Another possible way to determine the neutrino mass hierarchy

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We show that by combining high precision measurements of the atmospheric δm^2 in both the electron and muon neutrino (or antineutrino) disappearance channels one can determine the neutrino mass hierarchy. The required precision is a very challenging fraction of one per cent for both measurements. At even higher precision, sensitivity to the cosine of the *CP* violating phase is also possible. This method for determining the mass hierarchy of the neutrino sector does not depend on matter effects.

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Neutrino flavor transitions have been observed in atmospheric, solar, reactor and accelerator neutrino experiments. Transitions for at least two different E/L's (neutrino energy divided by baseline) are seen. To explain these transitions, extensions to the standard model of particle physics are required. The simplest and most widely accepted extension is to allow the neutrinos to have masses and mixings, similar to the quark sector, then these flavor transitions can be explained by neutrino oscillations.

This picture of neutrino masses and mixings has recently come into sharper focus with the latest salt data presented by the SNO collaboration [1]. When combined with the latest KamLAND experiment [2] and other solar neutrino experiments [3,4] the range of allowed values for the solar mass squared difference, δm^2_{21}, and the mixing angle, θ_{12}, are[1]

$$+7.3 \times 10^{-5}\ \mathrm{eV}^2 < \delta m^2_{21} < +9.0 \times 10^{-5}\ \mathrm{eV}^2$$
$$0.25 < \sin^2\theta_{12} < 0.37 \tag{1}$$

at the 90 % confidence level. Maximal mixing, $\sin^2\theta_{12} = 0.5$, has been ruled out at greater than 5 σ. The solar neutrino data is consistent with $\nu_e \to \nu_\mu$ and/or ν_τ.

The atmospheric neutrino data from Super Kamiokande has changed only slight in the last few years [6] and the latest results from the K2K long baseline experiment [7] are consistent with SK. The range of allowed values for the atmospheric mass squared difference, δm^2_{32}, and the mixing angle, θ_{23}, are

$$1.5 \times 10^{-3}\ \mathrm{eV}^2 < |\delta m^2_{32}| < 3.4 \times 10^{-3}\ \mathrm{eV}^2$$
$$0.36 < \sin^2\theta_{23} \leq 0.64 \tag{2}$$

at the 90% confidence level. The atmospheric data is consistent with $\nu_\mu \to \nu_\tau$ oscillations and the sign of δm^2_{32} is unknown. This sign is positive (negative) if the doublet of neutrino mass eigenstates, 1 and 2, which are responsible for the solar neutrino oscillations have a smaller (larger) mass than the 3rd mass eigenstate. This is the mass hierarchy question.

The best constraint on the involvement of the ν_e at the atmospheric δm^2 comes from the Chooz reactor experiment [8] and this puts a limit on the mixing angle associated with these oscillations, θ_{13}, reported as

$$0 \leq \sin^2\theta_{13} < 0.04 \tag{3}$$

at the 90 % confidence level at $|\delta m^2_{31}| = 2.5 \times 10^{-3}$ eV^2. This constraint depends on the precise value of $|\delta m^2_{31}|$ with a stronger (weaker) constraint at higher (lower) allowed values of $|\delta m^2_{31}|$.

So far the inclusion of genuine three flavor effects has not been important because these effects are controlled by the two small parameters

$$\frac{\delta m^2_{21}}{|\delta m^2_{32}|} \approx 0.03 \quad \text{and} \quad \sin^2\theta_{13} \leq 0.04. \tag{4}$$

However as the accuracy of the neutrino data improves it will become inevitable to take into account genuine three flavor effects including *CP* and T violation.

One of the goals of the next generation neutrino experiments is to establish the atmospheric mass hierarchy. Many authors have studied how to exploit matter effects in future conventional long baseline experiments [9], in supernova explosions [10] or in experiments using non conventional neutrino beams produced in a muon collider facility [11] to unravel the mass hierarchy. Here we discuss how to make this determination using precision disappearance experiments.

