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B meson wave function from the $B \to \gamma l \nu$ decay

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We show that the leading-power B meson wave function can be extracted reliably from the photon energy spectrum of the $B \to \gamma l \nu$ decay up to $O(1/m_b^2)$ and $O(\alpha_s^2)$ uncertainty, m_b being the b quark mass and α_s the strong coupling constant. The $O(1/m_b)$ corrections from heavy-quark expansion can be absorbed into a redefined leading-power B meson wave function. The two-parton $O(1/m_b)$ corrections cancel exactly, and the three-parton B meson wave functions turn out to contribute at $O(1/m_b^2)$. The constructive long-distance contribution through the $B \to V \to \gamma$ transition, V being a vector meson, almost cancels the destructive $O(\alpha_s)$ radiative correction. Using models of the leading-power B meson wave function available in the literature, we obtain the photon energy spectrum in the perturbative QCD framework, which is then compared with those from other approaches.

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I. INTRODUCTION

The two-parton leading-power (LP) B meson wave function (distribution amplitude) ϕ_+ plays an essential role in a perturbative analysis of exclusive B meson decays based on the k_T factorization theorem [1–3] (collinear factorization theorem [4–9]). Its behavior certainly matters and has been investigated in various approaches recently. Models of the distribution amplitude $\phi_+(x)$ with an exponential tail in the large x region have been proposed [10], where x is the longitudinal momentum fraction carried by the light spectator quark. Neglecting three-parton distribution amplitudes in a study by means of the equations of motion [11,12], $\phi_+(x)$ was found to be proportional to a step function with a sharp drop at large x [13]. The wave function $\phi_+(x, k_T)$, where k_T is the transverse momentum carried by the light spectator quark, was also derived in the same framework [13]. All these models depend on at least one shape parameter, whose determination requires experimental inputs from exclusive B meson decays.

In this paper we shall show that the radiative decay $B \to \gamma l \nu$ provides the cleanest information of the LP B meson wave function ϕ_+. This mode has been widely studied in [3,8,14–28] due to different motivations: for extracting the B meson decay constant f_B and the Cabibbo-Kobayashi-Maskawa matrix element $|V_{ub}|$, for demonstrating the next-to-leading-order (NLO) calculation and the proof of the QCD factorization theorem, for deriving resummation of large logarithmic corrections, for studying long-distance effect and the annihilation mechanism, etc. The subject on the extraction of the B meson wave function from the $B \to \gamma l \nu$ data has not yet been discussed. It will be shown that two-parton next-to-leading-power (NLP) [$O(1/m_b)$] corrections cancel exactly, m_b being the b quark mass. The contributions from higher Fock states, the three-parton B meson wave functions, turn out to be of $O(1/m_b^2)$. The constructive long-distance contribution through the $B \to V \to \gamma$ transition, V being a vector meson, almost cancels the destructive $O(\alpha_s)$ radiative correction, α_s being the strong coupling constant. The effect from bremsstrahlung photon emissions vanishes like the lepton mass because of helicity suppression. Therefore, the extraction of ϕ_+ from the measured photon energy spectrum of the $B \to \gamma l \nu$ decay suffers only $O(1/m_b^2)$ and $O(\alpha_s^2)$ uncertainty.

We identify and discuss the higher-power corrections to the $B \to \gamma l \nu$ decay in Sec. II, and calculate the long- and short-distance effects in Sec. III. Section IV is the conclusion. The hard kernel associated with the three-parton distribution amplitudes is derived in the Appendix, whose explicit expression is necessary for demonstrating the smallness of the higher-Fock-state contribution. Our conclusion differs from that drawn in [29], in which the semileptonic decay $B \to \pi l \nu$ was regarded as a more ideal process for extracting the B meson wave function. The argument is that the radiative decay $B \to \gamma l \nu$, receiving a large long-distance uncertainty, does not serve the purpose. As stated above, this long-distance effect is in fact canceled by the $O(\alpha_s)$ short-distance one almost exactly.

II. HIGHER-POWER CORRECTIONS

In this section we identify and discuss higher-power corrections to the $B \to \gamma l \nu$ decay. The B meson momentum P_1 and the photon momentum P_2 are parametrized, in the light-cone coordinates, as

$$P_1 = \frac{m_B}{\sqrt{2}}(1, 1, \mathbf{0}_T), \qquad P_2 = \frac{m_B}{\sqrt{2}}(0, \eta, \mathbf{0}_T), \qquad (1)$$

respectively, where $\eta \equiv 2E_\gamma/m_B$, m_B being the B meson mass, denotes the photon energy fraction. The decay amplitude is decomposed into

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$$\frac{1}{e}\langle\gamma(P_2,\epsilon_T)|\bar{u}\gamma_\mu(1-\gamma_5)b|\bar{B}(P_1)\rangle = \epsilon_{\mu\nu\alpha\beta}\epsilon_T^{*\nu}v^\alpha P_2^\beta F_V(q^2) + i[\epsilon^*_{T\mu}(v\cdot P_2) - (\epsilon_T^*\cdot v)P_{2\mu}]F_A(q^2), \tag{2}$$

where e is the electron charge, ϵ_T the polarization vector of the photon, $v = P_1/m_B$ the B meson velocity, and $q^2 \equiv (P_1 - P_2)^2 = (1-\eta)m_B^2$ the lepton-pair invariant mass. The decay spectrum is then given, in terms of the form factors $F_{V,A}$, by

$$\frac{d\Gamma}{d\eta} = \frac{\alpha G_F^2|V_{ub}|^2}{96\pi^2}m_B^5(1-\eta)\eta^3[F_V^2(q^2)+F_A^2(q^2)], \tag{3}$$

with $\alpha \equiv e^2/(4\pi)$ and the Fermi constant G_F.

