Minimal U(1)′ extension of the minimal supersymmetric standard model

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Motivated by the apparent need for extending the minimal supersymmetric standard model (MSSM) and perhaps mitigating naturalness problems associated with the μ parameter and fine-tuning of the soft masses, we augment the MSSM spectrum by a SM gauge singlet chiral superfield, and enlarge the gauge structure by an additional U(1)′ invariance, so that the gauge and Higgs sectors are relatively secluded. One crucial aspect of U(1)′ models is the existence of anomalies, the cancellation of which may require the inclusion of exotic matter which in turn disrupts the unification of the gauge couplings. In this work we pursue the question of canceling the anomalies with a minimal matter spectrum and no exotics. This can indeed be realized provided that U(1)′ charges are family dependent and the soft-breaking sector includes nonholomorphic operators for generating the fermion masses. We provide the most general solutions for U(1)′ charges by taking into account all constraints from gauge invariance and anomaly cancellation. We analyze various laboratory and astrophysical bounds ranging from fermion masses to relic density, for an illustrative set of parameters. The U(1)′ charges admit patterns of values for which family nonuniversality resides solely in the lepton sector, though this does not generate leptonic flavor-changing neutral currents due to the U(1)′ gauge invariance.

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I. INTRODUCTION

Supersymmetric models extending the minimal supersymmetric standard model (MSSM) are generally motivated for stabilizing the μ parameter at the electroweak scale, and for incorporating right-handed neutrinos into the spectrum. The extension of the MSSM may or may not involve additional gauge groups. Concerning the former, the most conservative approach is to extend the gauge structure of the MSSM by an extra Abelian group factor U(1)′ along with an additional chiral superfield $\hat{S}$ whose scalar component generates an effective μ parameter upon spontaneous U(1)′ breakdown. The U(1)′ symmetry in question is essentially the gauging of the global Peccei-Quinn invariance of the MSSM. What it actually does is to forbid a bare μ parameter thereby providing a dynamical solution to the μ problem [1]. Extra U(1) symmetries arise as low-energy manifestations of grand unified [2], of string [3], and of dynamical electroweak breaking [4] theories.

An important property of U(1)′ models is that the lightest Higgs boson weighs significantly more than M_Z even at tree level with small $\tan\beta$. Hence the existing CERN LEP bounds are satisfied with almost no need for large radiative corrections [5–7]. Besides, they offer a rather wide parameter space for facilitating the electroweak baryogenesis [8].

An important issue about extra U(1)′ models concerns the cancellation of anomalies. Indeed, for making the theory anomaly free the usual approach to U(1)′ models is to add several exotics to the spectrum [9]. This not only causes a significant departure from the minimal structure but also disrupts the gauge coupling unification—one of the fundamental predictions of the MSSM with weak scale soft masses.

The prime goal of the present work is to construct an anomaly-free U(1)′ model without exotics. We accomplish this by allowing family-nonuniversal U(1)′ invariance. It is known that when different fermion families posses different U(1)′ charges generally large Z'-mediated flavor-changing neutral currents (FCNC) arise [10]. However, there are exceptions to this, especially when Z' FCNC effects reside in the lepton sector. For example, if the U(1)′ charges forbid the off-diagonal terms in the fermion mass matrix (in the family space), the mass eigenstates will coincide with the gauge eigenstates. Therefore, there will be no FCNC induced by the Z' gauge boson. The family dependence of the U(1)′ invariance necessarily forbids certain Yukawa couplings in the superpotential, leading to massless fermions. The requisite fermion masses, however, can be induced at the loop level via nonholomorphic operators in the soft sector [11,12]. In addition to being allowed, these nonholomorphic terms can appear in intersecting brane models with certain types of fluxes turned on [13]. Therefore, as we will describe in the text, a minimal U(1)′ model can be realized with family-dependent charges and nonholomorphic terms.

The paper is organized as follows. In Sec. II below we introduce nonholomorphic terms and discuss how the fermion masses as well as other chirality-changing operators such as the magnetic moments are induced. In Sec. III we discuss in detail the construction of an anomaly-free U(1)′ model with minimal matter content. We also determine the flavor structures of the Yukawa matrices and of the non-

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holomorphic terms therein. In Sec. IV we survey phenomenological tests of the U(1)′ models by briefly discussing fine-tuning, the Higgs sector, Z' couplings, collider signatures, neutrino masses, muon $g-2$, and the relic density of the universe. In Sec. V we conclude the work.

II. U(1)′ MODELS WITH NONHOLOMORPHIC SUSY BREAKING

In U(1)′ models the MSSM gauge group is extended to include an extra Abelian group factor: $\mathrm{SU}(3)_c \times \mathrm{SU}(2)_L \times \mathrm{U}(1)_Y \times \mathrm{U}(1)'$ with respective gauge couplings g_3, g_2, g_Y and g_1'. This gauge structure survives all the energy scales from $M_{\mathrm{GUT}} \approx 2 \times 10^{16}$ GeV down to a TeV. The particle spectrum of the model is that of the MSSM plus a MSSM gauge singlet S charged under only the U(1)′ invariance. Clearly, the family universality of the MSSM gauge charges is not necessarily respected by the U(1)′ group. Hence we employ a general family-dependent charge assignment as tabulated in Table I.

The superpotential takes the form:

$$\hat{W} = h_s \hat{S}\hat{H}_d\hat{H}_u + h_u^{ij}\hat{U}_j^c\hat{Q}_i\hat{H}_u + h_d^{ij}\hat{D}_j^c\hat{Q}_i\hat{H}_d + h_e^{ij}\hat{E}_j^c\hat{L}_i\hat{H}_d. \tag{1}$$

The first term of the superpotential induces an effective μ parameter $h_s\langle S\rangle$ below the scale of U(1)′ breaking. This provides a dynamical solution to the μ problem when $\langle S\rangle \sim \mathcal{O}$ (TeV). The rest of the operators in (1) describes the Yukawa interactions of leptons and quarks.

The most general holomorphic structures which break supersymmetry (SUSY) softly are

$$\begin{aligned} -\mathcal{L}_{\mathrm{soft}} = &\Big(\sum_i M_i\lambda_i\lambda_i - A_S h_s S H_d H_u - A_u^{ij}h_u^{ij}U_j^c Q_i H_u \\ &- A_d^{ij}h_d^{ij}D_j^c Q_i H_d - A_e^{ij}h_e^{ij}E_j^c L_i H_d + \mathrm{H.c.}\Big) \\ &+ m_{H_u}^2|H_u|^2 + m_{H_d}^2|H_d|^2 + m_S^2|S|^2 + m_{Q_{ij}}^2\tilde{Q}_i\tilde{Q}_j^* \\ &+ m_{U_{ij}}^2\tilde{U}_i^c\tilde{U}_j^{c*} + m_{D_{ij}}^2\tilde{D}_i^c\tilde{D}_j^{c*} + m_{L_{ij}}^2\tilde{L}_i\tilde{L}_j^* \\ &+ m_{E_{ij}}^2\tilde{E}_i^c\tilde{E}_j^{c*} + \mathrm{H.c.} \end{aligned} \tag{2}$$

TABLE I. The gauge quantum numbers of chiral fields in the U(1)′ model. The index i runs over three families of matter. Each family can acquire a different charge under the U(1)′ group.

	$\mathrm{SU}(3)_c$	$\mathrm{SU}(2)_L$	$\mathrm{U}(1)_Y$	$\mathrm{U}(1)'$
Q_i	**3**	**2**	1/6	Q_{Q_i}
U_i^c	$\bar{\mathbf{3}}$	**1**	−2/3	$Q_{U_i^c}$
D_i^c	$\bar{\mathbf{3}}$	**1**	1/3	$Q_{D_i^c}$
L_i	**1**	**2**	−1/2	Q_{L_i}
E_i^c	**1**	**1**	1	$Q_{E_i^c}$
H_u	**1**	**2**	1/2	Q_{H_u}
H_d	**1**	**2**	−1/2	Q_{H_d}
S	**1**	**1**	0	Q_S

where the sfermion mass-squared $m^2_{Q,\ldots,E^c}$ and the trilinear couplings $A_{u,\ldots,e}$ are 3×3 matrices in flavor space. All these soft masses will be taken here to be diagonal. Moreover, all gaugino masses M_i and trilinear couplings $A_{S,\ldots,e}$ will be taken real since the (important and interesting) question of CP violation is beyond the scope of the present work (interested readers can refer to [6]).

