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Effects of final-state interactions on mixing-induced *CP* violation in penguin-dominated *B* decays

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Motivated by the recent indications of the possibility of sizable deviations of the mixing-induced *CP* violation parameter, S_f, in the penguin-dominated $b \to sq\bar{q}$ transition decays such as $B^0 \to (\phi, \omega, \rho^0, \eta', \eta, \pi^0, f_0)K_S$ from $\sin 2\beta$ determined from $B \to J/\psi K_S$, we study final-state rescattering effects on their decay rates and *CP* violation. Recent observations of large direct *CP* asymmetry in modes such as $B^0 \to K^+\pi^-$, $\rho^-\pi^+$ suggest that final-state phases in two-body *B* decays may not be small. It is therefore important to examine these long-distance effects on penguin-dominated decays. Such long-distance effects on S_f are found to be generally small [i.e. $\mathcal{O}(1-2\%)$] or negligible except for the ωK_S and $\rho^0 K_S$ modes where S_f is lowered by around 15% for the former and increased by the same percentage for the latter. However, final-state rescattering can enhance the ωK_S, ϕK_S, $\eta' K_S$, $\rho^0 K_S$, and $\pi^0 K_S$ rates significantly and flip the signs of direct *CP* asymmetries of the last two modes. Direct *CP* asymmetries in ωK_S and $\rho^0 K_S$ channels are predicted to be $\mathcal{A}_{\omega K_S} \approx -0.13$ and $\mathcal{A}_{\rho^0 K_S} \approx 0.47$, respectively. However, direct *CP* asymmetry in all the other $b \to s$ penguin-dominated modes that we study is found to be rather small ($\lesssim$ a few percent), rendering these modes a viable place to search for the *CP*-odd phases beyond the standard model. Since ΔS_f ($\equiv -\eta_f S_f - S_{J/\psi K_S}$, with η_f being the *CP* eigenvalue of the final state f) and $\mathcal{A}_f$ are closely related, the theoretical uncertainties in the mixing-induced parameter S_f and the direct *CP* asymmetry parameter $\mathcal{A}_f$ are also coupled. Based on this work, it seems difficult to accommodate $|\Delta S_f| > 0.10$ within the standard model for $B^0 \to (\phi, \omega, \rho^0, \eta', \eta, \pi^0)K_S$; in particular, $\eta' K_S$ is especially clean in our picture. For $f_0 K_S$, at present we cannot make reliable estimates. The sign of the central value of ΔS_f for all the modes we study is positive but quite small, compared to the theoretical uncertainties, so that at present conclusive statements on the sign are difficult to make.

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I. INTRODUCTION AND MOTIVATION

Possible new physics beyond the standard model (SM) is being intensively searched via the measurements of time-dependent *CP* asymmetries in neutral *B* meson decays into final *CP* eigenstates defined by

$$\frac{\Gamma(\overline{B}(t) \to f) - \Gamma(B(t) \to f)}{\Gamma(\overline{B}(t) \to f) + \Gamma(B(t) \to f)} = S_f \sin(\Delta m t) + \mathcal{A}_f \cos(\Delta m t), \quad (1.1)$$

where Δm is the mass difference of the two neutral *B* eigenstates, S_f monitors mixing-induced *CP* asymmetry, and $\mathcal{A}_f$ measures direct *CP* violation (in terms of the *BABAR* notation, $C_f = -\mathcal{A}_f$). The *CP*-violating parameters $\mathcal{A}_f$ and S_f can be expressed as

$$\mathcal{A}_f = -\frac{1-|\lambda_f|^2}{1+|\lambda_f|^2}, \qquad S_f = \frac{2\,\mathrm{Im}\lambda_f}{1+|\lambda_f|^2}, \quad (1.2)$$

where

$$\lambda_f = \frac{q_B}{p_B}\frac{A(\overline{B}^0 \to f)}{A(B^0 \to f)}. \quad (1.3)$$

In the standard model $\lambda_f \approx \eta_f e^{-2i\beta}$ [see Eq. (2.14) below] for $b \to s$ penguin-dominated or pure penguin modes with $\eta_f = 1\,(-1)$ for final *CP*-even (odd) states. Therefore, it is expected in the standard model that $-\eta_f S_f \approx \sin 2\beta$ and $\mathcal{A}_f \approx 0$ with β being one of the angles of the unitarity triangle.

The mixing-induced *CP* violation in *B* decays has already been observed in the golden mode $B^0 \to J/\psi K_S$ for several years. The current average of *BABAR* [1] and Belle [2] measurements is

$$\sin 2\beta \approx S_{J/\psi K_S} = 0.726 \pm 0.037. \quad (1.4)$$

However, the time-dependent *CP* asymmetries in the $b \to sq\bar{q}$-induced two-body decays such as $B^0 \to (\phi, \omega, \pi^0, \eta', f_0)K_S$ are found to show some indications of deviations from the expectation of the SM. The *BABAR* [3] and Belle [4] results and their averages are shown in Table I. In the SM, *CP* asymmetry in all above-mentioned modes should be equal to $S_{J/\psi K}$ with a small deviation *at most* $\mathcal{O}(0.1)$ [5]. As discussed in [5], this may originate from the $\mathcal{O}(\lambda^2)$ truncation and from the subdominant (color-suppressed) tree contribution to these processes. From Table I we see some possibly sizable deviations from the SM, especially in the $\eta' K_S$ mode in which the discrepancy $\Delta S_{\eta' K_S} = -0.30 \pm 0.11$ is a 2.7σ effect where

$$\Delta S_f \equiv -\eta_f S_f - S_{J/\psi K_S}. \quad (1.5)$$

TABLE I. Mixing-induced and direct CP asymmetries $-\eta_f S_f$ (first entry) and $\mathcal{A}_f$ (second entry), respectively, for various penguin-dominated modes with η_f being the CP eigenvalue of the final state. Experimental results are taken from [3,4].

Final state	*BABAR* [3]	Belle [4]	Average
ϕK_S	$0.50 \pm 0.25^{+0.07}_{-0.04}$	$0.08 \pm 0.33 \pm 0.09$	0.35 ± 0.20
	$-0.00 \pm 0.23 \pm 0.05$	$0.08 \pm 0.22 \pm 0.09$	0.04 ± 0.17
ωK_S	$0.50^{+0.34}_{-0.38} \pm 0.02$	$0.76 \pm 0.65^{+0.13}_{-0.16}$	$0.55^{+0.30}_{-0.32}$
	$0.56^{+0.29}_{-0.27} \pm 0.03$	$0.27 \pm 0.48 \pm 0.15$	0.48 ± 0.25
$\eta' K_S$	$0.30 \pm 0.14 \pm 0.02$	$0.65 \pm 0.18 \pm 0.04$	0.43 ± 0.11
	$0.21 \pm 0.10 \pm 0.02$	$-0.19 \pm 0.11 \pm 0.05$	0.04 ± 0.08
$\pi^0 K_S$	$0.35^{+0.30}_{-0.33} \pm 0.04$	$0.32 \pm 0.61 \pm 0.13$	$0.34^{+0.27}_{-0.29}$
	$-0.06 \pm 0.18 \pm 0.03$	$-0.11 \pm 0.20 \pm 0.09$	-0.08 ± 0.14
$f_0 K_S$	$0.95^{+0.23}_{-0.32} \pm 0.10$	$-0.47 \pm 0.41 \pm 0.08$	0.39 ± 0.26
	$0.24 \pm 0.31 \pm 0.15$	$-0.39 \pm 0.27 \pm 0.09$	-0.14 ± 0.22

If this deviation from $S_{J/\psi K}$ is confirmed and established in the future, it may imply some new physics beyond the SM.

In order to detect the signal of new physics unambiguously in the penguin $b \to s$ modes, it is of great importance to examine how much of the deviation of S_f from $S_{J/\psi K}$ is allowed in the SM [5–10]. In all these previous studies and estimates of ΔS_f, effects of final-state interactions (FSI) were not taken into account. In view of the striking observation of large direct CP violation in $B^0 \to K^\pm \pi^\mp$, it is clear that final-state phases in two-body B decays may not be small. It is therefore important to understand their effects on ΔS_f.

The decay amplitude for the pure penguin or penguin-dominated charmless B decay in general has the form

$$M(\overline{B}^0 \to f) = V_{ub}V_{us}^* F^u + V_{cb}V_{cs}^* F^c + V_{tb}V_{ts}^* F^t. \quad (1.6)$$

Unitarity of the Cabibbo-Kobayashi-Maskawa quark-mixing matrix (CKM) elements leads to

$$\begin{aligned} M(\overline{B}^0 \to f) &= V_{ub}V_{us}^* A_f^u + V_{cb}V_{cs}^* A_f^c \\ &\approx A\lambda^4 R_b e^{-i\gamma} A_f^u + A\lambda^2 A_f^c, \end{aligned} \quad (1.7)$$

where $A_f^u = F^u - F^t$, $A_f^c = F^c - F^t$, $R_b \equiv |V_{ud}V_{ub}/(V_{cd}V_{cb})| = \sqrt{\bar\rho^2 + \bar\eta^2}$ with $\bar\rho$, $\bar\eta$, A, λ being the Wolfenstein parameters [11]. The first term is suppressed by a factor of λ^2 relative to the second term. For a pure penguin decay such as $B^0 \to \phi K^0$, it is naively expected that A_f^u is, in general, comparable to A_f^c in magnitude. Therefore, to a good approximation $-\eta_f S_f \approx \sin 2\beta \approx S_{J/\psi K}$. For penguin-dominated modes such as ωK_S, $\rho^0 K_S$, $\pi^0 K_S$, A_f^u also receives tree contributions from the $b \to u\bar u s$ tree operators. Since the Wilson coefficient for the penguin operator is smaller than the one for the tree operator, A_f^u could be significantly larger than A_f^c. As the first term carries a weak phase γ, it is possible that S_f is subject to a significant "tree pollution." To quantify the deviation, it is known that to the first order in $r_f \equiv (\lambda_u A_f^u)/(\lambda_c A_f^c)$ [8,12]

$$\begin{aligned} \Delta S_f &= 2|r_f| \cos 2\beta \sin\gamma \cos\delta_f, \\ \mathcal{A}_f &= 2|r_f| \sin\gamma \sin\delta_f, \end{aligned} \quad (1.8)$$

with $\delta_f = \arg(A_f^u/A_f^c)$. Hence, the magnitude of the CP asymmetry difference ΔS_f and direct CP violation are both governed by the size of A_f^u/A_f^c. However, for the aforementioned penguin-dominated modes, the tree contribution is color suppressed and hence in practice the deviation of S_f is expected to be small [5]. (However, we shall see below that a sizable ΔS_f can occur in ωK_S and $\rho^0 K_S$ modes. For a review of model calculations of ΔS_f, see [13].)

Since the penguin loop contributions are sensitive to high virtuality, new physics beyond the SM may contribute to S_f through the heavy particles in the loops (for a review of the new physics sources contributing to S_f, see [14]). Another possibility is that final-state interactions are the possible tree pollution sources to S_f. Both A_f^u and A_f^c will receive long-distance (LD) tree and penguin contributions from rescattering of some intermediate states. In particular, there may be some dynamical enhancement of light u-quark loop. If tree contributions to A_f^u are sizable, then final-state rescattering will have the potential of pushing S_f away from the naive expectation. Take the penguin-dominated decay $\overline{B}^0 \to \omega \overline{K}^0$ as an illustration. It can proceed through the weak decay $\overline{B}^0 \to K^{*-}\pi^+$ followed by the rescattering $K^{*-}\pi^+ \to \omega\overline{K}^0$. The tree contribution to $\overline{B}^0 \to K^{*-}\pi^+$, which is color allowed, turns out to be comparable to the penguin one because of the absence of the chiral enhancement characterized by the a_6 penguin term. Consequently, even within the framework of the SM, final-state rescattering may provide a mechanism of tree pollution to S_f. By the same token, we note that although $\overline{B}^0 \to \phi\overline{K}^0$ is a pure penguin process at short distances, it does receive tree contributions via long-distance rescattering.

In this work, we shall study the effects of final-state interactions on the time-dependent CP asymmetries S_f and $\mathcal{A}_f$. In [15] we have studied the final-state rescattering effects on the hadronic B decays and examined their impact on direct CP violation. The direct CP-violating partial rate asymmetries in charmless B decays to $\pi\pi/\pi K$ and $\rho\pi$ are significantly affected by final-state rescattering and their signs are generally different from that predicted by the short-distance (SD) approach such as QCD factorization (QCDF) [16–18]. Evidence of direct CP violation in the decay $\overline{B}^0 \to K^-\pi^+$ is now established, while the combined *BABAR* and Belle measurements of $\overline{B}^0 \to \rho^\pm\pi^\mp$ imply a 3.6σ direct CP asymmetry in the $\rho^+\pi^-$ mode [19]. In fact, direct CP asymmetries in these channels are a lot bigger than expectations (of many people) and may be indicative of appreciable LD rescattering effects, in general, in B decays. Our predictions for CP violation agree with experiment in both magnitude and sign, whereas the QCD factorization predictions (especially for $\rho^+\pi^-$) [18] seem to have some difficulties with the data.

Besides some significant final-state rescattering effects on direct CP violation, another motivation for including FSIs is that there are consistently 2 to 3 σ deviations between the central values of the QCDF predictions for penguin-dominated modes such as $B \to K^*\pi, K\phi, K^*\phi, K\eta'$ and the experimental data [18]. This discrepancy between theory and experiment for branching ratios may indicate the importance of subleading power corrections such as FSI effects and/or the annihilation topology.

Since direct CP violation in charmless B decays can be significantly affected by final-state rescattering, it is clearly important to try to take into account the FSI effect on the mixing-induced and direct CP asymmetries S_f and $\mathcal{A}_f$ of these penguin-dominated modes. The layout of the present paper is as follows. In Sec. II we discuss the short-distance contributions to the $b \to sq\bar{q}$ transition decays $B^0 \to (\phi, \omega, \rho^0, \pi^0, \eta', \eta, f_0)K_S$ within the framework of QCD factorization. We then proceed to study the final-state rescattering effects on CP asymmetries S_f and $\mathcal{A}_f$ in Sec. III. Section IV contains our conclusions.

