

Re(ϵ'/ϵ_K) versus $B_d \rightarrow \phi K_S$ CP asymmetry

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(Received 15 March 2005; published 21 July 2005)

In a SUSY GUT seesaw scenario, the largeness of the atmospheric neutrino mixing can reflect itself into an enhanced flavor changing mixing of beauty and strange right-handed scalar quarks. If the CP violating phase in such down-type squark RR insertion is the main source of CP asymmetry in $B_d \rightarrow \phi K_S$ and the gluino contributions to $K^0-\bar{K}^0$ and $B^0-\bar{B}^0$ mixing are negligible, there is a correlation between $\text{Re}(\epsilon'/\epsilon_K)$ and $B_d \rightarrow \phi K_S$ CP asymmetry, in addition to that with the strange quark CEDM. The current data on $\text{Re}(\epsilon'/\epsilon_K) = (16.7 \pm 2.6) \times 10^{-4}$ imply that $S_{\phi K}$ should be greater than $\sim 0.5(0.25)$ for $\mu \tan\beta = 1(5)$ TeV, assuming the RR dominance in $b \rightarrow s$ transition and the minimal supergravity type boundary conditions for soft parameters.

DOI: [10.1103/PhysRevD.72.016004](https://doi.org/10.1103/PhysRevD.72.016004)

PACS numbers: 11.30.Er, 12.10.Dm, 12.60.Jv

I. INTRODUCTION

The flavor changing neutral current (FCNC) processes induced by $b \rightarrow s$ transitions have played a major role in probing proposals of new physics at the electroweak scale. While the study of $b \rightarrow s\gamma$ keeps being the highlight in such investigations, more recently the relevance of $b \rightarrow s$ induced purely hadronic decays has been emphasized, in particular, in relation with the issue of testing CP violation in B physics. Among such decays, the process $B_d \rightarrow \phi K_S$ has aroused much interest [1] at least for two reasons: (i) the process can occur only at the loop level in the Standard Model (SM), hence making it particularly suitable to spot sizeable contributions coming from new physics; (ii) in the SM the CP asymmetry in such decay arises only from the indirect CP violation of the B mixing, hence one can safely state that within the SM the golden mode $B_d \rightarrow J/\psi K_S$ and the decay $B_d \rightarrow \phi K_S$ yield the same amount of CP asymmetry, namely $S_{\phi K} = S_{\psi K}$; if the new physics entails the presence of direct CP asymmetry in the decay amplitude of $B_d \rightarrow \phi K_S$, this can be revealed by a departure from the equality between the two mentioned CP asymmetries [2–6]. Indeed, the first data on the CP asymmetry in $B_d \rightarrow \phi K_S$ in 2002 [7] gave rise to a wave of interest on this potentially very interesting signature of new physics which still continues today [8–16].

Indeed, even though the discrepancy of the data with respect to the SM predictions has been constantly decreasing in the two years elapsed from the first results in 2002, still the current world averages of $S_{\phi K}$ and $S_{\psi K}$ are [17]

$$S_{\phi K} = (0.34 \pm 0.21), S_{\psi K} = (0.726 \pm 0.037),$$

namely $S_{\phi K}$ is about 2σ lower than the SM prediction $S_{\phi K}^{\text{SM}} \simeq S_{\psi K}^{\text{SM}}$.

Notice that even if one does not wish to speculate much on the single above discrepancy, it is interesting to note that the data concerning $b \rightarrow q\bar{q}s$ processes with $q = u, d, s$ exhibit an overall discrepancy in the values of the CP asymmetry with respect to the SM predictions (indeed, the CP asymmetries in $B \rightarrow \phi K_S$ and $B_d \rightarrow \eta' K_S$ turn out to be smaller than that in $B_d \rightarrow J/\psi K_S$ which is measured from the tree level process $b \rightarrow c\bar{c}s$). This discrepancy might be a signal for physics beyond the SM [18–21].

Assuming that the current low value of $S_{\phi K}$ is a signal of new physics, we need a new CP violating phase in $b \rightarrow s$ transition. An attractive possibility for such new physics beyond the SM occurs in supersymmetric grand unified theories (SUSY GUT's) scenarios with seesaw mechanism for neutrino masses and mixings. In such scenarios, the large atmospheric neutrino oscillation can be related with a large $b \rightarrow s$ transition through down-type squark and gluino loop effects. This flavor changing effect is parametrized by a mixing parameter $(\delta_{23}^d)_{RR}$ with a CP phase $\sim O(1)$. We follow Ref. [22] for the definitions of the mass insertion parameters $(\delta_{ij}^d)_{AB}$'s. Let us note that the following relations

$$(\delta_{ij}^d)_{LL} = (\delta_{ji}^d)_{LL}^*, (\delta_{ij}^d)_{RR} = (\delta_{ji}^d)_{RR}^*, (\delta_{ij}^d)_{LR} = (\delta_{ji}^d)_{RL}^*,$$

will be useful, when we relate $(\delta_{23}^d)_{RR}$ and $(\delta_{32}^d)_{RR}$ in this work. For low $\tan\beta$, the single RR insertion can lead to some deviation in $S_{\phi K}$, if gluinos and squarks are relatively light. For large $\tan\beta$ case, the double mass insertion can

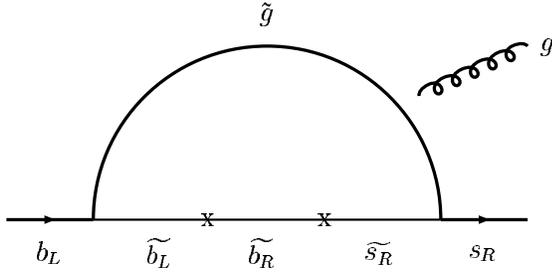
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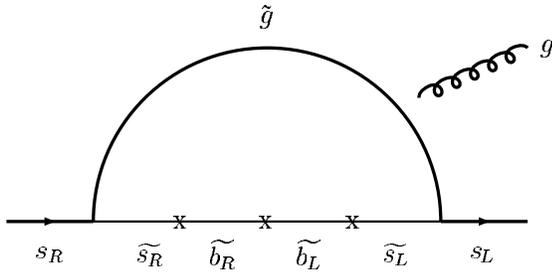
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lead to an effective RL insertion of $O(10^{-2})$, leading to a significant deviation in $S_{\phi K}$ from the SM prediction. In Fig. 1(a), we show the Feynman diagram for $b \rightarrow sg$ involving a CP violating $(\delta_{23}^d)_{RR}$.