The collinear factorization theorem for the form factors $F_{V,A}$ in the large η region has been proved in [24,27], which are expressed as the convolution of hard kernels with the B meson distribution amplitudes in the momentum fractions x of the light spectator quark. A hard kernel, being infrared finite, is calculable in perturbation theory. The B meson distribution amplitudes, collecting the soft dynamics in exclusive B meson decays, are not calculable but universal. In the framework of the factorization theorem, there are four sources of higher-power corrections to the $B \to \gamma l\nu$ decay:

(1) The heavy-quark expansion of the heavy-light current in Eq. (2),

$$\bar{u}\gamma_\mu(1-\gamma_5)b \to \bar{u}\gamma_\mu(1-\gamma_5)h + \frac{1}{2m_b}\bar{u}\gamma_\mu \times(1-\gamma_5)i\not{D}h + O(1/m_b^2), \tag{4}$$

where the operator D represents the covariant derivative, and the rescaled b quark field h is related to the full field b by

$$h(z) = \frac{1+\not{v}}{2}e^{im_b v\cdot z}b(z). \tag{5}$$

The factorization of the transition matrix element associated with the first (second) term in the above expansion leads to the LP (NLP) B meson distribution amplitudes.

(2) The higher-power interactions in the Lagrangian of the heavy-quark effective theory (HQET). The insertion of the HQET interactions,

$$O_1 = \frac{1}{m_b}\bar{h}(iD)^2h, \qquad O_2 = \frac{g}{2m_b}\bar{h}\sigma^{\mu\nu}G_{\mu\nu}h, \tag{6}$$

into the transition matrix element associated with the first term in Eq. (4) yields $O(1/m_b)$ corrections. We mention that there exists an alternative heavy-quark effective theory, in which the higher-power corrections are formulated in a different way [30].

(3) The higher Fock states of the B meson. The nonlocal matrix element,

$$\langle 0|\bar{u}(z)gG_{\alpha\beta}(uz)h(0)|\bar{B}(P_1)\rangle, \tag{7}$$

defines the three-parton distribution amplitudes, where $G_{\alpha\beta}(uz)$ is the gluon field strength evaluated at the coordinate uz, $0 \le u \le 1$. The additional valence gluon, attaching internal off-shell quark lines, introduces one more hard propagator, i.e., one more power of $1/m_b$.

(4) The subleading parton-level diagrams (hard kernels). The two-parton lowest-order hard kernels are displayed in Fig. 1, where the upper quark line represents a b quark. It is easy to observe that Fig. 1(a) [1(b)] represents the LP (NLP) hard kernel, since the internal quark line is off shell by $m_b\bar{\Lambda}$ (m_b^2) with $\bar{\Lambda}$ being a hadronic scale, such as the mass difference $m_B - m_b$.

A. Heavy-quark expansion

The factorization of soft dynamics from the transition matrix element associated with the first term on the right-hand side of Eq. (4),

$$\langle\gamma(P_2,\epsilon_T)|\bar{u}\gamma_\mu(1-\gamma_5)h|\bar{B}(P_1)\rangle, \tag{8}$$

leads to the nonlocal matrix element [10],

$$\int\frac{dz^-d^2z_T}{(2\pi)^3}e^{i(k^+z^- - \mathbf{k}_T\cdot\mathbf{z}_T)}\langle 0|\bar{u}_\rho(z)h_\delta(0)|\bar{B}(P_1)\rangle = i\frac{f_B}{\sqrt{2}}\{(\not{P}_1+m_B)\gamma_5[\not{n}_+\Phi_+(k)+\not{n}_-\Phi_-(k)]\}_{\delta\rho}, \tag{9}$$

which defines the two-parton LP B meson wave functions $\Phi_\pm$, with the null vectors $n_+ = (1,0,0_T)$ and $n_- = (0,1,0_T)$, and the light quark momentum k. Because the photon momentum P_2 has been chosen in the minus direction, the hard kernels for the form factors $F_{V,A}$ are independent of the component k^-, which becomes irrelevant. We construct the B meson distribution amplitudes $\phi_\pm(x)$, $x \equiv k^+/P_1^+$, from the B meson wave functions $\phi_\pm(x,k_T) \equiv P_1^+\Phi_\pm(xP_1^+,k_T)$ by integrating the latter over k_T,

$$\phi_\pm(x) = \int d^2k_T\phi_\pm(x,k_T). \tag{10}$$

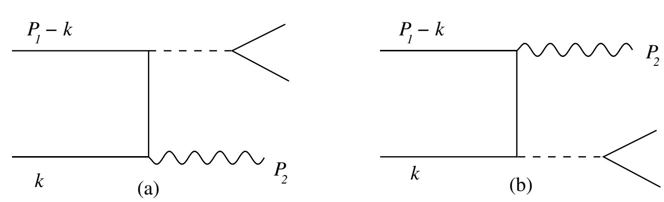

FIG. 1. Lowest-order diagrams for the $B \to \gamma l\nu$ decay.

The dependence of $\phi_\pm(x)$ and of $\phi_\pm(x, k_T)$ on the renormalization scale μ has been suppressed.

Define the moments of the B meson distribution amplitude $\phi_+(x)$,

$$\Lambda_0 \equiv \int dx \frac{\phi_+(x)}{x}, \qquad \Lambda_1 \equiv \int dx \phi_+(x). \tag{11}$$

The asymptotic behavior of $\phi_+(x)$ has been extracted from a renormalization-group equation, which exhibits a decrease slower than $1/x$ [31,32]. That is, the normalization Λ_1 of the B meson distribution amplitude is divergent after taking into account the evolution effect. It has been argued that a non-normalizable B meson distribution amplitude does not cause trouble in practice [33], since only the inverse moment Λ_0 matters at LP [25,34], which is convergent. Note that a hard kernel would not be as simple as $1/x$ at higher orders in α_s, and information of more moments is also necessary. In the following discussion we shall neglect the evolution effect, and assume that $\phi_+(x)$ is normalized to unity, i.e., $\Lambda_1 = 1$. Since the B meson distribution amplitudes absorb soft dynamics, the light quark momentum k is of $O(\bar{\Lambda})$. We then have the relative importance $\Lambda_1/\Lambda_0 \sim \bar{\Lambda}/m_b$ for $x \sim O(\bar{\Lambda}/m_b)$.

