Bounds on cosmic strings from WMAP and SDSS

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We find the constraints from Wilkinson microwave anisotropy probe (WMAP) and Sloan digital sky survey (SDSS) data on the fraction of cosmological fluctuations sourced by local cosmic strings using a Markov Chain Monte Carlo (MCMC) analysis. In addition to varying the usual 6 cosmological parameters and the string tension (μ), we also varied the amount of small-scale structure on the strings. Our results indicate that cosmic strings can account for up to 7 (14)% of the total power of the microwave anisotropy at 68 (95)% confidence level. The corresponding bound on the string mass per unit length, within our string model, is $G\mu < 3.4(5) \times 10^{-7}$ at 68 (95)% C.L. We also calculate the B-type polarization spectra sourced by cosmic strings and discuss the prospects of their detection.

I. INTRODUCTION

Cosmic strings were proposed as potential seeds for structure formation in the early 1980's [1,2] and extensively studied in this context until the late 1990's (see [3] for a review). The interest was fueled, in part, by the fact that the cosmic microwave background (CMB) temperature anisotropy discovered by COBE [4] was of the same order of magnitude (10^{-5}) as the density fluctuations that would be produced by cosmic strings formed around grand unified theory (GUT) epoch. However, by 1997 it was already apparent that strings could not explain the distribution of large-scale structure [5]. Finally, when Boomerang [6] and Maxima [7] revealed the existence of acoustic peaks in the CMB angular spectrum, strings were excluded as a viable alternative to inflation as a model for seeding the formation of structure. The shape and location of the peaks, currently measured to a high precision by WMAP [8], strongly indicate that structures grew out of initial fluctuations with adiabatic initial conditions and with a scale-invariant spectrum, just as prescribed by the inflationary paradigm [9]. In contrast, strings predict nearly featureless CMB spectra[1] and, when normalized to COBE or WMAP, do not produce enough clustering on large scales.

While strings could not have seeded all of the structure in the Universe, they could have created a subdominant yet non-negligible fraction of the primordial cosmological fluctuations [12–18]. This idea has recently received renewed attention with the realization that cosmic strings are produced in a wide class of string theory models of the inflationary epoch [19–26]. In these models, inflation can arise during the collisions of branes that coalesce to form, ultimately, the brane on which we live [27–30]. Brane inflation predicts adiabatic temperature and dark matter fluctuations capable of reproducing all currently available observations. In addition, the collision at the conclusion of brane inflation can produce a network of local cosmic strings [19,20], whose effects on cosmological observables range from negligible to substantial, depending on the specific scenario [21,22]. It has also been shown that strings could form at an observationally acceptable level at the end of the D-term inflation in supersymmetric GUT models [18]. As the precision of cosmological observations increases, one might hope to distinguish among the numerous presently-viable models of inflation by studying and constraining the properties of the cosmic strings they predict.

The properties of the strings produced at the end of brane inflation are similar to those of local strings. They intercommute and break off loops like networks of usual strings, implying that they will settle into a scaling solution. However, because of their higher-dimensional nature, they are expected to intercommute at a reduced rate and thus approach a scaling solution at a higher string density. There are also other, qualitatively new aspects of cosmic superstrings, the most important of which is that string theory models generically predict the existence of at least two fundamentally different kinds of cosmic strings. These are the so-called fundamental, or F-strings, and another kind of string formed by wrapping of all but one of the dimensions of a higher-dimensional D-brane around compact dimensions, or D-strings. The relative tensions of these string types are determined by the superstring coupling, g_s. Because these different string types interact through binding, rather than intercommutation, they can form higher-tension (p, q) states composed of p F-strings and q D-strings [25,31]. A first model of these networks has been developed recently, and shows that these binding

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[1]This is true for all "conventional" cosmic strings networks. Technically, it is possible to construct active sources of fluctuations leading to CMB temperature spectra in agreement with observations, if, for example, one allows the time- and space-correlations of strings to violate the bounds imposed by causality [10,11].

reactions allow network scaling [32]. Because of the complexity introduced by this kind of new physics, the amount of small-scale structure on strings—the string "wiggliness"—can vary depending on the particular brane inflation scenario. To make our constraints applicable to a wide range of models, we have included the string wiggliness as one of the parameters in the MCMC simulation. The effect of a higher string density can be roughly modeled by appropriately adjusting the normalization of the string-generated spectra. This then translates into a simple adjustment of the bound on $G\mu$.

The aim of this paper is to constrain the properties of cosmic strings by using the power spectrum data from the WMAP and SDSS experiments. There are other ways to constrain cosmic strings—some of them promising to produce much tighter bounds than those that will ever be possible with power spectrum data. We give a brief review of other methods in the summary section, Sec. IV.

The rest of this paper is organized as follows. In Sec. II we give a detailed account of the model and the methods used. We show the results in Sec. III and conclude with a discussion in Sec. IV.

II. THE MODEL AND ANALYSIS

The fluctuations resulting from brane inflation are expected to be an incoherent superposition of contributions from adiabatic perturbations initiated by curvature fluctuations and active perturbations induced by the decaying cosmic-string network. The resulting CMB angular spectra can be written as a sum of the adiabatic and string contributions:

$$C_l = C_l^{\mathrm{ad}} + C_l^{\mathrm{cs}}. \tag{1}$$

Analogous expressions hold for matter density spectra. We restrict our study to a flat FRW universe and vary the following cosmological parameters: the Hubble constant h, the matter density $\Omega_M h^2$, the baryon density $\Omega_b h^2$, and the reionization optical depth τ. In addition, we vary the galaxy bias factor F_b, the amplitude A_s and the spectral index n_s of the primordial scalar power spectrum, as well as the string mass per unit length, $G\mu$, and the string wiggliness parameter α_r (to be defined in Sec. II A).