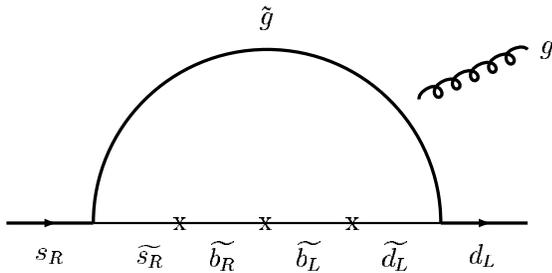
The new CP violating phase in the RR insertion can affect the strange quark chromo-electric dipole moment (CEDM) through triple mass insertions, if there is an LL insertion between third and second generation down



(a)



(b)



(c)

FIG. 1. Feynman diagrams for (a) $b \rightarrow sg$ with double mass insertions, (b) strange quark CEDM with triple mass insertions, and (c) $s \rightarrow dg$ with triple mass insertions, involving $(\delta_{23}^d)_{RR}$ as the dominant source of new CP violating parameter contributing to $S_{\phi K}$ and $\text{Re}(\epsilon'/\epsilon_K)$.

squarks. [Fig. 1(b)] [15,16,23]. Since the LL insertion is generically present in the minimal supergravity (mSUGRA) case, the strange quark CEDM puts a strong constraint on the possible deviation of $S_{\phi K}$ from the SM prediction. However, the substantial theoretical uncertainties occurring when one relates the quark CEDM's with the hadronic EDM's suggest that it would be preferable to have some other observable at disposal in addition to the strange quark CEDM in order to constrain $S_{\phi K}$.

In this letter, we point out that the phase in the $(\delta_{23}^d)_{RR}$ mixing parameter that would affect $S_{\phi K}$ can also contribute to direct CP violation within the neutral kaon system, namely, $\text{Re}(\epsilon'/\epsilon_K)$ through triple mass insertion. The Feynman diagram for $\text{Re}(\epsilon'/\epsilon_K)$ with triple mass insertion [Fig. 1(c)] is very similar to the Feynman diagram for the strange quark CEDM [Fig. 1(b)]. Needless to say, making use of the observable $\text{Re}(\epsilon'/\epsilon_K)$ to constrain some SUSY soft breaking parameters also entails theoretical uncertainties mainly ascribed to our ignorance in the evaluation of the relevant hadronic matrix elements. Our discussion shows that, even taking into account such a huge degree of uncertainty, $\text{Re}(\epsilon'/\epsilon_K)$ still constitutes a precious tool in constraining the interesting flavor changing mass insertion parameter $(\delta_{23}^d)_{RR}$, and, in any case, it plays at least a complementary role to the strange quark CEDM in performing such a task. Although the large RR mixing is well motivated in SUSY GUT plus seesaw mechanism, we should emphasize that the link between $\text{Re}(\epsilon'/\epsilon_K)$ and $S_{\phi K}$ is a generic feature of RR dominance scenario which in general can arise in other contexts.

For definiteness, we will work in the mSUGRA boundary condition at the reduced Planck scale $M_* \simeq 2.4 \times 10^{18}$ GeV, namely, the flavor universal scalar mass parameter m_0 , and the trilinear couplings A which is proportional to the Yukawa couplings. Then flavor changing off-diagonal squark masses will be induced by the renormalization group (RG) evolution from M_* down to $M_{\text{GUT}} \simeq 2 \times 10^{16}$ GeV, and subsequently from there to the M_W scale [24]:

$$(m_{\tilde{q}}^2)_{ij} = -\frac{1}{8\pi^2} [V^\dagger \lambda_u^2 V]_{ij} (3m_0^2 + A^2) \times \left(3 \log \frac{M_*}{M_{\text{GUT}}} + \log \frac{M_{\text{GUT}}}{M_W} \right), \quad (1)$$

from which one can estimate [13,15]

$$(\delta_{13}^d)_{LL} \simeq 8 \times 10^{-3} \times e^{-i2.7}.$$

The LR or RL mixing will be small in this limit, and we ignore them in the following.

For the RR insertion, it is also generated by the RG evolution. In the presence of a seesaw mechanism to give rise to neutrino masses, there will be the new Yukawa couplings responsible for the Dirac entries in the neutrino mass matrix. In a unified context, such couplings intervene also in the evolution of the right-handed squark masses in

the interval from the scale of appearance of the soft breaking terms down to the scale of right-handed neutrino masses. If at least some of such couplings are large (for instance one of the Dirac neutrino couplings could be of the order of the top Yukawa couplings), then one can expect sizeable contributions to the RG induced off-diagonal entries in right-handed squark mass matrix. Let us work in the basis where the down quark and the charged lepton mass matrices are diagonal. In this particular basis, we get [3]

$$\begin{aligned} (m_{\tilde{d}}^2)_{ij} &\simeq -\frac{1}{8\pi^2} [Y_N^\dagger Y_N]_{ij} (3m_0^2 + A^2) \log \frac{M_*}{M_{\text{GUT}}} \\ &\simeq -\frac{1}{8\pi^2} e^{-i(\phi_i^{(L)} - \phi_j^{(L)})} y_{\nu_k}^2 [V_L^*]_{ki} [V_L]_{kj} (3m_0^2 + A^2) \\ &\quad \times \log \frac{M_*}{M_{\text{GUT}}}. \end{aligned} \quad (2)$$