The factorization of soft dynamics from the transition matrix element associated with the second term on the right-hand side of Eq. (4) gives the nonlocal matrix element,

$$\langle 0|\bar{u}_\rho(z) i\not{D} h_\delta(0)|\bar{B}(P_1)\rangle. \tag{12}$$

The factorization of the transition matrix elements with the insertion of the $O(1/m_b)$ interactions in Eq. (6) into Eq. (8) leads to

$$\langle 0|i \int d^4y T[\bar{u}_\rho(z) h_\delta(0) O_{1,2}(y)]|\bar{B}(P_1)\rangle. \tag{13}$$

The contributions from Eqs. (12) and (13) can be absorbed into the nonlocal matrix element,

$$\langle 0|\bar{u}_\rho(z) b_\delta(0)|\bar{B}(P_1)\rangle, \tag{14}$$

where the rescaled b quark field h has been replaced by the full field b. It is easy to check that the heavy-quark expansion of Eq. (14) generates Eqs. (12) and (13). This absorption makes sense, because Eqs. (12) and (13), concerning only the initial b quark, are universal for all exclusive B meson decays. The decomposition in Eq. (9) still holds, but the B meson distribution amplitudes $\Phi_\pm$, redefined by Eq. (14) in terms of the full field b, exhibit a renormalization-group evolution different from that in Eq. (9) [35].

B. Three-parton distribution amplitudes

We explain that the nonlocal matrix element in Eq. (7) is negligible in the current accuracy: the three-parton distribution amplitudes, whose contributions to the form factors are supposed to be of $O(1/m_b)$, turn out to appear at $1/m_b^2$. The three-parton distribution amplitudes $\tilde{\Phi}_V$, $\tilde{\Phi}_A$, $\tilde{X}_A$, and $\tilde{Y}_A$ in coordinate space are defined via the decomposition,

$$\begin{aligned}\langle 0|\bar{u}_\rho(z) g G_{\alpha\beta}(uz) n_-^\beta h_\delta(0)|\bar{B}(P_1)\rangle \\ = f_B\Big\{(\not{P}_1 + m_B)\gamma_5\Big[(v_\alpha \not{n}_- - v\cdot n_- \gamma_\alpha)(\tilde{\Phi}_V(t,u) \\ - \tilde{\Phi}_A(t,u)) - i\sigma_{\alpha\beta} n_-^\beta \tilde{\Phi}_V(t,u) - n_{-\alpha}\tilde{X}_A(t,u) \\ + \frac{n_{-\alpha}}{v\cdot n_-}\not{n}_- \tilde{Y}_A(t,u)\Big]\Big\}_{\delta\rho},\end{aligned} \tag{15}$$

with the variable $t = v \cdot z$. The corresponding hard kernels arise from the contraction of all the structures $\Gamma = v_\alpha \not{n}_-$, $v \cdot n_- \gamma_\alpha, \ldots$, in Eq. (15) with Fig. 2, written as

$$\mathcal{M}_a^{(3)} \propto \frac{\mathrm{tr}\{\not{\epsilon}_T^*[u\not{n}_+\gamma^\alpha(\not{P}_2 - \not{k}_1 - u\not{k}_2) - \bar{u}(\not{P}_2 - \not{k}_1 - u\not{k}_2)\gamma^\alpha \not{n}_+]\gamma_\mu(1-\gamma_5)(\not{P}_1 + m_B)\gamma_5\Gamma\}}{[(P_2 - k_1 - uk_2)^2]^2}, \tag{16}$$

where k_1 (k_2) is the momentum carried by the light quark (gluon). The derivation of the above expression is referred to the Appendix.

For $\Gamma = v \cdot n_- \gamma_\alpha$, Eq. (16) vanishes because of $\epsilon_T^* \cdot n_+ = \epsilon_T^* \cdot (P_2 - k_1 - uk_2) = 0$. Express $\sigma_{\alpha\beta} n_-^\beta = i(n_{-\alpha} - \not{n}_- \gamma_\alpha)$, in which the first term has the same structure as of $\tilde{X}_A$. The second term $\not{n}_-\gamma_\alpha$ renders Eq. (16) vanish for the same reason. For the other structures $v_\alpha \not{n}_-$, $n_{-\alpha}$, and $n_{-\alpha}\not{n}_-$, we always have $\gamma^\alpha = \gamma^+$. Once $\gamma^\alpha = \gamma^+$, Eq. (16) is proportional to

$$\mathcal{M}_a^{(3)} \sim \frac{P_1 \cdot (k_1 + uk_2)}{[(P_2 \cdot (k_1 + uk_2)]^2}. \tag{17}$$

Note that k_1^+ and k_2^+ are of $O(\bar{\Lambda})$, and that the moments of the three-parton B meson distribution amplitudes are at

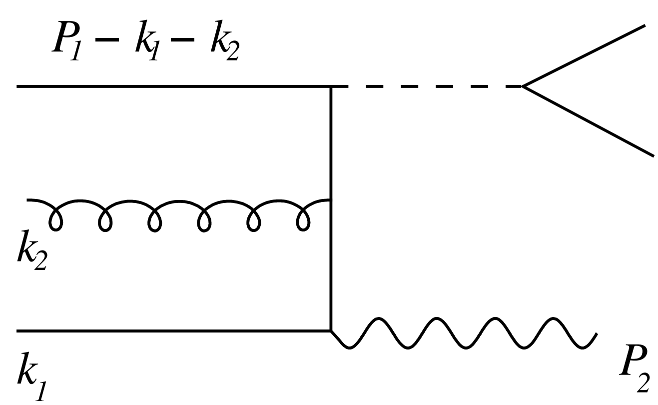

FIG. 2. Three-parton contribution to the $B \to \gamma l \nu$ decay.