We used a suite of different codes to produce and analyze the spectra. The model we employed for the cosmic-string generated perturbations is described in the subsection below. The string CMB and matter spectra were calculated using a modification [33,34] of CMBFAST [35] (see Figs. 1 and 2 for representative string-induced spectra). We first evaluated and stored the string spectra on a grid of parameters. During our calculations, the spectra were obtained by interpolation on the grid. The adiabatic matter spectra were also stored on a grid after having been evaluated using a publicly-available version of CMBFAST. For the adiabatic CMB spectra, we used the package CMBWARP [36].
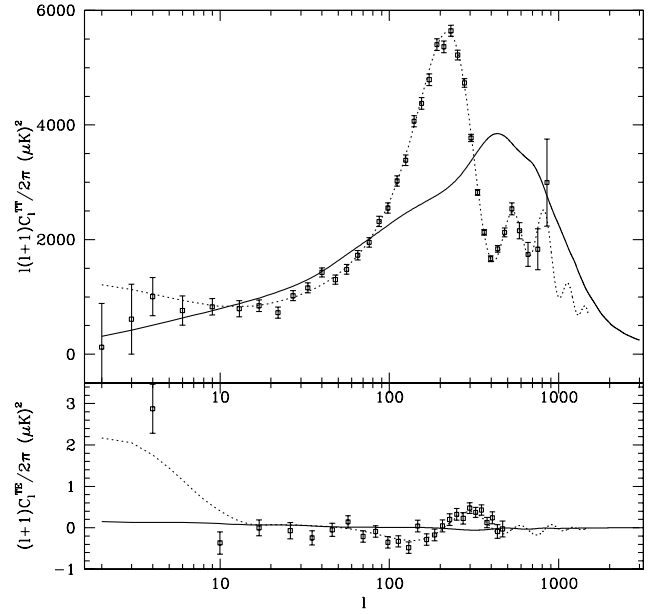

FIG. 1. The CMB TT, and TE spectra (solid lines) sourced by cosmic strings with wiggliness parameter $\alpha_r = 1.9$, as well as the adiabatic spectra for the same cosmological parameters (dashed lines) and WMAP's first year data. The string spectra are normalized so that the *total* TT power is the same for the two lines, which corresponds to $B = 1$.

To compare the theoretical linear matter power spectrum $P^{\mathrm{L}}(k)$ generated by our code with the SDSS results, we first applied the halo-fitting procedure of Smith *et al.* [37] to obtain the nonlinear spectra, $P^{\mathrm{NL}}(k)$. This procedure is
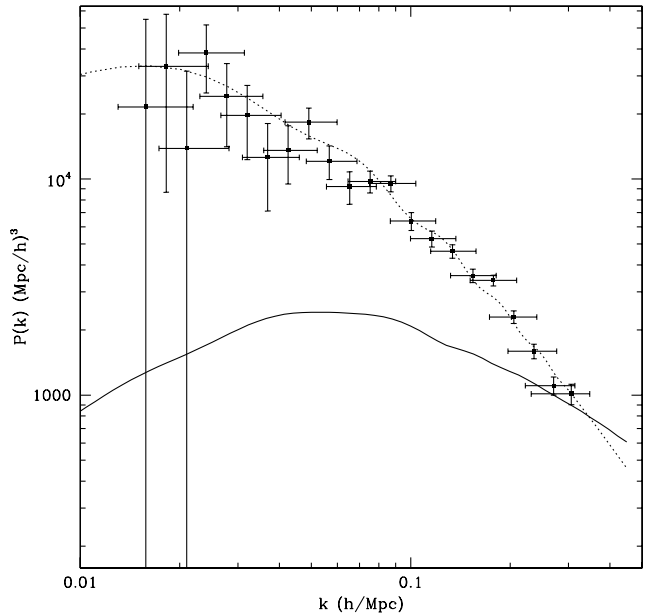

FIG. 2. The string-generated matter power spectrum (solid line) for the same parameters as in Fig. 1, i.e., for $B = 1$. The dashed line represents the linear spectrum from adiabatic perturbations at $z = 0$.

only valid for the adiabatic contribution to the $P(k)$ spectrum, so using it for the cosmic-string contribution introduces some inaccuracy into our model. However, as we can see in Fig. 2, the string power spectrum is considerably weaker than the adiabatic power spectrum on all but the smallest length scales even in models where strings account for all the large-scale microwave anisotropy. Thus, since only a small portion of the relevant values of k are affected by the halo-fitting procedure, and since the effect is small on the scales we consider, we expect any inaccuracy introduced by our approximate treatment of nonlinearity to be unimportant. Next, we assumed that the galaxy bias factor and the correction due to peculiar velocities on the scales of our interest ($k < 0.2$ h/Mpc) are approximately redshift- and scale-independent and can be combined into a single constant factor, F_b, multiplying the matter spectrum:

$$P(k) = F_b P^{\mathrm{NL}}(k). \tag{2}$$

We then fit $P(k)$ to the SDSS power spectrum data (for $k < 0.2$ h/Mpc) with F_b as a free parameter.

The likelihood of the spectra for a given set of parameters was obtained using the publicly-available likelihood codes produced by the WMAP [38] and SDSS [39] teams.

A. The cosmic-string model

Unlike the adiabatic inflationary perturbations, which are set as initial conditions in the equations of motion of linear perturbations, cosmic strings act as a continual source of fluctuations as the universe evolves. The density and temperature fluctuations created in the immediate locality of a cosmic string are highly nonlinear, e.g. the density contrast $\delta\rho/\rho$ is significantly larger than unity in the wake formed behind a moving string. However, the effect of strings on cosmological scales is that of a small perturbation to the evolution of the average cosmic energy-momentum tensor. Hence, for the purpose of computing the CMB and large scale structure (LSS) spectra, the metric and density perturbations caused by strings are described by the system of linearized Einstein-Boltzmann equations with strings acting as active sources.

Evaluation of the CMB and LSS spectra sourced by strings requires knowing their energy-momentum tensor (or its unequal time correlation functions [40]) for the entire dynamical range of the calculation, which is approximately four orders of magnitude in scale factor. Realistic simulations of cosmic-string networks have so far been limited either in their dynamical range [41] or their resolution [42]. Hence, until full-scale simulations become available, one is forced to resort to approximate methods to model the string sources.

Numerical simulations [43–45] show that during the radiation and matter dominated eras the string network evolves according to a scaling solution, which on sufficiently large scales can be described by two length scales.

