

Lepton charge and neutrino mixing in pion decay processesMassimo Blasone,¹ Antonio Capolupo,¹ Francesco Terranova,² and Giuseppe Vitiello¹¹*Dipartimento di Fisica and INFN, Università di Salerno, I-84100 Salerno, Italy*²*Laboratori Nazionali di Frascati dell'INFN, I-00044, Frascati, Italy*

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We consider neutrino mixing and oscillations in quantum field theory and compute the neutrino lepton charge in decay processes where neutrinos are generated. We also discuss the proper definitions of flavor charge and states and clarify the issues of the possibility of different mass parameters in field mixing.

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I. INTRODUCTION

In the context of quantum field theory (QFT) a rich nonperturbative vacuum structure associated with the mixing of fermion and boson fields has been revealed, [1–15] and the exact formulas for fermion and boson field oscillations [2,7,10–12] are now established. In particular, in a full QFT treatment particle mixing exhibits new features with respect to the usual formulas in quantum mechanics (QM) [16]. The phenomenological analysis for meson mixing has shown that, while for most of mixed systems the nonperturbative structure of the vacuum produces negligibly small effects, for strongly mixed systems such as $\omega - \phi$ or $\eta - \eta'$ nonperturbative corrections can be as large as 5%–20% [14]. Moreover the nonperturbative field theory effects may contribute in a crucial way in other physical contexts. For example, as shown in [17], the neutrino mixing may contribute to the value of the cosmological constant exactly because of the nonperturbative effects.

There are, however, several aspects which still need to be fully developed. For example, how to deal, in the presence of mixing, with those decay processes where neutrinos are generated.

Since the time of the introduction of the Pontecorvo mixing transformations [16], it is well known that the mixed (flavor) neutrinos are not mass eigenstates. This implies that flavor neutrinos are not representations of the Poincaré group; one cannot think of them as asymptotic fields in the frame of the Lehmann-Symanzik-Zimmerman (LSZ) formalism [18]. The QFT analysis of the mixing phenomenon has indeed clarified [1] that flavor neutrino field operators do not have the mathematical characterization necessary in order to be defined as asymptotic field operators acting on the massive neutrino vacuum. The origin of this is related to the fact that the vacuum for the massive neutrinos turns out to be unitarily inequivalent to the vacuum for the mixed neutrino fields.

Previous works were mainly focused on the determination of the oscillation probability through the analysis of the expectation value of the flavor charge operator

$$\langle \nu_\rho(t) | Q_{\nu_\sigma} | \nu_\rho(t) \rangle \quad (1)$$

Q being a function of the flavor annihilation/creation op-

erators [19] and $\rho, \sigma = e, \mu, \tau$ flavor indices. In fact, flavor states are produced in weak interaction processes and we are left to the question whether Eq. (1) can be consistently extended to include the neutrino production vertex and what is the explicit form of

$$\langle \Pi(t) | Q_{\nu_\sigma} | \Pi(t) \rangle, \quad (2)$$

where $|\Pi(t)\rangle$ represents the evolution at time t of the parent state for the neutrino.

The computation of the matrix element (2) is not a trivial matter since it simultaneously involves LSZ states and flavor states, which are not LSZ. This paper is devoted to the study of such matrix element and of several related topics. In Sec. II we review the formalism of neutrino mixing in QFT and discuss the proper definition of flavor charges and states; in Sec. III we clarify the issues of the possibility of different mass parameters in field mixing, which has recently attracted some attention [5,7,9,10,20]. In Sec. IV we perform a careful analysis of (2). Conclusions are drawn in Sec. V. For the reader's convenience, we present in Appendix A the proof of orthogonality of the flavor states at different times. The Appendix B contains the explicit form of some equations derived in the text.

II. FLAVOR FIELDS AND STATES

Let us start by introducing the general frame for our discussion, which is also useful to set up our notations. For a detailed review see [13]. For simplicity we consider only two Dirac neutrino fields. The Pontecorvo mixing transformations are [16]

$$\begin{aligned} \nu_e(x) &= \nu_1(x) \cos\theta + \nu_2(x) \sin\theta, \\ \nu_\mu(x) &= -\nu_1(x) \sin\theta + \nu_2(x) \cos\theta, \end{aligned} \quad (3)$$

where θ is the mixing angle and ν_1 and ν_2 are massive noninteracting, free fields, anticommuting with each other at any space-time point. The fields ν_1 and ν_2 have nonzero masses $m_1 \neq m_2$ and are explicitly given by

$$\nu_i(x) = \frac{1}{\sqrt{V}} \sum_{\mathbf{k}, r} [u_{\mathbf{k}, i}^r(t) \alpha_{\mathbf{k}, i}^r + v_{-\mathbf{k}, i}^r(t) \beta_{-\mathbf{k}, i}^{r\dagger}] e^{i\mathbf{k}\cdot\mathbf{x}}, \quad i = 1, 2. \quad (4)$$

with $u_{\mathbf{k},i}^r(t) = e^{-i\omega_{k,i}t}u_{\mathbf{k},i}^r(0)$, $v_{\mathbf{k},i}^r(t) = e^{i\omega_{k,i}t}v_{\mathbf{k},i}^r(0)$ and $\omega_{k,i} = \sqrt{\mathbf{k}^2 + m_i^2}$. Here and in the following we use $t \equiv x_0$, when no misunderstanding arises. The vacuum for the α_i and β_i operators is denoted by $|0\rangle_{1,2}$: $\alpha_{\mathbf{k},i}^r|0\rangle_{1,2} = \beta_{\mathbf{k},i}^r|0\rangle_{1,2} = 0$. The anticommutation relations are the usual ones (see Ref. [1]). The orthonormality and completeness relations are

$$u_{\mathbf{k},i}^{r\dagger}u_{\mathbf{k},i}^s = v_{\mathbf{k},i}^{r\dagger}v_{\mathbf{k},i}^s = \delta_{rs}, \quad u_{\mathbf{k},i}^{r\dagger}v_{-\mathbf{k},i}^s = v_{-\mathbf{k},i}^{r\dagger}u_{\mathbf{k},i}^s = 0, \\ \sum_r (u_{\mathbf{k},i}^r u_{\mathbf{k},i}^{r\dagger} + v_{-\mathbf{k},i}^r v_{-\mathbf{k},i}^{r\dagger}) = 1. \quad (5)$$

The fields ν_e and ν_μ are completely determined through Eq. (3), which can be rewritten in the form (we use $(\sigma, i) = (e, 1), (\mu, 2)$):

$$\nu_\sigma(x) = G_\theta^{-1}(t)\nu_i(x)G_\theta(t) \\ = \frac{1}{\sqrt{V}} \sum_{\mathbf{k},r} [u_{\mathbf{k},i}^r(t)\alpha_{\mathbf{k},\nu_\sigma}^r(t) + v_{-\mathbf{k},i}^r(t)\beta_{-\mathbf{k},\nu_\sigma}^{r\dagger}(t)]e^{i\mathbf{k}\cdot\mathbf{x}}, \quad (6)$$

with $G_\theta(t)$ the generator of the mixing transformations

$$\begin{pmatrix} \alpha_{\mathbf{k},\nu_e}^r(t) \\ \alpha_{\mathbf{k},\nu_\mu}^r(t) \\ \beta_{-\mathbf{k},\nu_e}^{r\dagger}(t) \\ \beta_{-\mathbf{k},\nu_\mu}^{r\dagger}(t) \end{pmatrix} = \begin{pmatrix} c_\theta & s_\theta U_{\mathbf{k}}^*(t) & 0 & s_\theta V_{\mathbf{k}}(t) \\ -s_\theta U_{\mathbf{k}}(t) & c_\theta & s_\theta V_{\mathbf{k}}(t) & 0 \\ 0 & -s_\theta V_{\mathbf{k}}(t) & c_\theta & s_\theta U_{\mathbf{k}}^*(t) \\ -s_\theta V_{\mathbf{k}}(t) & 0 & -s_\theta U_{\mathbf{k}}(t) & c_\theta \end{pmatrix} \begin{pmatrix} \alpha_{\mathbf{k},1}^r \\ \alpha_{\mathbf{k},2}^r \\ \beta_{-\mathbf{k},1}^{r\dagger} \\ \beta_{-\mathbf{k},2}^{r\dagger} \end{pmatrix}, \quad (9)$$

where $c_\theta \equiv \cos\theta$, $s_\theta \equiv \sin\theta$ and

$$U_{\mathbf{k}}(t) \equiv u_{\mathbf{k},2}^{r\dagger}(t)u_{\mathbf{k},1}^r(t) = v_{-\mathbf{k},1}^{r\dagger}(t)v_{-\mathbf{k},2}^r(t) \quad (10)$$