Here Y_N denotes the 3×3 Yukawa matrix of the term inducing the neutrino Dirac mass, which is diagonalized in the form,

$$\begin{aligned} Y_N &= U_N^\dagger \hat{Y}_N V_L \hat{\Theta}_L, \quad \hat{Y}_N = \text{diag}(y_{\nu_1}, y_{\nu_2}, y_{\nu_3}), \\ \hat{\Theta}_L &= \text{diag}(e^{i\phi_1^{(L)}}, e^{i\phi_2^{(L)}}, e^{i\phi_3^{(L)}}), \end{aligned} \quad (3)$$

where V_L is a unitary matrix with a single phase in the standard parameterization, U_N is a general unitary matrix, and the phases are subject to the constraint, $\phi_1^{(L)} + \phi_2^{(L)} + \phi_3^{(L)} = 0$. Hereafter, we assume that the right-handed neutrino mass matrix M_N is diagonal in the basis where Y_N is diagonal,

$$M_N = U_N^\dagger \hat{M}_N U_N^*, \quad \hat{M}_N = \text{diag}(M_{N_1}, M_{N_2}, M_{N_3}). \quad (4)$$

The simplest case of this sort would arise for M_N proportional to a unit matrix. A case with nondegenerate eigenvalues is also possible. Such a texture with simultaneously diagonalizable Y_N and M_N may result from simple U(1) family symmetries. Under this assumption, V_L coincides with hermitian conjugate of the usual definition of PMNS lepton mixing matrix, which is not always the case for arbitrary M_N .

From Eq. (2), we can estimate $(\delta_{ij}^d)_{RR}$. In particular, having in mind a large Yukawa coupling of the third generation, the entry $(\delta_{23}^d)_{RR}$ would be largely affected:

$$(\delta_{23}^d)_{RR} \simeq 2 \times 10^{-2} \left(\frac{M_{N_3}}{10^{14} \text{ GeV}} \right). \quad (5)$$

Its size depends on the right-handed neutrino mass scale, because $y_{\nu_3}^2$ in Eq. (2) grows with M_{N_3} for a fixed value of neutrino mass. Sizes of $(\delta_{12}^d)_{RR}$ and $(\delta_{13}^d)_{RR}$ crucially depend on y_{ν_1} , y_{ν_2} , and $[V_L]_{31}$ as well. In general, they can be large enough to influence ϵ_K , $\text{Re}(\epsilon'/\epsilon_K)$, and $B^0 - \bar{B}^0$ mixing significantly. Suppose that the neutrino mass spectrum has normal hierarchy with negligible lightest neutrino mass. In this case, Eq. (2) implies that $(\delta_{12}^d)_{RR}$ and

$(\delta_{13}^d)_{RR}$ scale as

$$|(\delta_{12}^d)_{RR}| \sim |(\delta_{13}^d)_{RR}| \sim \max(y_{\nu_2}^2/y_{\nu_3}^2, |[V_L]_{31}|) |(\delta_{23}^d)_{RR}|, \quad (6)$$

where we regard all the elements of V_L as $O(1)$ except for $[V_L]_{31}$. Maximal size of $(\delta_{23}^d)_{RR}$ that can be expected from Eq. (5) is $O(0.1)$ for neutrino Yukawa couplings to be perturbative. If the right-handed neutrino masses are degenerate so that $y_{\nu_2}^2/y_{\nu_3}^2 \simeq 0.2$, the resulting $(\delta_{12}^d)_{RR}$ can result in a huge contribution to ϵ_K [22,25]. Also $[V_L]_{31}$ close to the current upper bound ~ 0.15 [26] can give rise to a similar size. For large $\tan\beta$, this can lead to contribution to $\text{Re}(\epsilon'/\epsilon_K)$ as well, which is bigger than its experimental value [27,28]. The other mass insertion $(\delta_{13}^d)_{RR}$ may modify $B^0 - \bar{B}^0$ mixing to a sizeable extent.

One can make an observation from Eq. (6) that $(\delta_{12}^d)_{RR}$ and $(\delta_{13}^d)_{RR}$ will be highly suppressed provided that neutrino Yukawa couplings have strong hierarchy and that $[V_L]_{31}$ is vanishingly small. In fact, the former condition is naturally realized in a scenario with SO(10) unification [5]. In this scenario, the neutrino Yukawa couplings are unified with the up-type quark Yukawa couplings, thereby resulting in $y_{\nu_2}^2/y_{\nu_3}^2 \sim 10^{-4}$. Since $y_{\nu_2}^2/y_{\nu_3}^2$ is much smaller than m_{ν_2}/m_{ν_3} , eigenvalues of M_N should be split accordingly for the seesaw formula to yield correct neutrino mass spectrum. Then the RG induced $(\delta_{12}^d)_{RR}$ and $(\delta_{13}^d)_{RR}$ will be small, and SUSY contributions to $\text{Re}(\epsilon'/\epsilon_K)$, and $K^0 - \bar{K}^0$ and $B^0 - \bar{B}^0$ mixings can be safely neglected. We will consider the correlation between $S_{\phi K}$ and $\text{Re}(\epsilon'/\epsilon_K)$ in such a case, relegating the more general case with sizable $(\delta_{12}^d)_{RR}$ and $(\delta_{13}^d)_{RR}$ insertions to a future study [29].

This paper is organized as follows. In Sec. II, we give basic formulas for the SUSY contributions for $\Delta B = 1$ effective Hamiltonian which are relevant in the mSUGRA boundary conditions, and discuss how to include hadronic uncertainties in $S_{\phi K}$. In Sec. III, we give the relevant information on $\Delta S = 1$ effective Hamiltonian and hadronic uncertainties related with $\text{Re}(\epsilon'/\epsilon_K)$. The numerical analysis is given in Sec. IV, and the results are summarized in Sec. V.