The first scale, $\xi(t)$, is the coherence length of strings, i.e. the distance beyond which directions along the string are uncorrelated. The second scale, $L(t)$, is the average inter-string separation. Scaling implies that both length scales grow in proportion to the horizon. Simulations indicate that $\xi(t) \sim t$, while $L(t) = \gamma t$, with $\gamma \approx 0.8$ in the matter era [43,44]. The one-scale model [46,47], in which the two length scales are assumed to be equal, has been quite successful in describing the general properties of cosmic-string networks inferred from numerical simulations. These simulations assume that cosmic strings reconnect on every intersection. It is of interest to us, however, to also consider the case when the reconnection probability is less than one. If strings can move and interact in extra dimensions then, while appearing to intersect in our three dimensions, they may actually miss each other. Hence, the effective intercommutation rate of these strings will generally be less than unity. As a consequence, one would expect more strings per horizon in these theories [19,22]. Because of the straightening of wiggles on subhorizon scales due to the expansion of the universe, one would still expect $\xi(t) \sim t$. However, the string density would increase, therefore reducing the interstring distance. Hence, smaller intercommutation probabilities imply smaller γ.

In addition, numerical simulations show that long strings possess a great deal of small-scale structure in the form of kinks and wiggles on scales much smaller than the horizon. To an observer who cannot resolve this structure, the string will appear to be smooth, but with a larger effective mass per unit length $\tilde{\mu}$ and a smaller effective tension $\tilde{T}$. An unperturbed string (with $\mu = T$) exerts no gravitational force on nearby particles. In contrast, a wiggly string with $\tilde{\mu} > \tilde{T}$ attracts particles like a massive rod. The effective equation of state of a wiggly strings is $\tilde{\mu}\tilde{T} = \mu^2$ [48,49]. Depending on the brane inflation model, the presence of extra dimensions could mean that even more small-scale structure would be present on the strings [22].

To calculate the sources of perturbations we use an updated version of the cosmic-string model first introduced by Albrecht *et al.* [5] and further developed in Refs. [33,50], where the wiggly nature of strings was taken into account. In this model, the string network is represented by a collection of uncorrelated straight string segments produced at some early epoch, moving with uncorrelated, random velocities. At every subsequent epoch, a certain fraction of the number of segments decays in a way that maintains network scaling. The length of each segment at any time is taken to be equal to the correlation length of the network. This length and the root-mean-square (r.m.s.) velocity of segments are computed from the velocity-dependent one-scale model of Martins and Shellard [51]. The positions of segments are drawn from a uniform distribution in space, and their orientations are chosen from a uniform distribution on a two-sphere. This model is a rather crude approximation of a realistic string

network. However, there are good reasons to believe that its predictions for the CMB and LSS spectra are close to what one would obtain by using full-scale simulations. Its parameters have been calibrated to produce source correlation functions in agreement with those in [52], where comparison to a full simulation was possible. Also, the *shape* of the spectra obtained using this model are in good agreement with results of other groups [53–55], who used different methods that are also approximate. Finally, on the cosmological scales probed by the CMB measurements, the fine details of the string evolution do not play a major role. It is the large-scale properties—such as the scaling distance, the equation of state (wiggliness), the r.m.s. velocity, and how all these characteristics evolve through the radiation-matter equality—that determine the shape of the string-induced spectra. All of these effects are accounted for in our model and can, in principle, be adjusted to match any specific cosmic-string model. We choose to work with this model because one can easily calculate the sources for different cosmological parameters and because it allows us to include the effect of the wiggliness [33], which could be one of the distinguishing features of strings produced in theories with extra dimensions [22]. The other main effect of the presence of extra dimensions, the increased string density, can be approximately factored in by multiplying the spectra by $N_s \sim \gamma^{-2}$.

For technical details of the model, the reader is referred to [33] and references therein. The wiggly nature of strings is accounted for by modifying the string energy-momentum tensor so that it corresponds to the wiggly string equation of state:

$$\tilde{\mu} = \alpha\mu, \qquad \tilde{T} = \alpha^{-1}\mu, \qquad (3)$$

where α is a parameter describing the wiggliness, $\tilde{\mu}$ and $\tilde{T}$ are the mass per unit length and the string tension of the wiggly string, and μ is the tension (or, equivalently, the mass per unit length) of the smooth string. In addition to modifying the equation of state, the presence of small-scale structure slows strings down on large scales. We account for this by dividing the root mean squared string velocity by the parameter α. The wiggliness of the strings remains approximately constant during the radiation and matter eras, but changes its value during the transition between the two. We take the radiation era value, α_r, to be a free parameter that we vary, and set the matter era value to be $\alpha_m = (1 + \alpha_r)/2$, with a smooth interpolation between the two values (as prescribed in [33]). For conventional strings, this roughly agrees with results of numerical simulations [43,44] which show a decrease from $\alpha_r \sim 1.8 - 1.9$ in the radiation era to $\alpha_m \sim 1.4 - 1.6$ in the matter era.

The way the wiggliness modifies the shape of the spectra can be understood qualitatively, once one examines the main physical processes that contribute to CMB anisotropy on various scales. The spectrum can be approximately divided into two main components. One is a roughly flat

(scale-invariant) component that arises from the combined Kaiser-Stebbins effect [56] of many strings in a scaling string network; this component is produced *after* last scattering. The second component is the bump peaked at $\ell \sim 450$, which is primarily determined by the state of density and velocity perturbations *at the time* of last scattering. CMB photons experience Doppler shifts when they (last-) scatter off the velocity flows created by the string wakes that exist at this epoch [57]. In addition, all the density fluctuations that existed at the time of last scattering are imprinted on the CMB via the Sachs-Wolfe effect [58]; these density fluctuations are caused by the CDM wakes created by strings during the time between radiation-matter equality and last scattering. If we change the string substructure by introducing string wiggliness, there is a two-fold effect on the flat part of the spectrum that arises from the Kaiser-Stebbins effect. The first effect is simple: by nature, heavier strings have larger deficit angles, and thus produce larger CMB discontinuities. However, the size of the discontinuities is also proportional to string velocity and, since wiggly strings move more slowly than straight ones, the net increase in the size of the discontinuities is mitigated. The wakes behind a wiggly string are, nonetheless, always more prominent. This is because of wiggly strings having a nonzero Newtonian potential that attracts matter particles in the string's vicinity. The combined effect of wiggliness on the flat component of the spectrum is an overall increase in its amplitude, but a relatively weak one. The peak, on the other hand, is directly enhanced, since the wakes at the last scattering are more prominent for wiggly strings. The net result is an overall increase in the amplitude of the spectra coupled with an enhancement of the peak at $\ell \sim 450$. The characteristic scale of the fluctuations at decoupling is set by the size of the typical wake, or, in turn, by the coherence length $\xi(t)$ of the string network at that time. Independent of the string intercommutation probability, the coherence length $\xi(t)$ is always expected to be roughly equal to t. This is because of the straightening of wiggles on subhorizon scales caused by the expansion of the universe. Therefore, the qualitative features of the spectrum are quite independent of the details of the model. Hence, any observational constraint that we obtain based only on the *shape* of the spectrum (such as the constraint on the parameter f defined below) can be viewed as less model-dependent than our other results.