$$V_{\mathbf{k}}(t) \equiv \epsilon^r u_{\mathbf{k},1}^{r\dagger}(t)v_{-\mathbf{k},2}^r(t) = -\epsilon^r u_{\mathbf{k},2}^{r\dagger}(t)v_{-\mathbf{k},1}^r(t), \quad (11)$$

with $\epsilon^r = (-1)^r$. We have

$$U_{\mathbf{k}}(t) = |U_{\mathbf{k}}|e^{i(\omega_{k,2}-\omega_{k,1})t}, \quad V_{\mathbf{k}}(t) = |V_{\mathbf{k}}|e^{i(\omega_{k,2}+\omega_{k,1})t} \quad (12)$$

$$|U_{\mathbf{k}}| = \left(\frac{\omega_{k,1} + m_1}{2\omega_{k,1}}\right)^{1/2} \left(\frac{\omega_{k,2} + m_2}{2\omega_{k,2}}\right)^{1/2} \\ \times \left(1 + \frac{|\mathbf{k}|^2}{(\omega_{k,1} + m_1)(\omega_{k,2} + m_2)}\right) \quad (13)$$

$$|V_{\mathbf{k}}| = \left(\frac{\omega_{k,1} + m_1}{2\omega_{k,1}}\right)^{1/2} \left(\frac{\omega_{k,2} + m_2}{2\omega_{k,2}}\right)^{1/2} \\ \times \left(\frac{|\mathbf{k}|}{(\omega_{k,2} + m_2)} - \frac{|\mathbf{k}|}{(\omega_{k,1} + m_1)}\right), \quad (14)$$

$$|U_{\mathbf{k}}|^2 + |V_{\mathbf{k}}|^2 = 1. \quad (15)$$

As discussed in Ref. [3], in the two flavor mixing case the group structure associated with mixing transformations

Eq. (3)

$$G_\theta(t) = \exp\left[\theta \int d^3\mathbf{x}(\nu_1^\dagger(x)\nu_2(x) - \nu_2^\dagger(x)\nu_1(x))\right]. \quad (7)$$

Equation (6) provides an expansion of the flavor fields ν_e and ν_μ in the same basis of ν_1 and ν_2 . The flavor annihilation operators are then identified with

$$\begin{pmatrix} \alpha_{\mathbf{k},\nu_\sigma}^r(t) \\ \beta_{-\mathbf{k},\nu_\sigma}^{r\dagger}(t) \end{pmatrix} = G_\theta^{-1}(t) \begin{pmatrix} \alpha_{\mathbf{k},i}^r \\ \beta_{-\mathbf{k},i}^{r\dagger} \end{pmatrix} G_\theta(t) \quad (8)$$

The flavor vacuum is defined as $|0(t)\rangle_{e,\mu} \equiv G_\theta^{-1}(t)|0\rangle_{1,2}$ and turns out to be orthogonal to the vacuum for the mass eigenstates $|0\rangle_{1,2}$ in the infinite volume limit. Note the time dependence of $|0(t)\rangle_{e,\mu}$: it turns out that flavor vacua taken at different times are orthogonal in the infinite volume limit (see Appendix A). In the following for simplicity we will use the notation $|0\rangle_{e,\mu} \equiv |0(0)\rangle_{e,\mu}$ to denote the flavor vacuum state at the reference time $t = 0$.

The explicit expression of the flavor annihilation/creation operators for $\mathbf{k} = (0, 0, |\mathbf{k}|)$ is

is $SU(2)$ and one can define the following charges in the mass basis:

$$Q_{m,j}(t) = \frac{1}{2} \int d^3\mathbf{x} \nu_m^\dagger(x) \tau_j \nu_m(x), \quad j = 1, 2, 3, \quad (16)$$

where $\nu_m^T = (\nu_1, \nu_2)$ and $\tau_j = \sigma_j/2$ with σ_j being the Pauli matrices. The $U(1)$ Noether charges associated with ν_1 and ν_2 can be then expressed as

$$Q_1 \equiv \frac{1}{2}Q + Q_{m,3}; \quad Q_2 \equiv \frac{1}{2}Q - Q_{m,3}. \quad (17)$$

with Q total (conserved) charge. As usual, we need to normal order such charge operators:

$$:\mathcal{Q}_i: \equiv \int d^3\mathbf{x} : \nu_i^\dagger(x) \nu_i(x) : \\ = \sum_r \int d^3\mathbf{k} (\alpha_{\mathbf{k},i}^{r\dagger} \alpha_{\mathbf{k},i}^r - \beta_{-\mathbf{k},i}^{r\dagger} \beta_{-\mathbf{k},i}^r), \quad (18) \\ i = 1, 2.$$

where the $:\dots:$ denotes normal ordering with respect to the vacuum $|0\rangle_{1,2}$.

It is then clear that the neutrino states with definite masses defined as

$$|\nu_{\mathbf{k},i}^r\rangle = \alpha_{\mathbf{k},i}^{r\dagger}|0\rangle_{1,2}, \quad i = 1, 2, \quad (19)$$

are eigenstates of the above conserved charges, which can be identified with the lepton charges in the absence of mixing.

The situation changes when we turn to the flavor basis [3]

$$Q_{f,j}(t) = \frac{1}{2} \int d^3\mathbf{x} \nu_f^\dagger(x) \tau_j \nu_f(x), \quad f = e, \mu, \quad j = 1, 2 \quad (20)$$

where $\nu_f^T = (\nu_e, \nu_\mu)$. Observe that the diagonal $SU(2)$ generator $Q_{f,3}(t)$ is time-dependent in the flavor basis.

Thus the flavor charges defined as

$$Q_{\nu_e}(t) = \frac{1}{2} Q + Q_{f,3}(t); \quad Q_{\nu_\mu}(t) = \frac{1}{2} Q - Q_{f,3}(t), \quad (21)$$

are now time-dependent and are the lepton charges in presence of mixing [3]. Indeed their expectation values in the flavor state (see Eq. (26) below) give the oscillation formulas [2,7,10–12].

Particular attention has to be paid now to the normal ordering issue. We define the normal ordered charges $::Q_{\nu_\sigma}(t)::$ with respect to the vacuum $|0\rangle_{e,\mu}$ as

$$\begin{aligned} ::Q_{\nu_\sigma}(t):: &\equiv \int d^3\mathbf{x} ::\nu_\sigma^\dagger(x) \nu_\sigma(x):: \\ &= \sum_r \int d^3\mathbf{k} (\alpha_{\mathbf{k},\nu_\sigma}^{r\dagger}(t) \alpha_{\mathbf{k},\nu_\sigma}^r(t) - \beta_{-\mathbf{k},\nu_\sigma}^{r\dagger}(t) \\ &\quad \times \beta_{-\mathbf{k},\nu_\sigma}^r(t)), \\ \sigma &= e, \mu; \end{aligned} \quad (22)$$

where the new symbol $::\dots::$ for the normal ordering was introduced to remember that it refers to the flavor vacuum. The definition for any operator A , is the following

$$::A:: \equiv A -_{e,\mu} \langle 0|A|0\rangle_{e,\mu} \quad (23)$$

Note that

$$\begin{aligned} ::Q_{\nu_\sigma}(t):: &= G_\theta^{-1}(t) :Q_j: G_\theta(t), \\ \text{with } (\sigma, j) &= (e, 1), (\mu, 2), \end{aligned} \quad (24)$$

and

$$::Q_{\nu_e}(t):: + ::Q_{\nu_\mu}(t):: = :Q_1: + :Q_2: = :Q:. \quad (25)$$

We define the flavor states as eigenstates of the flavor charges Q_{ν_σ} at a reference time $t = 0$

$$|\nu_{\mathbf{k},\sigma}^r\rangle \equiv \alpha_{\mathbf{k},\nu_\sigma}^{r\dagger}(0)|0(0)\rangle_{e,\mu}, \quad \sigma = e, \mu \quad (26)$$

and similar ones for antiparticles. We have

$$\begin{aligned} ::Q_{\nu_e}(0):: |\nu_{\mathbf{k},e}^r\rangle &= |\nu_{\mathbf{k},e}^r\rangle; & ::Q_{\nu_\mu}(0):: |\nu_{\mathbf{k},\mu}^r\rangle &= |\nu_{\mathbf{k},\mu}^r\rangle \\ & & & (27) \end{aligned}$$

and $::Q_{\nu_e}(0):: |\nu_{\mathbf{k},\mu}^r\rangle = ::Q_{\nu_\mu}(0):: |\nu_{\mathbf{k},e}^r\rangle = 0$. Moreover

$$::Q_{\nu_\sigma}(0):: |0\rangle_{e,\mu} = 0. \quad (28)$$

These results are far from being trivial since the usual Pontecorvo states [16]

$$|\nu_{\mathbf{k},e}^r\rangle_P = \cos\theta |\nu_{\mathbf{k},1}^r\rangle + \sin\theta |\nu_{\mathbf{k},2}^r\rangle \quad (29)$$

$$|\nu_{\mathbf{k},\mu}^r\rangle_P = -\sin\theta |\nu_{\mathbf{k},1}^r\rangle + \cos\theta |\nu_{\mathbf{k},2}^r\rangle, \quad (30)$$

are *not* eigenstates of the flavor charges, as can be easily checked.