most of $O(\bar{\Lambda}^2)$ [10]. Therefore, when convoluting Eq. (17) with the three-parton distribution amplitudes, the resultant contribution to the form factors $F_{V,A}$ is of $O(\bar{\Lambda}^2/m_b^2)$ compared to the LP one from Fig. 1(a). The same higher Fock state has been shown to give a power-suppressed correction to the $B \to \gamma l \nu$ decay in the framework of the soft-collinear effective theory (SCET) [23]. With a similar reasoning, the three-parton B meson wave functions also contribute at $O(1/m_b^2)$ in k_T factorization theorems. We emphasize that the three-parton B meson wave functions are relevant in the NLP analysis of the $B \to \pi$ transition form factors. This is the reason the $B \to \gamma l \nu$ decay is a cleaner mode than the $B \to \pi l \nu$ decay for determining the LP B meson wave function.

C. NLP hard kernels

Contracting Fig. 1 with the two structures in Eq. (9), we get the quark-level amplitudes,

$$\mathcal{M}_a^+ = \frac{i}{4\sqrt{2}} \frac{\mathrm{tr}[\not\epsilon^*(\not P_2 - \not k)\gamma_\mu(1-\gamma_5)(\not P_1 + m_B)\gamma_5 \not n_+]}{(P_2 - k)^2},$$

$$\mathcal{M}_b^+ = \frac{i}{4\sqrt{2}} \times \frac{\mathrm{tr}[\gamma_\mu(1-\gamma_5)(\not q - \not k + m_b)\not\epsilon^*(\not P_1 + m_B)\gamma_5 \not n_+]}{(q-k)^2 - m_b^2}, \tag{18}$$

and $\mathcal{M}_{a,b}^-$ with the null vector n_+ in $\mathcal{M}_{a,b}^+$ being replaced by n_-. As stated above, Fig. 1(a) is LP, because of $(P_2 - k)^2 = -2P_2 \cdot k \sim O(m_b\bar{\Lambda})$, and Fig. 1(b) is NLP, because of $(q-k)^2 - m_b^2 = -2P_1 \cdot P_2 \sim O(m_b^2)$. The contribution from Fig. 1(b) has not yet been considered in the literature. We shall neglect the mass difference between the B meson and the b quark in $\mathcal{M}_b^{+,-}$ in our analysis accurate up to NLP.

The collinear factorization formulas for $F_{V,A}$ are written as

$$F_{V(A)}(q^2) = f_B \int dx[\phi_+(x)H_{V(A)}^+(x,\eta) + \phi_-(x)H_{V(A)}^-(x,\eta)], \tag{19}$$

where the hard kernels H are extracted according to Eq. (2) by keeping only the longitudinal component k^+ in Eq. (18). In terms of the LP and NLP moments in Eq. (11), Eq. (19) becomes

$$F_{V,A}(q^2) = \frac{f_B}{\eta m_B}\left[\Lambda_0 \pm \left(1 + \frac{1}{\eta}\right)\Lambda_1\right], \tag{20}$$

in which the coefficient 1 of Λ_1 comes from Fig. 1(a) and $1/\eta$ from Fig. 1(b). It has been mentioned that the equality of F_V and F_A at LP is attributed to the spin symmetry in the large-recoil region [20]. The coefficient $1/\eta$ implies the increase of the subleading-power correction with the decrease of the photon energy. This is why a perturbation theory is reliable only in the large η region. The distribution amplitude $\phi_-(x)$, contributing only through the normalization of the combination,

$$\int dx[\phi_+(x) - \phi_-(x)] = 0, \tag{21}$$

disappears from Eq. (20). As shown in Eq. (20), the normalization Λ_1 does appear at NLP, which is divergent under the evolution. This is another example that the QCD-improved factorization (QCDF) approach based on collinear factorization theorem breaks down at NLP [34,36].

The decay spectrum in Eq. (3) becomes

$$\frac{d\Gamma}{d\eta} = \frac{\alpha G_F^2 |V_{ub}|^2}{48\pi^2} f_B^2 m_B^3 (1-\eta)\eta\left[\Lambda_0^2 + \left(1 + \frac{1}{\eta}\right)^2 \Lambda_1^2\right]. \tag{22}$$

The above expression indicates that the NLP terms for the spectrum have canceled, and only the $O(1/m_b^2)$ term Λ_1^2 is left. In this case we can estimate the $O(1/m_b^2)$ effect using the models for the B meson distribution amplitudes available in the literature [13,37],

$$\phi_\pm(x) = \frac{\lambda \pm (x - \lambda)}{2\lambda^2}\theta(x)\theta(2\lambda - x), \tag{23}$$

with the shape parameter $\lambda \equiv \bar{\Lambda}/m_b$. The value of $\bar{\Lambda}$ has been found to range between 0.5 and 0.7 GeV [25,38,39], which corresponds to $\lambda = 0.1$–0.15 approximately. Certainly, there are other models of the B meson distribution amplitudes (see [40]).