Clearly, the U(1)′ charge assignments of chiral superfields put stringent constraints on the Yukawa textures [14]. For instance, if the U(1)′ charges satisfy

$$Q_{Q_1} + Q_{U_i} + Q_{H_u} \neq 0 \quad \text{for } i = 1,2,3 \tag{3}$$

then the up quark can acquire a mass neither at tree level nor at any loop level with holomorphic soft terms. Therefore, for avoiding massless fermions it is necessary to introduce nonholomorphic SUSY-breaking operators, the nonholomorphic terms [11–13,15,16]. Historically, the nonholomorphic terms have not been classified as "soft" since they might give rise to quadratic divergences [17]. However, such operators are perfectly soft when no gauge singlets are contained in the theory. Indeed, nonholomorphic terms are soft in the MSSM and its U(1)′ extensions. Concerning the origin of the nonholomorphic terms, one notes that they are generated by spontaneous SUSY breaking within gravity mediation [18]. In addition to this, they arise naturally in strongly coupled SUSY gauge theories [19]. Moreover, the effective potentials of $N=2$ and $N=4$ SUSY gauge theories are endowed with radiatively generated nonholomorphic soft terms [20].

For the U(1)′ model under concern the nonholomorphic SUSY-breaking Lagrangian takes the form

$$-\mathcal{L}_c = C_E^{ij}H_u^*\tilde{L}^i\tilde{E}_R^{cj} + C_U^{ij}H_d^*\tilde{Q}^i\tilde{U}_R^{cj} + C_D^{ij}H_u^*\tilde{Q}^i\tilde{D}_R^{cj} + \mathrm{c.c.} \tag{4}$$

and needs to be added to the holomorphic ones in (2). Clearly, a down-type quark, for instance, develops a finite mass via triangular diagrams proceeding with $\tilde{D}_L$ $\tilde{D}_R^c$ and a neutral gaugino λ, and the result is necessarily proportional to C_D. This radiative induction of the fermion masses is rather generic. Notice that coupling to the "wrong" Higgs doublet in (4) is essential for giving mass to fermions. Indeed, a fermion f obtains the mass [12,21]

$$m_f = (C_f v_\alpha)\Big[\frac{\alpha_s}{2\pi}\xi_f m_{\tilde{g}} I_m(m^2_{\tilde{f}_1}, m^2_{\tilde{f}_2}, m^2_{\tilde{g}}) + \frac{\alpha_Y}{2\pi}\sum_{j=1}^{6} K_f^j m_{\tilde{\chi}_j^0} I_m(m^2_{\tilde{f}_1}, m^2_{\tilde{f}_2}, m^2_{\tilde{\chi}_j^0})\Big] \tag{5}$$

where $v_\alpha = \langle H_u\rangle(\langle H_d\rangle)$ for down-type (up-type) fermions. Here the first term refers to SUSY-QCD contribution ($\xi_f = 4/3, 0$ for quarks and leptons, respectively), and the second term summarizes the contributions of all neutral Higgsinos and gauginos. C_f is the corresponding nonholomorphic terms in (4), and $\alpha_Y = g_Y^2/(4\pi)$. The triangular loop function I_m is defined by

$$I_m(m_1^2, m_2^2, m_\lambda^2) = \frac{1}{m_1^2 - m_2^2}\left(\frac{\ln\beta_1}{\beta_1 - 1} - \frac{\ln\beta_2}{\beta_2 - 1}\right) \quad (6)$$

where $\beta_i = m_\lambda^2/m_i^2$ with $i = 1, 2$. This function approaches $1/2m^2$ when $m_1 \sim m_2 \sim m_\lambda \equiv m$. The coupling of the jth neutralino to mass-eigenstate sfermions ($\tilde{f}_i$ with masses $m_{\tilde{f}_i}$) is given by

$$K_f^j = \left[Y_{f_R} N_{jB} + \left(\frac{g_1'}{g_Y}\right) Q_{f_R} N_{jB'} \right] \times \left[Y_{f_L} N_{jB} + \left(\frac{g_1'}{g_Y}\right) Q_{f_L} N_{jB'} + \cot\theta_W N_{jW} T_{3f_L} \right] \quad (7)$$

where Q_f is the U(1)′ charge of the fermion f, $Y_f = Q_{em}^f - T_3^f$, and g_1' and g_Y stand for the U(1)′ and hypercharge gauge couplings, respectively. Here $N_{jB'}$, N_{jB} and N_{jW} are the Z', b-ino and W-ino components of the jth neutralino. Note that the fermion masses in (5) are of the form $m_f = \kappa_f \langle H_\alpha \rangle$ where the dimensionless coupling in front involves gauge couplings and sparticle mixing angles as well as the ratios of the trilinear couplings to sparticle masses. Hence, various soft-breaking parameters must conspire to generate fermion masses in agreement with experiment. It might be useful to dwell on this point briefly. For reproducing the correct hierarchy of the light fermion masses (i.e. $m_u < m_d$, $m_s < m_c$, $m_e < m_\mu$) one can tune the sfermion masses, the nonholomorphic trilinear couplings C_f or the U(1)′ charges. As a simple case study let us examine the u-d mass hierarchy in the limit of degenerate $\tilde{u}$ and $\tilde{d}$ squarks. One finds

$$\frac{m_u}{m_d} = \frac{C_U^{11}}{C_D^{11}} \frac{1 + \frac{3\alpha_Y}{4\alpha_s} \sum_{j=1}^6 K_u^j R_j}{1 + \frac{3\alpha_Y}{4\alpha_s} \sum_{j=1}^6 K_d^j R_j} \quad (8)$$

where $R_j \equiv m_{\tilde{\chi}_j^0} I_m(m_{\tilde{f}_1}^2, m_{\tilde{f}_2}^2, m_{\tilde{\chi}_j^0}^2)/m_{\tilde{g}} I_m(m_{\tilde{f}_1}^2, m_{\tilde{f}_2}^2, m_{\tilde{g}}^2) \sim m_{\tilde{\chi}_j^0}/m_{\tilde{g}}$ is identical for up and down squarks. In case $C_U^{11} \simeq C_D^{11}$ the u-d hierarchy can be saturated if $\sum_j (0.5K_d^j - K_u^j) R_j \sim 10$ which is too large to be satisfied unless the gluino is exceedingly light, $m_{\tilde{g}} \sim 1$ GeV. Other fermion masses can be analyzed in a similar way. Therefore, the hierarchy among the fermion masses rests largely on the hierarchy of the nonholomorphic trilinears. On the other hand, the generation of the correct values of the individual fermion masses requires a judicious choice of the soft masses and U(1)′ couplings.

As was shown in [12], it is difficult to generate masses for the top quark and tau lepton if the nonholomorphic terms are not much larger than the other soft masses. Therefore, the U(1)′ charge assignments must be such that these fermions can obtain masses already at tree level. However, the rest of the fermions can acquire masses through (5) with no obvious contradiction with experiments.

The sparticle virtual effects which give rise to nonvanishing fermion masses (5) induce also chirality-violating operators pertaining to radiative transitions of the fermions. Among these are the electric and magnetic dipole moments. In fact, for a fermion with radiatively induced mass the magnetic dipole moment takes the form [12]

$$a_f^{\rm SUSY} = 2m_f^2 \frac{\sum_j K_f^j m_{\tilde{\chi}_j^0} I_{g-2}(m_{\tilde{f}_1}^2, m_{\tilde{f}_2}^2, m_{\tilde{\chi}_j^0}^2)}{\sum_j K_f^j m_{\tilde{\chi}_j^0} I_m(m_{\tilde{f}_1}^2, m_{\tilde{f}_2}^2, m_{\tilde{\chi}_j^0}^2)} \quad (9)$$

where

$$I_{g-2}(m_1^2, m_2^2, m_\lambda^2) = \frac{1}{m_\lambda^2} \frac{1}{m_2^2 - m_1^2} \times \left\{ \frac{\beta_1(\beta_1^2 - 1 - 2\beta_1 \log\beta_1)}{2(\beta_1 - 1)^2} - (1 \to 2) \right\} \quad (10)$$

with the same parametrization used for I_m. If $\tilde{m} = \max(m_{\tilde{f}_1}, m_{\tilde{f}_2}, m_{\tilde{\lambda}})$ then

$$a_f^{\rm SUSY} = 2m_f^2 \frac{\sum K_f^j m_{\tilde{\chi}_j^0} I_{g-2}(m_{\tilde{f}_1}^2, m_{\tilde{f}_2}^2, m_{\tilde{\chi}_j^0}^2)}{\sum K_f^j m_{\tilde{\chi}_j^0} I_m(m_{\tilde{f}_1}^2, m_{\tilde{f}_2}^2, m_{\tilde{\chi}_j^0}^2)} \sim \frac{m_f^2}{3\tilde{m}^2} \quad (11)$$

so that the larger the heaviest sparticle mass, the smaller the magnetic moment. One notes that the expression of the magnetic moment (9) contains no loop suppression factor $1/(4\pi)^2$ due to the fact that the fermion mass itself is generated radiatively. Hence, when the fermion mass is generated solely by nonholomorphic soft terms the magnetic moment, in particular, the muon magnetic moment a_μ, tends to be large. The most stringent bound is from the measured a_μ. Indeed, if the muon mass follows from nonholomorphic terms (as will be the case in our model mentioned below) then for saturating the existing experimental bounds on $g_\mu - 2$ the scalar muon $\tilde{\mu}$ must weigh $\mathcal{O}$(TeV).