It is instructive to consider the expectation values of the flavor charges onto the Pontecorvo states, in order to better appreciate how much the lepton charge is violated in the usual quantum mechanical states. We find

$$\begin{aligned} {}_P\langle \nu_{\mathbf{k},e}^r | ::Q_{\nu_e}(0):: | \nu_{\mathbf{k},e}^r \rangle_P &= \cos^4\theta + \sin^4\theta \\ &\quad + 2|U_{\mathbf{k}}| \sin^2\theta \cos^2\theta \\ &\quad + \sum_r \int d^3\mathbf{k}, \end{aligned} \quad (31)$$

and

$${}_{1,2}\langle 0 | ::Q_{\nu_e}(0):: | 0 \rangle_{1,2} = \sum_r \int d^3\mathbf{k}, \quad (32)$$

Equations (31) and (32) clearly are both infinite.

One may think that the problem with infinity is due to the normal ordering with respect to the flavor vacuum and consider the expectation values of $:Q_{\nu_\sigma}(t):$, i.e. the normal ordered flavor charges with respect to the mass vacuum $|0\rangle_{1,2}$. One has then

$$\begin{aligned} {}_P\langle \nu_{\mathbf{k},e}^r | :Q_{\nu_e}(0): | \nu_{\mathbf{k},e}^r \rangle_P &= \cos^4\theta + \sin^4\theta \\ &\quad + 2|U_{\mathbf{k}}| \sin^2\theta \cos^2\theta < 1, \end{aligned} \quad (33)$$

$$\forall \theta \neq 0, \quad m_1 \neq m_2, \quad \mathbf{k} \neq 0,$$

$${}_{1,2}\langle 0 | :Q_{\nu_e}(0): | 0 \rangle_{1,2} = 0, \quad (34)$$

and

$${}_{1,2}\langle 0 | (:Q_{\nu_e}(0):)^2 | 0 \rangle_{1,2} = 4\sin^2\theta \cos^2\theta \int d^3\mathbf{k} |V_{\mathbf{k}}|^2, \quad (35)$$

$$\begin{aligned} {}_P\langle \nu_{\mathbf{k},e}^r | (:Q_{\nu_e}(0):)^2 | \nu_{\mathbf{k},e}^r \rangle_P &= \cos^6\theta + \sin^6\theta + \sin^2\theta \cos^2\theta \\ &\quad \times \left[2|U_{\mathbf{k}}| + |U_{\mathbf{k}}|^2 + 4 \right. \\ &\quad \left. \times \int d^3\mathbf{k} |V_{\mathbf{k}}|^2 \right], \end{aligned} \quad (36)$$

which are both infinite, thus making the corresponding quantum fluctuations divergent.

Hence, we conclude that the correct flavor state and normal ordered operators are those defined in Eqs. (26) and (23) respectively.

III. MASS PARAMETERS AND FIELD MIXING

In Eq. (6) $u_{\mathbf{k},i}^r$ and $v_{-\mathbf{k},i}^r$ are the spinor wavefunctions of the massive neutrinos ν_i , $i = 1, 2$. As already observed in the previous section, Eq. (6) provides an expansion of the flavor fields ν_σ , $\sigma = e, \mu$, in the same basis of ν_i , $i = 1, 2$. However, it was noticed in Ref. [5] that expanding the flavor fields in the same basis as the (free) fields with definite masses is actually a special choice, and that in principle a more general possibility exists. Indeed, in the expansion (6) one could use eigenfunctions with arbitrary masses μ_σ . In other words, the transformation (8) can be generalized by writing the flavor fields as [5]

$$\nu_\sigma(x) = \frac{1}{\sqrt{V}} \sum_{\mathbf{k},r} [u_{\mathbf{k},\sigma}^r(t) \tilde{\alpha}_{\mathbf{k},\nu_\sigma}^r(t) + v_{-\mathbf{k},\sigma}^r(t) \tilde{\beta}_{-\mathbf{k},\nu_\sigma}^{r\dagger}(t)] e^{i\mathbf{k}\cdot\mathbf{x}}, \quad (37)$$

where u_σ and v_σ are the helicity eigenfunctions with mass μ_σ (the use of such helicity eigenfunctions as a basis simplifies calculation with respect to the choice of Ref.[1]). In Eq. (37) the generalized flavor operators are denoted by a tilde in order to distinguish them from the ones defined in Eq. (8). The expansion Eq. (37) is more general than the one in Eq. (6) since the latter corresponds to the particular choice $\mu_e \equiv m_1$, $\mu_\mu \equiv m_2$.

Since the issue of the arbitrary mass parameters in the field mixing formalism has attracted some attention [5,7,9,10,20], it is worth clarifying some basic facts about the choice of mass parameters within QFT in general, independently from the occurrence of the field mixing phenomenon.

We will refer to fermion fields since in this paper we are interested in neutrinos, but the conclusions can be also extended to boson fields.

First of all, it is worth noting that the mass parametrization problem can be revealed also in the free field case. Indeed, one may still consider the change of mass parametrization $m \rightarrow \mu$, which correspond to choosing $\tilde{u}_{\mathbf{k}}^r$, $\tilde{v}_{\mathbf{k}}^r$ as free field amplitudes with the arbitrary mass parameter μ .

Consider the set of free (fermion) field operators composed, for simplicity, by only two elements, i.e. assume our operators are

$$\begin{pmatrix} \alpha_{\mathbf{k},i}^r \\ \beta_{-\mathbf{k},i}^{r\dagger} \end{pmatrix}, \quad \text{with } i = 1, 2. \quad (38)$$

Let the nonzero masses be m_1 and m_2 , with $m_1 \neq m_2$. The wave functions u_i and v_i (we omit the index \mathbf{k} whenever no confusion arises) satisfy the free Dirac equations

$$(i\not{k} + m_i)u_i = 0, \quad (i\not{k} - m_i)v_i = 0, \quad (39)$$

respectively. Let $|0\rangle_{1,2}$ be the vacuum state annihilated by $\alpha_{\mathbf{k},i}^r$ and $\beta_{-\mathbf{k},i}^{r\dagger}$.

Since in QFT there exist infinitely many unitarily inequivalent representations of the canonical (anti-)commutation relations [18,21], one could consider another

fermion set of operators, say

$$\begin{pmatrix} \tilde{\alpha}_{\mathbf{k},i}^r(t) \\ \tilde{\beta}_{-\mathbf{k},i}^{r\dagger}(t) \end{pmatrix}, \quad i = 1, 2, \quad (40)$$

related to Eq. (38) by a Bogoliubov transformation (see below). The freedom of choosing another set of operators is, for example, typically exploited in QFT at finite temperature, or more generally when one introduces the irreducible set of ‘‘bare’’ field operators in terms of which the Lagrangian of the theory is written. In such a case the set of bare fields is not necessarily composed by the same number of elements as the one of the set of physical (asymptotic) fields satisfying free field equations and in terms of which observables are expressed. In general, indeed, bound states of bare fields may also belong to the set of physical fields. The mapping between the bare fields and the asymptotic fields is called the Haag expansion [18,21].

Suppose the wave functions of the field operators in Eq. (40) also satisfy the free Dirac equations

$$(i\not{k} + \mu_i)\tilde{u}_i = 0, \quad (i\not{k} - \mu_i)\tilde{v}_i = 0, \quad (41)$$

respectively. The mass parameter μ_i in Eqs. (41) represents now the mass of the corresponding arbitrarily chosen fields in Eq. (40) and therefore it represents an arbitrary parameter. Our two sets of operators are related by the transformation

$$\begin{pmatrix} \tilde{\alpha}_{\mathbf{k},i}^r(t) \\ \tilde{\beta}_{-\mathbf{k},i}^{r\dagger}(t) \end{pmatrix} = I_\mu^{-1}(t) \begin{pmatrix} \alpha_{\mathbf{k},i}^r \\ \beta_{-\mathbf{k},i}^{r\dagger} \end{pmatrix} I_\mu(t), \quad (42)$$

with

$$I_\mu(t) = \prod_{\mathbf{k},r} \exp \left\{ i \sum_i \xi_i^{\mathbf{k}} [\alpha_{\mathbf{k},i}^{r\dagger} \beta_{-\mathbf{k},i}^{r\dagger} e^{2i\omega_i t} + \beta_{-\mathbf{k},i}^r \alpha_{\mathbf{k},i}^r e^{-2i\omega_i t}] \right\} \quad (43)$$

where $\xi_i^{\mathbf{k}} \equiv (\tilde{\chi}_i - \chi_i)/2$ and $\cot \tilde{\chi}_i = |\mathbf{k}|/\mu_i$, $\cot \chi_i = |\mathbf{k}|/m_i$. Notice that for $\mu_1 \equiv m_1$, $\mu_2 \equiv m_2$ one has $I_\mu(t) = 1$, as it must be for the identity transformation.