Employing the inputs $\alpha = 1/137$, $G_F = 1.16639 \times 10^{-5}\ \mathrm{GeV}^{-2}$, $|V_{ub}| = 3.9 \times 10^{-3}$, $f_B = 190$ MeV, and $m_B = 5.28$ GeV, we derive the photon energy spectra of the $B \to \gamma l \nu$ decay for $\lambda = 0.1$ and for $\lambda = 0.15$ in Fig. 3. The specific models in Eq. (23) lead to the relation $\Lambda_1/\Lambda_0 = \lambda$. Therefore, the subleading-power term is in-

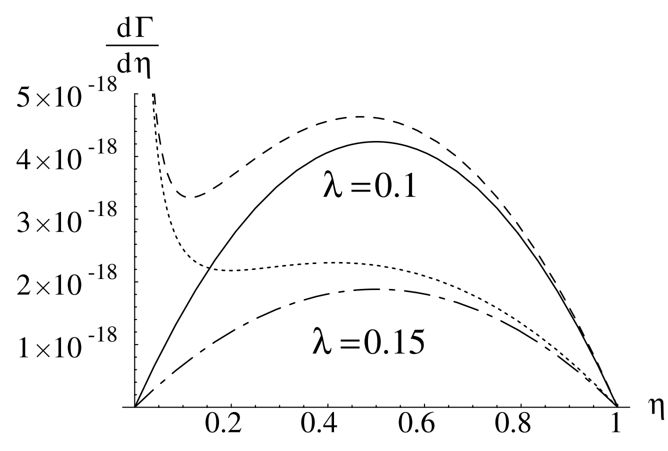

FIG. 3. Spectra in units of GeV^{-1} from the collinear factorization with the solid (dash-dotted) line corresponding to the LP contribution for $\lambda = 0.1$ ($\lambda = 0.15$), and the dashed (dotted) line to the inclusion of the NLP contribution for $\lambda = 0.1$ ($\lambda = 0.15$).

deed negligible at large η, whose contribution is around 5%. However, this term diverges quickly at small η, breaking the perturbative expansion in $1/m_b$. The form factors $F_{V,A}$ in Eq. (20) contain a dominant monopole component proportional to Λ_0/η, and a small dipole component proportional to Λ_1/η^2, which is important only at small η. This is the reason one always obtains a symmetric spectrum in η at LP from a perturbation theory [20] as shown in Fig. 3. To generate an asymmetric spectrum, the dipole component must be enhanced as postulated in [16,26]. Therefore, an asymmetric spectrum signals an important NLP contribution, i.e., a breakdown of the factorization theorem.

It has been explained that the undesirable feature of the B meson distribution amplitude under evolution is a consequence of collinear factorization, which can be removed in k_T factorization [35]. The evolution effect on the k_T-dependent B meson wave function was also studied in [41]. Moreover, applying the k_T factorization theorem to the $B \to \gamma l \nu$ decay, which has been proved in [3], we can extend the spectrum to lower η as demonstrated below. Keeping both the longitudinal momentum k^+ and the transverse momentum k_T in Eq. (18), the hard kernels in the k_T factorization theorem are derived. Defining the LP and NLP functions,

$$\Lambda_0(\eta) \equiv m_B^2 \int dx \int d^2k_T \frac{\phi_+(x, k_T)}{\eta x m_B^2 + k_T^2},$$
$$\Lambda_1(\eta) \equiv m_B^2 \int dx \int d^2k_T \left[\frac{\phi_+(x, k_T)}{\eta m_B^2 + k_T^2} + \frac{x\phi_-(x, k_T)}{\eta(\eta x m_B^2 + k_T^2)}\right], \tag{24}$$

respectively, we obtain the form factors,

$$F_{V,A}(q^2) = \frac{f_B}{m_B}[\Lambda_0(\eta) \pm \Lambda_1(\eta)]. \tag{25}$$

Because of $k_T \sim O(\bar{\Lambda})$ in the B meson, $\Lambda_1(\eta)$ is of $O(\bar{\Lambda}/m_b)$ relative to $\Lambda_0(\eta)$ in the large η region. Again, only a single B meson wave function is relevant in the LP analysis of the $B \to \gamma l \nu$ decay, consistent with the observation in [42]. Compared to Eq. (20), both $\phi_\pm$ appear in the k_T factorization theorem at NLP.

The decay spectrum is then given, according to Eq. (3), by

$$\frac{d\Gamma}{d\eta} = \frac{\alpha G_F^2 |V_{ub}|^2}{48\pi^2} f_B^2 m_B^3 (1-\eta)\eta^3[\Lambda_0^2(\eta) + \Lambda_1^2(\eta)]. \tag{26}$$

Similarly, the NLP terms have canceled, and only the $O(\bar{\Lambda}^2/m_b^2)$ term $\Lambda_1^2(\eta)$ is left. We adopt the models for the B meson wave functions in [13], whose k_T dependence is coupled to the x dependence through a δ function,

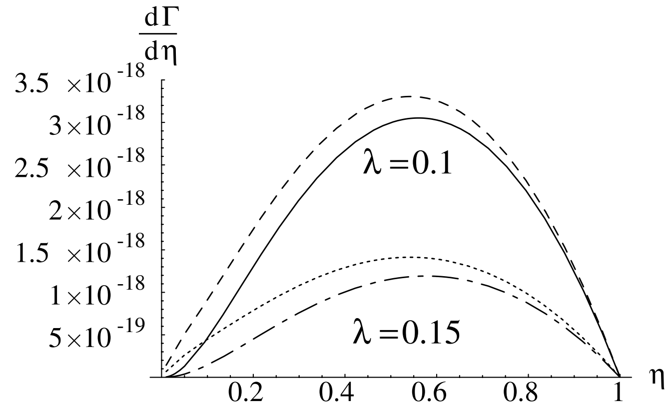

FIG. 4. Spectra in units of GeV^{-1} from the k_T factorization with the solid (dash-dotted) line corresponding to the LP contribution for $\lambda = 0.1$ ($\lambda = 0.15$), and the dashed (dotted) line to the inclusion of the NLP contribution for $\lambda = 0.1$ ($\lambda = 0.15$).

$$\phi_\pm(x, k_T) = \phi_\pm(x)\frac{1}{\pi}\delta(k_T^2 - x(2\lambda - x)m_B^2). \tag{27}$$

Using the same input parameters, we obtain the photon energy spectra from the k_T factorization theorem in Fig. 4 for $\lambda = 0.1$ and for $\lambda = 0.15$. These spectra are symmetric in η, and modified only slightly by the higher-power correction. Hence, the higher-power correction is under control in the k_T factorization theorem compared to that in the collinear factorization theorem: the power behavior $1/\eta$ of the spectrum in the small η region has been smeared into $\eta \ln^2 \eta$. It implies that the perturbative QCD (PQCD) approach based on the k_T factorization theorem [43–46] has a better convergence at the subleading level.