II. CP ASYMMETRY IN $B_d \rightarrow \phi K_S$

Let us start with $B_d \rightarrow \phi K_S$ decay by recapitulating the effective Hamiltonian for $\Delta B = 1$ relevant to $B_d \rightarrow \phi K_S$ ($S_{\phi K}$). With the operator basis in Eqs. (9) of Ref. [9], it is given by

$$\begin{aligned} H_{\text{eff}}^{\Delta B=1} &= \frac{G_F}{\sqrt{2}} \sum_{p=u,c} \lambda_p^{(s)} \left[C_1 O_1^p + C_2 O_2^p + \sum_{i=3}^{10} C_i(\mu) O_i(\mu) \right. \\ &\quad \left. + C_{7\gamma} O_{7\gamma} + C_{8g} O_{8g} \right] + \text{H.c.} \end{aligned} \quad (7)$$

One also has tilded operators $\tilde{O}_{i=3,\dots,10,7\gamma,8g}$, which are obtained from O_i 's by making chirality flip $L \leftrightarrow R$. Then

the Wilson coefficients for the tilded (chromo)magnetic operators, up to the second order of mass insertion approximation, [Fig. 1(a)], read

$$\begin{aligned}
 -\lambda_t \frac{G_F}{\sqrt{2}} \tilde{C}_{7\gamma} &= \frac{\pi\alpha_s}{\tilde{m}^2} \left[-\frac{4}{9} M_4(x) (\delta_{23}^d)_{RR} + \frac{4}{9} \frac{m_{\tilde{g}}}{m_b} M_2(x) \right. \\
 &\quad \left. \times (\delta_{23}^d)_{RR} (\delta_{33}^d)_{RL} \right], \\
 -\lambda_t \frac{G_F}{\sqrt{2}} \tilde{C}_{8g} &= \frac{\pi\alpha_s}{\tilde{m}^2} \left[\left(\frac{3}{2} M_3(x) - \frac{1}{6} M_4(x) \right) (\delta_{23}^d)_{RR} - \frac{m_{\tilde{g}}}{m_b} \right. \\
 &\quad \left. \times \left(-\frac{1}{6} M_2(x) + \frac{3}{2} M_1(x) \right) (\delta_{23}^d)_{RR} (\delta_{33}^d)_{RL} \right].
 \end{aligned} \tag{8}$$

The loop functions for single and double mass insertions can be found in Refs. [22,28], respectively. We also include the contributions from $\tilde{C}_{3,\dots,6}$. We have ignored the terms depending on $(\delta_{23}^d)_{RL}$ in the tilded Wilson coefficients, and $(\delta_{23}^d)_{LL}$ and $(\delta_{23}^d)_{LR}$ in the untilded Wilson coefficients, because they are all small within mSUGRA scenarios [24].

We calculate $S_{\phi K}$ using the QCD factorization in the BBNS approach [30]. There are theoretical uncertainties from the divergent integral in the hard-scattering (H) and the weak annihilation (A) contributions such as $\int_0^1 dy/y$. We adopt the suggestion by BBNS as follows [31]:

$$\int_0^1 dy/y \rightarrow (1 + \varrho_{H,A} e^{i\varphi_{H,A}}) \log(m_B/\lambda_h)$$

with

$$0 \leq \varrho_H, \varrho_A \leq 1, \quad 0 \leq \varphi_H, \varphi_A < 2\pi. \tag{9}$$

This prescription is an intrinsic limitation of the BBNS approach, and the associated uncertainties cannot be reduced at the moment. It turns out that these uncertainties are not very large if squarks and gluinos are relatively heavy, $350 \text{ GeV} \lesssim \tilde{m}, m_{\tilde{g}}$, but can be large for lighter squarks and gluinos close to the current lower bounds. If we assume the gaugino mass unification within SUSY GUT's, the LEP bound on chargino ($m_{\chi^+} > 94 \text{ GeV}$) implies that $m_{\tilde{g}} > 400 \text{ GeV}$. In the numerical analysis, we use $m_{\tilde{g}} = \tilde{m} = 500 \text{ GeV}$, and the hadronic uncertainties become smaller.

Finally, we will impose the usual bounds that any new physics involving $b \rightarrow s$ transitions should satisfy [32]:

$$\begin{aligned}
 2.0 \times 10^{-4} \leq B(B \rightarrow X_s \gamma) \leq 4.5 \times 10^{-4}, \\
 \Delta M_s \geq 14.9 \text{ ps}^{-1}.
 \end{aligned} \tag{10}$$

We should comment on constraints from $b \rightarrow s \ell^+ \ell^-$, another important ingredient. The branching ratio of the exclusive decay mode $B \rightarrow K \ell^+ \ell^-$ is [33]

$$B(B \rightarrow K \ell^+ \ell^-) = (5.4 \pm 0.8) \times 10^{-7}, \tag{11}$$

which is consistent with the SM prediction [34]. A model

independent analysis in Ref. [34] has shown that the region on the plane of $(C_9^{\text{NP}}, C_{10}^{\text{NP}})$ allowed by this decay at the 90% C.L. is an annulus with radius ~ 7 and thickness ~ 5 including the origin, where C_9^{NP} and C_{10}^{NP} are new physics contributions to the Wilson coefficients of the $b \rightarrow s \ell^+ \ell^-$ four fermion operators under their convention. On the other hand, for an $O(1)$ value of $|(\delta_{23}^d)_{LL}|$ and $m_{\tilde{g}} = \tilde{m} = 500 \text{ GeV}$, a maximal gluino-squark loop contribution to C_9^{NP} has the size ~ 0.2 , which is much smaller than the extent of the allowed annulus. Although the data quoted in the above reference is less precise than the present one, the overall feature should remain the same. Also, incorporation of the inclusive mode does not make much difference. From this we can deduce that the effect of an RR insertion, which we are considering in this work, should be equally insignificant. As for the effective RL insertion, it is much more severely constrained by $B \rightarrow X_s \gamma$ than $B \rightarrow X_s \ell^+ \ell^-$. For these reasons, we do not include the $b \rightarrow s \ell^+ \ell^-$ constraints explicitly in our analysis.

