Relaxing constraints on inflation models with curvaton

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We consider the effects of the curvaton, late-decaying scalar condensation, to observational constraints on inflation models. From current observations of cosmic density fluctuations, severe constraints on some classes of inflation models are obtained, in particular, on the chaotic inflation with higher-power monomials, the natural inflation, and the new inflation. We study how the curvaton scenario changes (and relaxes) the constraints on these models.

I. INTRODUCTION

Inflation, a superluminal expansion at the early stage of the Universe [1], is one of the most promising ideas to solve the flatness and horizon problems which are serious drawbacks of the standard hot big bang model. Inflation can be caused by the potential energy of a scalar field called inflaton which slowly rolls down the potential. Since quantum fluctuation of the inflaton during inflation is usually assumed to be the origin of cosmic density perturbation, models of inflation can be tested by observations of cosmic microwave background (CMB), large scale structure, and so on. Recent results from the Wilkinson microwave anisotropy probe (WMAP) [2] gave severe constraints on inflation models [3]. For example, the chaotic inflation model [4] with quartic potential $V_{\rm inf} \propto \chi^4$, where χ is the inflaton, is on the verge of exclusion by observations assuming the standard thermal history of the Universe.

However, today's cosmic fluctuation can originate from some source other than the inflaton. In the curvaton scenario [5–7], adiabatic fluctuation can be given by primordial fluctuation of some late-decaying scalar field, which is called curvaton. Importantly, there are a sizable number of well-motivated candidates of the curvaton. Indeed, various possibilities have been extensively discussed: for example, a pseudo-Nambu-Goldstone boson [8], late-decaying moduli fields [7], right-handed sneutrino in supersymmetric models [9], (string) axion [5,10], flat directions in the supersymmetric standard model [11], and so on. Thus, it is important to study how the possible primordial condensation of these fields may modify the constraints on the inflation models from the particle physicists' point of view as well as from the astrophysicists'.

In the curvaton scenario, the superluminal expansion is caused by the inflaton while (some part of) the fluctuation responsible for the cosmic fluctuation today is provided by the curvaton field. Thus, in this case, constraints on models of inflation can be alleviated to some extent. Even if the fluctuation of the inflaton is partly relevant to today's fluctuation, we can expect that constraints on models of inflation can be relaxed compared to the case without the

curvaton. Some authors have discussed the curvaton scenario along this line. In Ref. [12], the authors discussed inflation models which can be liberated by the curvaton paying attention to the so-called Cosmic Background Explorer (COBE) normalization. There have been subsequent works related to this issue [13,14].

In this paper, we investigate this issue in detail for various inflation models which, in particular, are on the verge or already excluded in the standard inflationary scenario. We will show that some inflation models can be made viable in the curvaton scenario.

The organization of this paper is as follows. In the next section, we give a brief review of the scenario where both inflaton and curvaton fluctuations give today's cosmic fluctuations and define our notations. In addition, we also describe our analysis method. In Sec. III, we discuss to what extent constraints on inflation models are alleviated in the curvaton scenario starting from the chaotic inflation model, the natural inflation model, and the new inflation model. The conclusion of our study is given in the final section.

II. FORMALISM

A. Background evolution

The aim of this paper is to investigate to what extent constraints on inflation models can be relaxed in the curvaton scenario. Thus, we need to discuss density perturbations. However, before discussing density perturbations, first we discuss the background evolution and fix our notations.

We start with the inflationary era. During inflation, there are assumed to be two scalar fields: one is the inflaton χ and the other is the curvaton ϕ, which follow the following equations of motion:

$$\ddot{\chi} + 3H\dot{\chi} + \frac{dV_{\rm inf}}{d\chi} = 0, \qquad (2.1)$$

$$\ddot{\phi} + 3H\dot{\phi} + \frac{dV_{\rm curv}}{d\phi} = 0, \qquad (2.2)$$

where the dot represents derivative with respect to time, V_{inf} and V_{curv} are the potentials for the inflaton and the curvaton, respectively, H is the Hubble parameter which is given by

$$H^2 = \left(\frac{\dot{a}}{a}\right)^2 = \frac{1}{3M_{\text{pl}}^2}\rho_{\text{tot}}, \qquad (2.3)$$

where a is the scale factor, $M_{\text{pl}} \simeq 2.4 \times 10^{18}$ GeV is the reduced Planck scale, and ρ_{tot} is the total energy density of the Universe. During inflation, it is given by the sum of contributions from the inflaton and the curvaton as

$$\rho_{\text{tot}} = \rho_\chi + \rho_\phi = \tfrac{1}{2}\dot{\chi}^2 + V_{\text{inf}} + \tfrac{1}{2}\dot{\phi}^2 + V_{\text{curv}}. \qquad (2.4)$$

For the potential of the inflaton, we consider various possibilities which we will describe later. For the curvaton, for simplicity, we consider the quadratic potential in this paper:

$$V(\phi) = \tfrac{1}{2}m_\phi^2\phi^2, \qquad (2.5)$$

where m_ϕ is the mass of the curvaton.

After the inflation, the inflaton begins to oscillate around the minimum of the potential. We call this epoch a χ-dominated or χD era. Then the inflaton decays when $H \sim \Gamma_\chi$ (with Γ_χ being the decay rate of the inflaton). In this period, the evolution of the inflaton is described by Eq. (2.1) with the decay term being included. Once the expansion rate H becomes much smaller than the oscillation frequency of χ, however, it is rather convenient to consider the averaged value of the energy density of χ and to follow its evolution. Indeed, for the inflaton potential $V_{\text{inf}} \propto \chi^n$, ρ_χ follows the equation

$$\dot{\rho}_\chi = -3(1 + w_\chi)H\rho_\chi - \Gamma_\chi\rho_\chi, \qquad (2.6)$$

with $w_\chi = (n-2)/(n+2)$. Thus, the energy density of the oscillating inflaton behaves as [15]

$$\rho_\chi \propto a^{-[6n/(n+2)]}. \qquad (2.7)$$

In our study, we consider only the case where Γ_χ is small enough so that the inflaton decays when H is much smaller than the oscillation frequency of χ. Thus, the reheating processes by the inflaton are studied by using Eq. (2.6). In addition, the energy density of the radiation evolves as

$$\dot{\rho}_{\text{rad}} = -4H\rho_{\text{rad}} + \Gamma_\chi\rho_\chi + \Gamma_\phi\rho_\phi, \qquad (2.8)$$

with Γ_ϕ being the decay rate of the curvaton. Here we included the effect of the curvaton decay, which will become important in the following discussion. When $H \sim \Gamma_\chi$, the inflaton decays into radiation; then the Universe is reheated to become radiation-dominated. We call this radiation-dominated era "RD1."