II. SHORT-DISTANCE CONTRIBUTIONS

A. Decay amplitudes in QCD factorization

We shall use the QCD factorization approach [16–18] to study the short-distance contributions to the decays $\overline{B}^0 \to (\phi, \omega, \rho^0, \pi^0, \eta', \eta, f_0)\overline{K}^0$. In QCD factorization, the factorization amplitudes of the above-mentioned decays are given by

$$\langle\phi\overline{K}^0|H_{\rm eff}|\overline{B}^0\rangle = \frac{G_F}{\sqrt{2}}(p_B\cdot\varepsilon_\phi^*)\sum_{p=u,c}\lambda_p\bigg\{\bigg[(a_3+a_5)(\overline{K}^0\phi)+(a_4^p+r_\chi^\phi a_6^p)(\overline{K}^0\phi)-\frac{1}{2}(a_7+a_9)(\overline{K}^0\phi)\bigg]2f_\phi m_\phi F_1^{BK}(m_\phi^2) + f_Bf_Kf_\phi\bigg[b_3(\overline{K}^0\phi)-\frac{1}{2}b_3^{\rm EW}(\overline{K}^0\phi)\bigg]\frac{2m_\phi}{m_B^2}\bigg\},$$

$$\langle\omega\overline{K}^0|H_{\rm eff}|\overline{B}^0\rangle = \frac{G_F}{2}(p_B\cdot\varepsilon_\omega^*)\sum_{p=u,c}\lambda_p\bigg\{\bigg[a_2(\overline{K}^0\omega)\delta_u^p+2(a_3+a_5)(\overline{K}^0\omega)+\frac{1}{2}(a_7+a_9)(\overline{K}^0\omega)\bigg]2f_\omega m_\omega F_1^{BK}(m_\omega^2) + \bigg[(a_4^p-\mu_\chi^K a_6^p)(\omega\overline{K}^0)-\frac{1}{2}(a_{10}^p-\mu_\chi^K a_8^p)(\omega\overline{K}^0)\bigg]2f_K m_\omega A_0^{B\omega}(m_K^2) + f_Bf_Kf_\omega\bigg[b_3(\omega\overline{K}^0)-\frac{1}{2}b_3^{\rm EW}(\omega\overline{K}^0)\bigg]\frac{2m_\omega}{m_B^2}\bigg\},$$

$$\langle\rho^0\overline{K}^0|H_{\rm eff}|\overline{B}^0\rangle = \frac{G_F}{2}(p_B\cdot\varepsilon_\rho^*)\sum_{p=u,c}\lambda_p\bigg\{\bigg[a_2(\overline{K}^0\rho)\delta_u^p+\frac{3}{2}(a_7+a_9)(\overline{K}^0\rho)\bigg]2f_\rho m_\rho F_1^{BK}(m_\rho^2) - \bigg[(a_4^p-\mu_\chi^K a_6^p)(\rho\overline{K}^0) - \frac{1}{2}(a_{10}^p-\mu_\chi^K a_8^p)(\rho\overline{K}^0)\bigg]2f_K m_\rho A_0^{B\rho}(m_K^2) - f_Bf_Kf_\rho\bigg[b_3(\rho\overline{K}^0)-\frac{1}{2}b_3^{\rm EW}(\rho\overline{K}^0)\bigg]\frac{2m_\rho}{m_B^2}\bigg\},$$

$$\langle\pi^0\overline{K}^0|H_{\rm eff}|\overline{B}^0\rangle = i\frac{G_F}{2}\sum_{p=u,c}\lambda_p\bigg\{\bigg[a_2(\overline{K}^0\pi^0)\delta_u^p+\frac{3}{2}\alpha_{3,{\rm EW}}(\overline{K}^0\pi^0)\bigg]f_\pi F_0^{BK}(m_\pi^2)(m_B^2-m_K^2) - \bigg[\alpha_4^p(\pi^0\overline{K}^0)-\frac{1}{2}\alpha_{4,{\rm EW}}^p(\pi^0\overline{K}^0)\bigg]f_K F_0^{B\pi}(m_K^2)(m_B^2-m_\pi^2) - f_Bf_Kf_\pi\bigg[b_3(\pi^0\overline{K}^0)-\frac{1}{2}b_3^{\rm EW}(\pi^0\overline{K}^0)\bigg]\bigg\},$$

$$\langle\eta'\overline{K}^0|H_{\rm eff}|\overline{B}^0\rangle = i\frac{G_F}{\sqrt{2}}\sum_{p=u,c}\lambda_p\bigg\{F_0^{BK}(m_{\eta'}^2)(m_B^2-m_K^2)\bigg\{\frac{f_{\eta'}^q}{\sqrt{2}}\bigg[a_2(\overline{K}^0\eta'_q)\delta_u^p+2\alpha_3(\overline{K}^0\eta'_q)+\frac{1}{2}\alpha_{3,\rm EW}(\overline{K}^0\eta'_q)\bigg]$$
$$+f_{\eta'}^c[a_2(\overline{K}^0\eta'_c)\delta_c^p+\alpha_3(\overline{K}^0\eta'_c)]$$
$$+f_{\eta'}^s\bigg[\alpha_3(\overline{K}^0\eta'_s)+\alpha_4^p(\overline{K}^0\eta'_s)-\frac{1}{2}\alpha_{3,\rm EW}(\overline{K}^0\eta'_s)-\frac{1}{2}\alpha_{4,\rm EW}^p(\eta'_s\overline{K}^0)\bigg]\bigg\}$$
$$+F_0^{B\eta'}(m_K^2)(m_B^2-m_{\eta'}^2)\frac{f_K}{\sqrt{2}}\bigg[\alpha_4^p(\eta'\overline{K}^0)-\frac{1}{2}\alpha_{4,\rm EW}^p(\eta'\overline{K}^0)\bigg]$$
$$+f_Bf_K\bigg(\frac{1}{\sqrt{2}}f_{\eta'}^q+f_{\eta'}^s\bigg)\bigg[b_3(\eta'\overline{K}^0)-\frac{1}{2}b_3^{\rm EW}(\eta'\overline{K}^0)\bigg]\bigg\},$$

$$\langle f_0\overline{K}^0|H_{\rm eff}|\overline{B}^0\rangle = -\frac{G_F}{\sqrt{2}}\sum_{p=u,c}\lambda_p\bigg\{\bigg[(a_4^p-r_\chi^Ka_6^p)(f_0\overline{K}^0)-\frac{1}{2}(a_{10}^p-r_\chi^Ka_8^p)(f_0\overline{K}^0)\bigg]f_KF_0^{Bf_0}(m_K^2)(m_B^2-m_{f_0}^2)$$
$$+(2a_6^p-a_8^p)(\overline{K}^0f_0)\bar{f}_s\frac{m_{f_0}}{m_b}F_0^{BK}(m_{f_0}^2)(m_B^2-m_K^2)-f_Bf_K(\bar{f}_s+\bar{f}_u)\bigg[b_3(f_0\overline{K}^0)-\frac{1}{2}b_3^{\rm EW}(f_0\overline{K}^0)\bigg]\bigg\}, \quad (2.1)$$

where $F_{1,0}$ and A_0 denote pseudoscalar and vector form factors in the standard convention [20], $\lambda_p \equiv V_{pb}V_{ps}^*$,

$$\begin{aligned}\alpha_3(M_1M_2) &= a_3(M_1M_2)-a_5(M_1M_2),\\ \alpha_4^p(M_1M_2) &= a_4^p(M_1M_2)+r_\chi^{M_2}a_6^p(M_1M_2),\\ \alpha_{3,\rm EW}(M_1M_2) &= -a_7(M_1M_2)+a_9(M_1M_2),\\ \alpha_{4,\rm EW}^p(M_1M_2) &= a_{10}^p(M_1M_2)+r_\chi^{M_2}a_8^p(M_1M_2),\end{aligned} \quad (2.2)$$

and

$$r_\chi^\phi(\mu)=\frac{2m_\phi}{m_b(\mu)}\frac{f_\phi^\perp(\mu)}{f_\phi}, \qquad r_\chi^\pi(\mu)=\frac{2m_\pi^2}{2m_b(\mu)m_q(\mu)},$$
$$r_\chi^K(\mu)=\frac{2m_K^2}{m_b(\mu)[m_s(\mu)+m_q(\mu)]},$$
$$r_\chi^{\eta'}(\mu)=\frac{2m_{\eta'}^2}{2m_b(\mu)m_s(\mu)}\bigg(1-\frac{f_{\eta'}^q}{\sqrt{2}f_{\eta'}^s}\bigg), \quad (2.3)$$

with the scale dependent transverse decay constant $f_V^\perp$ being defined as

$$\langle V(p,\varepsilon^*)|\bar{q}\sigma_{\mu\nu}q'|0\rangle = f_V^\perp(p_\mu\varepsilon_\nu^*-p_\nu\varepsilon_\mu^*). \quad (2.4)$$

Note that the a_6 penguin term appears in the decay amplitude of $\overline{B}^0\to\phi\overline{K}^0$ owing to the nonvanishing transverse decay constant of the ϕ meson. The scalar decay constant $\bar{f}_q$ of $f_0(980)$ in Eq. (2.1) is defined by $\langle f_0|\bar{q}q|0\rangle = m_{f_0}\bar{f}_q$. The decay amplitude for $\overline{B}^0\to\eta\overline{K}^0$ is obtained from $\overline{B}^0\to\eta'\overline{K}^0$ by replacing $\eta'\to\eta$. Note that the use of nonzero q^2 in the argument of form factors in Eq. (2.1) means that some corrections quadratic in the light quark masses are automatically incorporated.

The effective parameters a_i^p with $p=u,c$ can be calculated in the QCD factorization approach [16]. They are basically the Wilson coefficients in conjunction with short-distance nonfactorizable corrections such as vertex corrections and hard spectator interactions. In general, they have the expressions [16,18]

$$a_i^p(M_1M_2)=c_i+\frac{c_{i\pm1}}{N_c}+\frac{c_{i\pm1}}{N_c}\frac{C_F\alpha_s}{4\pi}\times\bigg[V_i(M_2)+\frac{4\pi^2}{N_c}H_i(M_1M_2)\bigg]+P_i^p(M_2), \quad (2.5)$$

where $i=1,\ldots,10$, the upper (lower) signs apply when i is odd (even), c_i are the Wilson coefficients, $C_F=(N_c^2-1)/(2N_c)$ with $N_c=3$, M_2 is the emitted meson, and M_1 shares the same spectator quark with the B meson. The quantities $V_i(M_2)$ account for vertex corrections, $H_i(M_1M_2)$ for hard spectator interactions with a hard gluon exchange between the emitted meson and the spectator quark of the B meson, and $P_i(M_2)$ for penguin contractions. The explicit expressions of these quantities together with the annihilation quantities b_3 and $b_3^{\rm EW}$ can be found in [16,18].

For $B\to\eta^{(\prime)}K$ decay, $\eta_q^{(\prime)}$ and $\eta_s^{(\prime)}$ in Eq. (2.1) refer to the nonstrange and strange quark states, respectively, of $\eta^{(\prime)}$. The decay constants $f_{\eta^{(\prime)}}^{q,s,c}$ are defined by

$$\begin{aligned}\langle\eta^{(\prime)}(p)|\bar{q}\gamma_\mu\gamma_5q|0\rangle &= -\frac{i}{\sqrt{2}}f_{\eta^{(\prime)}}^qp_\mu,\\ \langle\eta^{(\prime)}(p)|\bar{s}\gamma_\mu\gamma_5s|0\rangle &= -if_{\eta^{(\prime)}}^sp_\mu,\\ \langle\eta^{(\prime)}(p)|\bar{c}\gamma_\mu\gamma_5c|0\rangle &= -if_{\eta^{(\prime)}}^cp_\mu,\end{aligned} \quad (2.6)$$

with $q=u$ or d. Numerically, we shall use [21]

$$f^q_{\eta'} = 89\ \text{MeV}, \qquad f^s_{\eta'} = 131\ \text{MeV},$$
$$f^c_{\eta'} = -6.3\ \text{MeV}, \qquad f^q_{\eta} = 108\ \text{MeV}, \qquad (2.7)$$
$$f^s_{\eta} = -111\ \text{MeV}, \qquad f^c_{\eta} = -2.4\ \text{MeV},$$

recalling that, in our convention, $f_\pi = 132$ MeV. As for the $B \to \eta^{(\prime)}$ form factor, we shall follow [17] to use the relation $F_0^{B\eta^{(\prime)}} = (f^q_{\eta^{(\prime)}}/f^s_{\eta^{(\prime)}})F_0^{B\pi}$ to obtain the values of the form factors $F_0^{B\eta^{(\prime)}}$. The wave functions of the physical η' and η states are related to that of the SU(3) singlet state η_0 and octet state η_8 by

$$\eta' = \eta_8 \sin\phi + \eta_0 \cos\phi, \qquad \eta = \eta_8 \cos\phi - \eta_0 \sin\phi, \qquad (2.8)$$

with the mixing angle $\phi = -(15.4 \pm 1.0)°$ [21].