III. AN ANOMALY-FREE MINIMAL U(1)′ MODEL

One of the most important issues in U(1)′ models is the cancellation of gauge and gravitational anomalies. Indeed, for making the theory anomaly free one has been forced to augment the minimal spectrum by a number of exotics [9]. These additional fields usually disrupt the unification of the gauge couplings. In this section we will discuss the crucial role played by family-dependent U(1)′ charges in canceling the anomalies and hence in preserving the unification of gauge forces.

For the theory to be anomaly free the U(1)′ charges of chiral fields must satisfy

$$0 = \sum_i (2Q_{Q_i} + Q_{U_i^c} + Q_{D_i}), \quad (12)$$

$$0 = \sum_i (3Q_{Q_i} + Q_{L_i}) + Q_{H_d} + Q_{H_u}, \tag{13}$$

$$0 = \sum_i \left(\frac{1}{6}Q_{Q_i} + \frac{1}{3}Q_{D_i^c} + \frac{4}{3}Q_{U_i^c} + \frac{1}{2}Q_{L_i} + Q_{E_i^c}\right) + \frac{1}{2}(Q_{H_d} + Q_{H_u}), \tag{14}$$

$$0 = \sum_i (6Q_{Q_i} + 3Q_{U_i^c} + 3Q_{D_i^c} + 2Q_{L_i} + Q_{E_i^c}) + 2Q_{H_D} + 2Q_{H_u} + Q_s, \tag{15}$$

$$0 = \sum_i (Q_{Q_i}^2 + Q_{D_i^c}^2 - 2Q_{U_i^c}^2 - Q_{L_i}^2 + Q_{E_i^c}^2) - Q_{H_d}^2 + Q_{H_u}^2, \tag{16}$$

$$0 = \sum_i (6Q_{Q_i}^3 + 3Q_{D_i^c}^3 + 3Q_{U_i^c}^3 + 2Q_{L_i}^3 + Q_{E_i}^3) + 2Q_{H_d}^3 + 2Q_{H_u}^3 + Q_S^3 \tag{17}$$

which correspond to vanishing of U(1)′-SU(3)-SU(3), U(1)′-SU(2)-SU(2), U(1)′-U(1)$_Y$-U(1)$_Y$, U(1)′-graviton-graviton, U(1)′-U(1)′-U(1)$_Y$ and U(1)′-U(1)′-U(1)′ anomalies, respectively.

As mentioned before, the top quark and tau lepton masses must be generated already at tree level. Moreover, U(1)′ invariance must allow for SH_dH_u coupling for solving the μ problem. These conditions lead to

$$Q_{Q_3} + Q_{U_3^c} + Q_{H_u} = 0, \tag{18}$$

$$Q_{L_3} + Q_{E_3^c} + Q_{H_d} = 0, \tag{19}$$

$$Q_{H_u} + Q_{H_d} + Q_S = 0 \tag{20}$$

which should be added to Eqs. (12)–(17). The family-nonuniversal U(1)′ charges could lead to large Z' -mediated FCNCs [10]. One first observes that the very presence of the Cabibbo-Kobayashi-Maskawa (CKM) matrix implies that the physical quark states are achieved after a unitary rotation of the gauge-basis quarks. Therefore, for guaranteeing the suppression of FCNCs in the hadron sector it is good to keep quark U(1)′ charges family universal:

$$Q_{Q_1} = Q_{Q_2} = Q_{Q_3}, \qquad Q_{U_1^c} = Q_{U_2^c} = Q_{U_3^c}, \qquad Q_{D_1^c} = Q_{D_2^c} = Q_{D_3^c} \tag{21}$$

so that, depending on the charge assignments of the Higgs doublets, either the down or the up quark sector possesses tree-level Yukawa interactions. For the lepton sector one can relax the condition of family universality since it will lead to FCNCs only if mass- and gauge-eigenstate leptons are not identical. As will be seen below, U(1)′ charges can be assigned in such a way that the mass matrix of leptons is automatically flavor diagonal and hence leptonic FCNCs are absent.

We now want to illustrate the assignment of U(1)′ charges. There are 18 unknowns and 15 constraints (12)–(21) out of which (16) and (17) are nonlinear in charges. Using the linear constraints we first express 13 charges in terms of 5 charges which we choose to be

$$Q_{L_2}, \quad Q_{E_2^c}, \quad Q_{E_3^c}, \quad Q_{H_d} \quad \text{and} \quad Q_S. \tag{22}$$

The explicit expressions for charges read as

$$Q_{Q_1} = Q_{Q_2} = Q_{Q_3} = \frac{1}{9}(-3Q_{H_d} - 2Q_S), \tag{23}$$

$$Q_{D_1^c} = Q_{D_2^c} = Q_{D_3^c} = \frac{1}{9}(-6Q_{H_d} - 7Q_S), \tag{24}$$

$$Q_{U_1^c} = Q_{U_2^c} = Q_{U_3^c} = \frac{1}{9}(12Q_{H_d} + 11Q_S), \tag{25}$$

$$Q_{L_1} = Q_{E_3^c} - Q_{L_2} + 4Q_{H_d} + 3Q_S, \tag{26}$$

$$Q_{L_3} = -Q_{E_3^c} - Q_{H_d}, \tag{27}$$

$$Q_{E_1^c} = -Q_{E_2^c} - Q_{E_3^c} - 6Q_{H_d} - 5Q_S, \tag{28}$$

$$Q_{H_u} = -Q_{H_d} - Q_S \tag{29}$$

from which it follows that, for all i, j, $Q_{Q_i} + Q_{D_j^c} + Q_{H_d} = -1$ and $Q_{Q_i} + Q_{D_j^c} - Q_{H_u} = 0$. Hence, all of the down quarks get their masses from nonholomorphic terms via (5); they are not allowed to possess Yukawa structures h_d^{ij} in the superpotential. On the other hand, the up quarks obtain their masses from superpotential couplings only. Consider now the muon mass term. There are two alternatives[1]:

$$\begin{aligned} &\text{either} \quad Q_{L_2} + Q_{E_2^c} + Q_{H_d} = 0 \\ &\text{or} \quad Q_{L_2} + Q_{E_2^c} - Q_{H_u} = 0. \end{aligned} \tag{30}$$

The first option implies that the muon mass follows entirely from the Yukawa couplings. On the other hand, the second option restricts the muon mass to follow from nonholomorphic terms only. From (23)–(29), one can check that the first option leads to $Q_{L_1} + Q_{E_1^c} + Q_{H_d} = -2Q_S$ and $Q_{L_1} + Q_{E_1^c} - Q_{H_u} = -Q_S$. However, for solving the μ problem Q_S must be nonzero, and this implies

[1]When the determinant of a matrix is nonzero it cannot have a row or column with all zeroes. In fact, one can employ a rotation in the space of families to make all diagonal entries of the matrix nonzero. Hence, in the following we will assume that such a rotation has already been done such that whenever the determinant of a matrix is nonzero then no diagonal entry can vanish. In particular, one can employ a family redefinition to make (2,2) element of h_l nonzero.

that the electron is forbidden to acquire its mass from both the Yukawa couplings and the nonholomorphic terms. Hence, this option must be discarded; the muon cannot develop a mass from Yukawa couplings. The remaining alternative implies that $Q_{L_1} + Q_{E_1^c} - Q_{H_u} = 0$ so that both muon and electron receive their masses from nonholomorphic terms via (5). Using (29) and the second option in (30) it is easy to solve for Q_{H_d}:

$$Q_{H_d} = -Q_s - Q_{E_2^c} - Q_{L_2}, \tag{31}$$

so that 14 out of 18 charges get expressed in terms of

$$Q_{L_2}, \quad Q_{E_2^c}, \quad Q_{E_3^c} \quad \text{and} \quad Q_S. \tag{32}$$