The explicit matrix form of Eq. (42), written for both i values, is

$$\begin{pmatrix} \tilde{\alpha}_{\mathbf{k},1}^r(t) \\ \tilde{\alpha}_{\mathbf{k},2}^r(t) \\ \tilde{\beta}_{-\mathbf{k},1}^{r\dagger}(t) \\ \tilde{\beta}_{-\mathbf{k},2}^{r\dagger}(t) \end{pmatrix} = \begin{pmatrix} \rho_1^{\mathbf{k}*}(t) & 0 & i\lambda_1^{\mathbf{k}}(t) & 0 \\ 0 & \rho_2^{\mathbf{k}*}(t) & 0 & i\lambda_2^{\mathbf{k}}(t) \\ i\lambda_1^{\mathbf{k}*}(t) & 0 & \rho_1^{\mathbf{k}}(t) & 0 \\ 0 & i\lambda_2^{\mathbf{k}*}(t) & 0 & \rho_2^{\mathbf{k}}(t) \end{pmatrix} \times \begin{pmatrix} \alpha_{\mathbf{k},1}^r \\ \alpha_{\mathbf{k},2}^r \\ \beta_{-\mathbf{k},1}^{r\dagger} \\ \beta_{-\mathbf{k},2}^{r\dagger} \end{pmatrix} \quad (44)$$

where

$$\begin{aligned} \rho_i^{\mathbf{k}*}(t)\delta_{rs} &= \tilde{u}_{\mathbf{k},i}^{r\dagger}(t)u_{\mathbf{k},i}^s(t) = \tilde{v}_{-\mathbf{k},i}^{r\dagger}(t)v_{-\mathbf{k},i}^s(t) \\ &\equiv e^{i(\tilde{\omega}_{k,i}-\omega_{k,i})} \cos\xi_i^{\mathbf{k}} \delta_{rs} \end{aligned} \quad (45)$$

$$\begin{aligned} i\lambda_i^{\mathbf{k}*}(t)\delta_{rs} &= \tilde{u}_{\mathbf{k},i}^{r\dagger}(t)v_{-\mathbf{k},i}^s(t) = \tilde{v}_{-\mathbf{k},i}^{r\dagger}(t)u_{\mathbf{k},i}^s(t) \\ &\equiv ie^{-i(\tilde{\omega}_{k,i}+\omega_{k,i})} \sin\xi_i^{\mathbf{k}} \delta_{rs} \end{aligned} \quad (46)$$

with $i = 1, 2$ and $\tilde{\omega}_{k,i} = \sqrt{\mathbf{k}^2 + \mu_i^2}$. The vacuum state annihilated by the $(\tilde{\alpha}_{\mathbf{k},i}^r, \tilde{\beta}_{-\mathbf{k},i}^r)$ operators is

$$|\tilde{0}(t)\rangle_{1,2} \equiv I_{\mu}^{-1}(t)|0\rangle_{1,2}. \quad (47)$$

We observe that Eq. (42) is indeed nothing but the Bogoliubov transformation which relates the field operators $(\alpha_{\mathbf{k},i}^r, \beta_{-\mathbf{k},i}^r)$ and $(\tilde{\alpha}_{\mathbf{k},i}^r, \tilde{\beta}_{-\mathbf{k},i}^r)$, of masses m_i and μ_i , respectively. In the infinite volume limit, the Hilbert spaces where the operators $\alpha_{\mathbf{k},i}^r$ and $\tilde{\alpha}_{\mathbf{k},i}^r$ are, respectively, defined turn out to be unitarily inequivalent spaces. Moreover, the transformation parameter $\xi_i^{\mathbf{k}}$ acts as a label specifying Hilbert spaces unitarily inequivalent among themselves for each (different) value of the μ_i mass parameter.

We note that the vacuum $|\tilde{0}(t)\rangle_{1,2}$ is not annihilated by $\alpha_{\mathbf{k},i}^r$ and $\beta_{\mathbf{k},i}^r$ and it is not eigenstate of the number operators $N_{\alpha_i} = \sum_r \int d^3\mathbf{k} \alpha_{\mathbf{k},i}^{r\dagger} \alpha_{\mathbf{k},i}^r$ and $N_{\beta_i} = \sum_r \int d^3\mathbf{k} \beta_{\mathbf{k},i}^{r\dagger} \beta_{\mathbf{k},i}^r$. Similarly $|0\rangle_{1,2}$ is not annihilated by $\tilde{\alpha}_{\mathbf{k},i}^r$ and $\tilde{\beta}_{\mathbf{k},i}^r$ and it is not eigenstate of the number operators $\tilde{N}_{\alpha_i}(t) = \sum_r \int d^3\mathbf{k} \tilde{\alpha}_{\mathbf{k},i}^{r\dagger}(t) \tilde{\alpha}_{\mathbf{k},i}^r(t)$ and $\tilde{N}_{\beta_i}(t) = \sum_r \int d^3\mathbf{k} \tilde{\beta}_{\mathbf{k},i}^{r\dagger}(t) \tilde{\beta}_{\mathbf{k},i}^r(t)$. One obtains

$${}_{1,2}\langle\tilde{0}(t)|N_{\alpha_i}|\tilde{0}(t)\rangle_{1,2} = {}_{1,2}\langle\tilde{0}(t)|N_{\beta_i}|\tilde{0}(t)\rangle_{1,2} = \sin^2\xi_i^{\mathbf{k}}, \quad (48)$$

and

$${}_{1,2}\langle 0|\tilde{N}_{\alpha_i}(t)|0\rangle_{1,2} = {}_{1,2}\langle 0|\tilde{N}_{\beta_i}(t)|0\rangle_{1,2} = \sin^2\xi_i^{\mathbf{k}}. \quad (49)$$

In other words, the number operator, say N_{α_i} , is not an invariant quantity under the Bogoliubov transformation Eq. (42); it gets a dependence on the mass parameters. This is, however, not surprising since it is known that the Bogoliubov transformation Eq. (42) introduces a *new* set of canonical operators and a *new* (i.e. in the infinite volume limit unitarily, and therefore physically, inequivalent) Hilbert space. Stated differently, through the Bogoliubov transformation a *new* set of asymptotic fields (a *new* set of quasiparticles) is introduced, i.e. there are infinitely many sets of asymptotic fields, each set being associated to its specific representation. The choice of which one is the set to be used is then dictated by the physical conditions which are actually realized. For example, the mass values which have to be singled out in the renormalization procedure must be the observed physical masses.

Since the tilde quantities correspond to some new quasiparticle objects and the tilde number operator describes a different type of particles, then, the number operator aver-

age shall not be expected to remain the same under such transformations. Indeed, defined the state $|\tilde{\alpha}_{\mathbf{k},i}^r\rangle$ as $|\tilde{\alpha}_{\mathbf{k},i}^r(0)\rangle = \tilde{\alpha}_{\mathbf{k},i}^{r\dagger}(0)|\tilde{0}\rangle$, we have:

$$\begin{aligned} \langle\tilde{\alpha}_{\mathbf{k},i}^r(0)|\tilde{N}_{\alpha_i}(t)|\tilde{\alpha}_{\mathbf{k},i}^r(0)\rangle &= \{|\tilde{\alpha}_{\mathbf{k},i}^r(t), \tilde{\alpha}_{\mathbf{k},i}^{r\dagger}(0)\rangle\}^2 \\ &= \|\rho_i^{\mathbf{k}}\|^2 e^{i(\tilde{\omega}_{k,i}-\omega_{k,i})t} \\ &\quad + \|\lambda_i^{\mathbf{k}}\|^2 e^{i(\tilde{\omega}_{k,i}+\omega_{k,i})t}, \\ i &= 1, 2, \end{aligned} \quad (50)$$

which shows that the expectation value of the time-dependent number operator is not preserved by the transformation (42) applied to both states and operators. Nevertheless, in the cases of free fields, the charge operator is still conserved in transformation like Eq. (42):