Genuine three generation effects make the effective atmospheric neutrino δm^2 measured by disappearance experiments, in principle, flavor dependent even in vacuum and thus sensitive to the mass hierarchy and even to the *CP* phase. This observation suggests an alternative way to access the mass hierarchy by comparing precisely measured values for the atmospheric δm^2 in $\bar{\nu}_e \to \bar{\nu}_e$ (reactor) and $\nu_\mu \to \nu_\mu$ (accelerator) modes. To illuminate this

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[1]We use the notation of Ref. [5] throughout.

rather interesting but experimentally challenging possibility is the purpose of this paper. Sensitivity to how you define the hierarchy flip for the Chooz experiment was noted in Ref. [12]. A variant of this idea, using the solar δm^2 scale, can be found in Ref. [13].

Assuming three active neutrinos only, the survival probability for the α-flavor neutrino, in vacuum, is given by

$$\begin{aligned} P(\nu_\alpha \to \nu_\alpha) &= P(\bar{\nu}_\alpha \to \bar{\nu}_\alpha) \\ &= 1 - 4|U_{\alpha 3}|^2|U_{\alpha 1}|^2\sin^2\Delta_{31} \\ &\quad - 4|U_{\alpha 3}|^2|U_{\alpha 2}|^2\sin^2\Delta_{32} \\ &\quad - 4|U_{\alpha 2}|^2|U_{\alpha 1}|^2\sin^2\Delta_{21}, \end{aligned} \tag{5}$$

where $\Delta_{ij} = \delta m^2_{ij}L/4E$, $\delta m^2_{ij} = m^2_i - m^2_j$ and $U_{\alpha i}$ are elements of the MNS mixing matrix, [14]. The three Δ_{ij} are not independent since the δm^2_{ij}'s satisfy the constraint, $\delta m^2_{31} = \delta m^2_{32} + \delta m^2_{21}$.

If we define an effective atmospheric mass squared difference, δm^2_η, which depends linearly on the parameter η, as follows

$$\delta m^2_\eta \equiv \delta m^2_{31} - \eta\delta m^2_{21} = \delta m^2_{32} + (1-\eta)\delta m^2_{21}$$

$$\text{so that}\quad \Delta_\eta = \Delta_{31} - \eta\Delta_{21} = \Delta_{32} + (1-\eta)\Delta_{21} = \frac{\delta m^2_\eta L}{4E}, \tag{6}$$

then we can rewrite Eq. (5) using the two independent variables, Δ_η and Δ_{21}, as

$$\begin{aligned} 1 - P(\nu_\alpha \to \nu_\alpha) = 4|U_{\alpha 3}|^2(1 - |U_{\alpha 3}|^2)\Big[&\sin^2\Delta_\eta + \{r_1\sin^2(\eta\Delta_{21}) + r_2\sin^2((1-\eta)\Delta_{21})\}\cos 2\Delta_\eta \\ &+ \frac{1}{2}\{r_1\sin(2\eta\Delta_{21}) - r_2\sin(2(1-\eta)\Delta_{21})\}\sin 2\Delta_\eta\Big] + 4|U_{\alpha 2}|^2|U_{\alpha 1}|^2\sin^2\Delta_{21}, \end{aligned} \tag{7}$$

where

$$r_1 = \frac{|U_{\alpha 1}|^2}{|U_{\alpha 1}|^2 + |U_{\alpha 2}|^2} \quad \text{and} \quad r_2 = \frac{|U_{\alpha 2}|^2}{|U_{\alpha 1}|^2 + |U_{\alpha 2}|^2} = 1 - r_1. \tag{8}$$

Notice that the coefficient in front of $\sin 2\Delta_\eta$ is the derivative of the coefficient in front of $\cos 2\Delta_\eta$, with respect to $\eta\Delta_{21}$, up to a constant factor. Therefore by choosing η so as to set the coefficient in front of $\sin 2\Delta_\eta$ to zero one also minimizes the coefficient in front of $\cos 2\Delta_\eta$. That is, if η satisfies

$$\eta = \frac{1}{2\Delta_{21}}\arctan\left\{\frac{r_2\sin 2\Delta_{21}}{r_1 + r_2\cos 2\Delta_{21}}\right\} \approx r_2, \tag{9}$$

one minimizes the effects of both $\sin 2\Delta_\eta$ and $\cos 2\Delta_\eta$ terms and this δm^2_η with $\eta \approx r_2$ *is* truly the effective atmospheric δm^2, $\delta m^2_{\rm eff}|_\alpha$, measured in ν_α disappearance experiments. The approximation $\eta = r_2$ is excellent provided that $\Delta_{21} \ll 1$.