When the expansion rate of the Universe becomes comparable to m_ϕ, the curvaton also starts to oscillate; then it behaves like a matter component for the potential given in Eq. (2.5). Thus, at some epoch the Universe becomes curvaton-dominated. We call this epoch the "ϕD" era. After the ϕD era, the curvaton decays into radiation; then the Universe becomes radiation-dominated again. We call this epoch the "RD2" era. The time when the RD2 era starts depends on the decay rate of the curvaton Γ_ϕ. From the ϕD era to the RD2 era, the Universe consists of radiation and the oscillating curvaton field. Since the curvaton behaves like matter in this epoch, energy density of the curvaton is governed by the following equation:

$$\dot{\rho}_\phi = -3H\rho_\phi - \Gamma_\phi\rho_\phi. \qquad (2.9)$$

If the initial amplitude of the curvaton is as large as $\phi_{\text{init}} \sim M_{\text{pl}}$, the curvaton can cause the second inflation after the usual inflation provided by the inflaton. In this case, the second inflation epoch exists before the ϕD era mentioned above. Depending on the initial amplitude and mass of the curvaton, the second inflation era can start during the χD era or the RD1 era. Such an effect is also taken into account in our analysis (in particular, in our numerical study to evaluate the e-folding number during the inflation).

Here we summarize what we need to fix the background evolution. As for the background, the following parameters are necessary to determine the thermal history of the Universe except the potential of the inflaton (and the initial amplitude of the inflaton): the mass of the curvaton m_ϕ, the initial amplitude of the curvaton ϕ_{init}, the decay rates of the inflaton and the curvaton Γ_χ and Γ_ϕ.

B. Density perturbation

Next we discuss the density perturbation in the scenario. Since there are two sources of cosmic density fluctuation in this case (i.e., the inflaton and the curvaton), we need to take account of the effects of both of them. Since the primordial fluctuations of the inflaton and the curvaton are uncorrelated, we can study their effects separately. First we summarize the results for the standard case, i.e., the case with the inflaton only. Then we will discuss how the situation is modified in the curvaton scenario.

The quantum fluctuation of the inflaton $\delta\chi$ during inflation generates the curvature perturbation as $\mathcal{R} = -(H/\dot{\chi})\delta\chi$. Since $\delta\chi \simeq H/2\pi$, the power spectrum of the curvature perturbation from the inflaton fluctuation is

$$P_{\mathcal{R}}^{(\text{inf})} = \frac{1}{12\pi^2 M_{\text{pl}}^6} \frac{V_{\text{inf}}^3}{V_{\text{inf}}'^2}\Bigg|_{k=aH}, \qquad (2.10)$$

where the "prime" is the derivative with respect to χ. Here we used the slow-roll approximation. To discuss observational consequences, we have to set up the initial condition during the radiation-dominated era after the decay of the inflaton. For this purpose, we represent the primordial power spectrum with the (Bardeen's) gravitational potential Φ which appears in the perturbed metric in the con-

formal Newtonian (or longitudinal) gauge as

$$ds^2 = -a^2(1 + 2\Phi)d\tau^2 + a^2(1 - 2\Psi)dx^2, \quad (2.11)$$

with τ being the conformal time. During the radiation-dominated era, Φ and $\mathcal{R}$ are related as $\Phi = -(2/3)\mathcal{R}$. Thus, the power spectrum of Φ is given by $P_\Phi = (4/9)P_\mathcal{R}$. The spectral index of the primordial power spectrum is

$$n_s - 1 \equiv \frac{d\ln P_\Phi}{d\ln k}. \quad (2.12)$$

Using the slow-roll parameters defined as[1]

$$\epsilon \equiv \frac{1}{2}M_{pl}^2\left(\frac{V'_{inf}}{V_{inf}}\right)^2, \qquad \eta \equiv M_{pl}^2\frac{V''_{inf}}{V_{inf}}, \quad (2.13)$$

the spectral index can be written as

$$n_s^{(inf)} - 1 = -6\epsilon + 2\eta. \quad (2.14)$$

During inflation, the gravity wave can also be generated. The primordial gravity wave (tensor) power spectrum is given by

$$P_T^{(inf)} = \frac{2V_{inf}}{3\pi^2 M_{pl}^4}. \quad (2.15)$$

With this expression, the tensor-scalar ratio r is defined and given by

$$r^{(inf)} \equiv \frac{P_T^{(inf)}}{P_\mathcal{R}^{(inf)}} = \frac{4}{9}\frac{P_T^{(inf)}}{P_\Phi^{(inf)}} = 16\epsilon. \quad (2.16)$$

Now we consider the fluctuations generated by the curvaton fluctuation. Let us start with summarizing the equations relevant for the following discussion. From the Einstein equation, we obtain the perturbation equation for the metric perturbations:

$$-k^2\Phi = \frac{3}{2}\mathcal{H}^2\left[\delta_{tot} + \frac{3\mathcal{H}}{k}(1 + w_{tot})V_{tot}\right], \quad (2.17)$$

where $\mathcal{H} = (1/a)(da/d\tau)$ is the conformal Hubble parameter, $\delta_X = \delta\rho_X/\rho_X$, and V_X are density perturbation and velocity perturbation of a component X. In the above equation, "tot" denotes the total matter, and $w_{tot} = p_{tot}/\rho_{tot}$ is the equation-of-state parameter of the total matter. Assuming that the anisotropic stress is negligible, the metric perturbation variables are related as $\Phi = \Psi$. The equations for the density and velocity perturbations of the component whose equation-of-state parameter is w_X are

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[1] In this paper, we use so-called "potential" slow-roll parameters, which are defined using the value of the potential not using the Hubble parameter.