It is well known that properties and possible observational signatures of global and local strings can be very different. Global strings predict almost no power on small angular scales for CMB temperature anisotropy [40], while local strings—as we have argued above—produce a quite significant broad peak at $l \sim 450$ in a spatially flat universe [5,33,53–55,59]; this can be seen in Fig. 1. Global strings also induce a significantly larger vector component of metric perturbations [60]. Consequently, their prediction for the strength of the *B*-type polarization [61] is generally

higher than that of local strings. We will only concern ourselves with local strings. For the most recent constraints on global strings the reader is referred to [62,63].

Rather than working directly with the string parameter $G\mu$, we introduce a parameter B, defined as

$$B \equiv \left(\frac{\mu}{\mu_0}\right)^2, \tag{4}$$

where μ_0 is the tension that one obtains by setting the total power in string-induced CMB anisotropy to be equal to the total power observed by WMAP. That is, we set

$$I^{cs} \equiv \sum_l \frac{(2l + 1)}{4\pi} C_l^{cs}(\mu_0, \alpha_r^{(0)}, \vec{p}_0) = I^{WMAP}, \tag{5}$$

where we take $\alpha_r^{(0)} = 1.9$ and $\vec{p}_0$ is a fixed set of the remaining cosmological parameters that have taken to correspond to the the best-fit ΛCDM model of Tegmark *et al.* [64]. The value of $G\mu_0$ that we obtain with this prescription is 2×10^{-6}. Speaking very loosely, B can be said to measure the fraction of the anisotropy due to strings. Note, however, that this meaning is modified if, for instance, the strings have a different amount of small-scale structure ($\alpha_r \neq 1.9$) or if they have reduced inter-commutation probabilities. B is only really useful as an intermediate parameter. Our main results are the constraints on $G\mu$ that we obtain from B and the fraction f of the total CMB anisotropy due to strings [Eq. (6)].

B. Markov chain Monte Carlo

Because of the large size of our parameter space (nine parameters in total), we have used the Markov Chain Monte Carlo (MCMC) method for exploring the likelihood surface and for generating marginalized posterior distribution functions for the model parameters. We employed the MCMC algorithm described in the Appendix of [64].

We ran eight separate chains initialized at randomly generated initial positions within our prior range. The priors, given in Table I, were chosen with the expectation of the string contribution being subdominant and the best-fit parameters being close to their WMAP best-fit values [65][2]. Since we expected values of B near zero to be preferred, we also allowed B to range slightly below zero so as not to restrict artificially the ability of the chain's random steps to explore near $B = 0$; values below zero were discarded when the data were analyzed. One advantage to our use of multiple chains rather than a single, long chain was that we were able to verify directly that there was adequate mixing in each chain, since each successfully forgot its starting location and located the same maximum likelihood region of the parameter space.

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[2]The restriction on wiggliness, $\alpha_r < 4$, was to save computing time.

TABLE I. Prior constraints on the parameters.

$0 \leq B$
$1 \leq \alpha_r \leq 4$
$0 \leq A_s$
$0.92 \leq n_s \leq 1.07$
$0.019 \leq \Omega_B h^2 \leq 0.028$
$0.1 \leq \Omega_M h^2 \leq 0.2$
$0.5 \leq h \leq 0.8$
$0 \leq \tau \leq 0.23$
$0 \leq F_b$

III. RESULTS

A. The fraction in strings and $G\mu$

To test our MCMC code we first ran a chain with the string contribution set to zero ($B = 0$). The results are shown in Table II. Our results are consistent with those found in a similar analysis of the same data by members of the SDSS team [64].

Each of our eight chains allowed variation in all nine parameters. The results are summarized in Fig. 3 and Table II. In Fig. 3 we plot the marginalized 1-D posterior distribution functions for the nine parameters we allowed to vary in our analysis. The solid lines represent the posterior distribution functions (PDFs) for these parameters with cosmic strings included; the dashed lines show the results without cosmic strings ($B = 0$). We have shaded the regions excluded at 68% (light) and 95 % (dark) confidence. The peaks of each of these PDFs and the one-sigma error bars are given in Table II; for the parameter τ, where the results lack a clear peak, we have taken the midpoint of the 68% confidence region as our "peak" value.

The resulting cosmology with cosmic strings included is very close to the cosmology without cosmic strings. This verifies our hypothesis: a subdominant admixture of cosmic strings into the cosmological perturbation spectra gives a minor modification to the resulting cosmological parameters as determined in such a model. This justifies the approximation made in [66,67], where we fixed all of