Using this approximate solution for η, the effective atmospheric δm^2 for the α-flavor is[2]

$$\begin{aligned} \delta m^2_{\rm eff}|_\alpha &\equiv \frac{|U_{\alpha 1}|^2\delta m^2_{31} + |U_{\alpha 2}|^2\delta m^2_{32}}{|U_{\alpha 1}|^2 + |U_{\alpha 2}|^2} \\ &= r_1\delta m^2_{31} + r_2\delta m^2_{32}, \end{aligned} \tag{11}$$

then the full neutrino survival probability in vacuum, Eq. (5), can be rewritten as

$$\begin{aligned} 1 - P(\nu_\alpha \to \nu_\alpha) = 4|U_{\alpha 3}|^2(1 - |U_{\alpha 3}|^2)[&\sin^2\Delta_{\rm eff} + \{r_1\sin^2(r_2\Delta_{21}) + r_2\sin^2(r_1\Delta_{21})\}\cos 2\Delta_{\rm eff} \\ &- \frac{1}{2}\{r_2\sin(2r_1\Delta_{21}) - r_1\sin(2r_2\Delta_{21})\}\sin 2\Delta_{\rm eff}] + 4|U_{\alpha 2}|^2|U_{\alpha 1}|^2\sin^2\Delta_{21}. \end{aligned} \tag{12}$$

where $\Delta_{\rm eff} \equiv \delta m^2_{\rm eff}|_\alpha L/4E$. If the coefficients in front of the $\cos 2\Delta_{\rm eff}$ and $\sin 2\Delta_{\rm eff}$ terms are expanded in powers of Δ_{21}, one finds

[2] An alternative way to derive this is to notice that the first extremum, of the terms in Eq. (5) proportional to $|U_{\alpha 3}|^2$, occurs when

$$\frac{|U_{\alpha 1}|^2\Delta_{31} + |U_{\alpha 2}|^2\Delta_{32}}{|U_{\alpha 1}|^2 + |U_{\alpha 2}|^2} = \frac{\pi}{2}, \tag{10}$$

to first nontrivial order in Δ_{21}.

$$\{r_1 \sin^2(r_2\Delta_{21}) + r_2 \sin^2(r_1\Delta_{21})\} = r_1 r_2 \Delta_{21}^2 + \mathcal{O}(\Delta_{21}^4)\frac{1}{2}$$
$$\{r_2 \sin(2r_1\Delta_{21}) - r_1 \sin(2r_2\Delta_{21})\} = \frac{2}{3} r_1 r_2 (r_2 - r_1)\Delta_{21}^3 + \mathcal{O}(\Delta_{21}^5), \quad (13)$$

and one can see clearly that all terms linear in Δ_{21} have been absorbed into the $\Delta_{\rm eff}$ terms. This confirms that $\delta m^2_{\rm eff}$, Eq. (11), is the effective atmospheric δm^2 to first nontrivial order in δm^2_{21}. Note also that the first term odd in $\Delta_{\rm eff}$ occurs with a coefficient proportional to Δ^3_{21} which, at the first extremum, is a suppression factor of order 10^{-4}.

To understand the physical meaning of the effective atmospheric δm^2 it is useful to write it as follows

$$\delta m^2_{\rm eff}|_\alpha = m_3^2 - \langle m_\alpha^2\rangle_{12}, \quad \text{where} \quad \langle m_\alpha^2\rangle_{12} \equiv \frac{|U_{\alpha 2}|^2 m_2^2 + |U_{\alpha 1}|^2 m_1^2}{|U_{\alpha 1}|^2 + |U_{\alpha 2}|^2}. \quad (14)$$

Now $\langle m_\alpha^2\rangle_{12}$ has a clear interpretation, it is the α-flavor weighted average mass square of neutrino states 1 and 2. Thus the effective atmospheric δm^2 is the difference in the mass squared of the state 3 and this flavor average mass square of states 1 and 2 and is clearly flavor dependent.