$$\frac{d\delta_X}{d\tau} = -(1 + w_X)\left(kV_X - 3\frac{d\Psi}{d\tau}\right), \quad (2.18)$$

$$\frac{dV_X}{d\tau} = -\mathcal{H}(1 - 3w_X)V_X + \frac{w_X}{1 + w_X}k\delta_X + k\Phi. \quad (2.19)$$

Evolutions of the fluctuations are followed by solving the above equations (as well as those for the background quantities). Since there are two independent contributions to the cosmic density fluctuations, i.e., the primordial fluctuations of the inflaton and the curvaton, Φ_{RD2} in this case has two terms which are proportional to $\delta\chi_{init}$ and $\delta\phi_{init}$, respectively:

$$\Phi_{RD2} = -\frac{2}{3M_{pl}^2}\frac{V_{inf}}{V'_{inf}}\delta\chi_{init} - f(X)\frac{\delta\phi_{init}}{M_{pl}}, \quad (2.20)$$

where $\delta\phi_{init}$ is the primordial fluctuation of the curvaton, and

$$X = \frac{\phi_{init}}{M_{pl}}. \quad (2.21)$$

The first term of the right-hand side of Eq. (2.20) is the inflaton contribution which we have already discussed. The second term is the curvaton contribution; in our study, $f(X)$ is calculated by numerically solving the Einstein and Boltzmann equations. Here the curvaton contribution is parametrized by the function $f(X)$ [14]. Although we have used the numerical method to calculate $f(X)$, effects of the curvaton can be easily evaluated for the cases where the initial amplitude of the curvaton is much larger or much smaller than M_{pl} [14,16]; for these cases, $f(X)$ is given by

$$f(X) \simeq \begin{cases} \frac{4}{9X} & \phi_{init} \ll M_{pl} \\ \frac{1}{3}X & \phi_{init} \gg M_{pl}. \end{cases} \quad (2.22)$$

Assuming that the inflaton and the curvaton are uncorrelated, we find the power spectrum of Φ as [12,14]

$$P_\Phi = [1 + \tilde{f}^2(X)\epsilon]\frac{V_{inf}}{54\pi^2 M_{pl}^4\epsilon}, \quad (2.23)$$

where $\tilde{f} = (3/\sqrt{2})f$. Here the first and the second terms in the brackets are from the inflaton and curvaton fluctuations, respectively.

The inflaton and curvaton contributions have different scale dependence, which are parametrized by the following spectral indices:

$$n_s^{(inf)} - 1 \equiv \frac{d\ln P_\mathcal{R}^{(inf)}}{d\ln k}, \qquad n_s^{(curv)} - 1 \equiv \frac{d\ln P_\mathcal{R}^{(curv)}}{d\ln k}, \quad (2.24)$$

where $P_\mathcal{R}^{(curv)}$ is the power spectrum of the curvature perturbation from the curvaton fluctuation. With these parameters, the total CMB angular power spectrum can be calculated as $C_l = C_l^{(inf)} + C_l^{(curv)}$, where $C_l^{(inf)}$ is the in-

flaton contribution while $C_l^{(\text{curv})}$ is the curvaton contribution. (Notice that $C_l^{(\text{inf})}$ and $C_l^{(\text{curv})}$ are calculated with the spectral index $n_s^{(\text{inf})}$ and $n_s^{(\text{curv})}$, respectively.) However, the scale dependence of the total power spectrum is well parametrized by the single spectral index, which we call the "effective spectral index," given by

$$n_s - 1 \equiv \frac{d \ln(P_{\mathcal{R}}^{(\text{inf})} + P_{\mathcal{R}}^{(\text{curv})})}{d \ln k}. \tag{2.25}$$

This is because $n_s^{(\text{inf})}$ and $n_s^{(\text{curv})}$ are both close to 1 and also because the runnings of these indices are negligibly small.[2] Thus, we use this effective spectral index in our analysis in deriving the constraints.

The (effective) scalar spectral index is given by[3]

$$n_s - 1 = -2\epsilon + \frac{2\eta - 4\epsilon}{1 + \tilde{f}^2 \epsilon}. \tag{2.26}$$

The tensor power spectrum is not modified even with the curvaton. However, since the scalar perturbation spectrum is modified, the tensor-scalar ratio becomes

$$r = \frac{16\epsilon}{1 + \tilde{f}^2 \epsilon}. \tag{2.27}$$

III. RESULTS

Now we are ready to discuss how the constraints on inflation models are alleviated with curvaton. We investigate the following inflation models: chaotic inflation models for several monomials, the natural inflation model, and the new inflation model. For these models, observations of CMB anisotropy and large scale structure provide severe constraints, and some part of the parameter space in these models is excluded in the standard scenarios.

Including the curvaton contributions, we analyze the constraints on these inflation models, making use of the observational constraints on the scalar spectral index n_s and the tensor-scalar ratio r. In our analysis, we first calculate n_s and r for a given model. For this purpose, it is important to determine the amplitude of the inflaton field at the time of the horizon exit χ_*; once χ_* is given, n_s and r are calculated by using Eqs. (2.26) and (2.27). (In this paper, the subscript "*" is used for the quantities at the

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[2]We have checked that the difference between the CMB angular power spectrum from the exact calculation (with $n_s^{(\text{inf})}$ and $n_s^{(\text{curv})}$) and that with the effective spectral index is less than $\sim 0.1\%$ in our case.