With the solutions obtained so far, the two nonlinear anomaly cancellation conditions, (16) and (17), reduce to

$$0 = -2(2Q_{E_2^c} - Q_{E_3^c} + 2Q_{L_2} + Q_S)Q_S, \tag{33}$$

$$\begin{aligned} 0 = -3(2Q_{E_2^c} - Q_{E_3^c} + 2Q_{L_2} + Q_S)(3Q_{E_2^c}^2 - Q_{E_2^c}Q_{E_3^c} \\ + 10Q_{E_2^c}Q_{L_2} - 2Q_{E_3^c}Q_{L_2} + 8Q_{L_2}^2 + 3Q_{E_2^c}Q_S \\ + Q_{E_3^c}Q_S + 4Q_{L_2}Q_S) \end{aligned} \tag{34}$$

which are simultaneously satisfied when

$$2Q_{E_2^c} - Q_{E_3^c} + 2Q_{L_2} + Q_S = 0 \tag{35}$$

holds. One can eliminate $Q_{E_3^c}$ from this relation. Then 15 out of 18 charges get expressed in terms of the three independent ones,

$$Q_{L_2}, \quad Q_{E_2^c} \quad \text{and} \quad Q_S, \tag{36}$$

via the relations

$$\begin{aligned} &Q_{Q_1} = Q_{Q_2} = Q_{Q_3} = \frac{1}{9}(3Q_{E_2^c} + 3Q_{L_2} + Q_S), \\ &Q_{D_1^c} = Q_{D_2^c} = Q_{D_3^c} = \frac{1}{9}(6Q_{E_2^c} + 6Q_{L_2} - Q_S), \\ &Q_{U_1^c} = Q_{U_2^c} = Q_{U_3^c} = \frac{1}{9}(-12Q_{E_2^c} - 12Q_{L_2} - Q_S), \\ &Q_{L_1} = -2Q_{E_2^c} - 3Q_{L_2}, \qquad Q_{L_3} = -Q_{E_2^c} - Q_{L_2}, \\ &Q_{E_1^c} = 3Q_{E_2^c} + 4Q_{L_2}, \qquad Q_{E_3^c} = 2Q_{E_2^c} + 2Q_{L_2} + Q_S, \\ &Q_{H_d} = -Q_{E_2^c} - Q_{L_2} - Q_S, \qquad Q_{H_u} = Q_{E_2^c} + Q_{L_2} \end{aligned}$$

with which the theory becomes completely anomaly free. One can analyze all physical quantities of interest in terms of three free charges Q_{L_2}, $Q_{E_2^c}$ and Q_S without disrupting the unification of gauge couplings.

The U(1)′ charges entirely determine the Yukawa textures: they decide which flavors receive their masses at tree level and which ones at the loop level. In fact, the flavor structures of the Yukawa matrices can be determined via the charge matrices of the associated operators:

$$\begin{aligned} &(Q_{Q_i} + Q_{U_j^c} + Q_{H_u}) = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}_{ij}, \qquad (Q_{Q_i} + Q_{D_j^c} + Q_{H_d}) = -Q_S \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix}_{ij}, \\ &(Q_{L_i} + Q_{E_j^c} + Q_{H_d}) = \begin{pmatrix} -Q_S & -2Q_{E_2^c} - 4Q_{L_2} - Q_S & -Q_{E_2^c} - 2Q_{L_2} \\ 2Q_{L_2} + 4Q_{L_2} - Q_S & -Q_S & Q_{E_2^c} + 2Q_{L_2} \\ Q_{E_2^c} + 2Q_{L_2} - Q_S & -Q_{E_2^c} - 2Q_{L_2} - Q_S & 0 \end{pmatrix}_{ij}. \end{aligned} \tag{38}$$

It is clear that all of the up quarks get their masses from tree-level Yukawa interactions. On the other hand, none of the down-type quarks are allowed to have tree-level Yukawas, and only the tau lepton is permitted to have a direct tree-level mass. The massless fermions are to obtain their masses from nonholomorphic terms via (5). To see if this really happens it is necessary to examine the charge matrices determining the flavor structures of the nonholomorphic couplings:

$$\begin{aligned} &(Q_{Q_i} + Q_{U_j^c} - Q_{H_d}) = Q_S \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix}_{ij}, \qquad (Q_{Q_i} + Q_{D_j^c} - Q_{H_u}) = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}_{ij}, \\ &(Q_{L_i} + Q_{E_j^c} - Q_{H_u}) = \begin{pmatrix} 0 & -2Q_{E_2^c} - 4Q_{L_2} & -Q_{E_2^c} - 2Q_{L_2} + Q_S \\ 2Q_{E_2^c} + 4Q_{L_2} & 0 & Q_{E_2^c} + 2Q_{L_2} + Q_S \\ Q_{E_2^c} + 2Q_{L_2} & -Q_{E_2^c} - 2Q_{L_2} & Q_S \end{pmatrix}_{ij}. \end{aligned} \tag{39}$$

Obviously, the up-type squarks are unable to develop any nonholomorphic couplings: $C_U^{ij} = 0$ for all $i, j = 1, 2, 3$. The situation for down-type squarks is the opposite; they are allowed to develop generic nonholomorphic trilinears with no texture zeroes: $C_D^{ij} \neq 0$ for all i, j. The couplings of sleptons are interesting; when $Q_{E_2^c} + 2Q_{L_2} \neq 0$ and $-Q_{E_2^c} - 2Q_{L_2} + Q_S \neq 0$ they do not possess any flavor-changing nonholomorphic coupling: $C_E^{i\neq j} = 0$ for all i, j. However, the selectron and smuon still have nonholomorphic term couplings. Consequently, the tau lepton acquires its mass at tree level yet the electron and muon obtain their masses via (5) with no leptonic FCNCs. We summarize the

TABLE II. The mechanisms for fermion mass generation: Y means that mass is generated by tree-level Yukawa interactions, and R means that the mass is generated radiatively via (5).

	1st family	2nd family	3rd family
Up-type quarks	Y, H_u	Y, H_u	Y, H_u
Down-type quarks	R, H_u	R, H_u	R, H_u
Leptons	R, H_u	R, H_u	Y, H_d

mechanisms of mass generation for each fermion generation in Table II.

Given the allowed textures of Yukawa and nonholomorphic term matrices in (38) and (39), the effective Yukawa interactions below the soft-breaking scale take the form

$$-L_{\text{eff}} = h_u^{ij}(u_L)_i^c q_j H_u + \tilde{h}_d^{ij}(d_L)_i^c q_j H_u^c + \tilde{h}_e(e_L)^c L_1 H_u^c + \tilde{h}_\mu(\mu_L)^c L_2 H_u^c + h_\tau(\tau_L)^c L_3 H_d \quad (40)$$

where the superscript c stands for charge conjugation. The tilded Yukawa couplings are generated by nonholomorphic terms as in (5): $\tilde{h}_d^{ij} \propto C_D^{ij}$, $\tilde{h}_e \propto C_E^{11}$, $\tilde{h}_\mu \propto C_E^{22}$. One notes that the tau lepton is the only fermion which couples to H_d, in particular, it is very interesting that the entire quark sector behaves as in the SM (where H_u serves as the SM Higgs doublet H_{SM}) in contrast to its two-doublet origin encoded in the superpotential (1). It is clear from (40) that the entire hadronic FCNC is ruled by the CKM matrix as in the SM, and no leptonic FCNC exists. In this sense the family-nonuniversal U(1)′ model under consideration is highly conservative not only because of the minimality of the spectrum but also because of the SM-like couplings of all fermions but the tau lepton.

Since the model is already anomaly free with minimal matter content, $\text{SU}(3)_c$, $\text{SU}(2)_L$ and $\text{U}(1)_Y$ gauge couplings all unify into a common value $g_0 \simeq 1/\sqrt{2}$ at a scale $M_{\text{GUT}} \approx 2 \times 10^{16}$ GeV as in the MSSM. The U(1)′ gauge coupling reads at the weak scale as

$$g_1'^2(M_Z) = \frac{g_0^2}{1 - 2g_0^2 t_Z \,\text{Tr}[Q^2]} \quad (41)$$

where $t_Z = (4\pi)^{-2}\log(M_Z/M_{\text{GUT}})$, and clearly, $g_1'(M_Z)$ depends on what values are assigned to the independent charges Q_{L_2}, $Q_{E_2^c}$ and Q_S.

In the next section we will discuss some phenomenological implications of the minimal U(1)′ model under consideration.

IV. PHENOMENOLOGICAL TESTS

In general, one can analyze the phenomenological implications of our U(1)′ model as a function of the admissible values (e.g. $Q_S \neq 0$) of the charges Q_{L_2}, $Q_{E_2^c}$, Q_S. However, for simplicity we prefer to work with a representative point in the space of U(1)′ charges and all other model parameters. Therefore, we assign the following numerical values to the free charges:

$$Q_{L_2} = 2, \qquad Q_{E_2^c} = -3, \qquad Q_S = 3 \quad (42)$$

for which $g_1'(M_Z) = 0.196$ to be compared with $g_Y(M_Z) = 0.358$. With (42) the U(1)′ charges of chiral fields get fixed to values depicted in Table III. Note that the left-handed quarks are all singlets under U(1)′ and right-handed up and down quarks are charged oppositely under U(1)′. Furthermore, the left-handed electron does not couple to Z'.

TABLE III. The U(1)′ charges of chiral fields corresponding to the charge assignment in (42).