The three flavor average mass squares are[3]

$$\langle m_e^2\rangle_{12} = \frac{1}{2}[m_2^2 + m_1^2 - \cos 2\theta_{12}\delta m^2_{21}]$$
$$\langle m_\mu^2\rangle_{12} = \frac{1}{2}[m_2^2 + m_1^2 + (\cos 2\theta_{12} - 2\cos\delta \sin\theta_{13}\sin 2\theta_{12}\tan\theta_{23})\delta m^2_{21}] \quad (15)$$
$$\langle m_\tau^2\rangle_{12} = \frac{1}{2}[m_2^2 + m_1^2 + (\cos 2\theta_{12} + 2\cos\delta \sin\theta_{13}\sin 2\theta_{12}\cot\theta_{23})\delta m^2_{21}],$$

where the τ-flavor average is given for completeness only.

It is now obvious that ν_e and ν_μ disappearance experiments measure *different* $\delta m^2_{\rm eff}$'s. In fact the three disappearance $\delta m^2_{\rm eff}$ are[4]

$$\delta m^2_{\rm eff}|_e = \cos^2\theta_{12}\delta m^2_{31} + \sin^2\theta_{12}\delta m^2_{32} \quad (16)$$

$$\delta m^2_{\rm eff}|_\mu = \sin^2\theta_{12}\delta m^2_{31} + \cos^2\theta_{12}\delta m^2_{32} + \cos\delta \sin\theta_{13}\sin 2\theta_{12}\tan\theta_{23}\delta m^2_{21} \quad (17)$$

$$\delta m^2_{\rm eff}|_\tau = \sin^2\theta_{12}\delta m^2_{31} + \cos^2\theta_{12}\delta m^2_{32} - \cos\delta \sin\theta_{13}\sin 2\theta_{12}\cot\theta_{23}\delta m^2_{21}. \quad (18)$$

Note that if $\sin^2\theta_{12} \to 0$ then $\delta m^2_{\rm eff}|_e \to \delta m^2_{31}$ and $\delta m^2_{\rm eff}|_{\mu,\tau} \to \delta m^2_{32}$ as it must, as the mass eigenstate ν_1 is nearly 100% ν_e in this limit.

[3]Dropping terms of order $\sin^2\theta_{13}\delta m^2_{21}$.

[4]The effective atmospheric mass squared difference for the muon channel has been discussed in Ref. [15].

In Fig. 1 we show the survival probability in the $\bar{\nu}_e$ and ν_μ disappearance channels using three different choices of the atmospheric δm^2 whose sign flip, with constant magnitude, changes the hierarchy from normal to inverted. When we use $\delta m^2_{\rm eff}|_\alpha$ for the α flavor, the change in the survival probability is very small when we flip the hierarchy i.e. the magnitude of this $\delta m^2_{\rm eff}$ is insensitive to which hierarchy nature has chosen. Although δm^2_{31} (δm^2_{32}) works better for $\bar{\nu}_e$ (ν_μ) disappearance experiments neither choice is as good as $\delta m^2_{\rm eff}$. Thus, in summary, $\delta m^2_{\rm eff}|_e$, Eq. (16), is the atmospheric δm^2 measured by $\bar{\nu}_e$ disappearance experiments and $\delta m^2_{\rm eff}|_\mu$, Eq. (17), is the atmospheric δm^2 measured by ν_μ disappearance experiments upto corrections of $\mathcal{O}(\delta m^2_{21}/\delta m^2_{32})^2$.

Whether the absolute value of $\delta m^2_{\rm eff}|_e$ is larger or smaller than the absolute value of $\delta m^2_{\rm eff}|_\mu$ depends on whether $|\delta m^2_{31}|$ is larger or smaller than $|\delta m^2_{32}|$. The relative magnitude of these two δm^2 is determined by whether the mass squared of the 3-state is larger or smaller than the mass squared of the 1- and 2-states, i.e. by the neutrino mass hierarchy. It is easy to show that the difference in the absolute value of the e-flavor and μ-flavor $\delta m^2_{\rm eff}$'s is given by

$$|\delta m^2_{\rm eff}|_e - |\delta m^2_{\rm eff}|_\mu = \pm\delta m^2_{21}(\cos 2\theta_{12} - \cos\delta \sin\theta_{13}\sin 2\theta_{12}\tan\theta_{23}), \quad (19)$$

where the + sign (− sign) is for the normal (inverted) hierarchy. Thus by precision measurements of both of these $\delta m^2_{\rm eff}$ one can determine the hierarchy and possibly even $\cos\delta$ at very high precision. This identity, Eq. (19), is the principal observation of this paper.