[3]The spectral index for the curvaton fluctuation also acquires a contribution of the order of $m_\phi^2/H_{\text{inf}}^2$, with H_{inf} being the expansion rate during the inflation. (For the cases $X \ll 1$ and $X \gg 1$, such a contribution is given by $2\eta_{\phi\phi}$ and $-2\eta_{\phi\phi}$, respectively, where $\eta_{\phi\phi} = m_\phi^2 M_{\text{pl}}^2/V_{\text{total}}$ [6,17].) In this paper, however, we consider only the case where the mass of the curvaton is much smaller than H_{inf}, and, hence, such contribution is negligible.

time of the horizon exit.) Importantly, χ_* depends on various parameters. In order for the systematic and precise determination of χ_*, we followed the evolution of the inflaton and the curvaton field (as well as the energy density of radiation) from the inflationary era to the RD2 era by numerically solving the Einstein and Boltzmann equations given in the previous section. Since we evaluate the values of n_s and r at $k = 0.01 \text{ Mpc}^{-1}$ to compare a model with observations of CMB and large scale structure, χ_* is determined by the condition $H_*(a_*/a_0) = 0.01 \text{ Mpc}^{-1}$, where a_* and H_* are the scale factor and the expansion rate at $\chi = \chi_*$, respectively, and a_0 is the present scale factor. In our analysis, χ_* is calculated as a function of X, m_ϕ, and other model parameters (in particular, the decay rates of the inflaton and the curvaton). With χ_*, we calculate n_s and r. Then we compare them with the observational constraints on n_s and r obtained by many authors (see, e.g., [18]). In our analysis, an allowed region on the m_ϕ vs X plane is given by the region where (n_s, r) calculated with (X, m_ϕ) falls onto the 95% C.L. allowed region on the n_s vs r plane given in Ref. [18]. In the cases of the natural inflation and the new inflation, we show the constraints in other planes instead of the m_ϕ vs X plane replacing m_ϕ with another parameter in the inflaton sector. However, even in such a case, the analysis method is the same as above.

A. Chaotic inflation

We parametrize the potential for the chaotic inflation as

$$V_{\text{inf}} = \lambda M_{\text{pl}}^4 \left(\frac{\chi}{M_{\text{pl}}}\right)^\alpha. \tag{3.1}$$

Here we require α to be an even integer for the positivity and smoothness of the inflaton potential. We consider the cases with $\alpha \geq 4$, for which observations of CMB and large scale structure provide severe constraints for the case without the curvaton.

The slow-roll parameters in this model are given by

$$\epsilon = \frac{1}{2}\alpha^2 \frac{M_{\text{pl}}^2}{\chi_*^2}, \qquad \eta = \alpha(\alpha - 1)\frac{M_{\text{pl}}^2}{\chi_*^2}. \tag{3.2}$$

The moment when the present horizon scale exits the horizon during inflation is usually denoted with the e-folding number of inflation, which is defined as the logarithm of the ratio of the scale factors at the horizon crossing and the end of inflation: $N_e \equiv \ln(a_{\text{end}}/a_*)$. We can write this quantity with the amplitude of the inflaton

$$N_e = \frac{1}{M_{\text{pl}}^2} \int_{\chi_{\text{end}}}^{\chi_*} \frac{V_{\text{inf}}}{V_{\text{inf}}'} d\chi, \tag{3.3}$$

where χ_{end} is the inflaton amplitude when one of the slow-roll parameters becomes 1. Since the amplitude of the inflaton at the end of inflation is much smaller than χ_*, the e-folding number can be written as

$$N_e \simeq \frac{1}{2\alpha M_{\rm pl}^2} \chi_*^2. \qquad (3.4)$$

Although N_e depends on the thermal history of the Universe, it is often assumed that $N_e \sim 50$.[4] With $N_e = 50$, the scalar spectral index and the tensor-scalar ratio $(n_s - 1, r)$ are $(-0.06, 0.32)$ for $\alpha = 4$, which are marginally excluded by the WMAP result. For $\alpha = 6$ and 8, $(n_s - 1, r) = (-0.08, 0.48)$ and $(-0.1, 0.64)$, respectively, which are completely ruled out.

In the curvaton scenario, fluctuation of the curvaton also affects the primordial spectrum. As for the spectral index, we can see from Eq. (2.26) that $|n_s - 1|$ becomes smaller in the case with curvaton for the chaotic inflation. The tensor-scalar ratio also becomes smaller as we can see from Eq. (2.27). Thus, constraints on the inflation models based on n_s and r can be relaxed with curvaton.

Before going into the detail of our analysis, let us first describe how we treat the free parameters: In the chaotic inflation case, the free parameters are the coupling in the potential λ, the mass of the curvaton m_ϕ, the initial amplitude of the curvaton $\phi_{\rm init}$, and the decay rate of the curvaton Γ_ϕ. Properties of the density fluctuations depend on these parameters. One combination of these parameters is, however, precisely fixed in order to obtain properly normalized cosmic density fluctuations (i.e., the so-called COBE normalization). Normalization of the total density fluctuations is sensitive to the λ parameter, and, hence, in our analysis, we determine λ so that the normalization of the size of the density fluctuation becomes consistent with the COBE normalization. Since we discriminate the model using the constraints on n_s and r, we do not discuss λ in the following.[5] For other parameters, we regard them and derived the constraints on the model for several values of these parameters.

1. $\alpha = 4$ *case*

The chaotic inflation with $\alpha = 4$ is on verge of exclusion by cosmological observations as mentioned above. Here we consider how this constraint changes in the curvaton scenario.

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[4]In the following, we use $N_e = 50$ as a typical value of the e-folding number to make some estimations, although N_e depends on the model of inflation and on the properties of the curvaton (see [19]). Notice that, in our numerical calculations, the relation $N_e = 50$ is not assumed, but the e-folding number is calculated by numerically following the evolution of the Universe from the inflationary epoch.

[5]The parameter λ also affects the spectral index and the tensor-scalar ratio since, once we vary λ, the e-foldings during the inflation also slightly changes. In our analysis, we use the numerical method to follow the evolution of the Universe (even in the period of the inflation) and to calculate the e-folding number. Thus, the λ dependences of the n_s and r parameters are automatically taken into account.
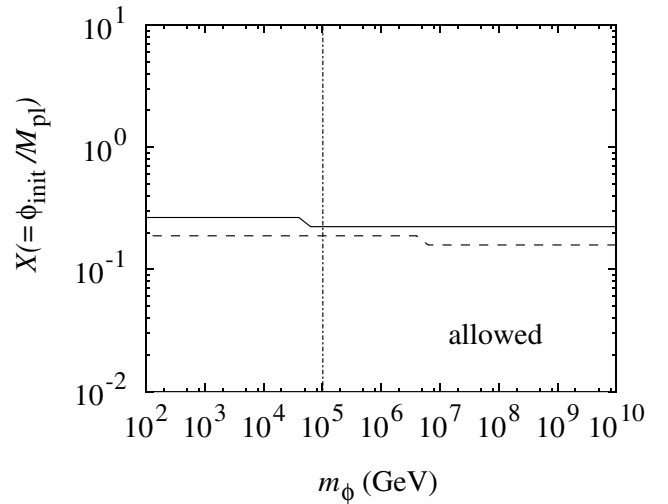

FIG. 1.   The 95% C.L. allowed regions are shown for $\alpha = 4$. The decay rate of the curvaton is taken to be $\Gamma_\phi = 10^{-6}$ GeV (solid line) and $\Gamma_\phi = 10^{-22}$ GeV (dashed line). The lower side of the horizontal line is allowed for the given Γ_ϕ. The vertical dashed-dotted line shows the value of $m_\phi^{\rm (grav)}$ given in Eq. (3.5) for the case of $\Gamma_\phi = 10^{-22}$ GeV.