	1st family	2nd family	3rd family
Q_{Q_i}	0	0	0
$Q_{U_i^c}$	1	1	1
$Q_{D_i^c}$	−1	−1	−1
Q_{L_i}	0	2	1
$Q_{E_i^c}$	−1	−3	1
	Q_{H_u}	Q_{H_d}	Q_S
	−1	−2	3

Of course, there is no known fundamental reason for the particular charge assignment in (42); one can adopt some other numerical representation as well. Hence, as a distinct case study consider another set of charges shown in Table IV. They satisfy all of the master relations in (38). In fact, Table IV has interesting properties in that the Z' boson couples to no lepton but the right-handed tau lepton and $\hat{H}_u$ is neutral under U(1)′. However, achieving such an extremely leptophobic Z' boson has a price: the leptonic Yukawa matrix and associated nonholomorphic terms are now allowed to have nonvanishing off-diagonal entries, and thus the Z' boson necessarily develops flavor-changing couplings to leptons which in turn facilitate the leptonic FCNC decays $\mu \to e\gamma$ or $\tau \to (\mu, e)\gamma$. However the rates of these processes depend on the rotation matrix which diagonalize the effective lepton Yukawa matrix. In the text we will not pursue this option any further except to comment on it occasionally. We will focus on the charge

TABLE IV. An alternative charge assignment leading to an extremely leptophobic Z'.

	1st family	2nd family	3rd family
Q_{Q_i}	1/3	1/3	1/3
$Q_{U_i^c}$	−1/3	−1/3	−1/3
$Q_{D_i^c}$	−1/3	−1/3	−1/3
Q_{L_i}	0	0	0
$Q_{E_i^c}$	0	0	3
	Q_{H_u}	Q_{H_d}	Q_S
	0	−3	3

assignments in Table III in discussing phenomenological implications of the Z' boson.

In assigning numerical values to the rigid and soft parameters of the theory we prefer to work at the weak scale. In fact, the renormalization group flow is not needed at all as one can always generate a given low-energy pattern from grand unified theory (GUT) scale parameters in the absence of constraints like universality of the scalar soft masses. Hence, we first fix the dominant Yukawa elements in the superpotential to

$$h_s = 0.6, \qquad h_t = 1.1 \tag{43}$$

for which their renormalization group equations develop no Landau pole up to $M_{\rm GUT}$. Concerning the soft-breaking sector, we choose gaugino masses and trilinear couplings as in Table V, and scalar soft mass squares as in Table VI.

Notice that the negative m_S^2 triggers the U(1)′ symmetry breaking. It is reasonable to expect that by adjusting other soft SUSY-breaking parameters one can get a positive m_S^2 at the unification scale so that the U(1)′ symmetry is radiatively broken, just like the radiative electroweak symmetry breaking in the MSSM. Investigating this possibility in detail is left for future work. Also notice that in Table V, only the largest two nonholomorphic terms, i.e. C_b and C_μ, are shown. As we already pointed out, due to the fact that $C_f \propto m_f$ for the fermions whose masses are due to the nonholomorphic terms, there is a hierarchy among the nonvanishing nonholomorphic terms, i.e. $m_b{:}m_s{:}m_d \approx C_b{:}C_s{:}C_d$ and $m_\mu{:}m_e \approx C_\mu{:}C_e$. Since $C_\mu \gg C_e$, the left-right mixing in the smuon sector is much large than the selectron sector, which tends to make $\tilde{\mu}_1$ lighter than $\tilde{e}_1$. This may have interesting consequences for collider signatures. For example, the chargino would more likely decay to $\mu\nu_\mu\tilde{N}_1$ than to $e\nu_e\tilde{N}_1$.

For the parameter values tabulated in Tables V and VI, the Higgs, Z, Z' and some of the fermion masses turn out to be as in Table VII for $\tan\beta = 2$. Notice that the Higgs mass agrees with the LEP bounds already at tree level. Moreover, the Z' boson weighs nearly a TeV and its mixing with the Z boson, $\alpha_{ZZ'}$, remains well inside the present experimental bounds. Furthermore, both b quark and muon masses agree with experiments though they originate from nonholomorphic terms rather than their Yukawa interactions with H_d.

TABLE V. The gaugino masses and trilinear couplings at the weak scale (in GeV).

M_1'	M_1	M_2	M_3	A_S	A_t	A_τ	C_b	C_μ
200	800	300	500	850	250	250	2000	1800

TABLE VI. The soft mass-squared parameters (in GeV^2) at the weak scale.

$m_{H_u}^2$	$m_{H_d}^2$	m_S^2	m_{Q,U^c,D^c}^2	m_{L,E^c}^2
$-(175)^2$	$(823)^2$	$-(565)^2$	$(1000)^2$	$(1400)^2$

TABLE VII. Some particle masses (in GeV) at the weak scale and Z-Z' mixing angle for $\tan\beta = 2$.

m_Z	$m_{Z'}$	m_t	m_b	m_μ	$\alpha_{ZZ'}$	$m_h^{\rm tree}$
91.2	800	175	2.9	0.101	-2.76×10^{-3}	114.7

TABLE VIII. The masses of light neutralinos and sfermions (in GeV).

$m_{\tilde{\chi}_1^0}$	$m_{\tilde{\chi}_2^0}$	$m_{\tilde{\chi}_3^0}$	$m_{\tilde{t}_1}$	$m_{\tilde{t}_2}$	$m_{\tilde{b}_1}$	$m_{\tilde{b}_2}$	$m_{\tilde{\mu}_1}$	$m_{\tilde{\mu}_2}$
281	577	588	999	1051	783	1177	1318	1725

Finally, for future use we also estimate the masses of the three light neutralinos together with those of the stops, sbottoms and smuons. The contributions from the D terms associated with the U(1)′ are taken into account in our calculation. The masses are shown in Table VIII. It is clear that the lightest supersymmetric partner (LSP) weighs 281 GeV and the light sbottom is the lightest sfermion in the spectrum. Below the scale of U(1)′ breakdown the model at hand resembles the MSSM in that there is an effective μ parameter induced: $\mu^{\rm eff} = h_s\langle S\rangle =$ 577 GeV which lies right at the weak scale.

The numerical predictions above show that the U(1)′ model under consideration does not have any obvious contradiction with the existing phenomenological bounds. As part of the ''new physics search'' program in laboratory and astrophysical environments, establishing or excluding the class of models we are developing will require analysis of various observables ranging from Higgs boson signatures to dark matter in the universe. In the following we will briefly discuss these observables, referring to the numerical predictions above where needed.

A. The Higgs sector

In the course of electroweak breaking Z and Z' bosons acquire their masses by eating, respectively, $\mathrm{Im}[-\sin\beta H_u^0 + \cos\beta H_d^0]$ and $\mathrm{Im}[\cos\alpha\cos\beta H_u^0 + \cos\alpha\sin\beta H_d^0 - \sin\alpha S]$ where $\cot\alpha = (v/\sqrt{2})\sin\beta\cos\beta/\langle S\rangle$ with $v^2/2 = \langle H_u^0\rangle^2 + \langle H_d^0\rangle^2$. The remaining neutral degrees of freedom $\mathcal{B} = \{\mathrm{Re}[H_u^0] - \langle H_u^0\rangle, \mathrm{Re}[H_d^0] - \langle H_d^0\rangle, \mathrm{Re}[S] - \langle S\rangle, \mathrm{Im}[\sin\alpha\cos\beta H_u^0 + \sin\alpha\sin\beta H_d^0 + \cos\alpha S]\}$ span the space of massive scalars. The physical Higgs bosons are given by $H_i = \mathcal{R}_{ij}\mathcal{B}_j$ where the mixing matrix $\mathcal{R}$ necessarily satisfies $\mathcal{R}\mathcal{R}^T = 1$, and it has already been computed up to one-loop order in [5–7,22]. In the CP-conserving limit the theory contains three CP-even, one CP-odd, and a charged Higgs boson. The CP-odd scalar is typically heavy as its mass squared goes like $A_S\langle S\rangle$. It differs from the MSSM spectrum by one

extra CP-even scalar. At tree level, the lightest Higgs mass is bounded as

$$m_{H_1}^2 \leq M_Z^2 \cos^2 2\beta + \frac{1}{2} h_s^2 v^2 \sin^2 2\beta + g_1'^2 (Q_{H_d} \cos^2\beta + Q_{H_u} \sin^2\beta)^2 v^2 \quad (44)$$

where the first term on the right-hand side is the MSSM bound where the lightest Higgs is lighter than the Z boson at tree level. The second term is an F-term contribution that also exists in the NMSSM [23]. The last term, the U(1)$'$ D-term contribution, enhances the upper bound in proportion to $g_1'^2$. Hence, rather generically the U(1)$'$ models have a sufficiently large m_{H_1} to higher values making it likely that the lightest Higgs lies beyond the LEP II kinematic reach.

Interestingly, the model favors specific values for $\tan\beta$ when $m_{Z'}$ approaches m_Z. Indeed, for such a light Z' boson the $Z - Z'$ mixing is suppressed by the mixing mass-squared term $M_{Z-Z'}^2 = (1/2) g_1' \sqrt{g_2^2 + g_Y^2}\, v^2 (Q_{H_d} \cos^2\beta - Q_{H_u} \sin^2\beta)$. This then requires $\tan\beta \sim \sqrt{Q_{H_d}/Q_{H_u}}$, so $\tan\beta$ is completely determined by the charge assignment. On the other hand, when Z' is sufficiently heavy, this constraint on $\tan\beta$ is absent.