In Fig. 2, we show the fractional difference in the effective atmospheric δm^2 for the normal and inverted hierarchy, as a function of $\sin^2\theta_{13}$. For the normal hierarchy, independently of δ, this normalized ratio is always positive, while for the inverted hierarchy, it is always negative. While the size of difference between the two hierarchies is smallest for $\cos\delta = 1$, for this value of δ, the difference between the two hierarchies increases as $\sin^2\theta_{13}$ goes to zero, as can be seen from Eq. (19).

What kind of precision is required? Given that

$$\frac{\delta m^2_{21}}{|\delta m^2_{32}|} \approx 0.03 \quad \text{and} \quad \cos 2\theta_{12} \approx 0.38, \quad (20)$$

the difference in the magnitude of the two effective atmos-

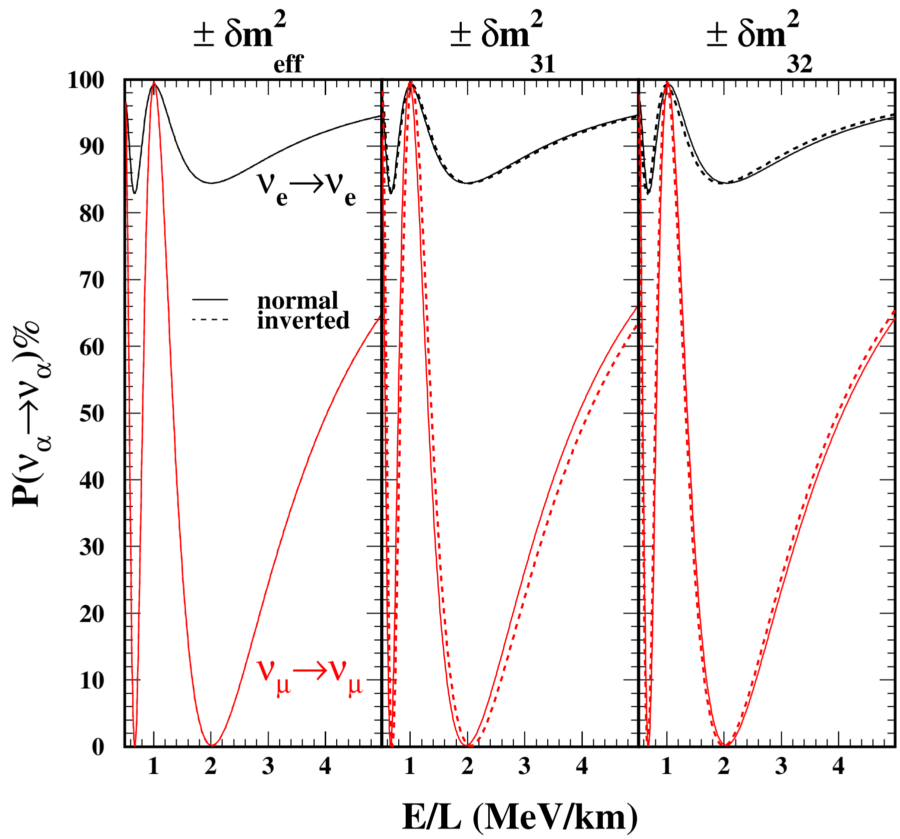

FIG. 1 (color online). The vacuum survival probability, $P(\nu_\alpha \to \nu_\alpha)$, as a function of E/L for the two mass hierarchies using three different choices of the atmospheric δm^2 whose sign flip, with constant magnitude, changes the hierarchy: $\delta m^2_{\rm eff}|_\alpha$ (left panel), δm^2_{31} (middle panel) and δm^2_{32} (right panel). The survival probability for the two different hierarchies coincide to high precision when the effective δm^2's, Eq. (16) and (17), are used (left panel) whereas they differ noticeably with the other two definitions. For this figure we have used $\sin^2\theta_{23} = 0.5$ (maximal mixing), $\sin^2\theta_{13} = 0.04$ (Chooz bound), $\sin^2\theta_{12} = 0.31$, $\delta m^2_{21} = +8.0 \times 10^{-5}$ eV^2 and the atmospheric δm^2 to be 2.5×10^{-3} eV^2.