The decay $B \to f_0(980)K$ has been discussed in detail in [22]. Since the scalar meson $f_0(980)$ cannot be produced via the vector current owing to charge conjugation invariance or conservation of vector current, the tree contribution to $\overline{B}^0 \to f_0\overline{K}^0$ vanishes under the factorization approximation. Just as the SU(3)-singlet η_0, $f_0(980)$ in the two-quark picture also contains strange and nonstrange quark content

$$|f_0(980)\rangle = |\bar{s}s\rangle \cos\theta + |\bar{n}n\rangle \sin\theta, \qquad (2.9)$$

with $\bar{n}n \equiv (\bar{u}u + \bar{d}d)/\sqrt{2}$. Experimental implications for the mixing angle θ have been discussed in detail in [22,23]; it lies in the ranges of $25° < \theta < 40°$ and $140° < \theta < 165°$. Based on the QCD sum-rule technique, the decay constants $\tilde{f}_s$ and $\tilde{f}_n$ defined by $\langle f_0^q|\bar{q}q|0\rangle = m_{f_0}\tilde{f}_q$ with $f_0^n = \bar{n}n$ and $f_0^s = \bar{s}s$ have been estimated in [22] by taking into account their scale dependence and radiative corrections. It turns out that $\tilde{f}_s(1\ \text{GeV}) \approx 0.33$ GeV [22], for example. In the two-quark scenario for $f_0(980)$, the decay constants $\bar{f}_{n,s}$ are related to $\tilde{f}_{n,s}$ via [22]

$$\bar{f}_s = \tilde{f}_s \cos\theta, \qquad \bar{f}_n = \tilde{f}_n \sin\theta. \qquad (2.10)$$

The hard spectator function relevant for $B \to f_0K$ decay has the form

$$H_i(f_0K) = \frac{\bar{f}_u f_B}{F_0^{Bf_0^u}(0)m_B^2}\int_0^1 \frac{d\rho}{\rho}\Phi_B(\rho)\int_0^1 \frac{d\xi}{\bar{\xi}}\Phi_K(\xi) \times \int_0^1 \frac{d\eta}{\bar{\eta}}\left[\Phi_{f_0}(\eta) + \frac{2m_{f_0}}{m_b}\frac{\bar{\xi}}{\xi}\Phi^p_{f_0}(\eta)\right], \qquad (2.11)$$

for $i = 1, 4, 10$, and $H_i = 0$ for $i = 6, 8$, where $\bar{\xi} \equiv 1 - \xi$ and $\bar{\eta} = 1 - \eta$. As for the parameters $a^{u,c}_{6,8}(\overline{K}^0 f_0)$ appearing in Eq. (2.1), they have the same expressions as $a^{u,c}_{6,8}(f_0\overline{K}^0)$ except that the penguin function $\hat{G}_K$ [see Eq. (55) of the second reference in [16]] is replaced by $\hat{G}_{f_0}$ and Φ^p_K by $\Phi^p_{f_0}$. For the distribution amplitudes Φ_{f_0}, $\Phi^p_{f_0}$ and the annihilation amplitudes, see [22] for details.

Although the parameters $a_i (i \neq 6, 8)$ and $a_{6,8}r_\chi$ are formally renormalization scale and γ_5 scheme independent, in practice there exists some residual scale dependence in $a_i(\mu)$ to finite order. To be specific, we shall evaluate the vertex corrections to the decay amplitude at the scale $\mu = m_b$. In contrast, as stressed in [16], the hard spectator and annihilation contributions should be evaluated at the hard-collinear scale $\mu_h = \sqrt{\mu\Lambda_h}$ with $\Lambda_h \approx 500$ MeV. There is one more serious complication about these contributions; that is, while QCD factorization predictions are model independent in the $m_b \to \infty$ limit, power corrections always involve troublesome endpoint divergences. For example, the annihilation amplitude has endpoint divergences even at twist-2 level and the hard spectator scattering diagram at twist-3 order is power suppressed and possesses soft and collinear divergences arising from the soft spectator quark. Since the treatment of endpoint divergences is model dependent, subleading power corrections generally can be studied only in a phenomenological way. We shall follow [16] to parametrize the endpoint divergence $X_A \equiv \int_0^1 dx/(1-x)$ in the annihilation diagram as

$$X_A = \ln\left(\frac{m_B}{\Lambda_h}\right)(1 + \rho_A e^{i\phi_A}) \qquad (2.12)$$

with $\rho_A \leq 1$. Likewise, the endpoint divergence X_H in the hard spectator contributions can be parametrized in a similar way.

B. Consideration of mixing-induced *CP* asymmetry

Consider the mixing-induced CP violation in the decay modes $(\phi, \omega, \rho^0, \pi^0, \eta', \eta, f_0)K_S$ mediated by $b \to sq\bar{q}$ transitions. Since a common final state is reached only via $K^0 - \overline{K}^0$ mixing,

$$\lambda_f = \frac{q_B}{p_B}\frac{q_K}{p_K}\frac{A(\overline{B}^0 \to M\overline{K}^0)}{A(B^0 \to MK^0)} \approx \frac{V^*_{tb}V_{td}}{V_{tb}V^*_{td}}\frac{V^*_{cd}V_{cs}}{V_{cd}V^*_{cs}}\frac{A(\overline{B}^0 \to M\overline{K}^0)}{A(B^0 \to MK^0)}. \qquad (2.13)$$

We shall use this expression for λ_f to compute CP asymmetries S_f and $\mathcal{A}_f$. For $M = V$, $A(B^0 \to VK^0)$ has the same expression as $-A(\overline{B}^0 \to V\overline{K}^0)$ with the CKM mixing angles $\lambda_p \to \lambda_p^*$ owing to $CP|VK^0\rangle = -|V\overline{K}^0\rangle$, while for $M = P$, $A(B^0 \to PK^0)$ is obtained from $A(\overline{B}^0 \to P\overline{K}^0)$ with $\lambda_p \to \lambda_p^*$. If the contributions from $V_{ub}V^*_{us}$ terms are neglected, then we will have

$$\frac{A(\overline{B}^0 \to M\overline{K}^0)}{A(B^0 \to MK^0)} \approx \eta_f \frac{V_{cd}V_{cs}^*}{V_{cd}^*V_{cs}} \quad \Rightarrow \lambda_f \approx \eta_f \frac{V_{td}}{V_{td}^*} = \eta_f e^{-2i\beta}$$
$$\Rightarrow -\eta_f S_f \approx \sin 2\beta. \tag{2.14}$$

Note that $\eta_f = 1$ for $f_0 K_S$ and $\eta_f = -1$ for $(\phi, \omega, \rho^0, \eta', \eta, \pi^0)K_S$.

From Eq. (2.1) we see that among the seven modes under consideration, only ωK_S, $\rho^0 K_S$, $\pi^0 K_S$, and $\eta^{(\prime)} K_S$ receive tree contributions from the tree diagram $b \to sq\bar{q}$ ($q = u, d$). However, since the tree contribution is color suppressed, the deviation of $-\eta_f S_f$ from $\sin 2\beta$ is expected to be small. Nevertheless, the large cancellation between a_4 and a_6 penguin terms in the amplitudes of $\overline{B}^0 \to \omega\overline{K}^0$ and $\overline{B}^0 \to \rho^0\overline{K}^0$ render the tree contribution relatively significant. Hence, ΔS_f is expected to be largest in the ωK_S and $\rho^0 K_S$ modes. Since the typical values of the effective Wilson parameters obtained from Eq. (2.5) are

$$\begin{aligned} &a_2 \approx 0.18 - 0.11i, &&a_3 \approx -0.003 + 0.005i,\\ &a_5 \approx 0.008 - 0.006i, &&a_4^u \approx -0.03 - 0.02i,\\ &a_4^c \approx -0.03 - 0.006i, &&a_6^u \approx -0.06 - 0.02i,\\ &a_6^c \approx -0.06 - 0.004i, \end{aligned} \tag{2.15}$$

and $r_\chi^K \approx 0.57$, it is not difficult to see from Eq. (2.1) that δ_f lies in the region $0 > \delta_f > -\pi/2$ for the ωK_S and $f_0 K_S$ modes, $\pi > \delta_f > \pi/2$ for $\rho^0 K_s$, and $\pi/2 > \delta_f > 0$ for the remaining three. Therefore, based purely on SD contributions, it is expected that $\Delta S_f > 0$ for all the modes except for $\rho^0 K_S$ and that $\mathcal{A}_f$ is negative for ωK_S, $f_0 K_S$ and positive for ϕK_S, $\rho^0 K_S$, $\pi^0 K_S$, $\eta^{(\prime)} K_S$.

C. Numerical results

To proceed with the numerical calculations, we shall follow [16,18] for the choices of the relevant parameters except for the form factors and CKM matrix elements. For form factors we shall use those derived in the covariant light-front quark model [24] and assign a common value of 0.03 for the form factor errors, e.g. $F^{B\pi}(0) = 0.25 \pm 0.03$. For CKM matrix elements, see the unitarity triangle analysis in [25]. For definiteness, we shall follow the first reference in [25] to use the Wolfenstein parameters $A = 0.801$, $\lambda = 0.2265$, $\bar{\rho} = 0.189$, and $\bar{\eta} = 0.358$ which correspond to $\sin 2\beta = 0.723$ and $\gamma = 63°$. We assign 15° error for the unitarity angle γ, recalling that two values $\gamma = (62^{+10}_{-12})°$ and $\gamma = (64 \pm 18)°$ are obtained in [25]. For endpoint divergences encountered in hard spectator and annihilation contributions we take the default values $\rho_A = \rho_H = 0$. We will return to this point below when discussing long-distance rescattering effects.

The obtained branching ratios for the decays $\overline{B}^0 \to (\phi, \omega, \rho^0, \pi^0, \eta', \eta, f_0)\overline{K}^0$ are shown in the second column of Table II, while the corresponding CP violation asymmetries S_f and $\mathcal{A}_f$ are depicted in Table III. In general, our results for branching ratios and direct CP asymmetries are in agreement with [18]. Some differences result from different inputs of the form factors and CKM parameters. It is evident that, as far as the central values are concerned, the predicted branching ratios by the SD QCD factorization approach are generally too low compared to experiment especially for $\omega\overline{K}^0$, $\rho^0 K_S$, and $\pi^0\overline{K}^0$. Note that $\mathcal{B}(\overline{B}^0 \to \omega\overline{K}^0) \lesssim 10^{-6}$ is predicted in the early QCD factorization calculation [26]. The very large (small) branching ratio for $\eta'\overline{K}^0$ ($\eta\overline{K}^0$) is understandable as follows. There are two distinct penguin contributions to $\eta^{(\prime)}\overline{K}^0$: one couples to the d quark content of the $\eta^{(\prime)}$, while the other is related to the s quark component of the $\eta^{(\prime)}$ [see also Eq. (2.1)]. If the $\eta - \eta'$ mixing angle is given by $-19.5°$, the expressions of the η' and η wave functions will become very simple:

TABLE II. SD and LD contributions to the branching ratios (in units of 10^{-6}) for various penguin-dominated modes. The first and second theoretical errors correspond to the SD and LD ones, respectively (see the text for details). The world averages of experimental measurements are taken from [19].

	SD	SD + LD	Expt
$\overline{B}^0 \to \phi\overline{K}^0$	$5.6^{+1.9}_{-1.8}$	$8.6^{+1.2+2.9}_{-1.2-1.8}$	$8.3^{+1.2}_{-1.0}$
$\overline{B}^0 \to \omega\overline{K}^0$	$2.0^{+3.5}_{-1.3}$	$5.6^{+2.9+3.7}_{-1.2-2.1}$	5.6 ± 0.9
$\overline{B}^0 \to \rho^0\overline{K}^0$	$2.8^{+3.2}_{-1.6}$	$5.2^{+3.2+2.6}_{-1.5-1.2}$	5.1 ± 1.6
$\overline{B}^0 \to \eta'\overline{K}^0$	$42.1^{+45.6}_{-19.4}$	$69.4^{+51.3+50.4}_{-21.4-19.2}$	68.6 ± 4.2
$\overline{B}^0 \to \eta\overline{K}^0$	$1.8^{+1.2}_{-0.9}$	$1.8^{+1.2+0.1}_{-0.8-0.0}$	<2.0
$\overline{B}^0 \to \pi^0\overline{K}^0$	$5.8^{+5.5}_{-3.1}$	$9.6^{+5.5+8.4}_{-2.9-3.0}$	11.5 ± 1.0
$\overline{B}^0 \to f_0\overline{K}^0$	$8.1^{+3.1}_{-2.6}$	$8.1^{+3.1+0.0}_{-2.7-0.0}$ [a]	11.3 ± 3.6

[a]Only the intermediate states $K^-\rho^+$ and π^+K^{*-} are taken into account; see Sec. III for details.

$$\begin{aligned} |\eta'\rangle &= \frac{1}{\sqrt{6}}|\bar{u}u + \bar{d}d + 2\bar{s}s\rangle,\\ |\eta\rangle &= \frac{1}{\sqrt{3}}|\bar{u}u + \bar{d}d - \bar{s}s\rangle. \end{aligned} \tag{2.16}$$

It is evident that the SD $\eta\overline{K}^0$ amplitude vanishes in the SU(3) limit, whereas the constructive interference between the penguin amplitudes accounts for the large rate of $\eta'\overline{K}^0$. In reality the $\eta - \eta'$ mixing angle is $-(15.4 \pm 1.0)°$ [21], but this does not affect the above physical picture.

Owing to the large cancellation between the a_4 and a_6 penguin terms, the main contribution to the decay $\overline{B}^0 \to f_0\overline{K}^0$ arises from the penguin diagram involving the strange quark content of $f_0(980)$, namely, the term with the scalar decay constant $\bar{f}_s$. Consequently, the maximal branching ratio 9.9×10^{-6} occurs near the zero mixing angle. The result of $\mathcal{B}(\overline{B}^0 \to f_0\overline{K}^0) = 8.1 \times 10^{-6}$ shown in Table II corresponds to $\theta = 150°$. Note that the decay $\overline{B}^0 \to f_0(980)\overline{K}^0$ was measured by *BABAR* [27] with the result

TABLE III. SD and LD contributions to the time-dependent CP asymmetry. The first and second theoretical errors correspond to the SD and LD ones, respectively (see the text for details).