$$\tilde{Q}_i = Q_i, \quad (51)$$

moreover the expectation value of the charge at time t on the state at time $t = 0$ is also free from mass parameters

$$\langle\tilde{\alpha}_{\mathbf{k},i}^r(0)|\tilde{Q}_i(t)|\tilde{\alpha}_{\mathbf{k},i}^r(0)\rangle = \langle\alpha_{\mathbf{k},i}^r(0)|Q_i(t)|\alpha_{\mathbf{k},i}^r(0)\rangle, \quad (52)$$

since we have

$$\begin{aligned} &\{|\tilde{\alpha}_{\mathbf{k},i}^r(t), \tilde{\alpha}_{\mathbf{k},i}^{r\dagger}(0)\rangle\}^2 + \{|\tilde{\beta}_{\mathbf{k},i}^r(t), \tilde{\alpha}_{\mathbf{k},i}^{r\dagger}(0)\rangle\}^2 \\ &= \{|\alpha_{\mathbf{k},i}^r(t), \alpha_{\mathbf{k},i}^{r\dagger}(0)\rangle\}^2 + \{|\beta_{\mathbf{k},i}^r(t), \alpha_{\mathbf{k},i}^{r\dagger}(0)\rangle\}^2. \end{aligned} \quad (53)$$

Similar results are obtained in the two flavor particle mixing case. Note, however, that in the case of three flavor mixing and in presence of CP violation, the charge operator and the flavor states are dependent on the arbitrary mass parameters and the quantity which is invariant under the transformation like the Eq. (42) is

$$\langle\alpha_{\mathbf{k},\nu_\sigma}^r|::Q_{\nu_\sigma}(t)::|\alpha_{\mathbf{k},\nu_\sigma}^r\rangle -_{e,\mu,\tau} \langle 0|::Q_{\nu_\sigma}(t)::|0\rangle_{e,\mu,\tau}, \quad (54)$$

with $|\alpha_{\mathbf{k},\nu_\sigma}^r\rangle = \alpha_{\mathbf{k},\nu_\sigma}^{r\dagger}(0)|0\rangle_{e,\mu,\tau}$ and $\sigma = e, \mu, \tau$ as observed in Ref.[10]

Having clarified that the possibility of different mass parameters is intrinsic to the very same structure of QFT and is independent of the occurrence or not of the field mixing, we may affirm that the mass parameters must be chosen not arbitrarily, but they must be justified on the ground of physical reasons [4,9,10].

In particular, in the mixing problem, the choice $\mu_e \equiv m_1$, $\mu_\mu \equiv m_2$, $\mu_\tau \equiv m_3$, is motivated by the fact that m_1 , m_2 and m_3 are the masses of the mass eigenfields and therefore such a choice is the only one physically relevant.

In our computations, instead of using the number operator, we use the charge operator Q which in the mixing phenomena describes the relative population densities of flavor particles in the beam and it is related with the oscillating observables that are: the lepton charge, in the case of neutrino mixing, the strange charge in meson systems like $K^0 - \bar{K}^0$ and $B_s^0 - \bar{B}_s^0$, the charmed charge

in the systems $D^0 - \bar{D}^0$, and so on. We note that, as shown in Refs. [12,14], the momentum operator for the mixing of neutral fields plays an analogous role to the one of the charge operator for charged fields.

IV. EXPECTATION VALUE OF NEUTRINO LEPTON CHARGE IN THE ONE PION STATE IN PION DECAY

In this Section we study the structure of the flavor charge expectation values in the case the production process of neutrinos is taken into account. This is done by using the flavor Hilbert space discussed above. In Ref. [22] a similar calculation was performed by using the mass Hilbert space, so neglecting flavor vacuum effects.

The final aim of the authors of Ref. [22] was to derive an oscillation formula in space, which is relevant for current experiments. On the other hand, a general oscillation formula with space-time dependence has been obtained in Ref. [11] in terms of expectation values of the flavor currents on the flavor neutrino states, exhibiting the corrections due to the flavor vacuum. In the following, we show in an explicit way how calculations can be performed with interacting fields on the flavor Hilbert space.

We consider the case where the flavor neutrinos are produced through the pion decay $\pi^+ \rightarrow \mu^+ + \nu_\mu$. We use the phenomenological approach to the pion decay [19] without referring to the quark structure of the pion. As initial π^+ state at time x_I^0 , we use $|\Pi(\langle \mathbf{k} \rangle, \mathbf{x}, x_I^0)\rangle \equiv |\pi(\langle \mathbf{k} \rangle, \mathbf{x}, x_I^0)\rangle \times |0(x_I^0)\rangle_\nu \times |0\rangle_\mu$ with $\langle \mathbf{k} \rangle$ the average k vector (see below). The neutrino vacuum is the flavor vacuum: $|0(x_I^0)\rangle_\nu = G_\theta^{-1}(x_I^0)|0\rangle_{1,2}$ (Note the change of notation for $|0(x_I^0)\rangle_{e,\mu}$ with respect to the previous sections.) We calculate the expectation values of neutrino flavor charge $::Q_{\nu_\mu}(x^0)::$ with respect to one pion state $|\Pi(\langle \mathbf{k} \rangle, \mathbf{x}, x^0)\rangle$ where $x_I^0 < x^0$

$$\begin{aligned} \langle ::Q_{\nu_\mu}(x^0):: \rangle &= \langle \Pi(\langle \mathbf{k} \rangle, \mathbf{x}, x^0) | ::Q_{\nu_\mu}(x^0):: | \Pi(\langle \mathbf{k} \rangle, \mathbf{x}, x^0) \rangle \\ &= \langle \Pi(\langle \mathbf{k} \rangle, \mathbf{x}, x_I^0) | S^{-1}(x^0, x_I^0) ::Q_{\nu_\mu}(x^0):: \\ &\quad \times S(x^0, x_I^0) | \Pi(\langle \mathbf{k} \rangle, \mathbf{x}, x_I^0) \rangle \end{aligned} \quad (55)$$

with

$$|\Pi(\langle \mathbf{k} \rangle, \mathbf{x}, x^0)\rangle = S(x^0, x_I^0) |\Pi(\langle \mathbf{k} \rangle, \mathbf{x}, x_I^0)\rangle \quad (56)$$

where, in the interaction representation, the normal ordered neutrino flavor charge $::Q_{\nu_\mu}(x^0)::$ is

$$\begin{aligned} ::Q_{\nu_\mu}(x^0):: &= \sum_r \int d^3\mathbf{k} [\alpha_{\mathbf{k}, \nu_\mu}^{r\dagger}(x^0) \alpha_{\mathbf{k}, \nu_\mu}^r(x^0) \\ &\quad - \beta_{-\mathbf{k}, \nu_\mu}^{r\dagger}(x^0) \beta_{-\mathbf{k}, \nu_\mu}^r(x^0)], \end{aligned} \quad (57)$$

and $S(x^0, x_I^0)$ is

$$\begin{aligned} S(x^0, x_I^0) &= \sum_{m=0} (-i)^m \int_{x_I^0}^{x^0} d^4y_1 \int_{x_I^0}^{y_1^0} d^4y_2 \dots \\ &\quad \times \int_{x_I^0}^{y_{m-1}^0} d^4y_m H_{\text{int}}(y_1) H_{\text{int}}(y_2) \dots H_{\text{int}}(y_m). \end{aligned} \quad (58)$$

The muon neutrino field and the muon field are expanded, in the interaction representation, as

$$\begin{aligned} \nu_\mu(x) &= \sum_r \int \frac{d^3\mathbf{q}}{(2\pi)^{3/2}} [u_{\mathbf{q},2}^r(x^0) \alpha_{\mathbf{q}, \nu_\mu}^r(x^0) e^{i\mathbf{q}\cdot\mathbf{x}} \\ &\quad + v_{\mathbf{q},2}^r(x^0) \beta_{\mathbf{q}, \nu_\mu}^{r\dagger}(x^0) e^{-i\mathbf{q}\cdot\mathbf{x}}], \end{aligned} \quad (59)$$

$$\begin{aligned} \mu(x) &= \sum_s \int \frac{d^3\mathbf{p}}{(2\pi)^{3/2}} [u_{\mathbf{p},\mu}^s(x^0) \alpha_{\mathbf{p},\mu}^s e^{i\mathbf{p}\cdot\mathbf{x}} \\ &\quad + v_{\mathbf{p},\mu}^s(x^0) \beta_{\mathbf{p},\mu}^{s\dagger} e^{-i\mathbf{p}\cdot\mathbf{x}}], \end{aligned} \quad (60)$$

with $\alpha_{\mathbf{q}, \nu_\mu}^r(x^0)$ flavor annihilation operator for neutrino. In Eq. (59), $u_{\mathbf{q},2}^r(x^0)$ is the spinor wave function for the massive neutrino field $\nu_2(x)$.