III. $\text{Re}(\epsilon'/\epsilon_K)$

The $\Delta S = 1$ effective Hamiltonian is given by

$$\begin{aligned}
 H_{\text{eff}}^{\Delta S=1} &= C_1 O_1 + C_2 O_2 + \sum_{i=3}^{10} C_i O_i + C_{7\gamma} O_{7\gamma} + C_{8g} O_{8g} \\
 &\quad + \text{h.c.},
 \end{aligned} \tag{12}$$

where the operators $O_{7\gamma}$ and O_{8g} are the same as in Eq. (7) except the replacements $(s, b) \rightarrow (d, s)$. The leading contributions to the Wilson coefficients of the (chromo)magnetic operators are provided by the triple mass insertions [Fig. 1(c)], reading as

$$\begin{aligned}
 C_{7\gamma} &= \frac{\pi\alpha_s}{\tilde{m}^2} \frac{4}{9} \frac{m_{\tilde{g}}}{m_s} N_1(x) (\delta_{13}^d)_{LL} (\delta_{33}^d)_{LR} (\delta_{32}^d)_{RR}, \\
 C_{8g} &= \frac{\pi\alpha_s}{\tilde{m}^2} \frac{m_{\tilde{g}}}{m_s} \left[\frac{1}{6} N_1(x) + \frac{3}{2} N_2(x) \right] (\delta_{13}^d)_{LL} (\delta_{33}^d)_{LR} \\
 &\quad \times (\delta_{32}^d)_{RR}.
 \end{aligned} \tag{13}$$

The loop functions $N_1(x)$ and $N_2(x)$ are available in Ref. [15]. In these expressions, we have omitted double and triple insertion terms from other mass insertion parameters as they are small compared to the terms we kept above. As in $C_{7\gamma}$ and C_{8g} , we ignore the $(\delta_{12}^d)_{LL}$ contributions in $C_{3,\dots,6}$. The importance of the single LR insertion and the double insertion contributions to $\text{Re}(\epsilon'/\epsilon_K)$ was pointed out in Refs. [35,27,28], respectively, in the general MSSM frameworks without relying on the flavor universal boundary conditions. In the following, we show that the triple mass insertion can give an important contribution to $\text{Re}(\epsilon'/\epsilon_K)$.

After accomplishing the computation of the relevant SUSY contributions in the effective hamiltonian responsible for $\text{Re}(\epsilon'/\epsilon_K)$, we now turn to the delicate issue of the

hadronic uncertainties in the evaluation of $\text{Re}(\epsilon'/\epsilon_K)$. For definiteness we will follow the treatment provided in Refs. [36,37]. In the SM contribution, the main uncertainties reside in the evaluation of the B parameters $B_6^{(1/2)}$, $B_8^{(3/2)}$ and in the estimate of the strange quark mass m_s . We define the nonperturbative parameters R_6 and R_8 as

$$\begin{aligned} R_6 &\equiv B_6^{(1/2)} \left[\frac{121 \text{ MeV}}{m_s(m_c) + m_d(m_c)} \right]^2, \\ R_8 &\equiv B_8^{(3/2)} \left[\frac{121 \text{ MeV}}{m_s(m_c) + m_d(m_c)} \right]^2. \end{aligned} \quad (14)$$

The value of $B_8^{(3/2)}$ is rather well estimated both from lattice QCD [38] and from analytic nonperturbative approaches [39]. In what follows, we employ the range of R_8 ,

$$R_8 = 1.0 \pm 0.2. \quad (15)$$

On the other hand, the situation of $B_6^{(1/2)}$ is very unclear, and there exist results from different approaches ranging within a factor of 2.2. For instance, the large- N_c limit predicts $R_6 = 1$. It is difficult to attribute a reliable uncertainty to such estimates. Taking $R_6 = R_8 = 1.0$ results in the SM prediction of $\text{Re}(\epsilon'/\epsilon_K)$, which is smaller than the observed value by about 3×10^{-4} . According to Ref. [36], the best fit from $\text{Re}(\epsilon'/\epsilon_K)$ yields $R_6 = 1.23$. However, let us emphasize that this is the case within the SM, and we cannot rely on a SM fit here in the presence of new physics affecting kaon decays. As a representative value, we use

$$R_6 = 1.0 \pm 0.2. \quad (16)$$

In any event, our conclusion does not depend significantly on the precise value of R_6 or R_8 because $\text{Re}(\epsilon'/\epsilon_K)$ is dominated by chromomagnetic contribution in most of the parameter space.

As for the strange quark mass, we make use of the range:

$$m_s(m_c) = 115 \pm 20 \text{ MeV}. \quad (17)$$

For completeness, we also specify our choice of the isospin breaking parameter Ω_{IB} . We use the value [40]

$$\Omega_{\text{IB}} = 0.06. \quad (18)$$

The uncertainty of $\Omega_{\text{IB}} \sim 0.08$ could shift $\text{Re}(\epsilon'/\epsilon_K)$ by about 0.8×10^{-4} , and neglecting it does not affect our conclusion.

It is time to come to the main uncertainty in the SUSY contribution to $\text{Re}(\epsilon'/\epsilon_K)$. This is related to the evaluation of the matrix element:

$$\langle Q_g^- \rangle_0 = \sqrt{\frac{3}{2}} \frac{11}{16\pi^2} \frac{\langle \bar{q}q \rangle}{F_\pi^3} m_\pi^3 B_G, \quad (19)$$

where

$$Q_g^- = \frac{1}{4m_s} (\tilde{O}_{8g} - O_{8g}). \quad (20)$$

The uncertainty in the above evaluation is encoded in the value of the parameter B_G . The result of Ref. [41] corresponds to $B_G = 1$. Unfortunately lattice computations are still unable to come up with a reliable estimate of the matrix element $\langle Q_g^- \rangle_0$; in fact, even the sign of this parameter is not certain yet, although the above reference estimated it to be positive. If we assume the opposite sign of B_G with the same magnitude, the SUSY contribution flips its sign. In spite of all this and even allowing for an uncertainty of a factor 4 in the estimate of $|B_G|$ between 1 and 4, we will show that the constraint on $(\delta_{23}^d)_{RR}$ from $\text{Re}(\epsilon'/\epsilon_K)$ still remains meaningful.