TABLE II. The best-fit results

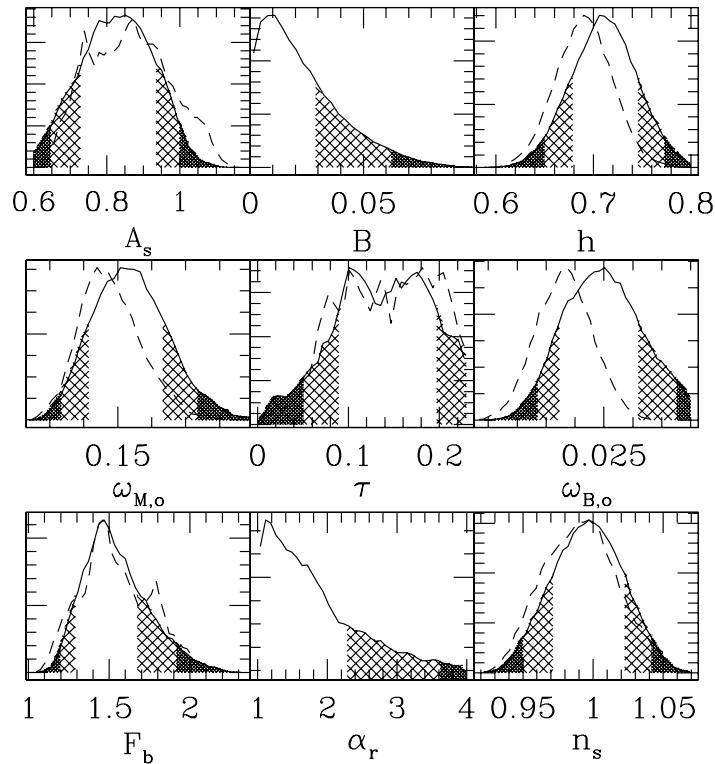| Parameter	$B = 0$	$B > 0$
f	—	$<0.068(68\%), <0.14(95\%)$
B	—	$<0.029(68\%), <0.062(95\%)$
α_r	—	$<2.3(68\%), <3.6(95\%)$
A_s	$0.87^{+0.08}_{-0.16}$	$0.85^{+0.09}_{-0.13}$
n_s	$1.0^{+0.02}_{-0.04}$	1.0 ± 0.026
$\Omega_B h^2$	0.024 ± 0.001	$0.025^{+0.0012}_{-0.0016}$
$\Omega_M h^2$	0.15 ± 0.01	$0.15^{+0.013}_{-0.01}$
h	0.69 ± 0.03	0.71 ± 0.034
τ	0.155 ± 0.057	0.143 ± 0.054
F_b	$1.46^{+0.24}_{-0.22}$	$1.47^{+0.2}_{-0.18}$

FIG. 3.   The one-dimensional projected PDFs for the 9 parameters varied by our Markov chain Monte Carlo code; note that $\omega_B = \Omega_B h^2$, $\omega_M = \Omega_M h^2$. The solid line represents the PDFs for models where cosmic strings are included; the dashed line represents the PDFs for models with $B = 0$, i.e. without cosmic strings. Each curve has been rescaled such that its area is unity. For each PDF the lightly shaded regions are excluded at the 68% confidence region; the dark regions are excluded at the 95% confidence region.

the cosmological parameters except for n_s to their WMAP best-fit values. The only noticeable shift caused by including cosmic strings comes in the peak likelihood in the PDF for $\Omega_B h^2$, and even this shift is small. The only potentially worrisome aspect of these PDFs is the fact that the PDF for τ appears to be running into the upper value of our prior range.

Let us examine the PDF for the cosmic-string weighting parameter, B, given in Fig. 4. The light and dark shaded regions again represent the 68 and 95% confidence intervals. Although the parameter name, B, is retained from our previous work [67], the fact that we have changed our string model has changed this parameter's meaning. Thus, Fig. 4 should not be directly compared with our earlier results. Indeed, we have added a new parameter for describing the strings, α_r, the string wiggliness. In our previous code, we had effectively fixed $\alpha_r = 1$. Since strings with $\alpha_r > 1$ have a larger effective mass per unit length than $\alpha_r = 1$ strings, the cosmological perturbations caused by $\alpha_r > 1$ strings are generally larger. Note that α_r and B were not degenerate parameters—the effect of a higher α_r, for a particular spectrum, was quite independent of B; see a (rough) contour plot in Fig. 5. This is because varying the wiggliness modifies the shape of the spectra in addition to their amplitude, as we described briefly in Sec. II A [33].
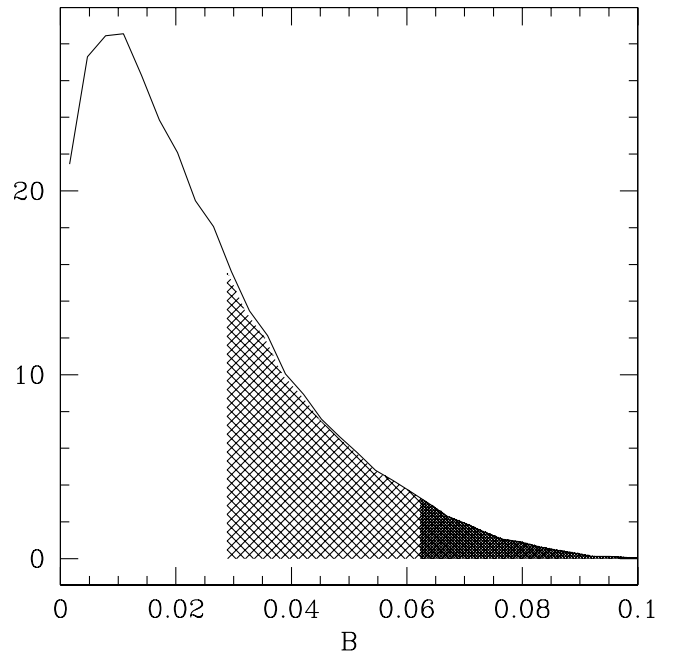

FIG. 4.   The PDF for the cosmic-string weighting parameter, $B \equiv (G\mu/2 \times 10^{-6})^2$.
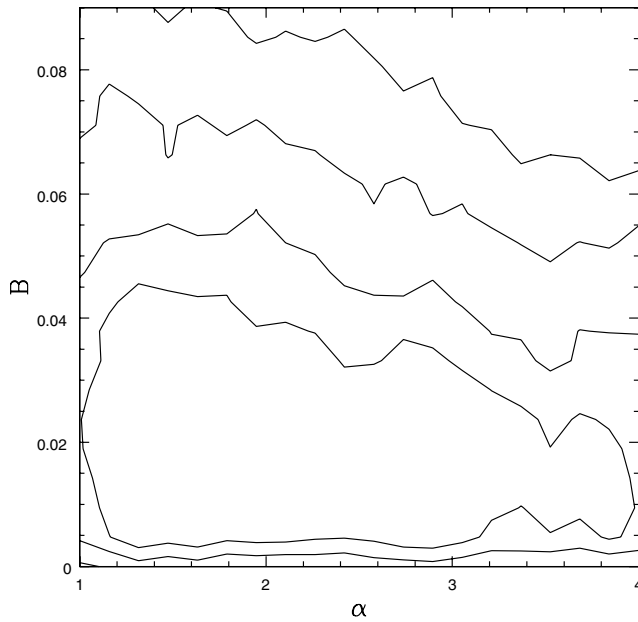
FIG. 5.   A contour plot of a separate set of MCMC results in the two-dimensional space defined by the parameters B and α. The contours are somewhat crude, but suggest that α is poorly constrained. At this level, there is no evidence for degeneracy between the parameters.