For the U(1)$'$ model example we analyze, the charge assignments in Table III ensure that the U(1)$'$ D-term contribution to the upper bound of the Higgs mass receives equal contributions from H_u and H_d. Moreover, for $\tan\beta \simeq \sqrt{2}$, $Z - Z'$ mixing would have been absent irrespective of the scale of the Z' mass. Notably, if one switches to charge assignments in Table IV then the U(1)$'$ D-term contribution to (44) gets significantly reduced at large values of $\tan\beta$.

B. The status of the fine-tuning problem

One crucial message conveyed by the relative heaviness of the lightest Higgs boson in U(1)$'$ models is that there is no need for large radiative corrections in order to agree with the LEP II lower bound. Indeed, when one-loop radiative corrections are included the Higgs mass obeys the upper bound

$$\overline{m}_{H_1}^2 \leq m_{H_1}^2 + \frac{3 m_t^4}{2\pi^2 v^2} \log\frac{m_{\tilde{t}}^2}{m_t^2} \quad (45)$$

where $m_{H_1}^2$ is the right-hand side of Eq. (44). The one-loop piece is an approximate result (note it does not depend on h_s) that holds when (i) the loop contributions are renormalized at $Q \sim m_{\tilde{t}}$, (ii) all terms involving the gauge couplings are neglected, and (iii) stop left-right mixing is much smaller than the diagonal terms such that the two physical stops are nearly degenerate with mass $m_{\tilde{t}}$ (see [6,22] for exact results). The radiatively corrected upper bound (45) can be used to place a lower bound on the stop mass

$$m_{\tilde{t}} \geq m_t e^{(\overline{m}_{H_1}^2 - m_{H_1}^2)(\pi^2 v^2/3 m_t^4)} \quad (46)$$

where $v \approx 246$ GeV is the electroweak breaking scale. Consequently, when $\overline{m}_{H_1} = 114$ GeV the SUSY-breaking scale has the lower bounds $m_{\tilde{t}}^2 \gtrsim 3 M_Z^2$ and $m_{\tilde{t}}^2 \gtrsim 4 M_Z^2$ for parameter values in Tables III and IV, respectively. A comparison of these results with the MSSM expectation, $m_{\tilde{t}}^2 \gtrsim 50 M_Z^2$ [24], demonstrates that in U(1)$'$ models the SUSY-breaking scale is well close to the top mass. This result, which demonstrates the absence of the little hierarchy problem in this class of models, stems from the fact that the tree-level upper bound (44) is already large enough to drag the Higgs mass near the LEP lower bound.

The results above, however, should be taken with care. The main reason is that the Z' boson should be heavy enough to satisfy the bounds from precision data. In particular, the Z-Z' mixing angle should be a few $\times 10^{-3}$, as mentioned and computed before. This may occur because of a somewhat heavy Z', or because there exists a selection rule that enforces approximately the interesting relation $Q_u v_u^2 - Q_d v_d^2 \simeq 0$ (as in the ''large trilinear vacuum'' of [5,25]).

We also want to mention that the recent analysis of the NMSSM [26] finds that fine-tuning [27] can be significantly reduced especially in parameter regions with a light pseudoscalar boson. The reason is that the invisible decay rate of the Higgs boson gets enhanced (and thus it escapes detection at LEP) via its decays into pairs of pseudoscalars.

C. The Z' couplings

The Z' boson mixes with $Z_\mu = \cos\theta_W W_\mu^3 - \sin\theta_W B_\mu$ after the electroweak breaking since Higgs fields are charged under both U(1)$_Y$ and U(1)$'$. On top of this B_μ and Z' can exhibit kinetic mixing [28]. In the presence of these mixings the mass-eigenstate gauge bosons assume varying electroweak and U(1)$'$ components and these reflect themselves in their interactions with matter species. For instance, the neutral vector boson observed in LEP experiments corresponds to

$$Z_\mu^{(1)} = \cos\alpha_{ZZ'} Z_\mu + \sin\alpha_{ZZ'} Z'_\mu \quad (47)$$

in the absence of kinetic mixing. The couplings of $Z_\mu^{(1)}$ to fermions deviate from their MSSM configuration in proportion to $\alpha_{ZZ'}$ and as a function of M_{Z_1}/M_{Z_2}. All such U(1)$'$ impurities can be conveniently represented by S, T and U parameters in a way useful for Z' searches in electroweak precision data [29].

In the following we will discuss the couplings of the Z' boson rather than those of $Z_\mu^{(1)}$ or the heavy one $Z_\mu^{(2)}$ as this is the crucial part of the information needed for U(1)$'$ phenomenology. Depending on the mixing scheme, kinetic or otherwise, one can always go to the physical basis for gauge bosons by appropriate rotations. The U(1)$'$ charges of the chiral fields shown in Table I are sufficient for specifying their interactions with the Z' boson. The physical bases for fermions are achieved by diagonalizing their

Yukawa matrices via the unitary transformations $h_d^{\rm diag} = V_R^d h_d V_L^{d\dagger}$, $h_u^{\rm diag} = V_R^u h_u V_L^{u\dagger}$ and $h_e^{\rm diag} = V_R^e h_e V_L^{e\dagger}$. Then the physical fermions couple to Z' as $g_1' J_\mu Z'^\mu$ + H.c. where

$$
\begin{aligned}
J_\mu = &\left[\overline{d}_L \gamma_\mu V_L^d \begin{pmatrix} Q_{Q_1} & 0 & 0 \\ 0 & Q_{Q_2} & 0 \\ 0 & 0 & Q_{Q_3} \end{pmatrix} V_L^{d\dagger} d_L - \overline{d}_R \gamma_\mu V_R^d \begin{pmatrix} Q_{D_1^c} & 0 & 0 \\ 0 & Q_{D_2^c} & 0 \\ 0 & 0 & Q_{D_3^c} \end{pmatrix} V_R^{d\dagger} d_R \right] \\
&+ \left[\overline{u}_L \gamma_\mu V_L^u \begin{pmatrix} Q_{Q_1} & 0 & 0 \\ 0 & Q_{Q_2} & 0 \\ 0 & 0 & Q_{Q_3} \end{pmatrix} V_L^{u\dagger} u_L - \overline{u}_R \gamma_\mu V_R^u \begin{pmatrix} Q_{U_1^c} & 0 & 0 \\ 0 & Q_{U_2^c} & 0 \\ 0 & 0 & Q_{U_3^c} \end{pmatrix} V_R^{u\dagger} u_R \right] \\
&+ \left[\overline{e}_L \gamma_\mu V_L^e \begin{pmatrix} Q_{L_1} & 0 & 0 \\ 0 & Q_{L_2} & 0 \\ 0 & 0 & Q_{L_3} \end{pmatrix} V_L^{e\dagger} e_L - \overline{e}_R \gamma_\mu V_R^e \begin{pmatrix} Q_{E_1^c} & 0 & 0 \\ 0 & Q_{E_2^c} & 0 \\ 0 & 0 & Q_{E_3^c} \end{pmatrix} V_R^{e\dagger} e_R \right]
\end{aligned} \tag{48}
$$

so that generically the Z' boson develops flavor-changing couplings if there are intergenerational mixings in the Yukawa matrices and/or if the U(1)′ charges are family dependent. A short glance at the effective Yukawa interactions in (40) reveals that the charged leptons are already in their physical bases whereas the quarks exhibit nontrivial mixing diagonalizations of which induce flavor violation in charged-current vertices via $V_{\rm CKM} = V_L^u V_L^{d\dagger}$. However, there are no flavor-changing Z' couplings to quarks at all. The reason is that U(1)′ charges of quarks are all family universal according to the anomaly-free solutions in (38). In conclusion, the Z' boson couples to fermions rather generically via

$$
J_\mu = \frac{1}{2}\sum_i \overline{\psi}_i \gamma_\mu [(Q^i_{\rm left} - Q^i_{\rm right}) - (Q^i_{\rm left} + Q^i_{\rm right})\gamma_5]\psi_i \tag{49}
$$

with no potential for tree-level flavor violation.

It is useful to discuss (49) in light of the charge assignments in Table III. First of all, one automatically concludes that J_μ is a $V + A$ current for quarks, that is, each quark couples to Z'_μ via $\overline{q}_R \gamma^\mu q_R$ current only. In particular, there is no involvement of the left-handed quark fields. On the other hand, leptons possess varying vector and axial couplings due to their family-nonuniversal U(1)′ charges. In fact, Z'_μ couples to the leptonic currents $(1/2)\overline{e}\gamma_\mu(1 + \gamma_5)e$, $(1/2)\overline{\mu}\gamma_\mu(5 + \gamma_5)\mu$ and $-(1/2)\overline{\tau}\gamma_\mu\gamma_5\tau$. Therefore, the electronic current is purely right handed as for quarks, the muonic current possesses a sizeable vector part, and the tauonic current is purely an axial-vector type. Moreover, the Z' boson does not couple to electron neutrinos at all, and its coupling to the muon neutrino current is twice larger than that to the tau neutrino current. These chirality and flavor sensitivities of the U(1)′ currents can have important implications for Z' searches at colliders. If one switches to charge assignments in Table IV the hadronic currents maintain their structure except for a resizing by 1/3, and the only surviving leptonic current turns out to be that of the right-handed tau lepton. Consequently, this particular charge assignment gives rise to an almost completely leptophobic Z'.

