(\rho + 3p), \tag{3}$$

where dot denotes time derivative and $H = \dot{a}/a$, is the Hubble constant, while ρ and p are, respectively, the total energy density and pressure of matter components in the universe. The deceleration parameter q which describes the deceleration of the scale factor $a(t)$ is defined as

$$q = -\frac{\ddot{a}}{aH^2}. \tag{4}$$

In a matter dominated universe, where $p = 0$, Eqs. (2) and (3) imply that

$$q = \frac{1}{2}\left(1 + \frac{\mathcal{K}}{a^2H^2}\right). \tag{5}$$

III. RENORMALIZATION OF THE LOCAL CURVATURE DUE TO LARGE-SCALE INHOMOGENEITIES

In this section we calculate the corrections to the local spatial curvature, $\mathcal{K}$, in a flat universe due to large-scale inhomogeneities. To obtain these corrections, we will start by expanding the metric to second order in perturbations in the synchronous gauge, the same metric that Barausse *et al.* use in [12], and then express all perturbative corrections in terms of the peculiar gravitational potential φ, as they did, while dropping all the terms of order higher than $\nabla^2\varphi$ in the gradient expansion (these are subdominant in the IR—i.e., long wavelength—limit):

$$ds^2 = -a^2 d\eta^2 + a^2\gamma_{ij}dx^i dx^j, \tag{6}$$

$$\begin{aligned}\gamma_{ij} = &\left(1 - \frac{10}{3}\varphi - \frac{\eta^2}{9}\nabla^2\varphi + \frac{50}{9}\varphi^2 + \frac{5\eta^2}{54}\varphi^{,k}\varphi_{,k}\right)\delta_{ij} \\ &- \frac{\eta^2}{3}\left(\varphi_{,ij} - \frac{1}{3}\delta_{ij}\nabla^2\varphi\right) \\ &- \frac{5\eta^2}{9}\left(\varphi^{,i}\varphi_{,j} - \frac{1}{3}\varphi^{,k}\varphi_{,k}\delta_{ij}\right).\end{aligned} \tag{7}$$

We can also write the Taylor expansion of φ around its value at the location of a particular observer. Assuming that φ is isotropic around this location [17], we have

$$\varphi \simeq \varphi_0 + \frac{1}{6}\nabla^2\varphi r^2. \tag{8}$$

Note that this constant φ_0 corresponds to the superhorizon modes of the potential fluctuation. As we will see, it is the interaction of these superhorizon modes with $\nabla^2\varphi$ which leads to a modification of the deceleration. Equation (8) further simplifies the γ_{ij} in the metric of Eq. (6) into

$$\begin{aligned}\gamma_{ij} = \bigg\{&1 - \frac{10}{3}\varphi_0 + \frac{50}{9}\varphi_0^2 - \frac{\eta^2}{9}\nabla^2\varphi - \frac{5}{9}\nabla^2\varphi r^2 \\ &+ \frac{50}{27}\varphi_0\nabla^2\varphi r^2\bigg\}\delta_{ij}.\end{aligned} \tag{9}$$

We now note that we can renormalize the scale factor $a(\eta)$ to reproduce the metric in Eq. (1). This can be done by taking

$$\tilde{a}(\eta) = a(\eta)\left[1 - \frac{10}{3}\varphi_0 + \frac{50}{9}\varphi_0^2 - \frac{\eta^2}{9}\nabla^2\varphi\right]^{1/2}, \tag{10}$$

leading to the following form for the metric

$$ds^2 = -dt^2 + \tilde{a}^2(\eta)\left(1 - \frac{5}{9}\nabla^2\varphi r^2\right)\delta_{ij}dx^i dx^j, \tag{11}$$

where the $\varphi_0\nabla^2\varphi$ terms cancel out and we have ignored the higher order terms. For small values of the curvature $\mathcal{K}$, the above metric is equivalent to the metric of Eq. (1), where the curvature term is now

$$\mathcal{K} = \frac{10}{9}\nabla^2\varphi + O[\varphi_0^2\nabla^2\varphi, (\nabla^2\varphi)^2]. \tag{12}$$

IV. IMPACT OF LARGE-SCALE INHOMOGENEITIES ON THE DECELERATION PARAMETER

We now compute the correction to the deceleration parameter q due to the renormalization of the local spatial curvature $\mathcal{K}$. Substituting from Eq. (12) into Eq. (5), we find

$$q = \frac{1}{2}\left(1 + \frac{10}{9}\frac{\nabla^2\varphi}{\dot{\tilde{a}}^2}\right). \tag{13}$$

Using Eq. (10), we find

$$\dot{\tilde{a}}^2 = \dot{a}^2\left[1 - \frac{10}{3}\varphi_0 - \frac{\eta^2}{9}\nabla^2\varphi - \frac{2}{9}\frac{a}{\dot{a}}\eta\nabla^2\varphi + \mathcal{O}(\varphi^2)\right]. \tag{14}$$

Substituting Eq. (14) into Eq. (13), we end up with corrections to the deceleration parameter

$$\begin{aligned}q = &\frac{1}{2}\left[1 + \left(\frac{5}{18}\nabla^2\varphi + \frac{25}{27}\varphi_0\nabla^2\varphi\right)\left(\frac{2}{\dot{a}}\right)^2\right] \\ &+ O[\varphi_0^2\nabla^2\varphi, (\nabla^2\varphi)^2]\end{aligned} \tag{15}$$

(neglecting derivatives larger than second order).