pheric δm^2 is 1 to 2%. Currently, the uncertainty on the size of this difference is dominated by the experimental uncertainty on the ratio of the solar to atmospheric δm^2's. To determine the hierarchy we need to determine whether $|\delta m^2_{\rm eff}|_e$ is larger, normal hierarchy, or smaller, inverted hierarchy, than $|\delta m^2_{\rm eff}|_\mu$. Thus determining the hierarchy with a confidence level near 90% one needs to measure *both* $\delta m^2_{\rm eff}$ to better than one per cent precision. These are very challenging levels of precision for atmospheric δm^2 measurements both within a given experiment and between two different experiments. In Fig. 3 we have calculated the required precision as function of the C.L., measured in sigmas, assuming that the two experiments have the same % precision. From this figure we see that for a 90% C.L. determination of the hierarchy one would require ~0.5% precision on *both* $\delta m^2_{\rm eff}$ measurements. Achieving such precision will require significant innovation.

So far our discussion has only been in vacuum. What about matter effects? How much do they shift the first extrema? For the ν_e disappearance channel the shift in the extrema is proportional to (aL) where $a = G_F N_e/\sqrt{2} \approx (4000\ \text{km})^{-1}$. Thus the expected shift is less than 0.1% for a baseline of a few kilometers. The size of this shift has been confirmed by a numerical calculation. For the ν_μ disappearance channel the shift in the extrema is again proportional to (aL) but here the baseline could go up to 1000 km. However the coefficient in front of (aL) is proportional to $\sin^2 2\theta_{13}$ and $\cos 2\theta_{23}/\cos^2\theta_{23}$ both of which are small numbers. Using an energy so that the first minimum occurs at 1000 km, we have calculate numerically the size of the shift assuming $\sin^2 2\theta_{13}$ is at the Chooz bound and found that the maximum shift is 0.4%. This maximum shift occurs when θ_{23} is as larger as is allowed by atmospheric neutrino data. If $\sin^2\theta_{23}$ and/or $\sin^2\theta_{13}$ are smaller than these maximum values then the shift is smaller. Also the shift at baselines smaller than 1000 km are proportionally smaller. Therefore, we conclude that in general matter effects can be safely ignored, or corrected for, in ν_μ disappearance experiments whose baseline is less than 1000 km.

In summary we have demonstrated that high precision measurements of the effective atmospheric δm^2 in both the