Final state	$-n_f S_f$			$\mathcal{A}_f(\%)$		
	SD	SD + LD	Expt	SD	SD + LD	Expt
ϕK_S	$0.747^{+0.002}_{-0.039}$	$0.759^{+0.007+0.005}_{-0.041-0.006}$	0.35 ± 0.20	$1.4^{+0.3}_{-0.5}$	$-2.6^{+0.8+1.1}_{-1.0-0.9}$	4 ± 17
ωK_S	$0.850^{+0.052}_{-0.055}$	$0.736^{+0.022+0.025}_{-0.035-0.014}$	$0.55^{+0.30}_{-0.32}$	$-7.3^{+3.5}_{-2.6}$	$-13.2^{+3.9+2.1}_{-2.8-2.6}$	48 ± 25
$\rho^0 K_S$	$0.635^{+0.028}_{-0.067}$	$0.761^{+0.071+0.073}_{-0.079-0.100}$	$\cdots$	$9.0^{+2.2}_{-4.6}$	$46.6^{+12.9+10.8}_{-14.7-5.9}$	$\cdots$
$\eta' K_S$	$0.737^{+0.002}_{-0.038}$	$0.725^{+0.004+0.005}_{-0.036-0.003}$	0.43 ± 0.11	$1.8^{+0.4}_{-0.4}$	$2.1^{+0.6+0.5}_{-0.3-0.2}$	4 ± 8
ηK_S	$0.793^{+0.017}_{-0.044}$	$0.802^{+0.025+0.002}_{-0.046-0.004}$	$\cdots$	$-6.1^{+5.1}_{-2.0}$	$-3.7^{+4.4+1.4}_{-1.8-2.4}$	$\cdots$
$\pi^0 K_S$	$0.787^{+0.018}_{-0.044}$	$0.770^{+0.006+0.015}_{-0.042-0.019}$	$0.34^{+0.27}_{-0.29}$	$-3.4^{+2.1}_{-1.1}$	$3.7^{+1.8+2.0}_{-2.0-0.4}$	-8 ± 14
$f_0 K_S$	$0.749^{+0.002}_{-0.039}$	$0.749^{+0.002+0.0}_{-0.039-0.0}$ [a]	0.39 ± 0.26	$0.77^{+0.13}_{-0.10}$	$0.75^{+0.14+0.01}_{-0.09-0.01}$ [a]	-14 ± 22

[a]Only the intermediate states $K^-\rho^+$ and $\pi^+ K^{*-}$ are taken into account (see Sec. III for details). This means that the prediction of the LD effects on the $f_0 K_S$ mode is less certain.

$$\mathcal{B}(B^0 \to f_0(980)K^0 \to \pi^+\pi^- K^0) = (6.0 \pm 0.9 \pm 1.3) \times 10^{-6}. \quad (2.17)$$

The absolute branching ratios for $B \to f_0 K$ depends critically on the branching fraction of $f_0(980) \to \pi\pi$. We use the results from the most recent analysis of [28] to obtain $\mathcal{B}(f_0(980) \to \pi\pi) = 0.80 \pm 0.14$ and $\mathcal{B}(B^0 \to f_0(980)K^0) = (11.3 \pm 3.6) \times 10^{-6}$ as shown in Table II.[1] In short, although the predicted branching ratios of $(\phi, \omega, \rho^0, \eta', \pi^0)\overline{K}^0$ are consistent with the data within the theoretical and experimental uncertainties, there are sizable discrepancies between the SD theory and experiment for the central values of their branching ratios. This may call for the consideration of subleading power corrections such as the annihilation topology and/or FSI effects.

In Tables II and III we have included the SD theoretical uncertainties arising from the variation of the unitarity angle $\gamma = (63 \pm 15)^\circ$, the renormalization scale μ from $2m_b$ to $m_b/2$, quark masses (especially the strange quark mass which is taken to be $m_s(2\text{ GeV}) = 90 \pm 20$ MeV), and form factors as mentioned before. To obtain the SD errors shown in Tables II and III, we first scan randomly the points in the allowed ranges of the above four parameters in two separated groups: the first one and the last three, and then add each error in quadrature. For example, for the decay $\overline{B}^0 \to \eta' K_S$ we obtain $2\mathcal{B} = (42.1^{+0.2+45.6}_{-0.2-19.4}) \times 10^{-6}$, $\mathcal{A} = 1.77^{+0.30+0.22}_{-0.18-0.30}\%$, and $S = 0.737^{+0.002+0.002}_{-0.038-0.004}$, where the first error is due to the variation of γ and the second error comes from the uncertainties in the renormalization scale, the strange quark mass and the form factors.

From Table III we obtain the differences between the CP asymmetry S_f^{SD} induced at short distances and the measured $S_{J/\psi K_S}$ to be

$$\begin{aligned} &\Delta S^{\text{SD}}_{\phi K_S} = 0.02^{+0.00}_{-0.04}, \qquad \Delta S^{\text{SD}}_{\omega K_S} = 0.12^{+0.05}_{-0.06}, \\ &\Delta S^{\text{SD}}_{\rho^0 K_S} = -0.09^{+0.03}_{-0.07}, \qquad \Delta S^{\text{SD}}_{\pi^0 K_S} = 0.06^{+0.02}_{-0.04}, \\ &\Delta S^{\text{SD}}_{\eta' K_S} = 0.01^{+0.00}_{-0.04}, \qquad \Delta S^{\text{SD}}_{\eta K_S} = 0.07^{+0.02}_{-0.04}, \\ &\Delta S^{\text{SD}}_{f_0 K_S} = 0.02^{+0.00}_{-0.04}, \end{aligned} \quad (2.18)$$

where the experimental error of $S_{J/\psi K_S}$ is not included. It should be noted that these results are not upset by the approximation of neglecting the error on $S_{J/\psi K_S}$. Our results for $\Delta S^{\text{SD}}_{\phi K_S}$ and $\Delta S^{\text{SD}}_{\eta' K_S}$ are consistent with that obtained in [18].[2] As expected before, the ωK_S and $\rho^0 K_S$ modes have the largest deviation of S_f from the naive expectation owing to the large tree pollution. In contrast, tree pollution in $\eta' K_S$ is diluted by the prominent $s\bar{s}$ content of the η'. As for direct CP violation, sizable direct CP asymmetries are predicted for ωK_S and $\rho^0 K_S$ based on SD contributions.

III. LONG-DISTANCE CONTRIBUTIONS

As noticed in passing, the predicted branching ratios for the decays $\overline{B}^0 \to (\phi, \omega, \rho^0, \pi^0)K_S$ by the short-distance QCD factorization approach are generally too low by a factor of 2 compared to experiment. Just like the perturbative QCD (pQCD) approach [29] where the annihilation topology plays an essential role for producing sizable strong phases and for explaining the penguin-dominated VP modes, it has been suggested in [18] that a favorable scenario (denoted as S4) for accommodating the observed penguin-dominated $B \to PV$ decays and the measured sign of direct CP asymmetry in $\overline{B}^0 \to K^-\pi^+$ is to have a large

[1]For comparison, the world average of the branching ratio for $B^- \to f_0 K^- \to \pi^+\pi^- K^-$ is $(8.49^{+1.35}_{-1.26}) \times 10^{-6}$ [19] and hence $\mathcal{B}(B^- \to f_0 K^-) \approx (15.9^{+3.8}_{-3.7}) \times 10^{-6}$.

[2]Note that unlike [18] we did not include the theoretical uncertainties arising from power corrections. Otherwise, there will be a double-counting problem when considering LD rescattering effects.

annihilation contribution by choosing $\rho_A = 1$, $\phi_A = -55°$ for PP, $\phi_A = -20°$ for PV, and $\phi_A = -70°$ for VP modes. The sign of ϕ_A is chosen so that the direct CP violation $A_{K^-\pi^+}$ agrees with the data. However, there are at least three difficulties with this scenario. First, the origin of these phases is unknown and their signs are not predicted. Second, since both annihilation and hard spectator scattering encounter endpoint divergences, there is no reason that soft gluon effects will only modify ρ_A but not ρ_H. Third, the annihilation topologies do not help enhance the $\pi^0\pi^0$ and $\rho^0\pi^0$ modes; both pQCD and QCDF approaches fail to describe these two color-suppressed tree-dominated modes. As stressed in [18], one would wish to have an explanation of the data without invoking weak annihilation.

As shown in [15], final-state rescattering can have significant effects on decay rates and CP violation. For example, the branching ratios of the penguin-dominated decay ϕK^* can be enhanced from $\sim 5 \times 10^{-6}$ predicted by QCDF to the level of 1×10^{-5} by FSIs via rescattering of charm intermediate states [15]. Indeed, it has been long advocated that charming-penguin long-distance contributions increase significantly the $B \to K\pi$ rates and yield better agreement with experiment [30,31]. The color-suppressed modes $D^0\pi^0$, $\pi^0\pi^0$, and $\rho^0\pi^0$ in B decays can also be easily enhanced by rescattering effects. Moreover, large nonperturbative strong phases can be generated from the final-state interactions through the absorptive part of rescattering amplitudes. We have shown explicitly in [15] that direct CP-violating partial rate asymmetries in $K^-\pi^+$, $\rho^+\pi^-$, and $\pi^+\pi^-$ modes are significantly affected by final-state rescattering and their signs, which are different from what is expected from the short-distance QCDF approach, and are correctly predicted. In order to avoid the double-counting problem, we will turn off the LD effects induced from the power corrections due to nonvanishing ρ_A and ρ_H; that is, we set $\rho_A = \rho_H = 0$ and $\phi_A = \phi_H = 0$; therefore it is important to note that we are not adding FSI on top of QCDF. We wish to stress that, in principle, LD rescattering effects can be included in the framework of QCDF, but that requires modeling of $\Lambda_{\rm QCD}/m_b$ power corrections and, in particular, one may then need to adopt nonvanishing values of ρ_A, ρ_H, ϕ_A, and ϕ_H [18], as mentioned above. In this work, we are providing a specific model for final-state rescattering to complement QCDF.

Besides direct CP violation, the mixing-induced CP asymmetry S_f also could be affected by final-state rescattering from some intermediate states. When the intermediate states are charmless, the relevant CKM matrix element is $V_{ub}V_{us}^* \approx A\lambda^4 R_b e^{-i\gamma}$ which carries the weak phase γ. In general, the charmless intermediate states will essentially not affect the decay rates but may have potentially sizable effect on S_f, whereas the charm intermediate states will affect both the branching ratios and S_f.

A. Final-state rescattering

At the quark level, final-state rescattering can occur through quark exchange and quark annihilation. In practice, it is extremely difficult to reliably calculate the FSI effects, but it may become amenable to estimate these effects at the hadron level where FSIs manifest as the rescattering processes with s-channel resonances and one particle exchange in the t channel. In contrast to D decays, the s-channel resonant FSIs in B decays is expected to be suppressed relative to the rescattering effect arising from quark exchange owing to the lack of the existence of resonances at energies close to the B meson mass. Therefore, we will model FSIs as rescattering processes of some intermediate two-body states with one particle exchange in the t channel and compute the absorptive part of the rescattering amplitude via the optical theorem [15].

Given the weak Hamiltonian in the form $H_{\rm W} = \sum_i \lambda_i Q_i$, where λ_i is the combination of the quark mixing matrix elements and Q_i is a T-even local operator (T: time reversal), the absorptive part of final-state rescattering can be obtained by using the optical theorem and time-reversal invariant weak decay operator Q_i. From the time-reversal invariance of $Q(= U_T Q^* U_T^\dagger)$, it follows that

$$\langle i;{\rm out}|Q|B;{\rm in}\rangle^* = \sum_j S_{ji}^* \langle j;{\rm out}|Q|B;{\rm in}\rangle, \qquad (3.1)$$

where $S_{ij} \equiv \langle i;{\rm out}|j;{\rm in}\rangle$ is the strong-interaction S-matrix element, and we have used $U_T|{\rm out(in)}\rangle^* = |{\rm in(out)}\rangle$ to fix the phase convention. Equation (3.1) implies an identity related to the optical theorem. Noting that $S = 1 + iT$, we find

$$2\mathcal{A}bs\langle i;{\rm out}|Q|B;{\rm in}\rangle = \sum_j T_{ji}^* \langle j;{\rm out}|Q|B;{\rm in}\rangle, \qquad (3.2)$$

where use of the unitarity of the S matrix has been made. Specifically, for two-body B decays, we have

$$\mathcal{A}\,bsM(p_B \to p_a p_b) = \frac{1}{2}\sum_j \left(\Pi_{k=1}^j \int \frac{d^3\vec{q}_k}{(2\pi)^3 2E_k}\right)(2\pi)^4 \delta^4\left(p_a + p_b - \sum_{k=1}^j q_k\right) M(p_B \to \{q_k\}) T^*(p_a p_b \to \{q_k\}). \qquad (3.3)$$

Thus the optical theorem relates the absorptive part of the two-body decay amplitude to the sum over all possible B decay final states $\{q_k\}$, followed by the strong rescattering $\{q_k\} \to p_a p_b$. In principle, the dispersive part of the rescattering amplitude can be obtained from the absorptive part via the dispersion relation

$$\mathcal{D}isA(m_B^2) = \frac{\mathcal{P}}{\pi}\int_s^\infty \frac{\mathcal{A}bsA(s')}{s' - m_B^2} ds'. \tag{3.4}$$

Unlike the absorptive part, it is known that the dispersive contribution suffers from the large uncertainties due to some possible subtractions and the complication from integrations. For this reason, we will assume the dominance of the rescattering amplitude by the absorptive part and ignore the dispersive part in the present work.