Notice that the corrections to q (in particular the third term including the coefficient), are the exact same corrections that [12,13] argue have statistical nature and could possibly be the reason for the apparent current acceleration of the universe. **However, our result implies that this correction arises due to the renormalization of the local spatial curvature, which in nature can never lead to an acceleration of the universe** (Note that as long as energy density is positive semi-definite, $1 + \mathcal{K}/(aH)^2 \geq 0$).

Furthermore, Wilkinson Microwave Anisotropy Probe [18] constraints on $\Omega_{\mathcal{K}}$ (based on the location of cosmic microwave background Doppler peaks) lead to a bound on the magnitude of these corrections:

$$\triangle q = \frac{1}{2}\Omega_{\mathcal{K}} = \frac{\mathcal{K}}{2\dot{\bar{a}}^2} < 0.02. \tag{16}$$

An even more severe constraint on the magnitude of these corrections is obtained in the context of inflationary models, which predict near scale-invariant power spectra of inhomogeneities. We notice that matter overdensity in a flat universe on large scales is in fact equal to $\Omega_{\mathcal{K}}$. Therefore, we find

$$\langle \Delta q^2 \rangle = \frac{1}{4}\Omega_{\mathcal{K}}^2 = \frac{1}{4}\Delta_m^2 \simeq 10^{-10} \text{ on Hubble scale}, \tag{17}$$

where the amplitude of matter overdensities for a scale-invariant power spectrum, $\Delta_m \sim 10^{-5}$, is observed in a host of cosmological observations (see e.g., [18]).

V. DISCUSSION

One deficiency of our argument is that we neglect higher order spatial gradients [19]. Unlike the second-order gradients, which in the long wavelength limit correspond to the local spatial curvature, there is no property of the homogeneous universe which can be (without averaging) associated with higher order spatial gradients of the metric perturbations. Hence, in principle, there may be a way to nonperturbatively arrange a large averaged correction to $\rho + 3p$ without disturbing $\mathcal{K}$ significantly [20].

In other words, if we integrate out UV degrees of freedom except modes with wave vector of order H_0, the renormalization to $\rho + 3p$ may be significant without significantly perturbing $\mathcal{K}$ [29]. Note that one need not integrate out IR degrees of freedom because approximate homogeneity and isotropy on cosmological scales of our Hubble patch is consistent with all observations. Of course, if the universe is extremely inhomogeneous outside of our horizon, we must also integrate out IR modes to reduce the approximate degree of freedom to ρ, p, and $\mathcal{K}$ [30]. Furthermore, a dynamical IR cutoff always exists due to the existence of a Hubble horizon. Despite this caveat, one unequivocal point of this paper is that this effect of renormalizing $\rho + 3p$ without disturbing $\mathcal{K}$ must occur through a pathologically nonuniformly convergent series or nonperturbative behavior (e.g., without resorting to derivative or small potential expansion) since perturbatively, the second gradient order term contributes to the spatial curvature which by itself cannot account for the acceleration of the universe (and is severely constrained observationally).

To reemphasize the need for nonperturbative corrections to have a possibility at explaining the acceleration of the universe, we can estimate the observational bounds on higher gradient order terms (assuming derivative expansions to be valid) in the context of inflationary models with near scale-invariant power spectra. For scale-invariant perturbations in φ, higher order corrections in the gradient expansion take the form

$$\Delta_n q \sim \varphi \nabla^{2n} \varphi \propto \lambda^{-2n}, \tag{18}$$

where λ is the physical scale at which $\Delta_n q$ is observed. However, fluctuations of $q = 0.5\rho/\bar{\rho}_c \propto \rho/H_0^2$ are well measured at sub-Hubble scales of say $\sim$50 Mpc (the scale of galaxy surveys) [31]

$$\Delta_n q|_{50\ \mathrm{Mpc}} < \Delta_{\mathrm{tot}} q|_{50\ \mathrm{Mpc}} = \frac{1}{2}\Delta_m|_{50\ \mathrm{Mpc}} \sim 0.1 \tag{19}$$

where $\Delta_m = \delta\rho_m/\rho_m$ is the matter density fluctuation. Therefore, on Hubble scales ($\lambda = H^{-1} \sim 5000$ Mpc) we find

$$\Delta_n q|_{H^{-1}} \sim \left(\frac{H^{-1}}{50\,\mathrm{Mpc}}\right)^{-2n} \Delta_n q|_{50\,\mathrm{Mpc}} \lesssim 10^{-5-4(n-1)}, \tag{20}$$

and thus, at least to this order of approximation, the corrections due to higher order terms are even more constrained. Note that averaging procedure will generically give the same order of magnitude as long as the process is perturbative. Hence, the nonlinear corrections appear to have a chance of explaining the acceleration of the universe only if nonperturbative (or pathological) effects take place.

VI. CONCLUSION

We computed the corrections to the local spatial curvature due to large-scale perturbations (up to second derivative expansion) and showed that they are the same corrections that [12,13] suggest may lead to the acceleration of the universe (as far as second derivative corrections are concerned). We conclude that as attractive as it may seem to have inhomogeneities resolve the dark energy problem, unfortunately, this term is insufficient due to the fact that spatial curvature can never lead to an acceleration of the universe (with energy density positive semi-definite). Furthermore, there are already severe bounds on this correction, implied from various cosmological observations, which indicate that this not only cannot serve as an alternative to the dark energy but also cannot change the value of the observed deceleration parameter significantly. Because our arguments are based on expanding the metric to second perturbative order in inhomogeneities and restricting to second order in derivative expansion, one way to evade these arguments is through nonperturbative effects.

ACKNOWLEDGMENTS

NA wishes to thank the Physics department at the University of Wisconsin-Madison for its hospitality. We thank Robert Brandenberger and Toni Riotto for helpful comments.

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