The kinetic terms of the Higgs fields completely determine the couplings of Z' to Higgs bosons. In close similarity to Z boson couplings one can have vertices involving two Z' and two Higgs bosons, or two Z' with a single CP-even Higgs boson, or a single Z' accompanied by one CP-even and one CP-odd Higgs boson. A single Z'_μ, for instance, couples to H_i and H_j via $(p_{H_i} - p_{H_j})^\mu$ times

$$
\begin{aligned}
&2g_1'(\mathcal{R})_{i4}[Q_{H_u}\cos\beta\sin\alpha(\mathcal{R})_{j1} + Q_{H_d}\cos\beta\sin\alpha(\mathcal{R})_{j2} \\
&\qquad + Q_S\cos\alpha(\mathcal{R})_{j3}]
\end{aligned} \tag{50}
$$

which vanishes unless H_i and H_j possess opposite CP compositions. Unlike this, however, the coupling of H_i to $Z'_\mu Z'^\mu$ involves only its CP-even component:

$$
2g_1'^2[Q^2_{H_u}\langle H_u^0\rangle(\mathcal{R})_{i1} + Q^2_{H_d}\langle H_d^0\rangle(\mathcal{R})_{i2} + Q_S^2\langle S\rangle(\mathcal{R})_{i3}]. \tag{51}
$$

Finally, H_i and H_j couple to $Z'_\mu Z'^\mu$ via

$$
\begin{aligned}
&g_1'^2[Q^2_{H_u}\{(\mathcal{R})_{i1}(\mathcal{R})_{j1} + \cos^2\beta\sin^2\alpha(\mathcal{R})_{i4}(\mathcal{R})_{j4}\} \\
&\quad + Q^2_{H_d}\{(\mathcal{R})_{i2}(\mathcal{R})_{j2} + \sin^2\beta\sin^2\alpha(\mathcal{R})_{i4}(\mathcal{R})_{j4}\} \\
&\quad + Q_S^2\{(\mathcal{R})_{i3}(\mathcal{R})_{j3} + \cos^2\alpha(\mathcal{R})_{i4}(\mathcal{R})_{j4}\}].
\end{aligned} \tag{52}
$$

The couplings of the Higgs bosons to distinct vector bosons, i.e. to $Z_\mu Z'_\nu$, are obtained by picking up both U(1)$_Y$ and U(1)′ contributions to Higgs kinetic terms. Clearly, once Z and Z' are rotated to their physical bases both $Z_\mu Z_\nu$ and $Z'_\mu Z'_\nu$ type structures will induce Higgs couplings to dissimilar vector bosons via operators of the form $Z_\mu^{(1)} Z_\nu^{(2)}$.

The expressions for couplings presented above are general enough to cover supersymmetric CP violation effects. In the CP-conserving theory, as was assumed in constructing the soft-breaking sector in Sec. II, the Higgs bosons possess definite CP quantum numbers, in particular, $\mathcal{R}_{4i} = 0$ for all $i \neq 4$ [6].

D. Z' searches at hadron colliders

From a phenomenological point of view, the U(1)$'$ model under concern differs from the MSSM by having one extra CP-even Higgs boson, one extra neutral gauge boson, and two extra neutral fermions. The ultimate confirmation of the model thus requires a complete construction of all these states in laboratory or astrophysical/cosmological environments. Here in this subsection we will provide a rather brief description of Z' signatures in accelerator experiments (see [30] for a review), in particular, in hadron colliders e.g. the Tevatron and upcoming CERN LHC. Needless to say, Z' signals at linear colliders are much cleaner than at hadron machines but presently the International Linear Collider is only being planned (presumably as a post-LHC precision measurement environment).

The LHC (Tevatron) is expected to probe Z' bosons as heavy as 4 TeV (0.8 TeV) depending on the model parameters, on the luminosity reach of the collider, and on the size of uncertainties coming from detector acceptances and systematic errors [30,31]. Z' production proceeds via various channels. It can be produced directly via quark-antiquark fusion giving rise to $pp/\overline{p}p \to Z'X$ or indirectly via Higgs or Z boson decays such as $H_1 \to Z'Z'$, $H_1 \to Z'H_4$ and $Z \to Z'H_1$. Each of these and similar contributions to Z' production can be analyzed by using the expressions for the couplings given in (49)–(52) in Sec. IV B above. Among all these production channels the dominant one is the quark-antiquark annihilation (at next-to-leading order in QCD gluon-quark scattering into Z' is also important), and it facilitates direct pp or $\overline{p}p$ fusion into Z'. The produced Z' boson will subsequently decay into leptons or jets. The latter are seldom useful for a Z' search due to large QCD background. The leptonic signals, however, are particularly promising due to their good momentum resolution and one's ability to suppress the MSSM background at high dilepton invariant masses [31]. When the subprocess center of mass energy $\simeq M_{Z'}$ the Z' propagator resonates to give

$$\sigma(pp \to Z'_{\searrow \ell^+\ell^-} X) = \sigma(pp \to Z'X)\mathrm{BR}(Z' \to \ell^+\ell^-) \tag{53}$$

with a similar expression for $p\overline{p}$ collisions. Here the Z' production rate is given by

$$\sigma(pp \to Z'X) = \sum_q \frac{4\pi^2}{3sM_{Z'}}\Gamma(Z' \to q\overline{q}) \times \int_{M_{Z'}^2/s}^1 \frac{dx}{x}\left[f_{\overline{q}}^p(x, M_{Z'}) f_q^p\left(\frac{M_{Z'}^2}{xs}, M_{Z'}\right) + f_q^p(x, M_{Z'}) f_{\overline{q}}^p\left(\frac{M_{Z'}^2}{xs}, M_{Z'}\right)\right] \tag{54}$$

where $f_x^y(a, b)$ stands for the probability of finding parton x in hadron y with a momentum fraction a at the relevant energy scale b of the scattering process. The partial fermionic width of the Z'

$$\Gamma(Z' \to \psi_i\overline{\psi}_i) = \frac{2N_c}{3}\alpha_1' M_{Z'}(Q_{\mathrm{left}}^{i2} + Q_{\mathrm{right}}^{i2}), \tag{55}$$

as follows from (49), collects all model parameters pertaining to the massless fermion sector. Presently, the CDF and D0 experiments continue to explore Z' signatures by projecting the measurement of (53) into possible values of $\alpha_1' Q_{\mathrm{left}}^{i2}\mathrm{BR}(Z' \to \ell^+\ell^-)$ in the plane of up and down quark couplings [32].

For the minimal U(1)$'$ model under consideration, the following properties could be important for collider searches for the Z' boson:

(i) At e^+e^- (or future $\mu^+\mu^-$) colliders running above the Z pole the Z' effects can be parametrized in terms of semielectronic four-fermion operators. The scales of such operators are $\mathcal{O}$ (10 TeV) at LEP II. The combined results of all four LEP collaborations [33] show that when Z' couples to electrons of one chirality only (either to left or right, not both) then bounds on $M_{Z'}$ are rather weak. This is indeed the case in our minimal U(1)$'$ model in which Z' couples to the right-handed electron current only. Consequently, it suffices to have $M_{Z'} \gtrsim 0.7$ TeV for LEP II bounds to be respected. Clearly, if one switches to the charge assignments in Table IV, there is no LEP (or future muon collider) bound to speak of (except for the precision measurements at the Z or Z' poles).

(ii) In the framework of the U(1)$'$ models under consideration, at hadron colliders the Z' boson is produced by the fusion of right-handed quarks. The decays of the produced Z' into leptons offer a rather clean signal for experimental purposes [30,31]. As suggested by (55) the larger the sum $Q_{\mathrm{left}}^{i2} + Q_{\mathrm{right}}^{i2}$ the larger the number of dilepton events. Therefore, the number of $\mu^+\mu^-$ events must be 13 times larger than e^+e^- events and 26 times larger than $\tau^+\tau^-$ events. This rather strong preference for muon production gives a clear signature of the model under concern. Of course, if one switches to U(1)$'$ charges in Table IV then Z' effects show up only in the $\tau^+\tau^-$ production.

At hadron colliders, one of the most important observables is the forward-backward asymmetry [30,31]. It is a measure of the angular distribution of the signal, and is proportional to the vector and axial couplings of both the initial and final state fermions in the process. For the U(1)$'$ charges in Table III it vanishes for $\tau^+\tau^-$ production, and is 5 times larger for $\mu^+\mu^-$ production than for the e^+e^- signal. For the alternative charge assignments in Table IV there is no asymmetry at all; the signal is distributed equally in forward and backward hemispheres.