For the case of $\alpha = 4$, there is one simplification. In general, the decay rate of the inflaton affects the number of e-foldings. In the case of $\alpha = 4$, however, the oscillating inflaton field behaves as radiation; namely, the energy density of the oscillating scalar field is given by $\rho_\chi \propto a^{-4}$. Hence, Γ_χ is irrelevant to determine the number of e-foldings in this model.

In Fig. 1, the 95% C.L. allowed regions are shown in the m_ϕ vs X plane.[6] In the figure, the decay rate of the curvaton is chosen to be $\Gamma_\phi = 10^{-6}$ and 10^{-22} GeV, which give the reheating temperature $T_{\rm reh} \sim 10^6$ GeV and 10 MeV, respectively. We also use these values for other models. (Here, $T_{\rm reh} \sim 10^6$ GeV is the upper bound for solving the gravitino problems [20], while $T_{\rm reh} \sim 10$ MeV is a possible lower bound not to spoil the big bang nucleosynthesis.) As discussed in the previous section, contribution from the curvaton fluctuation to the cosmic density fluctuations becomes larger for smaller $\phi_{\rm init}$. Thus, regions with small X become allowed. On the other hand, regions with large initial amplitude are excluded for most cases. In fact, in such regions, the contribution from the curvaton fluctuation is enhanced. However, with large $\phi_{\rm init}$, the second inflation occurs with the energy density of the curvaton, and the number of e-foldings during the first inflation is

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[6]When $\Gamma_\phi = 10^6$ GeV, in fact, the curvaton decays when the energy density of the curvaton becomes comparable to that of the radiation from the inflaton decay at the tiny lower-left corner of the figure (i.e., $m_\phi \sim 100$ GeV and $X \sim 10^{-2}$). When the value of X is as large as $O(0.1)$, however, the curvaton dominance is well realized when $H \sim \Gamma_\phi$. Thus, this fact does not affect the upper bounds on X.
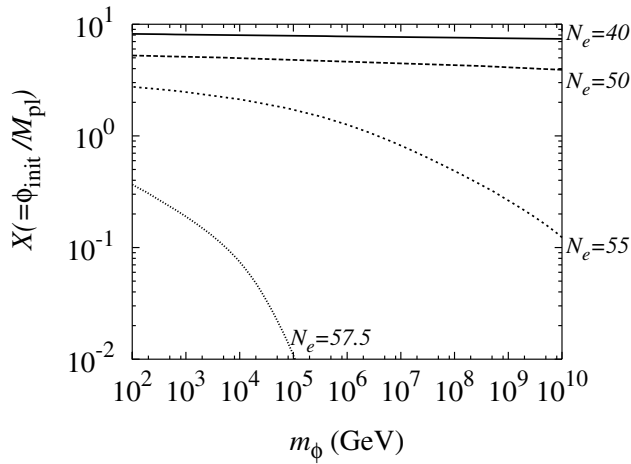
FIG. 2. Contours of constant N_e during inflation for the case with $\alpha = 4$. The decay rate of the curvaton is taken to be $\Gamma_\phi = 10^{-6}$ GeV.

suppressed. To see this more explicitly, in Fig. 2 we show contours of a constant number of e-foldings in this model. We can see that regions with large X give a small number of e-foldings. Consequently, the ϵ parameter becomes smaller when X is large since, in the chaotic inflation model, $\epsilon \propto 1/N_e$. Thus, in this case, the spectral index becomes too large since -2ϵ in the right-hand side of Eq. (2.26) cannot be suppressed. This is the reason why regions with large X cannot alleviate the inflation model.

Before closing this subsection, we comment on the possible value of m_ϕ. If one assumes that the curvaton is
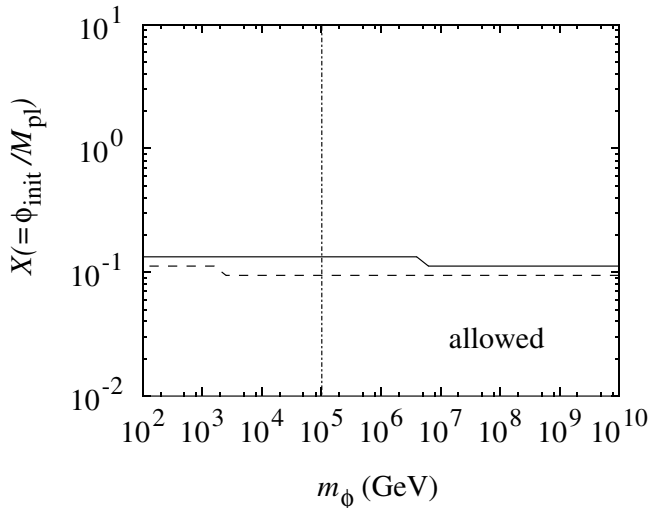
FIG. 3. The 95% C.L. allowed regions for $\alpha = 6$. The decay rate of the inflaton is taken to be $\Gamma_\chi = 10^8$ GeV. The decay rate of the curvaton is taken to be $\Gamma_\phi = 10^{-6}$ GeV (solid line) and $\Gamma_\phi = 10^{-22}$ GeV (dashed line). The lower side of the horizontal line is allowed for the given Γ_χ. The vertical dashed-dotted line shows the value of $m_\phi^{(\mathrm{grav})}$ given in Eq. (3.5) for the case of $\Gamma_\phi = 10^{-22}$ GeV.
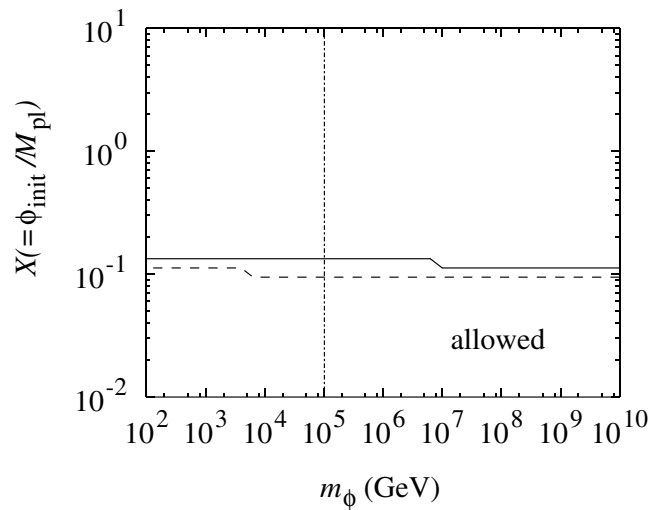
FIG. 4. Same as Fig. 3, except for $\Gamma_\chi = 10^6$ GeV.