IV. NUMERICAL ANALYSES

Now it is straightforward to calculate $\text{Re}(\epsilon'/\epsilon_K)$ and $S_{\phi K}$ from $s \rightarrow dg$ and $b \rightarrow sg$, when the $(\delta_{23}^d)_{RR}$ is the main new physics contribution beyond the SM contributions. Here we assume that SUSY contributions from $(\delta_{12}^d)_{RR}$ and $(\delta_{13}^d)_{RR}$ are negligible as is the case for hierarchical neutrino Yukawa couplings and vanishing $[V_L]_{31}$. If we relax this assumption, we should regard γ as a free variable that can be varied within a range compatible with other data such as $B \rightarrow X_d \gamma$. Then, the calculated value of $\text{Re}(\epsilon'/\epsilon_K)$ will change by a fraction of its SM prediction. Nevertheless, the qualitative feature remains true that the size of $\text{Re}(\epsilon'/\epsilon_K)$ bounds the deviation of $S_{\phi K}$ from $S_{\phi K}^{\text{SM}}$.

We would like to point out the main result of this work in a simple way, before we give a detailed numerical analysis. If the $(\delta_{23}^d)_{RR}$ mixing is the dominant new physics contribution to $B_d \rightarrow \phi K_S$, we find the following from Eqs. (13) and (8) in the previous sections :

$$\text{Re}(\epsilon'/\epsilon_K): C_{8g}^{\text{SUSY}}(\Delta S = 1) \propto f_1(x) (\delta_{13}^d)_{LL} (\delta_{33}^d)_{LR} (\delta_{32}^d)_{RR}, \quad (21)$$

$$\begin{aligned} S_{\phi K}: \tilde{C}_{8g}^{\text{SUSY}}(\Delta B = 1) &\propto f_2(x) (\delta_{23}^d)_{RR} \\ &+ f_3(x) \frac{m_{\tilde{g}}}{m_b} (\delta_{33}^d)_{RL} (\delta_{23}^d)_{RR}, \end{aligned} \quad (22)$$

where $f_{i=1,2,3}(x)$ are the loop functions obtained in the previous sections. Now, if the SUSY contribution saturates $\text{Re}(\epsilon'/\epsilon_K)$, then it is well known that one has to satisfy

$$|(\delta_{13}^d)_{LL} (\delta_{33}^d)_{LR} (\delta_{32}^d)_{RR}| \lesssim 10^{-5}$$

with an $O(1)$ phase [22]. Since the RG evolution generates $(\delta_{13}^d)_{LL} \sim \lambda^3$ within mSUGRA scenario, we can derive the following upper bound:

$$|(\delta_{33}^d)_{LR} (\delta_{32}^d)_{RR}| \lesssim 10^{-3}. \quad (23)$$

Note that this combination enters the calculation of $S_{\phi K}$ and $B \rightarrow X_s \gamma$ through $C_{8g(7\gamma)}(\Delta B = 1)$ along with $(\delta_{32}^d)_{RR}$. For a small $\mu \tan\beta$ (corresponding to a small $(\delta_{33}^d)_{RL}$), one can have larger $(\delta_{32}^d)_{RR}$, which is constrained by the lower bound on ΔM_s and the $B \rightarrow X_s \gamma$ branching ratio. For a large $\mu \tan\beta$ (corresponding to a large $(\delta_{33}^d)_{RL}$), $(\delta_{32}^d)_{RR}$ should be smaller in order to satisfy (23). In either case, we can expect that the deviation in $S_{\phi K}$ cannot be that large for such $(\delta_{32}^d)_{RR}$ satisfying the $\text{Re}(\epsilon'/\epsilon_K)$ constraint, (23).

Having described the qualitative features of our main points, we now provide the detailed analysis including theoretical uncertainties in $S_{\phi K}$ and $\text{Re}(\epsilon'/\epsilon_K)$ as summarized in Secs. II and III. We use the parameterization, $(\delta_{23}^d)_{RR} \equiv r e^{i\phi}$. We fix the modulus r at a maximal value compatible with $B(B \rightarrow X_s \gamma)$, and vary its phase ϕ from 0 to 2π . For $\mu \tan\beta = 1$ TeV, we use $r = 1$ which is dictated by requiring the validity of the mass insertion approximation. For $\mu \tan\beta = 5$ TeV, we set $r = 0.33$, which is defined by the upper bound on $B(B \rightarrow X_s \gamma)$. Some values of ϕ result in ΔM_s smaller than the lower bound [9], and they are discarded. For each value of ϕ consistent with the ΔM_s constraint, we plot a point in the plane $\text{Re}(\epsilon'/\epsilon_K) - S_{\phi K}$, following the procedures in Refs. [28] and [9,11], respectively. In Figs. 2(a) and 2(b), we show the plots for $\mu \tan\beta = 1$ and 5 TeV, respectively, with $\tilde{m} = m_{\tilde{g}} = 500$ GeV. The thick vertical error bar shows the current data on $S_{\phi K}$, and the two dashed vertical lines delimit the experimental value of $\text{Re}(\epsilon'/\epsilon_K)$ [33],