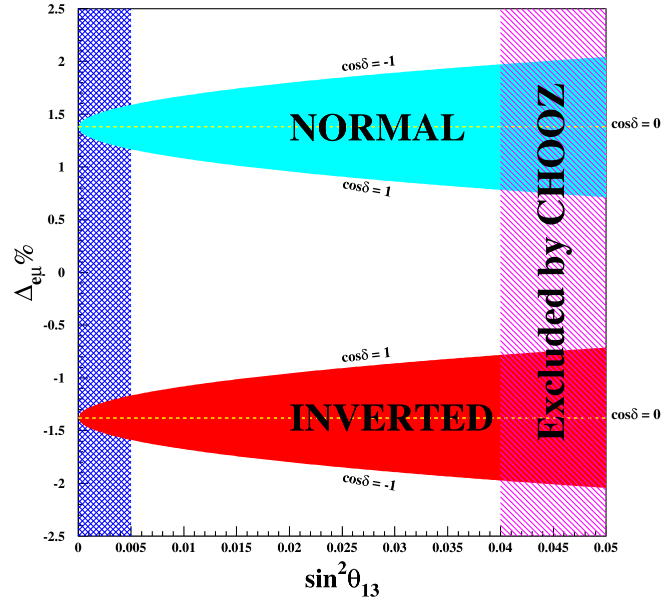

FIG. 2 (color online). The fractional difference of the electron and muon neutrino effective atmospheric δm^2, $\Delta_{e\mu} \equiv (|\delta m^2_{\rm eff}|_e - |\delta m^2_{\rm eff}|_\mu)/|\delta m^2_{\rm eff}|$, as a function of $\sin^2\theta_{13}$ for the normal and inverted hierarchies showing the dependence on $\cos\delta$. The vertical scale varies linearly with the not so well known ratio of $\delta m^2_{21}/|\delta m^2_{32}|$; here we have used $\delta m^2_{21} = 8.0 \times 10^{-5}$ eV^2 and $|\delta m^2_{32}| = 2.5 \times 10^{-3}$ eV^2. In a reactor $\bar{\nu}_e$ disappearance experiment, precision measurement of the effective atmospheric $\delta m^2_{\rm eff}|_e$ is probably very difficult unless $\sin^2\theta_{13} > 0.005$.

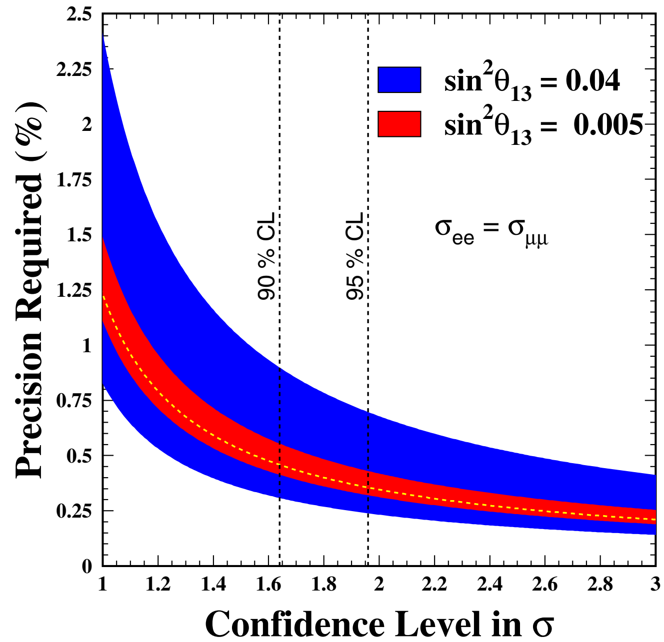

FIG. 3 (color online). The required percentage precision need to determine the neutrino mass hierarchy verses the confidence level of that determination. Here we have assumed both effective atmospheric δm^2 are measured with the same precision, $\sigma_{ee} = \sigma_{\mu\mu}$. The cosine of the *CP* violating phase is varied from +1 (bottom) through 0 (dashed line) to -1 (top). Again, the vertical scale varies linearly with the not so well known ratio of $\delta m^2_{21}/|\delta m^2_{32}|$. For this figure we have used 0.032, the same as in Fig. 2.

$\bar{\nu}_e \to \bar{\nu}_e$ (reactor) and $\nu_\mu \to \nu_\mu$ (long baseline accelerator) channels can determine the neutrino mass hierarchy independent of matter effects. The sign of the difference determines the hierarchy. For any reasonable confidence level determination the precision required in *both* channels is a very challenging fraction of 1%. The next generation of long baseline experiments such as T2K [16] and NOνA [17] estimate their precision on the effective atmospheric δm^2 at 2%. However, so far there has been no physics reason to push this to a precision measurement. For the reactor channel the emphasis so far has been on the observation of nonzero θ_{13} [18], very little effort has been made on a precision determination of the effective atmospheric δm^2. This kind of precision, can perhaps be achieved in beta beam facility [19]. We realize that to make these measurements to the precision suggested is very challenging experimentally. However we encourage our experimental colleagues to give this some thought especially since this method has a different dependence on the unknown *CP* violating phase, $\cos\delta$ versus $\sin\delta$, compared with long baseline experiments.

While we were completing this manuscript, Ref. [20] appeared which discusses the physics of this possibility in a pure 3-flavor frame work as well as discussing other possible ways of determining the hierarchy.

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