III. LONG- AND SHORT-DISTANCE CORRECTIONS

In this section we discuss the long-distance and short-distance corrections to the $B \to \gamma l \nu$ decay spectrum. For this purpose, the form factors are written, in the k_T factorization theorem, as

$$F_{V,A}(q^2) = \frac{f_B}{m_B}[\Lambda_0(\eta) + \Lambda_0^{(1)}(\eta)] + F_{V,A}^{\mathrm{LD}}(q^2), \tag{28}$$

where $\Lambda_0^{(1)}$ and $F_{V,A}^{\mathrm{LD}}$ denote the $O(\alpha_s)$ and long-distance correction to the leading result, respectively. We shall estimate the latter by considering the $B \to V \to \gamma$ transition. This correction is certainly significant in the small η (large q^2) region, where the internal quark becomes soft, and easily forms a resonance with the spectator quark. Hence, it could break the QCD factorization of the form factors $F_{V,A}$ at small η. At large η, the long-distance contribution may be suppressed by the values of the $B \to V$ transition form factors [15].

The long-distance amplitude is written as [47]

$$\frac{1}{e}\langle\gamma(P_2,\epsilon_T)|\bar{u}\gamma_\mu(1-\gamma_5)b|\bar{B}(P_1)\rangle = \sum_V \langle\gamma(P_2,\epsilon_T)|J^\alpha_{\rm em}|V(P_2,\epsilon_T)\rangle \frac{-i\epsilon^*_{T\alpha}}{P_2^2-m_V^2+im_V\Gamma_V} \times \langle V(P_2,\epsilon_T)|\bar{u}\gamma_\mu(1-\gamma_5)b|\bar{B}(P_1)\rangle, \tag{29}$$

with the vector mesons $V=\rho,\omega,\ldots$, and their masses m_V and widths Γ_V. Take the B meson transition into a transversely polarized ρ meson as an example, for which the first matrix element on the right-hand side of Eq. (29) gives

$$\langle\gamma(P_2,\epsilon_T)|J^\alpha_{\rm em}|\rho(P_2,\epsilon_T)\rangle = -\frac{i}{2}m_\rho f_\rho \epsilon^\alpha_T, \tag{30}$$

f_ρ being the ρ meson decay constant. The second matrix element is decomposed into

$$\langle\rho(P_2,\epsilon_T)|\bar{u}\gamma_\mu(1-\gamma_5)b|\bar{B}(P_1)\rangle = -\frac{2V(q^2)}{m_B+m_\rho}\epsilon_{\mu\nu\rho\sigma}\epsilon_T^{*\nu}P_1^\rho P_2^\sigma - i(m_B+m_\rho)A_1(q^2)\epsilon^*_{T\mu}, \tag{31}$$

with the $B\to\rho$ form factors $V(q^2)$ and $A_1(q^2)$. Combining Eqs. (30) and (31), we extract from Eq. (29),

$$F_V^{\rm LD}(q^2) = \frac{f_\rho}{m_\rho - i\Gamma_\rho}\frac{m_B}{m_B+m_\rho}V(q^2), \qquad F_A^{\rm LD}(q^2) = \frac{f_\rho}{m_\rho - i\Gamma_\rho}\frac{(m_B+m_\rho)}{\eta m_B}A_1(q^2). \tag{32}$$

For the long-distance contribution through the $B\to\omega$ transition, we have the similar expressions to Eq. (32), but with the charge factor $1/2$ in Eq. (30) being replaced by $1/6$, and the appropriate replacement of the vector meson mass and of the decay constant. The $B\to\psi$ transitions do not contribute in this case.

For the ρ and ω mesons, we employ the inputs [47]

$$m_\rho = 0.771\ {\rm GeV}, \quad \Gamma_\rho/m_\rho = 0.21, \quad f_\rho = 0.217\ {\rm GeV}, \quad m_\omega = 0.783\ {\rm GeV}, \quad \Gamma_\omega/m_\omega \approx 0, \quad f_\omega = 0.195\ {\rm GeV}. \tag{33}$$

For the $B\to\rho$, ω form factors, we adopt the models derived from the light-front QCD [48], which have been parametrized as

$$F(q^2) = \frac{F(0)}{1-a(q^2/m_B^2)+b(q^2/m_B^2)^2}, \tag{34}$$

with the constants,

$$V(q^2):\ F(0)=0.27, \quad a=1.84, \quad b=1.28, \qquad A_1(q^2):\ F(0)=0.22, \quad a=0.95, \quad b=0.21. \tag{35}$$

We restrict the above formalism in the region,

$$\eta > 1 - \frac{q^2_{\rm max}}{m_B^2} = 0.275, \tag{36}$$

with $q^2_{\rm max}$ being the maximal value of q^2 in the $B\to\omega$ transition, in which Eq. (34) holds. The long-distance contribution increases $F_{V,A}$ by about 30%–50% for $\lambda = 0.1$–0.15 at large η, consistent with the observations in [15,25,49]. Its effect to the decay spectrum is quite important, especially for $\eta < 0.8$, as shown in Fig. 5.