$$\text{Re}(\epsilon'/\epsilon_K) = (16.7 \pm 2.6) \times 10^{-4}. \quad (24)$$

The full black box shows our estimates of $S_{\phi K}$ and $\text{Re}(\epsilon'/\epsilon_K)$ within the SM. Its width and height are the uncertainties in $\text{Re}(\epsilon'/\epsilon_K)$ and $S_{\phi K}$, respectively. In Fig. 2(a), we show a curve for $\mu \tan\beta = 1$ TeV, and $r = 1$ which may be regarded to define the boundary of a region of the two observables generically predicted in this scenario. For this curve, we fix $B_G = 2$ and use the central values of $R_6, R_8,$ and m_s , as given in Sec. III. As mentioned previously, disconnected parts of the curve are excluded by the ΔM_s constraint. If we turn on hadronic uncertainties, this curve gets broadened into the gray region around it. We estimate the uncertainty of an observable by taking its maximum and minimum values reached while varying the relevant input parameters in the ranges quoted in the previous two sections. For $\text{Re}(\epsilon'/\epsilon_K)$, they are $R_6, R_8, m_s,$ and B_G , and for $S_{\phi K}$, they are $\varrho_H, \varphi_H, \varrho_A,$ and φ_A . Each of the uncertainties in $\text{Re}(\epsilon'/\epsilon_K)$ and $S_{\phi K}$ is displayed by the horizontal or vertical error bars at five selected points. The gray region is drawn by varying all the eight parameters simultaneously. It therefore covers wider space than is obtained by quadrature addition. Suppose that the sign of B_G is negative. The resulting curve and the region around it can easily be guessed by taking the mirror image of the present one around the vertical axis passing through the SM point. Even then the $\text{Re}(\epsilon'/\epsilon_K)$ data gives a strong constraint on the possible value of $S_{\phi K}$.

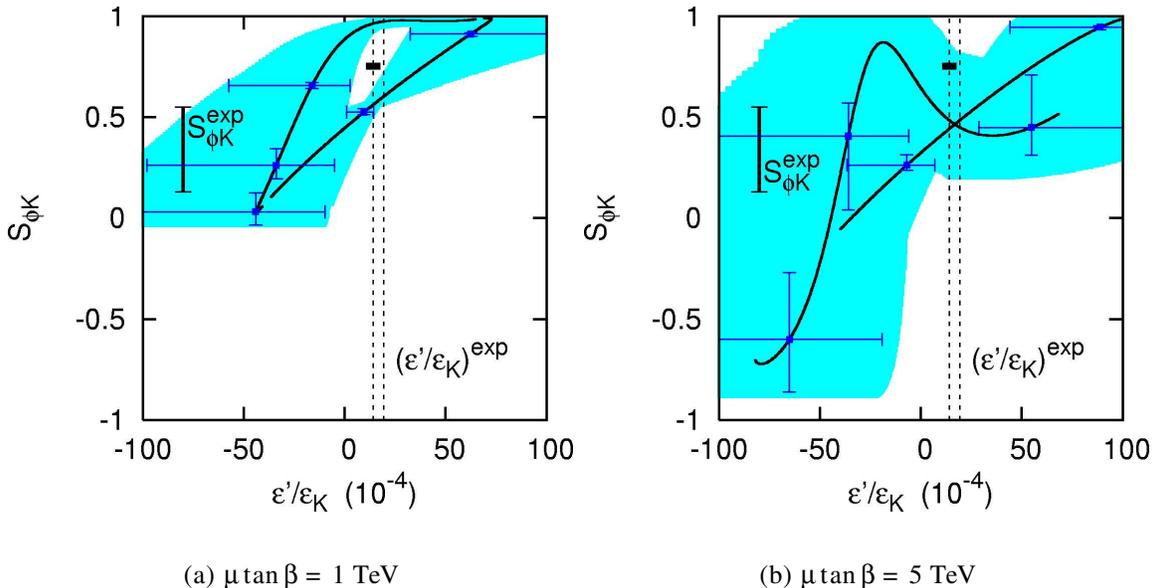


FIG. 2 (color online). $S_{\phi K}$ vs $\text{Re}(\epsilon'/\epsilon_K)$ for (a) $\mu \tan\beta = 1$ TeV and (b) $\mu \tan\beta = 5$ TeV, with $\tilde{m} = m_{\tilde{g}} = 500$ GeV. Experimental bounds on $\text{Re}(\epsilon'/\epsilon_K)$ and $S_{\phi K}$ are depicted by the vertical dashed lines and the thick vertical error bar, respectively. Their SM predictions are marked by the black box, whose extent indicates their uncertainties. The black curve does not include hadronic uncertainties, and the gray region includes them. The respective uncertainties in $\text{Re}(\epsilon'/\epsilon_K)$ and $S_{\phi K}$ are shown by the horizontal and vertical error bars at some selected points.

We repeat the same exercise for $\mu \tan\beta = 5$ TeV in Fig. 2(b). We find that $S_{\phi K} > 0.25$ if the data on Re(ϵ'/ϵ_K) is imposed.

Note that the constraint from Re(ϵ'/ϵ_K) is comparable to that from the strange quark CEDM. In particular the positive (negative) $S_{\phi K}$ is correlated with the positive (negative) Re(ϵ'/ϵ_K) within minimal SUGRA boundary conditions. Therefore, the old Belle data with the negative $S_{\phi K}$ implies a negative Re(ϵ'/ϵ_K) in the RR dominance scenario such as SUSY GUT models with right-handed neutrinos, which is clearly excluded by the data Re(ϵ'/ϵ_K) = $(16.7 \pm 2.6) \times 10^{-4}$. If the old Belle data were still valid, then the RR dominance scenario should have been discarded. Our results provide a meaningful correlation between $S_{\phi K}$ and Re(ϵ'/ϵ_K) despite large hadronic uncertainties in both quantities. This is independent of the strange quark CEDM constraint, and probably has less theoretical uncertainties.