To quantify the total string contribution to the CMB anisotropy for a given set of parameters $\vec{s}$ we define the fraction f as

$$f(\vec{s}) = \frac{I^{\rm cs}}{I^{\rm cs} + I^{\rm ad}}, \qquad (6)$$

where

$$I^{\rm cs} = \sum_l \frac{(2l+1)}{4\pi} C_l^{\rm cs}(\vec{s}),$$

and

$$I^{\rm ad} = \sum_l \frac{(2l+1)}{4\pi} C_l^{\rm ad}(\vec{s}),$$

where $C_l^{\rm ad,cs}$ are the temperature (TT) correlation spectra. We then compute the PDF for this new parameter, which we show in Fig. 6 along with the 68 and 95% probability regions. We find that about 7% (14%) of the CMB power can be sourced by strings at 68% (95%) confidence. Note that the dashed line, corresponding to the three parameter analysis of [67], shows a smaller allowed string contribution.

Figures 4 and 6 are the principal results of this paper. Limits on B alone give limits on the string tension itself. Using the results in Fig. 4, we find a cosmic string weight of $B \lesssim 0.029(0.06)$ allowed at the 68 (95)% confidence level. This corresponds to $G\mu \lesssim 3.4(5.0) \times 10^{-7}$. The peak of the PDF for B lies at $B = 0.01$, or $G\mu \sim 2 \times$
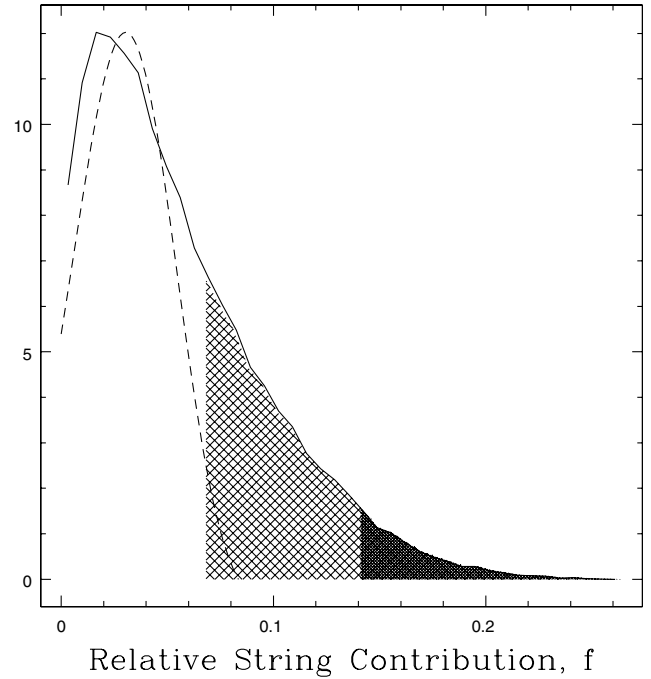

FIG. 6.   The PDF for the combination parameter f, which quantifies the fractional contribution of cosmic strings to the total $C_l^{\rm TT}$ spectrum. The solid line shows f from our full analysis, with the 68% (light) and 95% (dark) confidence regions shaded. The dotted line is the result for f from our previous three parameter analysis.

10^{-7}. These limits are relevant to searches for direct detections of cosmic strings, as the magnitudes of gravity wave and lensing events caused by cosmic strings depend directly on their tension.

The above bounds on $G\mu$ are model-dependent. In order to match the total observed CMB power, our string model requires $G\mu_0 \sim 2 \times 10^{-6}$. This value is on the upper end of other estimates in the literature, which means that our upper bound on $G\mu$ can be treated as a conservative upper bound. Our $G\mu_0$ is consistent with the COBE normalized values in [68] ($G\mu \approx 1.7 \times 10^{-6}$), in [69] ($G\mu = (1.5 \pm 0.5) \times 10^{-6}$), in [70] ($G\mu = (1.7 \pm 0.7) \times 10^{-6}$), and in [71] ($G\mu \approx 2 \times 10^{-6}$). It also agrees with [5] (for similar model parameters). Significantly different estimates of $G\mu_0$ were found in [54], where the COBE normalized value of $G\mu$ was $\sim 1.0 \times 10^{-6}$ (for their parameter $w^X = 1/3$), and similarly in [55]. The latest estimates of Landriau and Shellard [42] using realistic simulations of cosmic strings [42] (reliable up to $\ell \sim 20$) give the COBE normalized value of $G\mu = (0.74 \pm 0.2) \times 10^{-6}$ for a ΛCDM cosmology, which is consistent with results of a similar study in [72] where the value obtained was $G\mu = (1.05 \pm 0.3) \times 10^{-6}$. Note that most of the results obtained prior to 1999 assumed a CDM dominated cosmology—switching to ΛCDM leads to a $\sim 10\%$ increase in COBE normalized value of $G\mu$ [42].

Our bound on $G\mu$ would also be altered if the strings intercommute at a rate less than unity, as is expected in many string theory models of cosmic strings. The effect of reduced intercommutation would be to lower the upper limit on $G\mu$.

Our bound on the fraction of CMB power in strings, f, depends only on the *shapes* of the string-sourced CMB and LSS spectra. These shapes, as discussed in §II A, are largely independent of the details of the string model. The bound on f can be used to derive an approximate bound on $G\mu$, given one's favorite value of μ_0. For example, if one accepts the values in [42,72], i.e. $G\mu_0 \sim 10^6$, one obtains $G\mu' \sim f^{1/2}G\mu_0 \lesssim 2(3) \times 10^{-7}$ at 68 (95)% confidence level.