coupled to the standard-model particles at least at the gravitational strength, the decay rate of the curvaton is estimated to be $\Gamma_\phi \sim m_\phi^3/M_{\mathrm{pl}}^2$ or larger. If this is the case, the mass of the curvaton is constrained to be larger than $m_\phi^{(\mathrm{grav})}$, where

$$m_\phi^{(\mathrm{grav})} \sim (M_{\mathrm{pl}}^2 \Gamma_\phi)^{1/3}. \qquad (3.5)$$

In some part of the parameter space we consider, m_ϕ is smaller than such a bound. For the case with $\Gamma_\phi = 10^{-22}$ GeV, this bound becomes $\sim 10^5$ GeV. Thus, in this case, the constraint does not apply for $m_\phi \gtrsim 10^5$ GeV. For reference, we show the boundary of the region with vertical dashed-dotted lines in Fig. 1 (as well as in Figs. 3 and 4). It is, however, possible that the curvaton interaction is much more suppressed due to a symmetry or some other reason. Thus, we do not take the bound seriously.

2. $\alpha \geq 6$ case

Next we discuss the case with $\alpha = 6$. The situation is very similar to the case with $\alpha = 4$. One of the differences is that, once the inflaton starts to oscillate, its energy density evolves as $\rho_\chi \propto a^{-4.5}$, which is stiffer than that of radiation. Thus, the constraint on the m_ϕ vs X plane depends on the decay rate of the inflaton in this case. We present the results for $\Gamma_\chi = 10^8$ and 10^6 GeV in Figs. 3 and 4, respectively.[7] Similarly to the case with $\alpha = 4$,

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[7]Since the values of the decay rate of the inflaton are highly model-dependent and we do not have strong criterion for what value we should choose in the setup we are considering (i.e., the curvaton scenario), we take the decay rate of the inflaton just as an example. However, notice that the conclusion does not depend on the values of Γ_χ much.

regions with small X can relax the constraints on the inflation model.

So far, we have seen that, for $\alpha = 4$ and 6, the chaotic inflation models become viable when the initial amplitude of the curvaton is small enough. As one can expect, similar results are obtained for larger values of α. We have calculated the upper bounds on X which make the chaotic inflation models with $\alpha \gtrsim 6$ viable. The bound is given by $X \leq 0.137$ (0.088) for $\alpha = 6$ (8). Here, we have taken $\Gamma_\chi = 10^6$ GeV, $m_\phi = 10^2$ GeV, and $\Gamma_\phi = 10^{-10}$ GeV.

Since the model with larger values of α predicts larger $|n_s - 1|$, the upper bound on X becomes severer for a larger value of α. In particular, with the decay rates and the curvaton mass used above, models with $\alpha \gtrsim 10$ cannot become viable even with the curvaton. This is because, even if the curvaton contribution dominates, the spectral index $n_s \simeq 1 - 2\epsilon \simeq 1 - \alpha/2N_e$ becomes too small to be consistent with the observations. For $\alpha = 10$, for example, $n_s - 1 \simeq -0.1$ with $N_e \simeq 50$, which is excluded by the observations. One possibility to make the models viable is to increase the e-folding number N_e, since $n_s - 1$ is approximately proportional to N_e^{-1}. For the case with $V(\chi) \propto \chi^\alpha$, the e-folding number during inflation is approximately written as [21]

$$N_e \simeq -\ln\frac{k}{a_0 H_0} - \frac{\alpha + 2}{6\alpha}\ln\frac{\rho_{\text{end}}}{\rho_{\text{reh}}} + \frac{1}{3}\ln\frac{s_0}{s_{\text{reh}}}$$
$$+ \ln\frac{H_{\text{inf}}}{H_0} + \Delta N_e, \tag{3.6}$$

where s_{reh} and s_0 are the entropy density the time of the reheating and at the present epoch, respectively, while H_{inf} and H_0 are the expansion rate during inflation and that at present, respectively. In addition, ΔN_e is the modification to the standard result due to the fact that the Universe is once dominated by the curvaton and is given by [14,22]

$$\Delta N_e \simeq -\frac{1}{12}\ln\frac{\rho_{\phi D}}{\rho_{\text{reh}2}} \simeq -\frac{1}{6}\ln\frac{m_\phi}{\Gamma_\phi} - \frac{2}{3}\ln X, \tag{3.7}$$

where we have assumed $X \lesssim 1$. If we pick up the terms related to the decay rates of the inflaton and the curvaton, we can see that

$$N_e \simeq \frac{4 - \alpha}{6\alpha}\ln\Gamma_\chi + \frac{1}{6}\ln\Gamma_\phi + \cdots. \tag{3.8}$$

Thus, for $\alpha \gtrsim 6$, a larger value of N_e is realized with a smaller value of Γ_χ or a larger value of Γ_ϕ.