The relevant Lagrangian for final-state strong interactions is given by

$$\mathcal{L} = \mathcal{L}_l + \mathcal{L}_h, \tag{3.5}$$

where

$$\begin{aligned}\mathcal{L}_l &= -\frac{1}{4}\mathrm{Tr}[F_{\mu\nu}(V)F^{\mu\nu}(V)] + ig_{VPP}\mathrm{Tr}(V^\mu P \overleftrightarrow{\partial}_\mu P) + g_{VVP}\epsilon^{\mu\nu\alpha\beta}\,\mathrm{Tr}(\partial_\mu V_\nu \partial_\alpha V_\beta P)\\ &= -\frac{1}{4}\mathrm{Tr}[f_{\mu\nu}(V)f^{\mu\nu}(V)] - i\frac{g_V}{2}(\phi_{\mu\nu}K^{*-\mu}K^{*+\nu} + K^{*-}_{\mu\nu}K^{*+\mu}\phi^\nu + K^{*+}_{\mu\nu}K^{*-\nu}\phi^\mu + \ldots)\\ &\quad + ig_{VPP}[\phi^\mu K^-\overleftrightarrow{\partial}_\mu K^+ + \rho^{+\mu}(K^0\overleftrightarrow{\partial}_\mu K^- - \sqrt{2}\pi^0\overleftrightarrow{\partial}_\mu\pi^-) + K^{*-\mu}\pi^+\overleftrightarrow{\partial}_\mu K^0 + K^{*-}\Big(\frac{1}{\sqrt{2}}\eta'_q\overleftrightarrow{\partial}_\mu K^+ + K^+\overleftrightarrow{\partial}_\mu\eta'_s\Big)\\ &\quad + \overline{K}^{*0}\Big(\frac{1}{\sqrt{2}}\eta'_q\overleftrightarrow{\partial}_\mu K^0 + K^0\overleftrightarrow{\partial}_\mu\eta'_s\Big) + \ldots] + g_{VVP}\epsilon^{\mu\nu\alpha\beta}[K^-(\partial_\mu K^{*+}_\nu\partial_\alpha\phi_\beta) + K^+(\partial_\mu\phi_\nu\partial_\alpha K^{*-}_\beta) + K^0(\partial_\mu K^{*-}_\nu\partial_\alpha\rho^+_\beta)\\ &\quad + \sqrt{2}\pi^\pm(\partial_\mu\rho^\mp_\nu\partial_\alpha\omega_\beta) + \ldots],\end{aligned} \tag{3.6}$$

with P and V being the usual pseudoscalar and vector multiplets, respectively, $F_{\mu\nu} = f_{\mu\nu} + ig_V[V_\mu, V_\nu]/2$, $f_{\mu\nu} = \partial_\mu V_\nu - \partial_\nu V_\mu$ and

$$\begin{aligned}\mathcal{L}_h &= -ig_{D^*DP}(D^i\partial^\mu P_{ij}D^{*j\dagger}_\mu - D^{*i}_\mu\partial^\mu P_{ij}D^{j\dagger}) - \frac{1}{2}g_{D^*D^*P}\epsilon_{\mu\nu\alpha\beta}D^{*\mu}_i\partial^\nu P^{ij}\overleftrightarrow{\partial}^\alpha D^{*\beta\dagger}_j - ig_{DDV}D^\dagger_i\overleftrightarrow{\partial}_\mu D^j(V^\mu)^i_j\\ &\quad - 2f_{D^*DV}\epsilon_{\mu\nu\alpha\beta}(\partial^\mu V^\nu)^i_j(D^\dagger_i\overleftrightarrow{\partial}^\alpha D^{*\beta j} - D^{*\beta\dagger}_i\overleftrightarrow{\partial}^\alpha D^j) + ig_{D^*D^*V}D^{*\nu\dagger}_i\overleftrightarrow{\partial}_\mu D^{*j}_\nu(V^\mu)^i_j\\ &\quad + 4if_{D^*D^*V}D^{*\dagger}_{i\mu}(\partial^\mu V^\nu - \partial^\nu V^\mu)^i_j D^{*j}_\nu.\end{aligned} \tag{3.7}$$

Only those terms relevant for later purposes are shown in $\mathcal{L}_l$ and the convention $\epsilon^{0123} = 1$ has been adopted. For the coupling constants, we take $g_{\rho KK} \simeq g_{\rho\pi\pi} \simeq 4.28$,[3] $g_{\phi KK} = 4.48$, $\sqrt{2}g_{VVP} = 16\ \mathrm{GeV}^{-1}$ [32]. In the chiral and heavy quark limits, we have [33]

$$g_{D^*D^*\pi} = \frac{g_{D^*D\pi}}{m_{D^*}}, \qquad g_{DDV} = g_{D^*D^*V} = \frac{\beta g_V}{\sqrt{2}}, \qquad f_{D^*DV} = \frac{f_{D^*D^*V}}{m_{D^*}} = \frac{\lambda g_V}{\sqrt{2}}, \tag{3.8}$$

with $f_\pi = 132$ MeV. The parameters g_V, β, and λ (not to be confused with the Wolfenstein parameter λ) thus enter into the effective chiral Lagrangian describing the interactions of heavy mesons with low momentum light vector mesons (see e.g. [33]). The parameter g_V respects the relation $g_V = m_\rho/f_\pi = 5.8$ [33]. We shall follow [34] to use $\beta = 0.9$ and $\lambda = 0.56\ \mathrm{GeV}^{-1}$. The coupling $g_{D^*D\pi}$ has been extracted by CLEO to be $17.9 \pm 0.3 \pm 1.9$ from the measured D^{*+} width [35].

B. $B^0 \to \phi K_S$ as an example

We next proceed to study long-distance rescattering contributions to the $b \to sq\bar{q}$ transition-induced decays $\overline{B}^0 \to (\phi, \omega, \rho^0, \eta^{(\prime)}, \pi^0, f_0)K_S$. To illustrate the calculations of rescattering amplitudes, we shall take the ϕK_S mode as an example. Its major final-state rescattering diagrams are depicted in Fig. 1.

The absorptive parts of the $\overline{B}^0 \to K^-\rho^+ \to \phi\overline{K}^0$ amplitude via the $K^\pm$, $K^{*\pm}$ exchanges are given by

[3] The $\rho\pi\pi$ coupling defined here differs from that in [15] by a factor of $1/\sqrt{2}$.

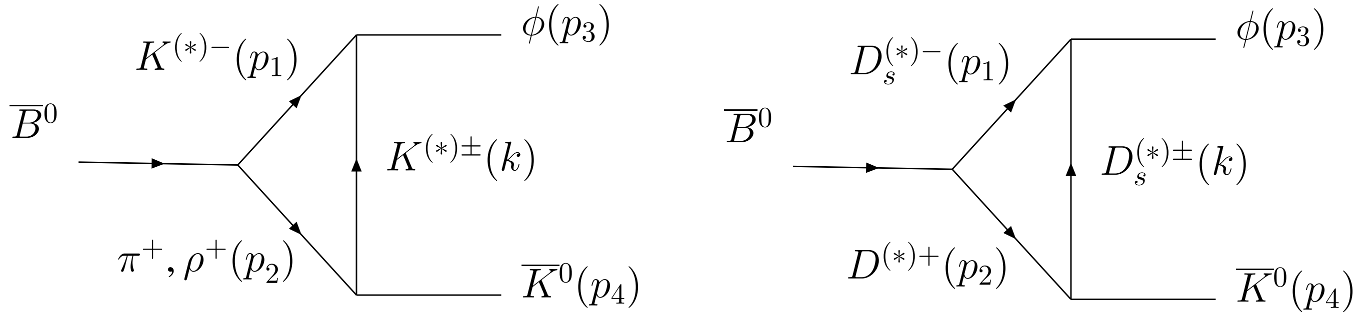

FIG. 1. Final-state rescattering contributions to the $\overline{B}^0 \to \phi \overline{K}^0$ decay.

$$\begin{aligned}\mathcal{A}bs(K^-\rho^+; K^\pm) &= \frac{1}{2}\int \frac{d^3\vec{p}_1}{(2\pi)^3 2E_1}\frac{d^3\vec{p}_2}{(2\pi)^3 2E_2}(2\pi)^4\delta^4(p_B - p_1 - p_2)\frac{A(\overline{B}^0 \to K^-\rho^+)}{2\varepsilon_2^* \cdot p_B} \\ &\quad \times \sum_{\lambda_2} 2\varepsilon_2^* \cdot p_B(-2i)g_{\phi KK}(\varepsilon_3^* \cdot p_1)\frac{F^2(t, m_K)}{t - m_K^2} 2ig_{\rho KK}(\varepsilon_2 \cdot p_4) \\ &= 2\varepsilon_3^* \cdot p_B \times \frac{1}{2}\int \frac{d^3\vec{p}_1}{(2\pi)^3 2E_1}\frac{d^3\vec{p}_2}{(2\pi)^3 2E_2}(2\pi)^4\delta^4(p_B - p_1 - p_2)\frac{A(\overline{B}^0 \to K^-\rho^+)}{2\varepsilon_2^* \cdot p_B} 4g_{\phi KK}\frac{F^2(t, m_K)}{t - m_K^2} \\ &\quad \times g_{\rho KK}(A_1^{(1)} - A_2^{(1)})\left(-p_1 \cdot p_4 + \frac{(p_1 \cdot p_2)(p_2 \cdot p_4)}{m_2^2}\right),\end{aligned} \tag{3.9}$$

$$\begin{aligned}\mathcal{A}bs(K^-\rho^+; K^{*\pm}) &= \frac{1}{2}\int \frac{d^3\vec{p}_1}{(2\pi)^3 2E_1}\frac{d^3\vec{p}_2}{(2\pi)^3 2E_2}(2\pi)^4\delta^4(p_B - p_1 - p_2)\frac{A(\overline{B}^0 \to K^-\rho^+)}{2\varepsilon_2^* \cdot p_B} \\ &\quad \times \sum_{\lambda_2} 2\varepsilon_2^* \cdot p_B(-i)g_{\phi K^* K}[p_{1,\mu}, p_3, \varepsilon_3^*]\left(-g^{\mu\nu} + \frac{k^\mu k^\nu}{m_{K^*}^2}\right)\frac{F^2(t, m_{K^*})}{t - m_{K^*}^2}(-i)g_{\rho K^* K}[p_{4,\nu}, p_2, \varepsilon_2] \\ &= 2\varepsilon_3^* \cdot p_B \times \frac{1}{2}\int \frac{d^3\vec{p}_1}{(2\pi)^3 2E_1}\frac{d^3\vec{p}_2}{(2\pi)^3 2E_2}(2\pi)^4\delta^4(p_B - p_1 - p_2)\frac{A(\overline{B}^0 \to K^-\rho^+)}{2\varepsilon_2^* \cdot p_B} g_{\phi K^* K}\frac{F^2(t, m_{K^*})}{t - m_{K^*}^2} \\ &\quad \times g_{\rho K^* K}(-2A_1^{(2)} m_3^2),\end{aligned} \tag{3.10}$$

where the dependence of the polarization vector in the amplitude $A(\overline{B}^0 \to K^-\rho^+)$ has been extracted and $k \equiv p_1 - p_3$. In order to avoid using too many dummy indices, we have defined $[A, B, C, D] \equiv \epsilon_{\alpha\beta\gamma\delta}A^\alpha B^\beta C^\gamma D^\delta$, $[A, B, C,_\mu] \equiv \epsilon_{\alpha\beta\gamma\mu}A^\alpha B^\beta C^\gamma$ and so on for later convenience. Moreover, we have applied the identities [15]

$$\begin{aligned}p_{1\mu} &\doteq P_\mu A_1^{(1)} + q_\mu A_2^{(1)}, \\ p_{1\mu}p_{1\nu} &\doteq g_{\mu\nu}A_1^{(2)} + P_\mu P_\nu A_2^{(2)} + (P_\mu q_\nu + q_\mu P_\nu)A_3^{(2)} \\ &\quad + q_\mu q_\nu A_4^{(2)},\end{aligned} \tag{3.11}$$

under the integration

$$\int \frac{d^3\vec{p}_1}{(2\pi)^3 2E_1}\frac{d^3\vec{p}_2}{(2\pi)^3 2E_2}(2\pi)^4\delta^4(p_B - p_1 - p_2)f(t) \times \{p_{1\mu}, p_{1\mu}p_{1\nu}\}, \tag{3.12}$$

with $P \equiv p_3 + p_4$, $q \equiv p_3 - p_4$. These identities follow from the fact that the above integration can be expressed only in terms of the external momenta p_3, p_4 with suitable Lorentz and permutation structures. The explicit expressions of $A_j^{(i)} = A_j^{(i)}(t, m_B^2, m_1^2, m_2^2, m_3^2, m_4^2)$ can be found in [15].

Before proceeding it should be stressed that we have applied the hidden gauge symmetry Lagrangian Eq. (3.6) for light vector mesons and the chiral Lagrangian Eq. (3.7), based on heavy quark effective theory (HQET) and chiral symmetry, for heavy mesons to determine the strong vertices in Fig. 1. This requires that the involved light pseudoscalar or vector mesons be soft. However, the final-state particles are necessarily hard and the particle exchanged in the t channel can be far off shell, especially for the t-exchanged D meson. This is beyond the applicability of the aforementioned chiral perturbation theory and HQET. Therefore, as stressed in [15], it is necessary to introduce the form factor $F(t, m)$ appearing in Eqs. (3.9) and (3.10) to take care of the off-shell effect of the t-channel exchanged particle and the hardness of the final particles. Indeed, if the off-shell effect is not considered, the long-distance rescattering contributions will become so large that perturbation theory is no longer trustworthy. For example, since $B \to D_s \bar{D}$ is CKM doubly enhanced relative to $B \to K\pi$, the rescattering process $B \to D_s \bar{D} \to K\pi$ will overwhelm the initial $B \to K\pi$ amplitude. Hence, form factors or cutoffs must be introduced to the strong vertices to render the calculation meaningful in perturbation theory.

The form factor $F(t, m)$ is usually parametrized as

$$F(t, m) = \left(\frac{\Lambda^2 - m^2}{\Lambda^2 - t}\right)^n, \tag{3.13}$$

normalized to unity at $t = m^2$ with Λ being a cutoff parameter which should not be far from the physical mass of the exchanged particle. To be specific, we write

$$\Lambda = m_{\rm exc} + \eta \Lambda_{\rm QCD}, \tag{3.14}$$

where the parameter η is expected to be of order unity and it depends not only on the exchanged particle but also on the external particles involved in the strong-interaction vertex. The monopole behavior of the form factor (i.e. $n = 1$) is preferred as it is consistent with the QCD sum-rule expectation [36]. Although the strong couplings are large in the magnitude, the rescattering amplitude is suppressed by a factor of $F^2(t) \sim m^2\Lambda^2_{\rm QCD}/t^2$. Consequently, the off-shell effect will render the perturbative calculation meaningful. Moreover, since in the heavy quark limit $t \sim m_B^2$, the final-state rescattering amplitude does vanish in the $m_B \to \infty$ limit, as it should.