It is also worth recognizing that f can serve as a measure of the goodness-of-fit of the paradigmatic inflationary scenario in comparison with a physically motivated model; isocurvature is another example of a model used in such a manner (e.g. [73]). Our results from cosmic strings, serving from this viewpoint merely as a self-consistent foil to the standard model, show that as much as 14% of the CMB TT-correlation power could be sourced by a radically different spectrum without destroying the close agreement of the resulting spectrum with the anisotropy data. Loosely speaking, we can conclude from this bound that there is a cumulative ambiguity in the uniqueness of the adiabatic C_ℓ spectrum shape, as determined from the WMAP data, of around 10%. As more CMB data become available in the future, repetition of this analysis might be worthwhile, if only to discover whether the intrinsic shape of the adiabatic spectrum is more uniquely picked out by the more-exact future data sets.

B. The *B*-polarization spectrum

In Fig. 7 we plot the *B*-mode polarization spectra in the case of smooth strings ($\alpha_r = 1$) and wiggly strings $\alpha_r = 1.9$ predicted by our string model for the case when the total contribution of strings to the CMB anisotropy is 10%. That is, for each of the curves, the value of $G\mu$ was adjusted separately to correspond to $f = 0.1$. For comparison, we also plot the *B*-mode spectra from a purely adiabatic cosmology. The light dotted line represents the *B*-mode polarization arising from gravitational lensing of *E*-mode polarization. The light dash-dotted line represents the *B*-mode arising from gravitational waves. Any direct detection of cosmic-string-generated *B*-modes will rely upon accurate predictions of the spectrum of *B*-mode polarization arising from lensing of *E*-mode polarization. Fortunately, several observational groups believe that it will be possible to "clean" as much as 90% of the apparent *E* to *B* lensed power through accurate *E*-mode observations. With gravity-wave generated *B*-modes peaking at a much lower ℓ, any excess power in observed *B*-mode spectra at $\ell \sim 1000$ could be a telling sign of cosmic-string activity. Two planned experiments, QUIET and QUaD
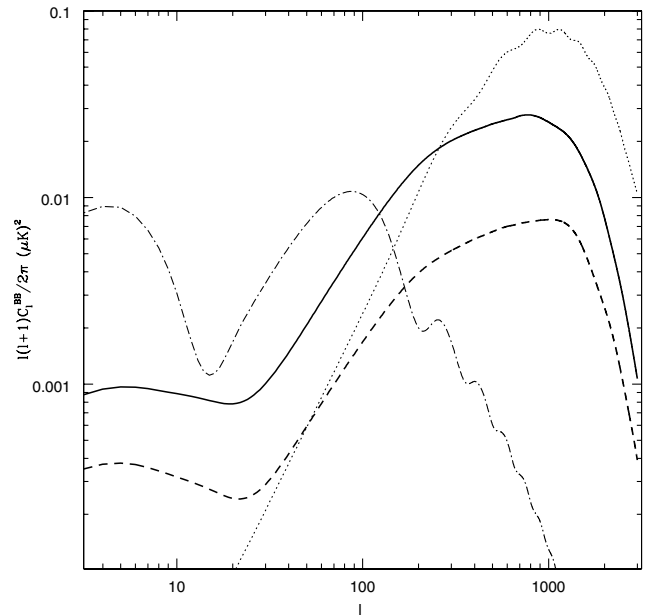

FIG. 7. The *B*-type polarization spectra for $\alpha_r = 1.9$ (dashed lined) and $\alpha_r = 1$ (solid line) both corresponding $f = 0.1$. The light dotted line is the *B*-mode expected from gravitational lensing of adiabatic *E*-mode polarization; the light dash-dotted line is the *B*-mode expected from gravity wave-sourced polarization, for tensor-to-scalar ratio $r = 0.1$.

[74], expect to be able to measure such high-ℓ polarization with great precision. It is also worth noting that the amplitude of gravity-wave generated *B*-mode polarization is intimately tied to the as-yet undetermined scalar-to-tensor ratio, r. We have used $r = 0.1$ in Fig. 7, which is usually regarded as an optimistically high value. Inflationary models frequently produce orders-of-magnitude lower estimates for r (in [24], for instance, investigations predict $10^{-8} \lesssim r \lesssim 10^{-3}$ for KKLMMT-motivated brane inflation [75]). For $r \ll 0.1$, cosmic strings could be the dominant source of *B*-mode polarization for low values of ℓ ($10 < \ell < 1000$), but with a spectrum that is recognizably distinct from the polarization generated by gravity waves. The proposed CLOVER experiment [76] and its space-based successors plan to focus their measurements on this region of ℓ-space.

IV. SUMMARY

We found two types of constraints on cosmic-string networks through our analysis of the WMAP and SDSS data. One is on the value of $G\mu$, which is sensitive to our normalization convention and to the string intercommutation rate, which we take to be unity in setting our bound. The other is on the fraction of total CMB power due to cosmic strings, f. The constraint on f depends chiefly on the general shape of the string-induced spectrum. The shape of the spectrum is quite generic – a plateau on large scales due to superposition of the Kaiser-Stebbins effect of

many strings (from a scaling network), followed by a broad peak on small scales. The peak is caused by the Doppler and Sachs-Wolfe effects produced by velocity and density perturbations caused by strings during the epoch of recombination. The rough position of the peak is set by the size of a typical wake at last scattering, while the wake size is set by the coherence (or curvature) length $\xi(t)$ at that time. We can call these features generic because they are agreed upon by all groups that have studied string-induced CMB spectra. The fine details of the shape, e.g. the sharpness of the peak, depend on many factors. Among the most important of those extra factors is the amount of small-scale structure on strings (the wiggliness), which we included as a parameter in our calculation.

In addition to quantifying the allowed fraction of the cosmic-string contribution, our parameter f can also be interpreted as a measure of the goodness-of-fit of the fiducial adiabatic CMB spectrum model. The fact that the data permit approximately 10% of the TT-correlation power to arise from a very different competitor model gives a useful measure of how uniquely the data pick out the shape of the adiabatic spectrum.