B. Natural inflation

The natural inflation model [23,24] is based on a (pseudo-)Nambu-Goldstone field which has a potential of the form

$$V_{\text{inf}} = \Lambda^4\left[1 - \cos\left(\frac{\chi}{F}\right)\right]. \tag{3.9}$$

In this case, the slow-roll parameters are given by

$$\epsilon = \frac{1}{2}\left(\frac{M_{\text{pl}}}{F}\right)^2 \frac{1}{\tan^2(\chi/2F)},$$
$$\eta = \frac{1}{2}\left(\frac{M_{\text{pl}}}{F}\right)^2\left[\frac{1}{\tan^2(\chi/2F)} - 1\right]. \tag{3.10}$$

The inflation ends when the slow-roll condition does not hold; namely, one of the slow-roll parameters becomes $\mathcal{O}(1)$. The amplitude of the inflaton at the end of inflation can be obtained by

$$\tan\left(\frac{\chi_{\text{end}}}{2F}\right) \simeq \frac{M_{\text{pl}}}{\sqrt{2}F}. \tag{3.11}$$

The number of e-foldings can be evaluated using the standard technique as

$$N_e = \frac{2F^2}{M_{\text{pl}}^2}\ln\left[\frac{\cos(\chi_{\text{end}}/2F)}{\cos(\chi_*/2F)}\right]. \tag{3.12}$$

After the inflation, the inflaton starts to oscillate around the minimum of the potential. Around the minimum, the potential can be well-approximated as a quadratic form $V_{\text{inf}} \simeq (1/2)(\Lambda^2/F)^2\chi^2$. Thus, the energy density of an oscillating inflaton field behaves the same as that of matter. The natural inflation model generally predicts a red-tilted primordial spectrum and small tensor-scalar ratio [25]. However, in the curvaton scenario, the spectral index can approach the scale-invariant value, which can alleviate the constraint on this model. The modification can be obtained using the formulas given in the previous section.

In Fig. 5, the 95% allowed regions are shown in the F vs X plane. In our analysis, we calculated the value of Λ
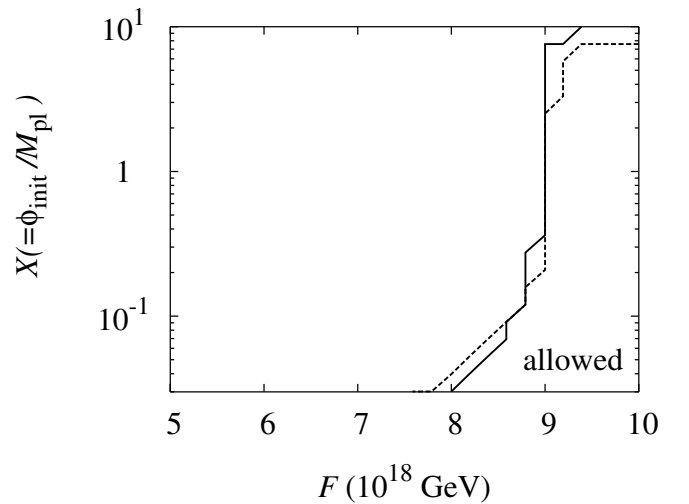
FIG. 5. The 95% C.L. allowed regions are shown for the natural inflation model in the F vs X plane. The lower region under the line is allowed. The decay rate of the inflaton is taken to be $\Gamma_\chi = 10^6$ GeV. The decay rate of the curvaton is taken to be $\Gamma_\phi = 10^{-6}$ GeV (solid line) and 10^{-22} GeV (dashed line). The mass of the curvaton is $m_\phi = 10^2$ GeV.

which gives the COBE normalized density amplitude. For this purpose, using the code provided by the WMAP group [26], we calculated the χ^2 variable from the WMAP data as a function of Λ and minimized it. In the parameter region shown in the figure, Λ is found to be $\sim 10^{16}$ GeV. It is important to note that the smallness of Λ compared to the symmetry breaking scale $F \sim M_{\rm pl}$ is protected by symmetry; χ is a pseudo-Nambu-Goldstone boson, and Λ is proportional to some soft breaking parameter of the corresponding symmetry. Thus, a small value of Λ can be natural in 't Hooft's sense [27]. The constraint also depends on the mass of the curvaton. However, we checked that the variation of the mass of the curvaton does not affect the constraint much, and, hence, we show only the result on the F vs X plane.

When fluctuation of the curvaton has a small contribution to the total curvature fluctuation, the spectral index is too small to be consistent with the observations in the natural inflation model. However, as is the case with the chaotic inflation models, a larger contribution from the curvaton fluctuation can liberate the model. In particular, the natural inflation predicts a smaller spectral index as F decreases. Correspondingly, the upper limit on X where the curvaton mechanism can liberate the model becomes smaller as can be seen from Fig. 5.

C. New inflation

Among various possibilities of the new inflation models, we chose to use the one proposed in Refs. [28,29] to be definite. The model is based on the supersymmetric models with Z_n R symmetry. We parametrize the superpotential with the parameters λ and v as

$$W = \frac{\lambda}{v^{n-2}}\left(v^n \hat{\chi} - \frac{1}{n+1}\hat{\chi}^{n+1}\right), \qquad (3.13)$$

where $\hat{\chi}$ is the superfield for the inflaton.

Using the fact that the inflaton χ is a real scalar field, the inflaton potential is given by[8]

$$V_{\rm inf} = \tilde{\lambda}^2 \tilde{v}^4\left[1 - 2\left(\frac{\chi}{\tilde{v}}\right)^n + \left(\frac{\chi}{\tilde{v}}\right)^{2n}\right], \qquad (3.14)$$

with

$$\tilde{\lambda} = \tfrac{1}{2}\lambda, \qquad \tilde{v} = \sqrt{2}v. \qquad (3.15)$$

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[8]In supergravity, an extra term may arise in the inflaton potential with nonminimal Kähler potential. We assume that the higher-order terms in the Kähler potential are small enough to be neglected. It is the case in, for example, the class of models with large cutoff scale [30]. In addition, if we naively use the superpotential given in Eq. (3.13) in supergravity, a negative cosmological constant arises at the minimum of the potential. We assume that the cosmological constant problem is somehow solved since it is not our main issue. Thus, we assume that the inflaton potential can be well approximated by Eq. (3.14) even near the minimum of the potential.