The Hamiltonian of weak interaction is

$$H_{\text{int}}(x) = -ig_\pi \bar{\nu}_\mu(x) \gamma^\lambda (1 + \gamma^5) \mu(x) \partial_\lambda \pi(x) + \text{h.c.}, \quad (61)$$

where $\pi(x)$, $\mu(x)$, $\nu_\mu(x)$, are the fields of pion, muon and flavor (muon) neutrino, respectively.

In the lowest order (i.e. the second order) of the weak interaction, we have

$$\begin{aligned} \langle ::Q_{\nu_\mu}(x^0):: \rangle &= \langle \Pi(\langle \mathbf{k} \rangle, \mathbf{x}, x_I^0) | \left[\int_{x_I^0}^{x^0} d^4z H_{\text{int}}(z) ::Q_{\nu_\mu}(x^0):: \right. \\ &\quad \left. \times \int_{x_I^0}^{x^0} d^4y H_{\text{int}}(y) \right] | \Pi(\langle \mathbf{k} \rangle, \mathbf{x}, x_I^0) \rangle \end{aligned} \quad (62)$$

and, in the case of positively charged pion π^+ , we have explicitly

$$\begin{aligned} \langle ::Q_{\nu_\mu}(x^0):: \rangle &= \int_{x_I^0}^{x^0} \int_{x_I^0}^{x^0} d^4z d^4y \langle \pi(\langle \mathbf{k} \rangle, \mathbf{x}, x_I^0) \\ &\quad \times [[ig_\pi^* \partial_\lambda \pi^\dagger(z)_\mu \langle 0 | \bar{\mu}(z) \gamma^\lambda (1 + \gamma_5) \\ &\quad \times \nu \langle 0(x_I^0) | \nu_\mu(z) ::Q_{\nu_\mu}(x^0):: \bar{\nu}_\mu(y) | 0(x_I^0)\rangle_\nu \\ &\quad \times \gamma^\sigma (1 + \gamma_5) \mu(y) | 0\rangle_\mu ig_\pi \partial_\sigma \pi(y)] \\ &\quad \times \pi(\langle \mathbf{k} \rangle, \mathbf{x}, x_I^0) \rangle. \end{aligned} \quad (63)$$

Expressing the charge operator as in Eq. (57), a straightforward calculation gives

$$\begin{aligned}
\langle ::\mathcal{Q}_{\nu_\mu}(x^0):: \rangle &= -|g_\pi|^2 \int_{x_I^0}^{x^0} \int_{x_I^0}^{x^0} d^4z d^4y \langle \pi(\langle \mathbf{k} \rangle, \mathbf{x}, x_I^0) | \partial_\lambda \pi^\dagger(z) \partial_\sigma \pi(y) | \pi(\langle \mathbf{k} \rangle, \mathbf{x}, x_I^0) \rangle \sum_{r,t,u,v} \int \frac{d^3\mathbf{p} d^3\mathbf{q} d^3\mathbf{q}' d^3\mathbf{q}''}{(2\pi)^6} \\
&\times (e^{i(\mathbf{p}+\mathbf{q})\cdot\mathbf{z}} e^{-i(\mathbf{p}+\mathbf{q}')\cdot\mathbf{y}} e^{-i(\omega_p+\omega_{q,2})z^0} e^{i(\omega_p+\omega_{q',2})y^0} \bar{v}_{\mathbf{p},\mu}^r \gamma^\lambda (1+\gamma_5) u_{\mathbf{q},2}^t \bar{u}_{\mathbf{q}',2}^u \gamma^\sigma (1+\gamma_5) v_{\mathbf{p},\mu\nu}^r \langle 0(x_I^0) | \alpha_{\mathbf{q},\nu_\mu}^t(z^0) \\
&\times [\alpha_{\mathbf{q}'',\nu_\mu}^{v\dagger}(x^0) \alpha_{\mathbf{q}'',\nu_\mu}^v(x^0) - \beta_{\mathbf{q}'',\nu_\mu}^{v\dagger}(x^0) \beta_{\mathbf{q}'',\nu_\mu}^v(x^0)] \alpha_{\mathbf{q}',\nu_\mu}^{u\dagger}(y^0) | 0(x_I^0) \rangle_\nu + \text{similar terms}, \quad (64)
\end{aligned}$$

being $\omega_p = \sqrt{\mathbf{p}^2 + m_\mu^2}$ and $\omega_{q,2} = \sqrt{\mathbf{q}^2 + m_\pi^2}$.

The explicit expression of Eq. (64) is given by Eq. (B1) in Appendix B.

A. Evaluation of the bosonic term

We consider first the bosonic part: $\langle \pi(\langle \mathbf{k} \rangle, \mathbf{x}, x_I^0) | \partial_\lambda \pi^\dagger(z) \partial_\sigma \pi(y) | \pi(\langle \mathbf{k} \rangle, \mathbf{x}, x_I^0) \rangle$. The pion state is defined as:

$$|\pi(\langle \mathbf{k} \rangle, \mathbf{x}, x_I^0)\rangle = \int d^3k \frac{A_\pi(\mathbf{k}, \langle \mathbf{k} \rangle) e^{-i\mathbf{k}\cdot\mathbf{x} + i\Omega_k x_I^0}}{\sqrt{2\Omega_k}} a_{\mathbf{k}\pi}^\dagger |0\rangle_\pi \quad (65)$$

with

$$A_\pi(\mathbf{k}, \langle \mathbf{k} \rangle) = \frac{1}{(\sqrt{2\pi\sigma_\pi^2})^{3/2}} \exp\left[-\frac{(\mathbf{k} - \langle \mathbf{k} \rangle)^2}{4\sigma_\pi^2}\right], \quad (66)$$

and $\Omega_k = \sqrt{\mathbf{k}^2 + m_\pi^2}$. The pion field in the interaction picture is:

$$\pi(x) = \int \frac{d^3\mathbf{k}}{(2\pi)^{3/2}} \frac{1}{\sqrt{2\Omega_k}} (a_{\mathbf{k}} e^{i(\mathbf{k}\cdot\mathbf{x} - \Omega_k t)} + b_{\mathbf{k}}^\dagger e^{-i(\mathbf{k}\cdot\mathbf{x} - \Omega_k t)}), \quad (67)$$

hence

$$\begin{aligned}
\langle \pi(\langle \mathbf{k} \rangle, \mathbf{x}, x_I^0) | \partial_\lambda \pi^\dagger(z) \partial_\sigma \pi(y) | \pi(\langle \mathbf{k} \rangle, \mathbf{x}, x_I^0) \rangle \\
= \int \frac{d^3\mathbf{k} d^3\mathbf{k}'}{4(2\pi)^3 \Omega_k \Omega_{k'}} A_\pi^*(\mathbf{k}, \langle \mathbf{k} \rangle) A_\pi(\mathbf{k}', \langle \mathbf{k} \rangle) \\
\times k_\lambda k'_\sigma e^{-i\mathbf{k}\cdot\mathbf{z}} e^{i\mathbf{k}'\cdot\mathbf{y}} e^{i\Omega_k(z^0 - x_I^0)} e^{-i\Omega_{k'}(y^0 - x_I^0)} e^{i(\mathbf{k} - \mathbf{k}')\cdot\mathbf{x}}, \quad (68)
\end{aligned}$$

thus Eq. (64) may be expressed as

$$\begin{aligned}
\langle ::\mathcal{Q}_{\nu_\mu}(x^0):: \rangle &= -|g_\pi|^2 \int_{x_I^0}^{x^0} \int_{x_I^0}^{x^0} d^4z d^4y \sum_{r,t,u,v} \int \frac{d^3\mathbf{k} d^3\mathbf{k}' d^3\mathbf{p} d^3\mathbf{q} d^3\mathbf{q}' d^3\mathbf{q}''}{4(2\pi)^9 \Omega_k \Omega_{k'}} A_\pi^*(\mathbf{k}, \langle \mathbf{k} \rangle) A_\pi(\mathbf{k}', \langle \mathbf{k} \rangle) k_\lambda k'_\sigma e^{i(\mathbf{k} - \mathbf{k}')\cdot\mathbf{x}} e^{-i(\Omega_k - \Omega_{k'})x_I^0} \\
&\times e^{-i(\mathbf{k} - \mathbf{p} - \mathbf{q})\cdot\mathbf{z}} e^{i(\mathbf{k}' - \mathbf{p} - \mathbf{q}')\cdot\mathbf{y}} e^{i(\Omega_k - \omega_p - \omega_{q,2})(z^0 - x_I^0)} e^{-i(\Omega_{k'} - \omega_p - \omega_{q',2})(y^0 - x_I^0)} \bar{v}_{\mathbf{p},\mu}^r \gamma^\lambda (1+\gamma_5) u_{\mathbf{q},2}^t \bar{u}_{\mathbf{q}',2}^u \gamma^\sigma (1+\gamma_5) \\
&\times v_{\mathbf{p},\mu\nu}^r \langle 0(x_I^0) | \alpha_{\mathbf{q},\nu_\mu}^t(z^0) [\alpha_{\mathbf{q}'',\nu_\mu}^{v\dagger}(x^0) \alpha_{\mathbf{q}'',\nu_\mu}^v(x^0) - \beta_{\mathbf{q}'',\nu_\mu}^{v\dagger}(x^0) \beta_{\mathbf{q}'',\nu_\mu}^v(x^0)] \alpha_{\mathbf{q}',\nu_\mu}^{u\dagger}(y^0) | 0(x_I^0) \rangle_\nu + \text{similar terms}, \quad (69)
\end{aligned}$$