Other recent constraints on local strings that also used WMAP data include Ref. [77], where the narrowness of the first peak was used to constraint the size of the incoherent string contribution, and Refs. [78,79], where the constraint was based on the expected non-Gaussian signatures induced by strings. Ref. [77] suggests an interesting way to obtain a rough bound the string contribution to the CMB spectrum at $\ell \sim 220$ from constraints on the width of the main peak. Our method has the advantage of including the information on all scales (not just the main peak) and the ability to account for changes in in cosmological parameters and the shape of the string spectra (by varying the wiggliness).

Our limit on the string tension, $G\mu < 5 \times 10^{-7}$, does not contradict a variety of recent claims of observational evidence for the existence of cosmic strings. The most prominent have been possible examples of cosmic-string lensing. In the first case [80], a pair of nearly identical images were found, the best explanation of which appears to be a direct lensing by a cosmic string with tension $G\mu \sim 4 \times 10^{-7}$. Another possible cosmic-string lensing observation [81] is the appearance of short time-scale variations in the brightness of the well-known gravitational lens system, Q0957 + 561, which could be explained by a passing cosmic-string loop with a tension in the range $10^{-8} \lesssim G\mu \lesssim 6 \times 10^{-7}$. The inferred time scale of these variations is so short, ~ 100 days, that very small-scale string structure would be required, making the claim somewhat problematic (for more on these events, see [82]). It is also possible that strings are responsible for the early reionization suggested by WMAP [83], since cosmic-string wakes can bring about star formation much earlier [84] than the standard Press-Schechter scenario [85]; con-

servative analysis of string-mediated early reionization suggests a bound of $G\mu \lesssim 5 \times 10^{-7}$ [86]. In the arena of gravitational wave observation, current pulsar timing bounds are still marginally consistent with $G\mu \sim 10^{-7}$ [87], while analysis of gravitational wave bursts from string loops suggests that $G\mu \sim 10^{-7}$ cosmic strings will be readily observable by both LIGO and LISA [88].

It is often thought that a key observational test for cosmic strings would be whether non-Gaussianity is found in the primordial perturbations seen by such experiments as WMAP. This is a natural assumption, since each string acting alone would perturb the CMB in a highly non-Gaussian manner. However, the central limit theorem tells us that the superposition of perturbations produced by many strings must be Gaussian. Therefore, one expects to see string-related non-Gaussian features only on scales that are sufficiently small not to have been crossed by more than a few strings during the entire period of time during which strings have produced their effects. It is not difficult to get a conservative estimate of this scale. The dominant contribution to the anisotropy on small (subdegree) scales comes from the Doppler and Sachs-Wolfe effects produced at the last scattering [89]. A natural length scale to start with is the angular size of the horizon at recombination, which corresponds to $\ell \sim 220/\sqrt{3}$. Numerical simulations [43,44] show that the typical distance between strings during matter domination is $L \sim 0.8t$. It is somewhat smaller in the radiation era, and can be much smaller if the intercommutation probability was less than unity, as may be the case in string theory models. A conservative estimate of the number of strings per horizon at any time in the matter era is $\sim (4\pi t^3/3)/L^3 \sim 10$. When projected onto the last scattering surface, about half of this number of strings would contribute. Hence, the CMB anisotropy in a patch corresponding to $\ell \sim 220/\sqrt{3}$ would be a superposition of the effects of about 5 strings, and to isolate the effect of one string, one would have to go to scales of order: $\ell \sim 220 \times 5/\sqrt{3} \sim 600$. In doing this rough estimate we have ignored the density perturbations created by wakes between the radiation-matter equality and last scattering, which contribute a non-negligible fraction of the power near the main peak [89]. This contribution would tend to make the scale at which non-Gaussianity appears even smaller, since the wakes started to from from the onset of matter domination. Thus one will likely need a resolution of at least $\ell \sim 1000$ to have any hope of seeing non-Gaussianity from strings. Furthermore, in the above argument we did not account for the fact that strings can produce only 10% of the total anisotropy. This makes the detection of their non-Gaussian signatures even more difficult. The possibility of low string intercommutation rates (high string density), in addition to making strings more Gaussian (via the central limit theorem), also strengthens the bound on their tension, hence further complicating the detection of their non-Gaussian properties. The analysis in

Refs. [78,79] assumed $\ell \sim 200$ as the scale for the onset of non-Gaussianity in the CMB caused by strings. A more realistic scale, as we have argued above, is likely to be an order of magnitude smaller, so the detection of string-sourced non-Gaussianity appears to be beyond the reach of WMAP, and quite possibly even Planck. The existing constraints on string-sourced CMB non-Gaussianity, such as those obtained in Refs. [78,79], appear to reflect the limitation that, given the variance of a CMB map on a certain scale, σ_ℓ, one naturally has difficulty resolving any detailed features on those scales that have amplitudes comparable to σ_ℓ. In light of this, it is not surprising that the upper bound on $G\mu$ obtained in [78,79] ($G\mu \lesssim 10^{-5}$) roughly corresponds to the variance of the WMAP CMB map on subdegree scales.

We find that cosmic strings with tensions of $G\mu = 5 \times 10^{-7}$ are still allowed by the data from the WMAP and SDSS experiments, and that strings can account for as much as 14% of the the temperature fluctuations in a cosmic microwave background radiation dominated by adiabatic fluctuations without any significant changes in the underlying cosmology. Indeed, the excess power at very small angular scales seen in CMB observations may already suggest the presence of cosmic-string generated perturbations, which may dominate on such scales [90]. Strings with allowed tensions are produced in brane infla-tion models, implying that such models are still viable and that the strings produced by them may be observable, giving us hope of an observational window on string theory. One promising signature of cosmic strings with these tensions in the early universe would be their creation of observable *B*-mode polarization in the CMB with spec-tra distinct both from those created by *E*-mode lensing and by gravity waves from primordial tensor modes. If primor-dial tensor modes are weak, cosmic strings could be the principal new physics seen by *B*-mode observations; suc-cessful observation of such *B*-modes would in turn be our first direct observational probe into the physics of string theory.

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