Here, we neglect the higher-order terms which are allowed by the Z_n symmetry. Assuming that the higher-order terms are suppressed by the inverse powers of the Planck scale, the scalar potential given above is relevant when $\tilde{v} \ll M_{\rm pl}$. In the following, we first analyze the model using the above superpotential, allowing the v parameter to be as large as $M_{\rm pl}$. After deriving the constraints, we take account of the constraint $\tilde{v} \ll M_{\rm pl}$ and discuss the implications.

In this model, for a given value of $\tilde{v}$, the $\tilde{\lambda}$ parameter is determined so that the cosmic density fluctuation has the correct size. However, as we see below, the spectral index and the tensor-scalar ratio are insensitive to $\tilde{\lambda}$.

Now we evaluate the slow-roll parameters in the model. During inflation, the last term in Eq. (3.14) is irrelevant to the dynamics of the inflaton. Thus, we neglect the last term for the moment. The slow-roll parameters are

$$\epsilon = 2n^2\left(\frac{M_{\rm pl}}{\tilde{v}}\right)^2\left(\frac{\chi_*}{\tilde{v}}\right)^{2(n-1)},$$
$$\eta = -2n(n-1)\left(\frac{M_{\rm pl}}{\tilde{v}}\right)^2\left(\frac{\chi_*}{\tilde{v}}\right)^{n-2}. \qquad (3.16)$$

The number of e-foldings is given by

$$N_e = \frac{1}{2n(n-2)}\left(\frac{\tilde{v}}{M_{\rm pl}}\right)^2\left(\frac{\chi_*}{\tilde{v}}\right)^{2-n}. \qquad (3.17)$$

Thus, in the standard inflationary scenario, the scalar spectral index can be written as

$$n_s - 1 = -2\frac{n-1}{n-2}\frac{1}{N_e}. \qquad (3.18)$$

For $N_e = 50$, for example, $n_s - 1$ becomes -0.08, -0.06, and -0.053 for $n = 3$, 4, and 5, respectively. In particular, the model with $n = 3$ is marginally excluded by observations.

The situation for $n = 3$ may be improved by the curvaton. In Fig. 6, we plot the 95% C.L. allowed region in the v vs X plane. Notice that this model predicts a very small value of the ϵ parameter, in particular, for the case with $\tilde{v} \ll M_{\rm pl}$. Using the e-folding number and the amplitude of the inflaton at the horizon crossing, ϵ is given as

$$\epsilon = 2n^2\left(\frac{M_{\rm pl}}{\tilde{v}}\right)^2\left[\frac{1}{2n(n-2)}\left(\frac{\tilde{v}}{M_{\rm pl}}\right)^2\frac{1}{N_e}\right]^{2(n-1)/(n-2)}. \qquad (3.19)$$

As one can see, in this class of model, the ϵ parameter becomes very small. For $n = 3$, for example, this equation becomes $\epsilon = 18(1/6N_e)^4(\tilde{v}/M_{\rm pl})^6$. For $N_e = 50$ and $\tilde{v} \sim 10^{16}$ GeV, the ϵ parameter is of order 10^{-21}. Importantly, the effects of the curvaton come in the combination of $\tilde{f}^2\epsilon$ [see Eq. (2.26)]. Thus, in the new inflation models, $\phi_{\rm init}$ should be much smaller than $M_{\rm pl}$ to make the curvaton contribution dominant: $\phi_{\rm init} \ll (\tilde{v}/M_{\rm pl})^{n/(n-2)}N_e^{-(n-1)/(n-2)}M_{\rm pl}$.
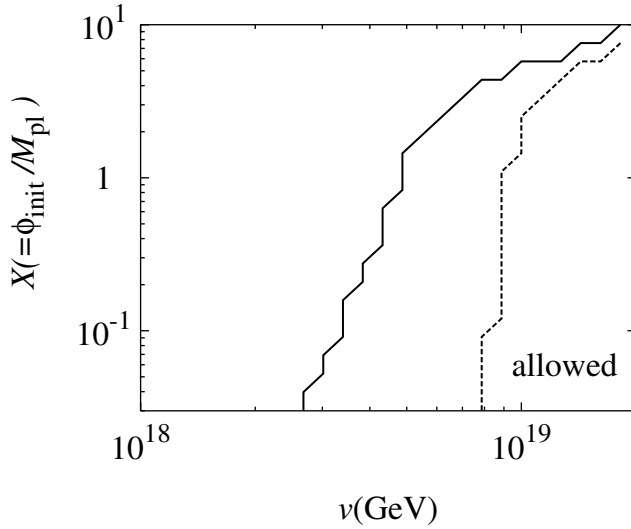
FIG. 6. The 95% C.L. allowed region is shown for the natural inflation model in the v vs X plane. The lower region under the line is allowed. The decay rate of the inflaton is taken to be $\Gamma_\chi = 10^6$ GeV. The decay rate of the curvaton is taken to be $\Gamma_\phi = 10^{-6}$ GeV (solid line) and 10^{-22} GeV (dashed line). The mass of the curvaton is $m_\phi = 10^2$ GeV.

IV. CONCLUSION

In this paper, we have studied the constraints on the inflaton potential taking account of the effects of the curvaton. We have followed the basic procedure given in Ref. [14]. We have considered several models of the inflation for which some part of the parameter space is excluded by the observations. We have seen that the constraints on these models can be relaxed once the curvaton is introduced.

For the case of the chaotic inflation, inflation models with the inflaton potential $V_{inf} \propto \chi^\alpha$ can become viable with curvaton, if α is small enough. Importantly, the curvaton contribution to the cosmic density fluctuation is inversely proportional to ϕ_{init} (when ϕ_{init} is less than M_{pl}). Thus, to make the models viable, upper bounds on ϕ_{init} are obtained. A similar result is obtained for the natural inflation model.

We also considered the new inflation model. If all the parameters in the inflaton potential [in particular, the v parameter defined in Eq. (3.13)] are free, the curvaton mechanism can relax the constraint on the new inflation model. However, for some class of the new inflation models, the ϵ parameter is supposed to be very small. In such a case, even the curvaton mechanism cannot liberate the constraint unless the initial amplitude of the curvaton is much smaller than M_{pl}.

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