Likewise, the absorptive part of the $\overline{B}^0 \to K^{*-}\pi^+ \to \phi\overline{K}^0$ amplitude via the $K^{*\pm}$ exchange is given by

$$\begin{aligned}
\mathcal{A}bs(K^{*-}\pi^+; K^{*\pm}) &= \frac{1}{2}\int \frac{d^3\vec{p}_1}{(2\pi)^3 2E_1}\frac{d^3\vec{p}_2}{(2\pi)^3 2E_2}(2\pi)^4\delta^4(p_B - p_1 - p_2)\frac{A(\overline{B}^0 \to K^{*-}\pi^+)}{2\varepsilon_1^*\cdot p_B} \\
&\quad \times \sum_{\lambda_1} 2\varepsilon_1^*\cdot p_B\, i\frac{g_V}{2}[\varepsilon^*_{3\mu}(2\varepsilon_1\cdot p_3) - \varepsilon_1\cdot\varepsilon_3^*(p_1+p_3)_\mu + \varepsilon_{1\mu}(2p_1\cdot\varepsilon_3)]\left(-g^{\mu\nu} + \frac{k^\mu k^\nu}{m^2_{K^*}}\right) \\
&\quad \times \frac{F^2(t, m_{K^*})}{t - m^2_{K^*}}(-i)g_{K^*K\pi}(p_2 + p_4)_\nu \\
&= 2\varepsilon_3^*\cdot p_B \times \frac{1}{2}\int \frac{d^3\vec{p}_1}{(2\pi)^3 2E_1}\frac{d^3\vec{p}_2}{(2\pi)^3 2E_2}(2\pi)^4\delta^4(p_B - p_1 - p_2)\frac{A(\overline{B}^0 \to K^{*-}\pi^+)}{2\varepsilon_1^*\cdot p_B}g_V\frac{F^2(t, m_{K^*})}{2(t - m^2_{K^*})} \\
&\quad \times g_{K^*K\pi}\left\{2\left[2 - A_1^{(1)} + A_2^{(1)} + (A_1^{(1)} - A_2^{(1)})\frac{m_2^2 - m_4^2}{m^2_{K^*}}\right]\left(p_2\cdot p_3 - \frac{(p_1\cdot p_2)(p_2\cdot p_3)}{m_1^2}\right)\right. \\
&\quad + \left[A_1^{(1)} - A_2^{(1)} - 1 + (A_1^{(1)} - A_2^{(1)})\frac{p_1\cdot p_2}{m_1^2}\right]\left[(p_1+p_3)\cdot(p_2+p_4) + \frac{1}{m^2_{K^*}}(m_1^2 - m_3^2)(m_2^2 - m_4^2)\right] \\
&\quad \left. + 2(A_1^{(1)} - A_2^{(1)})\left[\left(p_2\frac{m^2_{K^*} - m_2^2 + m_4^2}{m^2_{K^*}} + p_4\frac{m^2_{K^*} + m_2^2 - m_4^2}{m^2_{K^*}}\right)\cdot\left(p_2 - p_1\frac{p_1\cdot p_2}{m_1^2}\right)\right]\right\}.
\end{aligned} \tag{3.15}$$

Note that since the πKK vertex is absent, there is no contribution from the K^+ exchanged particles.

The $\overline{B}^0 \to \phi\overline{K}^0$ decay also receives contributions from charmless VV modes. The leading candidate is the $K^{*-}\rho^+$ mode via the p-wave configuration. However, as we have checked numerically, its amplitude is 1 or 2 orders of magnitude smaller than those from the previous two charmless VP modes and its effect on S_f is quite small. Given this, we will not go into any further detail on the rescattering from charmless VV modes. Thus far we have only considered the contributions where the two intermediate states originating from the weak vertex are on shell. There are additional contributions where one of the mesons coming from the weak vertex and the exchanged meson in the t channel is on shell. For example, in the diagram of Fig. 1(a), we can set K^- and K^+ on shell while keeping ρ^+ off shell. This corresponds to the three-body weak decay $\overline{B}^0 \to K^+K^-\overline{K}^0$ followed by the strong rescattering $K^+K^-\overline{K}^0 \to \phi\overline{K}^0$ where $\overline{K}^0$ behaves as a spectator. However, there are many possible pole contributions to $\overline{B}^0 \to K^+K^-\overline{K}^0$. In addition to $\overline{B}^0 \to K^-\rho^+ \to K^+K^-\overline{K}^0$ as inferred from Fig. 1(a), one can also have $\overline{B}^0 \to \overline{B}_s^*\overline{K}^0 \to K^+K^-\overline{K}^0$, for example. In the present work, we will only focus on two-body intermediate state contributions to the absorptive part. Since the analogous three-body contributions do not occur in Fig. 1(b) and since Fig. 1(a) is CKM doubly suppressed relative to Fig. 1(b), it is safe to neglect the additional three-body contributions for our purposes.

We next turn to the FSI contribution arising from the intermediate states $D_s^{(*)-}D^{(*)}$. Note that only the p-wave configurations of the $D_s^{(*)-}D^{(*)}$ systems can rescatter into the $\phi\overline{K}^0$ final state. The absorptive parts of $\overline{B}^0 \to D_s^{*-}D^+ \to \phi\overline{K}^0$ amplitudes via D_s^* exchanges, $\overline{B} \to D_s^{*-}D^+, D_s^{*-}D^{*+} \to \phi\overline{K}^0$ amplitudes via D_s and D_s^* exchanges, are given by

$$\begin{aligned}\mathcal{A}bs(D_s^{*-}D^+;D_s^{*\pm}) &= \frac{1}{2}\int\frac{d^3\vec{p}_1}{(2\pi)^3 2E_1}\frac{d^3\vec{p}_2}{(2\pi)^3 2E_2}(2\pi)^4\delta^4(p_B-p_1-p_2)\frac{A(\overline{B}^0\to D_s^{*-}D^+)}{2\varepsilon_1^*\cdot P}\\ &\quad\times\sum_{\lambda_1}2\varepsilon_1^*\cdot P(-4i)f_{D_s^*D_s^*\phi}\frac{F^2(t,m_{D_s^*})}{t-m_{D_s^*}^2}ig_{D_s^*DK}\varepsilon_1^\rho(\sigma g_{\rho\mu}p_1\cdot\varepsilon_3^*+p_{3\rho}\varepsilon_{3\mu}^*-\varepsilon_{3\rho}^*p_{3\mu})\\ &\quad\times\left(-g^{\mu\nu}+\frac{k^\mu k^\nu}{m_{D_s^*}^2}\right)p_{4\nu}\\ &= 2\varepsilon_3^*\cdot p_B\times\frac{1}{2}\int\frac{d^3\vec{p}_1}{(2\pi)^3 2E_1}\frac{d^3\vec{p}_2}{(2\pi)^3 2E_2}(2\pi)^4\delta^4(p_B-p_1-p_2)\frac{A(\overline{B}^0\to D_s^{*-}D^+)}{2\varepsilon_1^*\cdot P}\\ &\quad\times 4f_{D_s^*D_s^*\phi}\frac{F^2(t,m_{D_s^*})}{t-m_{D_s^*}^2}g_{D_s^*DK}\Bigg\{\sigma(A_1^{(1)}-A_2^{(1)})\left(p_2-\frac{p_1\cdot p_2}{m_1^2}p_1\right)\cdot\left(p_4-k\frac{k\cdot p_4}{m_{D_s^*}^2}\right)\\ &\quad+\left[1-(A_1^{(1)}-A_2^{(1)})\frac{k\cdot p_4}{m_{D_s^*}^2}\right]\left(p_2\cdot p_3-\frac{p_1\cdot p_2 p_1\cdot p_3}{m_1^2}\right)-\left[1-(A_1^{(1)}-A_2^{(1)})\frac{P\cdot p_1}{m_1^2}\right]\\ &\quad\times\left(p_3\cdot p_4-\frac{p_3\cdot kk\cdot p_4}{m_{D_s^*}^2}\right)\Bigg\},\end{aligned}$$

$$\begin{aligned}\mathcal{A}bs(D_s^-D^{*+};D_s^{\pm}) &= \frac{1}{2}\int\frac{d^3\vec{p}_1}{(2\pi)^3 2E_1}\frac{d^3\vec{p}_2}{(2\pi)^3 2E_2}(2\pi)^4\delta^4(p_B-p_1-p_2)\frac{A(\overline{B}^0\to D_s^-D^{*+})}{2\varepsilon_2^*\cdot P}\\ &\quad\times\sum_{\lambda_2}2\varepsilon_2^*\cdot P2ig_{D_sD_s\phi}\varepsilon_3^*\cdot p_1\frac{F^2(t,m_{D_s})}{t-m_{D_s}^2}(-i)g_{D^*D_sK}p_4\cdot\varepsilon_2\\ &= 2\varepsilon_3^*\cdot p_B\times\frac{1}{2}\int\frac{d^3\vec{p}_1}{(2\pi)^3 2E_1}\frac{d^3\vec{p}_2}{(2\pi)^3 2E_2}(2\pi)^4\delta^4(p_B-p_1-p_2)\frac{A(\overline{B}^0\to D_s^-D^{*+})}{2\varepsilon_2^*\cdot P}\\ &\quad\times\left\{2g_{D_sD_s\phi}\frac{F^2(t,m_{D_s})}{t-m_{D_s}^2}g_{D^*D_sK}(A_1^{(1)}-A_2^{(1)})\left(-p_1\cdot p_4+\frac{p_1\cdot p_2 p_2\cdot p_4}{m_2^2}\right)\right\},\end{aligned}$$

$$\begin{aligned}\mathcal{A}bs(D_s^-D^{*+};D_s^{*\pm}) &= \frac{1}{2}\int\frac{d^3\vec{p}_1}{(2\pi)^3 2E_1}\frac{d^3\vec{p}_2}{(2\pi)^3 2E_2}(2\pi)^4\delta^4(p_B-p_1-p_2)\frac{A(\overline{B}^0\to D_s^-D^{*+})}{2\varepsilon_2^*\cdot P}\\ &\quad\times\sum_{\lambda_2}2\varepsilon_2^*\cdot P4if_{D_s^*D_s\phi}\frac{F^2(t,m_{D_s^*})}{t-m_{D_s^*}^2}ig_{D^*D_s^*K}[p_3,\varepsilon_3^*,p_1,_\mu]\left(-g^{\mu\nu}+\frac{k^\mu k^\nu}{m_{D_s^*}^2}\right)[\varepsilon_2,p_4,p_2,_\nu]\\ &= 2\varepsilon_3^*\cdot p_B\times\frac{1}{2}\int\frac{d^3\vec{p}_1}{(2\pi)^3 2E_1}\frac{d^3\vec{p}_2}{(2\pi)^3 2E_2}(2\pi)^4\delta^4(p_B-p_1-p_2)\\ &\quad\times\frac{A(\overline{B}^0\to D_s^-D^{*+})}{2\varepsilon_2^*\cdot P}(-8)f_{D_s^*D_s\phi}\frac{F^2(t,m_{D_s^*})}{t-m_{D_s^*}^2}g_{D^*D_s^*K}m_3^2A_1^{(2)},\end{aligned}$$

$$\begin{aligned}\mathcal{A}bs(D_s^{*-}D^{*+};D_s^{\pm}) &= \frac{1}{2}\int\frac{d^3\vec{p}_1}{(2\pi)^3 2E_1}\frac{d^3\vec{p}_2}{(2\pi)^3 2E_2}(2\pi)^4\delta^4(p_B-p_1-p_2)(ic[_\mu,_\nu,P,p_2])\\ &\quad\times\sum_{\lambda_1,\lambda_2}\varepsilon_1^{*\mu}\varepsilon_2^{*\nu}(-4i)f_{D_s^*D_s\phi}\frac{F^2(t,m_{D_s})}{t-m_{D_s}^2}(-i)g_{D^*D_sK}[p_3,\varepsilon_3^*,p_1,\varepsilon_1]p_4\cdot\varepsilon_2\\ &= 2\varepsilon_3^*\cdot p_B\times\frac{1}{2}\int\frac{d^3\vec{p}_1}{(2\pi)^3 2E_1}\frac{d^3\vec{p}_2}{(2\pi)^3 2E_2}(2\pi)^4\delta^4(p_B-p_1-p_2)(-4ic)f_{D_s^*D_s\phi}\frac{F^2(t,m_{D_s})}{t-m_{D_s}^2}\\ &\quad\times g_{D^*D_sK}m_3^2A_1^{(2)},\end{aligned}$$

$$
\begin{aligned}
\mathcal{A}bs(D_s^{*-}D^{*+}; D_s^{*\pm}) &= \frac{1}{2}\int \frac{d^3\vec{p}_1}{(2\pi)^3 2E_1}\frac{d^3\vec{p}_2}{(2\pi)^3 2E_2}(2\pi)^4\delta^4(p_B - p_1 - p_2)(ic[_{\delta}, _{\beta}, P, p_2]) \\
&\quad \times \sum_{\lambda_1,\lambda_2} \varepsilon_1^{*\delta}\varepsilon_2^{*\beta}(-4i)f_{D_s^*D_s^*\phi}\frac{F^2(t, m_{D_s^*})}{t - m_{D_s^*}^2} i g_{D^*D_s^*K}\varepsilon_1^{\rho}(\sigma g_{\rho\mu} p_1\cdot\varepsilon_3^* + p_{3\rho}\varepsilon_{3\mu}^* - \varepsilon_{3\rho}^* p_{3\mu}) \\
&\quad \times \left(-g^{\mu\nu} + \frac{k^\mu k^\nu}{m_{D_s^*}^2}\right)[\varepsilon_2, p_4, p_{2,\nu}] \\
&= 2\varepsilon_3^*\cdot p_B \times \frac{1}{2}\int \frac{d^3\vec{p}_1}{(2\pi)^3 2E_1}\frac{d^3\vec{p}_2}{(2\pi)^3 2E_2}(2\pi)^4\delta^4(p_B - p_1 - p_2)(-4ic)f_{D_s^*D_s^*\phi}\frac{F^2(t, m_{D_s^*})}{t - m_{D_s^*}^2} \\
&\quad \times g_{D^*D_s^*K^*}[\sigma(A_1^{(1)} - A_2^{(1)})(p_1\cdot p_4 m_2^2 - p_1\cdot p_2 p_2\cdot p_4) + m_3^2 A_1^{(2)}],
\end{aligned} \tag{3.16}
$$

where the dependence of the polarization vectors in $A(\overline{B}^0 \to D_s^{*-}D^+, D_s^- D^{*+})$ has been extracted out explicitly and $\sigma \equiv g_{D^*D^*V}/(2f_{D^*D^*V})$ and the $\overline{B} \to D_s^{*-}D^{*+}$ decay amplitude has been denoted as

$$
A(\overline{B} \to D_s^*(p_1, \lambda_1)D^*(p_2, \lambda_2)) = \varepsilon_1^{*\mu}\varepsilon_2^{*\nu}(a g_{\mu\nu} + bP_\mu P_\nu + ic[_\mu, _\nu, P, p_2]). \tag{3.17}
$$