One may wonder why we are not considering triple mass insertion on a squark line in the box diagram for ϵ_K . The size of the effective insertion,

$$(\delta_{12}^d)_{LR}^{\text{eff}} \equiv (\delta_{13}^d)_{LL}(\delta_{33}^d)_{LR}(\delta_{32}^d)_{RR}, \quad (25)$$

is always smaller than 2×10^{-4} due to $B(B \rightarrow X_s \gamma)$. If we require that the SUSY contribution to Re(ϵ'/ϵ_K) be smaller than its experimental value, we get

$$|\text{Im}(\delta_{12}^d)_{LR}^{\text{eff}}| \lesssim 10^{-5}, \quad (26)$$

which implies that

$$\sqrt{|\text{Im}[(\delta_{12}^d)_{LR}^{\text{eff}}]^2|} \lesssim 6 \times 10^{-5}. \quad (27)$$

This limits the SUSY contribution to ϵ_K below 1/30 of its experimental value. In view of theoretical uncertainties in predicting ϵ_K , we may regard ϵ_K being always safe provided that Re(ϵ'/ϵ_K) constraint is satisfied.

Let us add a remark on the CP violation in $B_s - \bar{B}_s$ mixing, whose indirect CP asymmetry nearly vanishes in the SM. The large RR mixing leads to a considerable modification in the mixing amplitude. This is evident from the fact that part of the curves in Figs. 2 was excluded by the ΔM_s constraint. Since this new piece of amplitude has a phase different from the SM one in general, the indirect CP asymmetry in $B_s - \bar{B}_s$ mixing can have a value of $O(1)$ according to the phase of $(\delta_{23}^d)_{RR}$. This will show up in the decay channel $B_s \rightarrow J/\psi \phi$ as the time dependent CP asymmetry therein. For quantitative analyses, see Refs. [9–11] for instance.

In SU(5) SUSY GUT, the left-handed sleptons and the right-handed down-type squarks are tied to a $\bar{5}$, and therefore their flavor changing effects are related to each other. Off-diagonal elements of the left-handed slepton mass matrix are given by

$$(m_{\tilde{l}}^2)_{ij} \simeq -\frac{1}{8\pi^2} y_{\nu_k}^2 [V_L]_{ki} [V_L^*]_{kj} (3m_0^2 + A^2) \log \frac{M_*}{M_{N_k}}, \quad (28)$$

in a way similar to Eq. (2). In order to estimate lepton flavor violation from this mass matrix, we should go to the super CKM basis. If the down-type quark and the charged lepton Yukawa matrices are the same at the GUT scale as

$$Y_d = Y_l^T, \quad (29)$$

the right-handed down-type squark and the charged slepton mass insertion parameters are unified as well. Under this assumption, the current upper bound on the $\tau \rightarrow \mu \gamma$ branching ratio [42],

$$B(\tau \rightarrow \mu \gamma) < 6.8 \times 10^{-8}, \quad (30)$$

translates into the limit [14],

$$|(\delta_{23}^d)_{RR}| < 0.03, \quad (31)$$

for $\tan\beta = 10$. This is roughly 1/10 of the size of $(\delta_{23}^d)_{RR}$ we used for $\mu \tan\beta = 5$ TeV, and we cannot expect considerable change in $S_{\phi K}$ satisfying the $\tau \rightarrow \mu \gamma$ constraint. The assumption of Yukawa unification, however, leads to an incorrect mass relation,

$$\frac{m_d}{m_s} = \frac{m_e}{m_\mu}, \quad (32)$$

and Eq. (29) should be modified to account for the mass ratios of the first and second generation fermions. Even in this case, a mass insertion involving a third generation is not much affected, and $\tau \rightarrow \mu \gamma$ remains a strong constraint on $S_{\phi K}$.

In this work, we are considering SUSY GUT with right-handed neutrinos as an example of a scenario that gives rise to large RR mixing. This is why $\tau \rightarrow \mu \gamma$ constrains $S_{\phi K}$. Here, we would like to stress again that the interconnection between Re(ϵ'/ϵ_K) and $S_{\phi K}$, unlike $\tau \rightarrow \mu \gamma$, is a common consequence of large RR mixing, which is not specific to SUSY GUT. Suppose that there is a large RR mixing but no LL mixing in the squark sector at the reduced Planck scale due to a flavor symmetry and that we do not have a unified gauge group. Even in this case, one has a strong correlation between Re(ϵ'/ϵ_K) and $S_{\phi K}$, while $\tau \rightarrow \mu \gamma$ is unrelated to $S_{\phi K}$.

If we considered more general scalar masses at M_* with some flavor structures, then our results will be changed accordingly. The Wilson coefficients for C_{8g} 's for both $\Delta B(S) = 1$ have to include other mass insertion parameters such as $(\delta_{23}^d)_{LR}$, $(\delta_{12}^d)_{LL}$, $(\delta_{23}^d)_{LL}$, etc., which were neglected in Secs. II and III because they are small within the mSUGRA scenarios. Still we should make it sure that the new flavor physics that affects $S_{\phi K}$ does not contribute to Re(ϵ'/ϵ_K) too much, and this could make a strong constraint on new sources of flavor and CP violation despite theoretical uncertainties in Re(ϵ'/ϵ_K).

V. CONCLUSIONS

In summary, we showed that if the $RR\ b \rightarrow s$ transition is large with $O(1)$ phase, it can affect not only $S_{\phi K}$ through double mass insertion and the strange quark CEDM through triple mass insertion, but it also affects $\text{Re}(\epsilon'/\epsilon_K)$. The correlation between the two observables are strong despite large hadronic uncertainties in both observables, within mSUGRA boundary conditions with flavor universal scalar masses at M_* . The current data on $\text{Re}(\epsilon'/\epsilon_K)$ indicates that $S_{\phi K}$ should be in the range of

0.25–1.0, which is now in accord with the present world average of $S_{\phi K}$.

ACKNOWLEDGMENTS

We are grateful to Weon Jong Lee and A. I. Sanda for discussion on the $\text{Re}(\epsilon'/\epsilon_K)$. P. K. and A. M. thank Aspen Center for Physics for the hospitality during their stay where this work was initiated. P. K. is supported in part by the BK21 Haeksim Program, KOSEF through CHEP at Kyungpook National University, and by KOSEF Sundo Grant R02-2003-000-10085-0.

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