The $B\to\rho,\omega$ transition form factors at large recoil could be regarded as an $O(\alpha_s)$ object [42]. This observation hints that we should attempt to take into account the NLO short-distance correction to $F_{V,A}$. The NLO correction to the $B\to\gamma l\nu$ decay has been computed by several groups [22–24] in the collinear factorization theorem (SCET or QCDF). However, we need the result from the k_T factorization theorem (with the parton transverse mo-

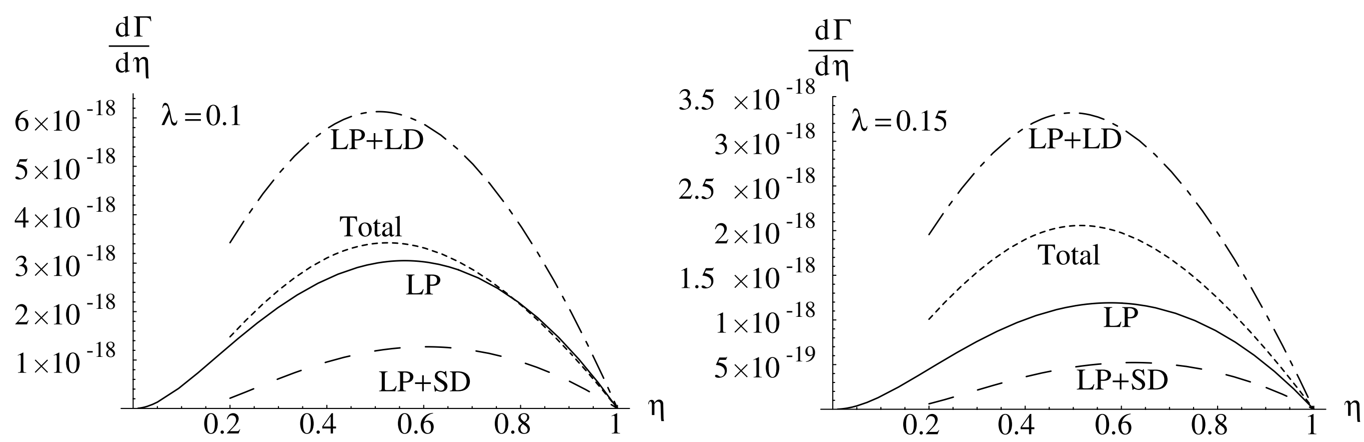

FIG. 5. Spectra in units of GeV^{-1} for $\lambda = 0.1$ and $\lambda = 0.15$ with the solid lines corresponding to the LP contribution only, the dash-dotted lines to the inclusion of the long-distance contribution, the dashed lines to the inclusion of the NLO correction, and the dotted lines to the inclusion of both the long-distance and the NLO contributions.

menta k_T being included), which is quoted from [20]:

$$\Lambda_0^{(1)}(\eta) = -\frac{\alpha_s(2E_\gamma)}{4\pi} C_F m_B^2 \int dx \int d^2k_T \frac{\phi_+(x, k_T)}{\eta x m_B^2 + k_T^2} \times \left[\ln^2\frac{\eta}{x} - \frac{5}{2}\ln\frac{\eta}{x} + \frac{4\pi^2}{3} - \ln^2\left(1 + \frac{k_T^2}{2k^{+2}}\right) + 2\pi i \ln\left(1 + \frac{k_T^2}{2k^{+2}}\right)\right]. \quad (37)$$

The weaker evolution of f_B will be neglected for simplicity. Because of the large negative double logarithm, the NLO correction to the form factors $F_{V,A}$ is destructive, and about 30% of the leading result for both $\lambda = 0.1$ and $\lambda = 0.15$ at large η. The resummation of this double logarithm to all orders has been discussed in [20,22–24,28].

We emphasize that the NLO hard kernel depends on a factorization scheme, in which the B meson wave function is defined [23]. Therefore, it is not very legitimate to adopt an expression straightforwardly from some other works in the literature. The calculation of the NLO hard kernel for the $B \to \gamma l \nu$ decay in the factorization scheme specified in [35] is in progress, which will be published elsewhere. The NLO correction in SCET has been further factorized into a function characterized by the scale m_b, and another by $\sqrt{m_b \bar{\Lambda}}$. As stated in [23], this further factorization is not numerically essential for $m_b \approx 5$ GeV. On the other hand, the model-dependent estimate of the long-distance contribution also suffers large uncertainty. Hence, we just intend to point out the potential strong cancellation between the long-distance and short-distance corrections in this mode. As shown in Fig. 5, after combining both subleading contributions, the net effect has been greatly reduced. Especially, for the shape parameter $\lambda = 0.1$, the cancellation is almost exact for $\eta > 0.8$. We conclude that the leading result in the large η region is stable under these corrections.

Using the lifetime of a charged B meson $\tau_{B^\pm} = 1.674 \times 10^{-12}$ s and considering only the leading contribution, we obtain the branching ratios for $\lambda = 0.15$–0.1,

$$B(B \to \gamma l \nu) = (1.8\text{–}4.8) \times 10^{-6}, \quad (38)$$

from Eq. (26) in the k_T factorization theorem (PQCD), with only the $O(\bar{\Lambda}^2/m_b^2)$ and $O(\alpha_s^2)$ uncertainty. The values in Eq. (38) are more or less consistent with other estimates in the literature: a model-dependent evaluation of the structure-dependent photon emission contribution gave the branching ratio 10^{-7}–10^{-6} [14]. Using the B meson bound-state wave function from a Salpeter equation, 0.9×10^{-6} has been obtained [17,21]. Both a simple nonrelativistic model and light-front QCD lead to 3.5×10^{-6} [18,19]. Light-cone sum rules and the pole-model calculation give 2×10^{-6} [16] and 2.26×10^{-6} [26], respectively. At last, the experimental upper bound at 90% confidence level is [50]