In order to perform a numerical study of the above analytic results, we need to specify the short-distance $A(\overline{B} \to D_s^{(*)}D^{(*)})$ amplitudes. In the factorization approach, we have

$$
\begin{aligned}
A(\overline{B} \to D_s^* D)_{\rm SD} &= \frac{G_{\rm F}}{\sqrt{2}} V_{cb}V_{cs}^* a_1 f_{D_s^*} m_{D_s^*} F_1^{BD}(m_{D_s^*}^2)(2\varepsilon_{D_s^*}^*\cdot p_B), \\
A(\overline{B} \to D_s D^*)_{\rm SD} &= \frac{G_{\rm F}}{\sqrt{2}} V_{cb}V_{cs}^* a_1 f_{D_s} m_{D^*} A_0^{BD^*}(m_{D_s}^2)(2\varepsilon_{D^*}^*\cdot p_B), \\
A(\overline{B} \to D_s^* D^*)_{\rm SD} &= -i\frac{G_{\rm F}}{\sqrt{2}} V_{cb}V_{cs}^* a_1 f_{D_s^*} m_{D_s^*}(m_B + m_{D^*})\varepsilon_{D_s^*}^{*\mu}\varepsilon_{D^*}^{*\nu}\left[A_1^{BD^*}(m_{D_s^*}^2)g_{\mu\nu} - \frac{2A_2^{BD^*}(m_{D_s^*}^2)}{(m_B + m_{D^*})^2}p_{B\mu}p_{B\nu}\right. \\
&\quad \left. - i\frac{2V^{BD^*}(m_{D_s^*}^2)}{(m_B + m_{D^*})^2}\epsilon_{\mu\nu\alpha\beta}p_B^\alpha p_{D^*}^\beta\right].
\end{aligned} \tag{3.18}
$$

Since the phase of the parameter a_1 originating from vertex corrections [see Eq. (2.5)] is very small, one can neglect the strong phase of the short-distance amplitudes.

The long-distance contribution to the $\overline{B}^0 \to \omega\overline{K}^0$ decay can be performed similarly. Because of the absence of the PPP vertex and the G-parity argument, the number of FSI diagrams from charmless intermediate states is greatly reduced compared to the previous case. For example, the $K^-\rho^+$ intermediate state does not contribute to the $\overline{K}^0\omega$ amplitude (through t-channel π and ρ exchanges) as both $KK\pi$ and $\rho\rho\omega$ vertices are forbidden. In fact, there is only one relevant rescattering diagram, namely, $\overline{B}^0 \to K^{*-}\pi^+ \to \overline{K}^0\omega$ via $\rho^\pm$ exchange, arising from charmless intermediate states and the corresponding absorptive part is given by

$$
\begin{aligned}
\mathcal{A}bs(K^{*-}\pi^+; \rho^\pm) &= \frac{1}{2}\int \frac{d^3\vec{p}_1}{(2\pi)^3 2E_1}\frac{d^3\vec{p}_2}{(2\pi)^3 2E_2}(2\pi)^4\delta^4(p_B - p_1 - p_2)\frac{A(\overline{B}^0 \to K^{*-}\pi^+)}{2\varepsilon_1^*\cdot p_B} \\
&\quad \times \sum_{\lambda_1} 2\varepsilon_1^*\cdot p_B(ig_{\rho K^*K})(i\sqrt{2}g_{\omega\rho\pi})[p_1, \varepsilon_1, k, _\mu][k, _\nu, p_4, \varepsilon_4^*]\left(-g^{\mu\nu} + \frac{k^\mu k^\nu}{m_{K^*}^2}\right)\frac{F^2(t, m_\rho)}{t - m_\rho^2} \\
&= 2\varepsilon_4^*\cdot p_B \times \frac{1}{2}\int \frac{d^3\vec{p}_1}{(2\pi)^3 2E_1}\frac{d^3\vec{p}_2}{(2\pi)^3 2E_2}(2\pi)^4\delta^4(p_B - p_1 - p_2)\frac{A(\overline{B}^0 \to K^{*-}\pi^+)}{2\varepsilon_1^*\cdot p_B} \\
&\quad \times (-2\sqrt{2})g_{\rho K^*K}\frac{F^2(t, m_\rho)}{(t - m_\rho^2)}g_{\omega\rho\pi}m_4^2 A_1^{(2)}.
\end{aligned} \tag{3.19}
$$

Furthermore, we have checked numerically that the $\overline{B}^0 \to K^{*-}\rho^+ \to \overline{K}^0\omega$ contribution is small. Hence we will skip further detail on the rescattering from charmless VV modes.

The rescattering amplitudes of $\overline{B}^0 \to D_s^{*-}D^+$, D^-D^{*+}, $D^{*-}D^{*+} \to \overline{K}^0\omega$ amplitudes via D, D^* exchanges can be evaluated in a similar way. The analytic expressions of their absorptive parts are similar to those for $\phi\overline{K}^0$. For example,

$$\mathcal{A}bs(D_s^{*-}D^+;D^\pm) = 2\varepsilon_4^*\cdot p_B \times \frac{1}{2}\int \frac{d^3\vec{p}_1}{(2\pi)^3 2E_1}\frac{d^3\vec{p}_2}{(2\pi)^3 2E_2}(2\pi)^4\delta^4(p_B-p_1-p_2)\frac{A(\overline{B}^0\to D_s^- D^{*+})}{2\varepsilon_2^*\cdot P} \times\left\{\sqrt{2}g_{D_s^*DK}\frac{F^2(t,m_D)}{t-m_D^2}g_{DD\omega}(1-A_1^{(1)}-A_2^{(1)})\left(-p_2\cdot p_3+\frac{p_1\cdot p_2 p_1\cdot p_3}{m_1^2}\right)\right\} \tag{3.20}$$

is the same as $\mathcal{A}bs(D_s^- D^{*+};D_s^\pm)$ in (3.16) after the replacements $g_{D^*D_sK}\to g_{D_s^*DK}$, $f_{D_sD_s\phi}\to f_{DD\omega}/\sqrt{2}$, $p_1\leftrightarrow p_2$, $p_3\leftrightarrow p_4$, $A_1^{(1)}-A_2^{(1)}\to 1-A_1^{(1)}-A_2^{(1)}$ and a suitable replacement of the source amplitude (note that the replacement of momentum should not be performed in $A_j^{(i)}$). Likewise, we can obtain $\mathcal{A}bs(D_s^{*-}D^+;D^{*\pm})$, $\mathcal{A}bs(D_s^-D^{*+};D^{*\pm})$, $\mathcal{A}bs(D_s^{*-}D^{*+},D^{(*)\pm})$ from $\mathcal{A}bs(D_s^-D^{*+};D_s^{*\pm})$, $\mathcal{A}bs(D_s^{*-}D^+;D_s^{*\pm})$, $\mathcal{A}bs(D_s^{*-}D^{*+}, D_s^{(*)\pm})$ in (3.16), respectively, with $A_1^{(2)}$ being unchanged and an additional overall minus sign for $D_s^{*-}D^{*+}$ contributions. This similarity is by no means accidental; it follows from the so-called CPS symmetry, i.e. CP plus $s\leftrightarrow d$ switch symmetry. A similar but more detailed discussion is given in [15] for the case of $B\to\phi K^*$ decay.

The final-state rescattering contributions to other penguin-dominated decays $B^0\to(\rho^0,\eta^{(\prime)},f_0)K_S$ can be worked out in a similar manner. The dominant intermediate states for each decay mode are summarized in Table IV.

C. Results and discussions

Writing $A = A^{\rm SD}+i\mathcal{A}bsA^{\rm LD}$ with $\mathcal{A}bsA^{\rm LD}$ obtained above and the form factors given in [24], the results of the final-state rescattering effects on decay rates, direct and mixing-induced CP violation parameters are shown in Tables II and III. As pointed out in [15], the long-distance rescattering effects are sensitive to the cutoff parameter Λ appearing in Eq. (3.14) or η in Eq. (3.14). Since we do not have first-principles calculations of η, we will determine it from the measured branching ratios and then use it to predict the CP-violating parameters $\mathcal{A}_f$ and S_f. As shown in [15], $\eta=0.69$ for the exchanged particles D and D^* is obtained from fitting to the $B\to K\pi$ rates. We take $\eta = 0.85$ for the exchanged particles $D_{(s)}$ and $D_{(s)}^*$ to fit the data of $\overline{B}^0\to\eta'\overline{K}^0$ rates and $\eta=1$ for other light exchanged particles such as π, K, K^*. As for the VP modes, namely, ϕK_S, ωK_S, and ρ^0K_S, we take $\eta_{D_{(s)}}=\eta_{D_{(s)}^*}=0.95$, 1.4, and 1.5, respectively, in order to accommodate their rates.[4] Note that the result $\eta_{D_{(s)}}=\eta_{D_{(s)}^*}=1.5$ for ρ^0K_S is consistent with the value of $\eta_D=\eta_{D^*}=1.6$ obtained for $B\to\rho\pi$ decays [15] within SU(3) symmetry. For the decay $\overline{B}^0\to f_0K_S$, since only the strong couplings of f_0 to $K\bar{K}$ and $\pi\pi$ are available experimentally, we shall only consider the intermediate states $K^-\rho^+$ and π^+K^{*-}. The LD theoretical uncertainties shown in Tables II and III originate from three sources: an assignment of 15% error in $\Lambda_{\rm QCD}$, the measured error in the coupling $g_{D^*D\pi}=17.9\pm0.3\pm1.9$ [35], and 5% error in the form factors for B to $D^{(*)}$ transitions. They are obtained by scanning randomly the points in the allowed ranges of the abovementioned three parameters. The calculations of hadronic diagrams for FSIs also involve many other theoretical uncertainties, some of which are already discussed in [15]. From Tables II and III it is clear that the SD errors are in general not significantly affected by FSI effects and that LD uncertainties are in general comparable to the SD ones.

We see from Table II that final-state rescattering will enhance the decay rates of ϕK_S, ωK_S, ρ^0K_S, $\eta'K_S$, and π^0K_S but it does not affect the ηK_S rate. The seemingly large disparity between $\eta'K_S$ and ηK_S for FSIs can be understood as follows. There are two types of exchanged particles in the rescattering processes, namely, $D^{(*)}$ and $D_s^{(*)}$ (see Table IV). The former (latter) couple to the d (s) quark component of the $\eta^{(\prime)}$. Since the η' and η wave functions are approximately given by Eq. (2.16), it is clear that the rescattering amplitudes due to the exchanged particles $D^{(*)}$ and $D_s^{(*)}$ interfere constructively for the $\eta'K_S$ production but compensate largely for ηK_S.

It should be stressed that although we have used the measured branching ratio of $\eta'K^0$ to fix the LD contribu-

TABLE IV. Dominant intermediate states contributing to various final states. For the ηK_S final state, the intermediate states are the same as that for $\eta'K_S$.

Final state	Intermediate state (exchanged particle)
ϕK_S	$K^{*-}\pi^+(K^*)$, $K^-\rho^+(K,K^*)$,
	$D_s^{*-}D^+(D_s^*)$, $D_s^-D^{*+}(D_s,D_s^*)$, $D_s^{*-}D^{*+}(D_s,D_s^*)$
ωK_S	$K^{*-}\pi^+(\rho)$,
	$D_s^{*-}D^+(D,D^*)$, $D_s^-D^{*+}(D^*)$, $D_s^{*-}D^{*+}(D,D^*)$
ρ^0K_S	$K^{*-}\pi^+(\rho)$,
	$D_s^{*-}D^+(D,D^*)$, $D_s^-D^{*+}(D^*)$, $D_s^{*-}D^{*+}(D,D^*)$
$\eta'K_S$	$\eta'\bar{K}^0(K^{*0})$, $K^{*-}\rho^+(K,K^*)$
	$D_s^-D^+(D^*,D_s^*)$, $D_s^{*-}D^{*+}(D,D_s,D^*,D_s^*)$
π^0K_S	$\eta'\bar{K}^0(K^{*0})$,
	$D_s^-D^+(D^*)$, $D_s^{*-}D^{*+}(D,D^*)$
f_0K_S	$\rho^+K^-(K,\rho)$, $\pi^+K^{*-}(\pi)$,
	$D_s^{*-}D^+(D,D_s^*)$, $D_s^-D^{*+}(D^*)$

[4] Note that a monopole momentum dependence is used for the form factors throughout this paper [see Eq. (3.13)]. If a dipole form of the momentum dependence is used for D^*-exchange diagrams as in [15], we will obtain $\mathcal{B}^{\rm SD+LD}(\phi K^0)=(6.1^{+2.0+0.4}_{-1.9-0.2})\times10^{-6}$, $S^{\rm SD+LD}_{\phi K_S}=0.723^{+0.004+0.009}_{-0.036-0.012}$, $\mathcal{A}^{\rm SD+LD}_{\phi K_S}=-0.070^{+0.015}_{-0.017}\pm0.019$, $\mathcal{B}^{\rm SD+LD}(\omega K^0)=(2.5^{+3.7+0.6}_{-1.4-0.3})\times10^{-6}$, $S^{\rm SD+LD}_{\omega K_S}=0.847^{+0.029+0.007}_{-0.054-0.013}$, and $\mathcal{A}^{\rm SD+LD}_{\omega K_S}=0.013^{+0.034+0.027}_{-0.078-0.029}$.

tions and the unknown cutoff parameter η, there exist some other possible mechanisms that can help explain its large rate. For example, the QCD anomaly effect manifested in the two gluon coupling with the η' may provide a dynamical enhancement of the $\eta' K^0$ production [37]. And it is likely that both final-state rescattering and the gluon anomaly are needed to account for the unexpectedly large branching fraction of $\eta' K$. Note that both contributions carry negligible CP-odd phase. Hence, whether the anomalously large branching ratio of $\eta' K$ comes from the QCD anomaly and/or from final-state rescattering, it will be very effective in diluting the $u\bar{u}$ tree contributions and rendering $\Delta S_{\eta' K_S}$ small.