(see Eq. (B2) in Appendix B) and then

$$\begin{aligned}
\langle ::\mathcal{Q}_{\nu_\mu}(x^0):: \rangle &= -|g_\pi|^2 \int_{x_I^0}^{x^0} \int_{x_I^0}^{x^0} dz^0 dy^0 \sum_{r,t,u,v} \int \frac{d^3\mathbf{k} d^3\mathbf{k}' d^3\mathbf{p} d^3\mathbf{q}}{4(2\pi)^3 \Omega_k \Omega_{k'}} A_\pi^*(\mathbf{k}, \langle \mathbf{k} \rangle) A_\pi(\mathbf{k}', \langle \mathbf{k} \rangle) e^{i(\mathbf{k} - \mathbf{k}')\cdot\mathbf{x}} e^{-i(\Omega_k - \Omega_{k'})x_I^0} \\
&\times e^{i(\Omega_k - \omega_p - \omega_{k-p,2})(z^0 - x_I^0)} e^{-i(\Omega_{k'} - \omega_p - \omega_{k'-p,2})(y^0 - x_I^0)} \bar{v}_{\mathbf{p},\mu}^r k_\lambda \gamma^\lambda (1+\gamma_5) u_{\mathbf{k}-\mathbf{p},2}^t \bar{u}_{\mathbf{k}'-\mathbf{p},2}^u k'_\sigma \gamma^\sigma (1+\gamma_5) \\
&\times v_{\mathbf{p},\mu\nu}^r \langle 0(x_I^0) | \alpha_{\mathbf{k}-\mathbf{p},\nu_\mu}^t(z^0) [\alpha_{\mathbf{q},\nu_\mu}^{v\dagger}(x^0) \alpha_{\mathbf{q},\nu_\mu}^v(x^0) - \beta_{\mathbf{q},\nu_\mu}^{v\dagger}(x^0) \beta_{\mathbf{q},\nu_\mu}^v(x^0)] \alpha_{\mathbf{k}'-\mathbf{p},\nu_\mu}^{u\dagger}(y^0) | 0(x_I^0) \rangle_\nu + \text{similar terms}, \quad (70)
\end{aligned}$$

(see Eq. (B3) in Appendix B).

B. Evaluation of terms like $\nu \langle 0(x_I^0) | \alpha_{\mathbf{k}-\mathbf{p},\nu_\mu}^t(z^0) [\alpha_{\mathbf{q},\nu_\mu}^{v\dagger}(x^0) \alpha_{\mathbf{q},\nu_\mu}^v(x^0) - \beta_{\mathbf{q},\nu_\mu}^{v\dagger}(x^0) \beta_{\mathbf{q},\nu_\mu}^v(x^0)] \alpha_{\mathbf{k}'-\mathbf{p},\nu_\mu}^{u\dagger}(y^0) | 0(x_I^0) \rangle_\nu$

We have

$$\begin{aligned}
&\nu \langle 0(x_I^0) | \alpha_{\mathbf{k}-\mathbf{p},\nu_\mu}^t(z^0) [\alpha_{\mathbf{q},\nu_\mu}^{v\dagger}(x^0) \alpha_{\mathbf{q},\nu_\mu}^v(x^0) - \beta_{\mathbf{q},\nu_\mu}^{v\dagger}(x^0) \beta_{\mathbf{q},\nu_\mu}^v(x^0)] \alpha_{\mathbf{k}'-\mathbf{p},\nu_\mu}^{u\dagger}(y^0) | 0(x_I^0) \rangle_\nu \\
&= {}_{1,2} \langle 0 | G(x_I^0) \alpha_{\mathbf{k}-\mathbf{p},\nu_\mu}^t(z^0) G^{-1}(x_I^0) G(x_I^0) [\alpha_{\mathbf{q},\nu_\mu}^{v\dagger}(x^0) \alpha_{\mathbf{q},\nu_\mu}^v(x^0) - \beta_{\mathbf{q},\nu_\mu}^{v\dagger}(x^0) \beta_{\mathbf{q},\nu_\mu}^v(x^0)] G^{-1}(x_I^0) G(x_I^0) \alpha_{\mathbf{k}'-\mathbf{p},\nu_\mu}^{u\dagger}(y^0) G^{-1}(x_I^0) | 0 \rangle_{1,2} \\
&= {}_{1,2} \langle 0 | \hat{\alpha}_{\mathbf{k}-\mathbf{p},\nu_\mu}^t(x_I^0, z^0) [\hat{\alpha}_{\mathbf{q},\nu_\mu}^{v\dagger}(x_I^0, x^0) \hat{\alpha}_{\mathbf{q},\nu_\mu}^v(x_I^0, x^0) - \hat{\beta}_{\mathbf{q},\nu_\mu}^{v\dagger}(x_I^0, x^0) \hat{\beta}_{\mathbf{q},\nu_\mu}^v(x_I^0, x^0)] \hat{\alpha}_{\mathbf{k}'-\mathbf{p},\nu_\mu}^{u\dagger}(x_I^0, y^0) | 0 \rangle_{1,2}, \quad (71)
\end{aligned}$$

Where the flavor annihilation operators are $\alpha_{\mathbf{q},\nu\sigma}^r(t, \theta) \equiv G_{\theta}^{-1}(t)\alpha_{\mathbf{q},i}^r G_{\theta}(t)$:

$$\begin{aligned}\alpha_{\mathbf{q},\nu e}^r(t, \theta) &= \cos\theta\alpha_{\mathbf{q},1}^r + \sin\theta\sum_s[u_{\mathbf{q},1}^{r\dagger}(t)u_{\mathbf{q},2}^s(t)\alpha_{\mathbf{q},2}^s + u_{\mathbf{q},1}^{r\dagger}(t)v_{-\mathbf{q},2}^s(t)\beta_{-\mathbf{q},2}^{s\dagger}], \\ \alpha_{\mathbf{q},\nu\mu}^r(t, \theta) &= \cos\theta\alpha_{\mathbf{q},2}^r - \sin\theta\sum_s[u_{\mathbf{q},2}^{r\dagger}(t)u_{\mathbf{q},1}^s(t)\alpha_{\mathbf{q},1}^s + u_{\mathbf{q},2}^{r\dagger}(t)v_{-\mathbf{q},1}^s(t)\beta_{-\mathbf{q},1}^{s\dagger}], \\ \beta_{-\mathbf{q},\nu e}^r(t, \theta) &= \cos\theta\beta_{-\mathbf{q},1}^r + \sin\theta\sum_s[v_{-\mathbf{q},2}^{s\dagger}(t)v_{-\mathbf{q},1}^r(t)\beta_{-\mathbf{q},2}^s + u_{\mathbf{q},2}^{s\dagger}(t)v_{-\mathbf{q},1}^r(t)\alpha_{\mathbf{q},2}^{s\dagger}], \\ \beta_{-\mathbf{q},\nu\mu}^r(t, \theta) &= \cos\theta\beta_{-\mathbf{q},2}^r - \sin\theta\sum_s[v_{-\mathbf{q},1}^{s\dagger}(t)v_{-\mathbf{q},2}^r(t)\beta_{-\mathbf{q},1}^s + u_{\mathbf{q},1}^{s\dagger}(t)v_{-\mathbf{q},2}^r(t)\alpha_{\mathbf{q},1}^{s\dagger}].\end{aligned}\tag{72}$$