$$B(B \to \gamma l \nu) < 2.0 \times 10^{-6}. \quad (39)$$

IV. CONCLUSION

In this paper we have studied the $B \to \gamma l \nu$ decay in the PQCD approach based on the k_T factorization theorem. This formalism is well defined at the subleading level, since the two-parton LP B meson wave functions remain normalizable even after including the evolution effect. Note that the QCDF approach based on the collinear factorization theorem fails at NLP. We have shown that the $O(1/m_b)$ corrections from the heavy-quark expansion can be absorbed into the LP B meson wave functions redefined by the nonlocal matrix element in Eq. (14). The NLP contributions from the hard kernels to the decay spectrum cancel. The three-parton B meson wave functions turn out to be suppressed by $1/m_b^2$ in this special mode. The constructive long-distance contribution almost cancels the destructive NLO radiative correction for both the form factors F_V and F_A. The B meson wave function ϕ_+ can then be extracted from the observed $B \to \gamma l \nu$ decay spectrum using the leading formalism, which suffers only the $O(1/m_b^2)$ and $O(\alpha_s^2)$ uncertainly. We conclude that the $B \to \gamma l \nu$ decay is the cleanest mode for determining this important nonperturbative input for the perturbation theories of exclusive B meson decays. The determination can be refined by including the evolution and resummation effects into the factorization formulas [20,22–24,28].

Measuring the $B \to \gamma l \nu$ spectrum in the lepton and photon energies [20],

$$\frac{d^2\Gamma}{d\eta dy} = \frac{\alpha G_F^2 |V_{ub}|^2 m_B^3}{64\pi^2}(1-\eta)\{[F_V^2(q^2) + F_A^2(q^2)] \times [2(1-y)(1-y-\eta) + \eta^2] - 2F_V(q^2)F_A(q^2)\eta(2-2y-\eta)\}, \quad (40)$$

with the lepton energy fraction $y = 2E_l/m_B$, $1 - \eta \le y \le 1$, we can extract the information of the form factors F_V and F_A separately. It is then possible to fix the two two-parton B meson wave functions $\phi_\pm$ simultaneously from Eq. (25). At this NLP level, the three-parton wave functions are still absent following the reasoning in Sec. II B. The long-distance contribution and the NLO corrections also cancel each other as indicated in Eq. (28). With the $B \to \gamma l \nu$ branching ratio around 10^{-6}, the above experimental determination is possible.

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APPENDIX: THREE-PARTON CONTRIBUTION

We start with Eq. (1.3) in Ref. [51]:

$$G^{(1)}(z) = \int d^4w \frac{i(\not{z} - \not{w})}{2\pi^2(z-w)^4} ig\not{A}(w)\frac{i\not{w}}{2\pi^2 w^4}, \quad \text{(A1)}$$

which describes the interaction of a quark with a gluon. In momentum space the above expression becomes

$$G^{(1)}(z) = \int \frac{d^4 l}{(2\pi)^4} \times \int \frac{d^4 k_2}{(2\pi)^4} e^{i(k_2+l)\cdot z}\frac{i(\not{k}_2 + \not{l})}{(k_2+l)^2}\gamma^\alpha \frac{i\not{l}}{l^2} ig\tilde{A}_\alpha(k_2), \quad \text{(A2)}$$

where l (k_2) is the momentum carried by the incoming quark (gluon). The Feynman parametrization gives

$$G^{(1)}(z) = -\int du \int \frac{d^4 l}{(2\pi)^4} e^{il\cdot z}\int \frac{d^4 k_2}{(2\pi)^4} \times e^{iuk_2\cdot z}\frac{(\not{l} + u\not{k}_2)\gamma^\alpha(\not{l} - \bar{u}\not{k}_2)}{(l^2)^2} ig\tilde{A}_\alpha(k_2), \quad \text{(A3)}$$

where the variable change $l + \bar{u}k_2 \to l$, $\bar{u} \equiv 1 - u$ has been applied.

In the case we are considering, the gluon momentum k_2 is of $O(\bar{\Lambda})$, since the B meson is dominated by soft dynamics. We expand the above expression up to $O(k_2)$:

$$\begin{aligned} G^{(1)}(x) &= -\int du \int \frac{d^4 l}{(2\pi)^4} e^{il\cdot z}\int \frac{d^4 k_2}{(2\pi)^4} e^{iuk_2\cdot z}\left\{\frac{\not{l}\gamma^\alpha \not{l}}{(l^2)^2} + \frac{u\not{k}_2\gamma^\alpha\not{l}}{(l^2)^2} - \frac{\bar{u}\not{l}\gamma^\alpha\not{k}_2}{(l^2)^2}\right\} ig\tilde{A}_\alpha(k_2) \\ &= -\int du \int \frac{d^4 l}{(2\pi)^4} e^{il\cdot z}\left\{\frac{\not{l}\gamma^\alpha\not{l}}{(l^2)^2} igA_\alpha(uz) + \frac{u\not{n}_+\gamma^\alpha\not{l} - \bar{u}\not{l}\gamma^\alpha\not{n}_+}{(l^2)^2} ig\partial_\beta A_\alpha(uz)n_-^\beta\right\}. \end{aligned} \quad \text{(A4)}$$

The first term on the right-hand side of Eq. (A4), contributing to a phase factor [51], will be dropped. For convenience, we work in the light-cone gauge $A^+ = 0$, in which the second and third terms are rewritten as

$$G^{(1)}(z) = i\int dugG_{\alpha\beta}(uz)n_-^\beta \int \frac{d^4 l}{(2\pi)^4} \times e^{il\cdot z}\frac{u\not{n}_+\gamma^\alpha\not{l} - \bar{u}\not{l}\gamma^\alpha\not{n}_+}{(l^2)^2}. \quad \text{(A5)}$$

It is clear that the field strength $gG_{\alpha\beta}(uz)n_-^\beta$ can be factored together with the rescaled b quark field h and the light quark field $\bar{u}$ into the nonlocal matrix element in Eq. (15). The integrand depending on l is then identified as the hard kernel in momentum space for the three-parton contribution. Employing Eq. (A5) for Fig. 2, and substituting $P_2 - k_1 - uk_2$ for l, we obtain Eq. (16).

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