We are not able to estimate the long-distance rescattering contributions to the $f_0 K_S$ rate from intermediate charm states due to the absence of information on $f_0 DD$ and $f_0 D^*_{(s)} D^*_{(s)}$ couplings.

Since the modes ωK_S, $\rho^0 K_S$, ϕK_S, and $\pi^0 K_S$ receive significant final-state rescattering contributions, it is natural to expect that their direct CP asymmetries will be affected accordingly. It is clear from Table III that the signs of $\mathcal{A}_f$ in the last two channels are flipped by final-state interactions. As for the mixing-induced CP violation S_f, we see from Table III that ωK_S and $\rho^0 K_S$ receive the largest corrections from final-state rescattering, while the long-distance correction to ϕK_S is not as large as what was originally expected. The underlying reason is as follows. The mixing CP asymmetries $S^{\rm SD}_{\omega K_s}$ and $S^{\rm SD}_{\rho^0 K_S}$ deviate from $\sin 2\beta$ as they receive contributions from the tree amplitude. The contribution from the tree amplitude is relatively enhanced as the QCD penguin amplitude is suppressed by a cancellation between penguin terms ($|a_4 - r_\chi a_6| \ll |a_4|, |a_6|$). Final-state rescattering from $D_s^{(*)} D^{(*)}$ states with vanishing weak phases will dilute the SD tree contribution and bring the asymmetry closer to $\sin 2\beta$. For the ϕK_S mode, the asymmetry $S^{\rm SD}_{\phi K_S}$ is slightly greater than $\sin 2\beta$. With rescattering from either charmless intermediate states, such as $K^*\pi$, or from $D_s^{(*)} D^{(*)}$ states, the asymmetry is reduced to $S_{\phi K_S} \simeq 0.73$. When both charmless and charmful intermediate states are considered, the asymmetry is enhanced to $S_{\phi K_S} \simeq 0.76$ owing to the interference effect from these two contributions.

Among the seven modes $(\phi, \omega, \rho^0, \pi^0, \eta^{(\prime)}, f_0) K_S$, the first three are the states where final-state interactions may have a potentially large effect on the mixing-induced CP asymmetry S_f. In order to maximize the effects of FSIs on S_f, one should consider rescattering from charmless intermediate states that receive sizable tree contributions. Most of the intermediate states such as $\overline{K}^0 \eta'$, $K^{*-}\rho^+, \ldots$, in B decays are penguin dominated and hence will not affect S_f. For the decay $\overline{B}^0 \to \phi K_S$, we have rescattering from $K^-\rho^+$ and $K^{*-}\pi^+$. Because of the absence of the penguin chiral enhancement in $\overline{B}^0 \to K^{*-}\pi^+$ and the large cancellation between a_4 and a_6 penguin terms in $\overline{B}^0 \to K^-\rho^+$, it follows that the color-allowed tree contributions in these two modes are either comparable to or slightly smaller than the penguin effects. As for the ωK_S mode, there is only one rescattering diagram, namely, $\overline{B}^0 \to K^{*-}\pi^+ \to \overline{K}^0 \omega$, arising from the charmless intermediate states. (The rescattering diagram from $K^{*-}\rho^+$ is suppressed as elucidated before for the ϕK_S mode.) As a result, one will expect that the final-state rescattering effect on S_f will be most prominent in $B \to \omega K_S$, $\rho^0 K_S$, and ϕK_S. Indeed, we see from Table III that FSI lowers $S_{\omega K_S}$ by 15% and enhances $S_{\rho^0 K_S}$ by 17% and $S_{\phi K_S}$ slightly. The theoretical predictions and experimental measurements for the differences between $S_f^{\rm SD+LD}$ and $S_{J/\psi K_S}$, $\Delta S_f^{\rm SD+LD}$, are summarized in Table V. It is evident that final-state interactions cannot induce large ΔS_f in any of these modes.

It is interesting to study the correlation between $\mathcal{A}_f$ and S_f for the penguin-dominated modes in the presence of FSIs. It follows from Eq. (1.8) that

$$\frac{\Delta S_f}{\mathcal{A}_f} = \cos 2\beta \cot\delta_f \approx 0.95 \cot\delta_f, \tag{3.21}$$

for $r_f = (\lambda_u A_f^u)/(\lambda_c A_f^c) \ll 1$. This ratio is independent of $|r_f|$ and hence it is less sensitive to hadronic uncertainties. Therefore, it may provide a better test of the SM even in the presence of FSIs. Writing $A_f^{u,c} = |A^{u,c}_{\rm SD}| e^{i\delta^{u,c}_{\rm SD}} + iA^{u,c}_{\rm LD}$, we have

$$|r_f| = \frac{|\lambda^u|}{|\lambda^c|} \frac{\sqrt{|A^u_{\rm SD}\cos\delta^u_{\rm SD}|^2 + (|A^u_{\rm SD}|\sin\delta^u_{\rm SD} + A^u_{\rm LD})^2}}{\sqrt{|A^c_{\rm SD}\cos\delta^c_{\rm SD}|^2 + (|A^c_{\rm SD}|\sin\delta^c_{\rm SD} + A^c_{\rm LD})^2}},$$

$$\delta_f = \tan^{-1}\left(\tan\delta^u_{\rm SD} + \frac{A^u_{\rm LD}}{|A^u_{\rm SD}|\cos\delta^u_{\rm SD}}\right) - \tan^{-1}\left(\tan\delta^c_{\rm SD} + \frac{A^c_{\rm LD}}{|A^c_{\rm SD}|\cos\delta^c_{\rm SD}}\right), \tag{3.22}$$

where the reality of $A_{\rm LD}$ has been used, and $\mathcal{A}_f$ and ΔS_f can be obtained by using Eq. (1.8). It is interesting to note that in the absence of LD contributions we have $|\delta^{\rm SD}_f| = |\delta^u_{\rm SD} - \delta^c_{\rm SD}| \lesssim \pi/4$ for some typical SD (perturbative) strong phases and, consequently, we expect $|\Delta S_f/\mathcal{A}_f| \gtrsim 1$. This result generally does not hold in the presence of FSI. For example, in the case of $|A^u_{\rm LD}| \ll |A^{u,c}_{\rm SD}| \lesssim |A^c_{\rm LD}|$, it is possible to have $|\Delta S_f/\mathcal{A}_f| \lesssim 1$. From Table V we obtain

$$\begin{aligned} \Delta S_{\phi K_S}/\mathcal{A}_{\phi K_S} &\approx -1.3(1.5), \\ \Delta S_{\omega K_S}/\mathcal{A}_{\omega K_S} &\approx -0.08(-1.7), \\ \Delta S_{\rho^0 K_S}/\mathcal{A}_{\rho^0 K_S} &\approx 0.08(-1.1), \\ \Delta S_{\eta' K_S}/\mathcal{A}_{\eta' K_S} &\approx -0.05(0.6), \\ \Delta S_{\eta K_S}/\mathcal{A}_{\eta K_S} &\approx -2.0(-1.1), \\ \Delta S_{\pi^0 K_S}/\mathcal{A}_{\pi^0 K_S} &\approx 1.2(-1.8), \end{aligned} \tag{3.23}$$

TABLE V. Direct CP asymmetry parameter $\mathcal{A}_f$ and the mixing-induced CP parameter ΔS_f^{SD+LD} for various modes. The first and second theoretical errors correspond to the SD and LD ones, respectively (see the text for details). The $f_0 K_S$ channel is not included as we cannot make reliable estimate of FSI effects on this decay.

Final state	ΔS_f			$\mathcal{A}_f(\%)$		
	SD	SD + LD	Expt	SD	SD + LD	Expt
ϕK_S	$0.02^{+0.00}_{-0.04}$	$0.03^{+0.01+0.01}_{-0.04-0.01}$	-0.38 ± 0.20	$1.4^{+0.3}_{-0.5}$	$-2.6^{+0.8+0.0}_{-1.0-0.4}$	4 ± 17
ωK_S	$0.12^{+0.05}_{-0.06}$	$0.01^{+0.02+0.02}_{-0.04-0.01}$	$-0.17^{+0.30}_{-0.32}$	$-7.3^{+3.5}_{-2.6}$	$-13.2^{+3.9+1.4}_{-2.8-1.4}$	48 ± 25
$\rho^0 K_S$	$-0.09^{+0.03}_{-0.07}$	$0.04^{+0.09+0.08}_{-0.10-0.11}$	$\cdots$	$9.0^{+2.2}_{-4.6}$	$46.6^{+12.9+3.9}_{-13.7-2.6}$	$\cdots$
$\eta' K_S$	$0.01^{+0.00}_{-0.04}$	$0.00^{+0.00+0.00}_{-0.04-0.00}$	-0.30 ± 0.11	$1.8^{+0.4}_{-0.4}$	$2.1^{+0.5+0.1}_{-0.2-0.1}$	4 ± 8
ηK_S	$0.07^{+0.02}_{-0.04}$	$0.07^{+0.02+0.00}_{-0.05-0.00}$	$\cdots$	$-6.1^{+5.1}_{-2.0}$	$-3.7^{+4.4+1.4}_{-1.8-2.4}$	$\cdots$
$\pi^0 K_S$	$0.06^{+0.02}_{-0.04}$	$0.04^{+0.02+0.01}_{-0.03-0.01}$	$-0.39^{+0.27}_{-0.29}$	$-3.4^{+2.1}_{-1.1}$	$3.7^{+3.1+1.0}_{-1.7-0.4}$	-8 ± 14

where the SD contributions are shown in parentheses. For the $f_0 K_S$ mode, $\Delta S/\mathcal{A} \approx 3.0$ but there is no reliable estimate of the FSI effect. From Eq. (3.23) we see that the relation $|\Delta S_f/\mathcal{A}_f| \gtrsim 1$ indeed holds at short distances except for the $\eta' K_S$ mode where $|\delta_f^{\rm SD}| \sim 65°$. The sign flip of $\Delta S_f/\mathcal{A}_f$ in the presence of LD rescattering is due to the sign switching of either $\mathcal{A}_f$ (for ϕK_S and $\pi^0 K_S$) or ΔS_f (for $\rho^0 K_S$).

IV. CONCLUSIONS AND COMMENTS

In the present work we have studied final-state rescattering effects on the decay rates and CP violation in the penguin-dominated decays $B^0 \to (\phi, \omega, \rho^0, \eta', \eta, \pi^0, f_0)K_S$. Our main goal is to understand to what extent indications of possibly large deviations of the mixing-induced CP violation seen in the above modes from $\sin 2\beta$ determined from $B \to J/\psi K_S$ can be accounted for by final-state interactions. Our main results are as follows:

(1) We have applied the QCD factorization approach to study the short-distance contributions to the above-mentioned seven modes. There are consistently 2 to 3 σ deviations between the central values of the QCDF predictions and the experimental data.

(2) The differences between the CP asymmetry $S_f^{\rm SD}$ induced at short distances and the measured $S_{J/\psi K_S}$ are summarized in Eq. (2.18). The deviation of $S_f^{\rm SD}$ in the ωK_S and $\rho^0 K_S$ modes from $\sin 2\beta$ is a 2 to 3 σ effect owing to a large tree pollution. In contrast, tree pollution in $\eta' K_S$ is diluted by the QCD anomaly and/or final-state rescattering both of which carry negligible CP-odd phase. The long-distance effects on S_f are generally negligible except for the ωK_S and $\rho^0 K_S$ modes where S_f is lowered by around 15% for the former and enhanced by the same percentage for the latter and $\Delta S^{\rm SD+LD}_{\omega K_S, \rho^0 K_S}$ become consistent with zero within errors.

(3) Final-state rescattering effects from charm intermediate states can account for the discrepancy between theory and experiment for the branching ratios of the modes ωK_S, $\eta' K_S$, ϕK_S, and $\pi^0 K_S$. Moreover, direct CP asymmetries in these modes are significantly affected; the signs of $\mathcal{A}_f$ in the last two modes are flipped by final-state interactions. Direct CP asymmetries in the ωK_S and $\rho^0 K_S$ channels are predicted to be $\mathcal{A}_{\omega K_S} \approx -0.13$ and $\mathcal{A}_{\rho^0 K_S} \approx 0.47$, respectively, which should be tested experimentally.

(4) For the $f_0(980)K_S$ mode, the short-distance contribution gives $\Delta S/\mathcal{A} \approx 3.0$, but at present we cannot make reliable estimates of FSI effects on this channel.

(5) Direct CP asymmetry in all the $(b \to s)$ penguin-dominated modes is rather small ($\lesssim$ a few %) except for ωK_S and $\rho^0 K_S$. This strengthens the general expectation that experimental search for direct CP violation in $b \to s$ modes may be a good way to look for possible effects of new CP-odd phases (see e.g. [37]).

(6) Since the mixing-induced CP parameter S_f (actually $\Delta S_f \equiv -\eta_f S_f - S_{J/\psi K_S}$) and the direct CP parameter $\mathcal{A}_f$ are closely related, so are their theoretical uncertainties. Based on this study, it seems rather difficult to accommodate $|\Delta S_f| > 0.10$ within the SM, at least in the modes we study in this paper (except for $f_0 K_S$, for which we cannot make reliable estimates).

(7) In particular, $\eta' K_S$ and (to some degree) ϕK_S appear theoretically cleanest in our picture; i.e. for these modes the central value of ΔS_f as well as the uncertainties on it are rather small. This also seems to be the case in QCDF [38]. Note also that the experimental errors on $\eta' K_S$ are the smallest (see Table III) and its branching ratio is the largest, making it especially suitable for faster experimental progress in the near future [39].

(8) The sign of ΔS_f at short distances is found to be positive except for the channel $\rho^0 K_S$. After including final-state rescattering effects, the central values

of ΔS_f are positive for all the modes under consideration, but they tend to be rather small compared to the stated uncertainties so that it is difficult to make reliable statements on the sign at present. However, since the S_f and $\mathcal{A}_f$ are strongly correlated, improved measurements could provide enough useful information that stronger statements on the sign could be made in the future.

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