In experiments with polarized proton beams one can define spin-dependent asymmetries which probe chiral

couplings of the initial and final state fermions separately [34]. The left-right asymmetry, defined with respect to the parent proton helicity, is proportional to the multiplication of the vector and axial couplings of the quarks, and it is universal for all quarks in either of the charge assignments Tables III and IV. On the other hand, forward-backward asymmetry for polarized protons measures the chiral couplings of the leptons in isolation in a way similar to the forward-backward asymmetry of unpolarized beams.

In this subsection we have discussed very briefly the prospects for Z' searches at colliders within our minimal U(1)′ extension of the MSSM. Clearly, for a complete determination of the Z' signatures it is necessary to perform a detailed study of all relevant processes. Notice that the particular model we showed at the beginning of this section has a Z' at 800 GeV. One can certainly lower the Z' mass to increase the chance of detectability at the Tevatron. Smaller Z' mass will typically increase the Z-Z' mixing angle. But as shown in Sec. IVA, special values of $\tan\beta$ can be chosen to reduce the mixing. We have found that it is possible to make Z' as light as around 500 GeV and the mixing angle close to the border line of the experimental bound.

E. The neutrino masses

By construction, the model analyzed in this work does not contain any fields necessary for inducing the neutrino masses and mixings. These can be generated via various mechanisms [16,35–37]. For a consistent analysis of the neutrino sector one has to import appropriate fields into the spectrum and analyze their consequences, especially for anomaly cancellation. Here, we simply take the seesaw contribution

$$\Delta W_\nu = Y_\nu^{ij} \frac{L^i H_u L^j H_u}{M} \tag{56}$$

to the superpotential as a basis for our brief discussion. Here Y_ν^{ij} are some $\mathcal{O}(1)$ couplings, and M represents the Majorana mass scale. In models with additional U(1)′ symmetry, some of the entries of Y_{ij} could be forbidden by the U(1)′ symmetry. Indeed, a short glance at the charge assignments in Table III reveals that Y_ν^{ij} should take the following form:

$$Y_\nu^{ij} = \begin{pmatrix} 0 & a & 0 \\ a & 0 & 0 \\ 0 & 0 & b \end{pmatrix} \tag{57}$$

where a and b are some coefficients. Clearly, this texture does not account for the observed oscillation data, and one has to invent some other way of inducing a viable Y_ν.

On the other hand, for the U(1)′ charge assignments in Table IV the seesaw mechanism alone suffices to induce all neutrino masses and mixings in full generality (at the expense of opening up the lepton flavor violation effects). Analyzing these patterns and constraints is left for further work.

F. The Muon $g-2$

We have already provided general expressions for $g_\mu - 2$ in Sec. II. Thanks to the nonholomorphic operator $C_E^{22} H_u^* \tilde{L}^2 \tilde{E}_R^{c2}$ one can induce both muon mass and $g_\mu - 2$ via a one-loop neutralino-smuon diagram. On the other hand, there is no similar chirality-flip operator on the $\tilde{\nu}_\mu$ line so that the chargino contribution is a two-loop effect and is thus negligible. Inducing the muon mass without violating $g_\mu - 2$ bounds is an important constraint [12], and for the parameter values listed before we find

$$a_\mu^{\mathrm{SUSY}} = 22 \times 10^{-10} \tag{58}$$

by using (9). This result is well inside the allowed room for a new physics contribution to the muon anomalous magnetic moment [38].

G. The cold dark matter

The mapping of the cosmic microwave background anisotropy provides precise information about the densities of matter and dark energy in the universe. It is now known with good precision that the matter distribution is dominated by a nonbaryonic nonrelativistic component whose candidate particle should be massive, stable, neutral and weakly interacting. Supersymmetric models with conserved R parity provide a natural candidate for cold dark matter (CDM) in the lightest superpartner i.e. the lightest neutralino χ_1^0. For the parameter values listed in Table V the LSP turns out to be W-ino dominated

$$\tilde{\chi}_1^0 = -0.015\tilde{Z}' - 0.019\tilde{B} + 0.967\tilde{W} - 0.197\tilde{H}_d + 0.158\tilde{H}_d - 0.004\tilde{S} \tag{59}$$

with a rather small singlino component. For W-ino LSPs, coannihilation during the freeze-out is highly efficient. In fact, the neutralino relic density turns out to be $\Omega_\chi h^2 \simeq 0.5 \times 10^{-2}$ which is smaller than the observed CDM density by an order of magnitude. Hence, as pointed out before [39], the W-ino LSP is far from being a viable CDM candidate. However, nonthermal production can provide the actual relic density, e.g., for the W-ino LSP, and decays of the moduli fields into gauginos can help in enhancing Ω_χ for saturating the correct value of Ω_{CDM} [40]. Clearly, if the LSP is dominated by other components i.e. singlino, b-ino or Z-ino then one can saturate the observed value of Ω_{CDM} since their annihilation rates are relatively smaller than those of the W-inos [41].

V. CONCLUSION

We have discussed ways of constructing an anomaly-free U(1)′ model (as needed for solving the μ problem and moderating the fine-tuning problem) with minimal matter

content in order to maintain the unification of gauge couplings. We have found and illustrated with some numerical examples that it is possible to achieve the cancellation of anomalies with no exotic matter by invoking (i) family-nonuniversal $U(1)'$ charge assignments and (ii) nonholomorphic soft-breaking operators.

The model discussed in this work is an anomaly-free version of the generic $U(1)'$ model analyzed in [5]. Indeed, the two models have identical matter spectrum. However, achieving anomaly freedom without exotic states requires the introduction of family-dependent $U(1)'$ charge assignments plus nonholomorphic soft-breaking terms. Of course, $U(1)'$ models that follow from E_6 breaking are anomaly free thanks to the exotic states present in the light spectrum [42]. In this sense, the model discussed here constitutes an anomaly-free minimal $U(1)'$ model.

From the experimental point of view, distinguishing the minimal $U(1)'$ model here from other $U(1)'$ models or from the MSSM requires measurement of a number of observables. In general, establishing the existence of a $U(1)'$-extended MSSM structure necessitates experimental evidence for the Z' boson, extra Higgs bosons and extra neutralino states. On the other hand, one might interpret certain phenomenological results as being evidences for an extended gauge sector. For instance, the electric dipole moments constraints generically require the phase of the μ parameter (in the MSSM) to be rather small, and this result can be naturally tied to the radiative nature of the μ parameter in $U(1)'$ models [6].

Distinguishing the minimal $U(1)'$ model here from other $U(1)'$ models in the literature requires certain signatures which could come from nonholomorphicity and a family-dependent nature of the Z' couplings. Concerning the latter, one recalls from Sec. IV D that Z' decays into a specific difermion state, e.g. $\mu^+\mu^-$, can be significantly enhanced compared to others due to the family dependence of the $U(1)'$ couplings displayed in Table III. In fact, the quarks which participate in production and hadronic decays of the Z' boson are right handed more often than is typical. These are signals that cannot be found in other $U(1)'$ models. The family nonuniversality implies several collider events that enable one to distinguish the minimal $U(1)'$ here from other models.

Being another important effect of family nonuniversality, one notes from Table I that Z' does not couple to left-handed squarks and left-handed selectrons, at all. In fact, its strongest coupling is to smouns, in particular, to the right-handed smuon. The dominance of the right-handed currents (except the stau states) is interesting since right-handed sfermions (of the first two generations, especially) decay preferably into b-ino and right-handed fermions. In particular, multilepton plus jet plus missing energy signals coming from left-handed squarks are now reduced. Besides these, dominance of the muon signal compared to others is a signal of the violation of lepton universality, and the Z' boson of Table III could be a viable source of this.

The nonholomorphicity of the soft-breaking terms affects certain observables in a distinct way. For instance, due to their radiative origin the Higgs-fermion couplings depend on the momentum transfer in a given scattering process, and thus, nonholomorphic structures may be tested by measuring various Higgs branching fractions into fermions [12]. Furthermore, the electric dipole moments (though not analyzed here) are naturally suppressed since dipole moments are aligned towards the fermion masses [12]. Finally, the heavier the fermion the larger the nonholomorphic trilinear, and hence, the sfermion left-right mixings are enhanced for relatively heavy fermions whose masses are due to the nonholomorphic terms.

In conclusion, we have analyzed the conditions for and phenomenological consequences of canceling the anomalies in $U(1)'$ models with minimal matter content. We have briefly discussed a number of observables ranging from fermion masses to dark matter in the universe. The model explored here is minimal in that it is a direct $U(1)'$ gauging of the MSSM plus a gauge singlet, and it needs to be extended to include right-handed neutrinos to induce neutrino masses and mixings. Moreover, the numerical examples provided here can be extended to a sufficiently dense sampling of the parameter space for determining the laboratory and astrophysical implications of the model.

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