By using $\hat{\alpha}_{\mathbf{q},\nu\sigma}^r(t', t) \equiv G_{\theta}(t')\alpha_{\mathbf{q},\nu\sigma}^r(t, \theta)G_{\theta}^{-1}(t')$, and $G_{\theta}^{-1}(t) = G_{-\theta}(t)$, we have

$$\begin{aligned}\hat{\alpha}_{\mathbf{q},\nu e}^r(t', t) &= \cos\theta\alpha_{\mathbf{q},\nu e}^r(t', -\theta) + \sin\theta\sum_s[u_{\mathbf{q},1}^{r\dagger}(t)u_{\mathbf{q},2}^s(t)\alpha_{\mathbf{q},\nu\mu}^s(t', -\theta) + u_{\mathbf{q},1}^{r\dagger}(t)v_{-\mathbf{q},2}^s(t)\beta_{-\mathbf{q},\nu\mu}^{s\dagger}(t', -\theta)], \\ \hat{\alpha}_{\mathbf{q},\nu\mu}^r(t', t) &= \cos\theta\alpha_{\mathbf{q},\nu\mu}^r(t', -\theta) - \sin\theta\sum_s[u_{\mathbf{q},2}^{r\dagger}(t)u_{\mathbf{q},1}^s(t)\alpha_{\mathbf{q},\nu e}^s(t', -\theta) + u_{\mathbf{q},2}^{r\dagger}(t)v_{-\mathbf{q},1}^s(t)\beta_{-\mathbf{q},\nu e}^{s\dagger}(t', -\theta)], \\ \hat{\beta}_{-\mathbf{q},\nu e}^r(t', t) &= \cos\theta\beta_{-\mathbf{q},\nu e}^r(t', -\theta) + \sin\theta\sum_s[v_{-\mathbf{q},2}^{s\dagger}(t)v_{-\mathbf{q},1}^r(t)\beta_{-\mathbf{q},\nu\mu}^s(t', -\theta) + u_{\mathbf{q},2}^{s\dagger}(t)v_{-\mathbf{q},1}^r(t)\alpha_{\mathbf{q},\nu\mu}^{s\dagger}(t', -\theta)], \\ \hat{\beta}_{-\mathbf{q},\nu\mu}^r(t', t) &= \cos\theta\beta_{-\mathbf{q},\nu\mu}^r(t', -\theta) - \sin\theta\sum_s[v_{-\mathbf{q},1}^{s\dagger}(t)v_{-\mathbf{q},2}^r(t)\beta_{-\mathbf{q},\nu e}^s(t', -\theta) + u_{\mathbf{q},1}^{s\dagger}(t)v_{-\mathbf{q},2}^r(t)\alpha_{\mathbf{q},\nu e}^{s\dagger}(t', -\theta)].\end{aligned}\tag{73}$$

By using Eqs. (72) in Eqs. (73) the last equality in Eqs. (71) can be finally expressed in terms of the massive neutrino operators $\alpha_{\mathbf{q},i}^r$, $\alpha_{\mathbf{q},i}^{r\dagger}$, $\beta_{-\mathbf{q},i}^r$, $\beta_{-\mathbf{q},i}^{r\dagger}$ which act on the massive neutrino vacuum $|0\rangle_{1,2}$. Thus our computation allows the expression of the matrix element $\langle\Pi(t)|Q_{\nu\sigma}|\Pi(t)\rangle$ as a function of states that have the proper mathematical characterization of the LSZ formalism.

We observe that the formula (69) is consistent with Eq. (14) of Ref. [22]: indeed, when the pion state is represented by a plane wave (as done in Ref. [22]), the two equations acquire the same form, except for the neutrino charge expectation value, which in our case includes the flavor vacuum contributions. If we neglect such flavor vacuum effect, we precisely recover Eq. (14) of Ref. [22].

Furthermore, when we consider the effect of finite lifetime of the pion and we take the neutrino oscillation time $x^0 - x_l^0$ much bigger than $1/\Gamma_{\pi}$, then $\alpha_{\mathbf{k},\nu\mu}^{r\dagger}(y^0)|0(x_l^0)\rangle_{\nu} \simeq |\nu_{\mathbf{k},\mu}^r(x_l^0)\rangle$ and we obtain a form which is similar (once integrated in time) to the one considered in Ref. [11] giving the space dependent oscillation formula, except for the fact that the pion state rather than the neutrino state is represented by a wave packet. The final expression ensuing the formula (70) is too lengthy to be discussed at this stage. Further analysis will be done elsewhere.

V. CONCLUSION

In this paper we have studied neutrino mixing and oscillations in quantum field theory and we discussed the determination of the oscillation probability including the neutrino production vertex. A crucial point in our analysis is the disclosure of the fact that in order to describe the neutrino oscillations we have to use the flavor states de-

finied as $|\nu_{\mathbf{k},\sigma}^r\rangle = \alpha_{\mathbf{k},\nu\sigma}^{r\dagger}(0)|0(0)\rangle_{\nu}$, with $\sigma = e, \mu, \tau$ and the flavor charge operators $::Q_{\nu\sigma}::$ normal ordered with respect to the flavor vacuum. Indeed, we have shown that the usual Pontecorvo states are not eigenstates of the flavor charges $:Q_{\nu\sigma}:$ and $::Q_{\nu\sigma}::$.

We showed that the possibility of different mass parameters is intrinsic to the very same structure of QFT and is independent of the occurrence or not of the field mixing; hence, as noted in [4,9,10], the mass parameters must be chosen not arbitrarily, but on the ground of physical reasons: $\mu_e \equiv m_1$, $\mu_{\mu} \equiv m_2$, $\mu_{\tau} \equiv m_3$.

Moreover, we have computed the neutrino lepton charge in decay processes where neutrinos are generated, proving that the corresponding lepton charge expectation value can be uniquely expressed in terms of LSZ states and in particular of the massive neutrino annihilation/creation operators acting on the massive neutrino vacuum.

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APPENDIX A: ORTHOGONALITY OF FLAVOR STATES AT DIFFERENT TIMES

The product of two vacuum states at different times $t \neq t'$ (we put for simplicity $t' = 0$) is

$${}_{\nu}\langle 0|0(t)\rangle_{\nu} = \prod_k C_k^2(t) = e^{2\sum_k \ln C_k(t)}\tag{A1}$$

with

$$\begin{aligned}
C_k(t) &\equiv (1 - \sin^2\theta|V_k|^2)^2 \\
&+ 2\sin^2\theta\cos^2\theta|V_k|^2 e^{-i(\omega_{k,2}+\omega_{k,1})t} \\
&+ \sin^4\theta|V_k|^2|U_k|^2(e^{-2i\omega_{k,1}t} + e^{-2i\omega_{k,2}t}) \\
&+ \sin^4\theta|V_k|^4 e^{-2i(\omega_{k,2}+\omega_{k,1})t}. \tag{A2}
\end{aligned}$$

In the infinite volume limit we obtain (note that $|C_k(t)| \leq 1$ for any value of k, t , and of the parameters θ, m_1, m_2):

$$\begin{aligned}
\lim_{V \rightarrow \infty} \langle 0|0(t) \rangle_\nu &= \lim_{V \rightarrow \infty} \exp \left[\frac{2V}{(2\pi)^3} \int d^3\mathbf{k} (\ln|C_k(t)| \right. \\
&\left. + i\alpha_k(t)) \right] = 0 \tag{A3}
\end{aligned}$$

with $|C_k(t)|^2 = \text{Re}[C_k(t)]^2 + \text{Im}[C_k(t)]^2$ and $\alpha_k(t) = \tan^{-1}(\text{Im}[C_k(t)]/\text{Re}[C_k(t)])$.

Thus we have orthogonality of the vacua at different times¹.

One can easily check that flavor neutrino states are also orthogonal at different times. Consider the electron neutrino state at time t with momentum \mathbf{k} :

$$|\nu_{\mathbf{k},e}^r(t)\rangle = \alpha_{\mathbf{k},\nu_e}^{r\dagger}(t)|0(t)\rangle_\nu. \tag{A4}$$

The flavor vacuum is explicitly given by

$$|0(t)\rangle_\nu = \prod_{\mathbf{p}} G_{\mathbf{p},\theta}^{-1}(t)|0\rangle_{1,2}, \tag{A5}$$

where, to be precise, the mass vacuum is to be understood as: $|0\rangle_{1,2} = |0\rangle_{1,2}^{k_1} \otimes |0\rangle_{1,2}^{k_2} \otimes |0\rangle_{1,2,\dots}^{k_3}$

Then, we have

$$\langle \nu_{\mathbf{k},e}^r(0) | \nu_{\mathbf{k},e}^r(t) \rangle = \langle 0 | \alpha_{\mathbf{k},\nu_e}^r(0) \alpha_{\mathbf{k},\nu_e}^{r\dagger}(t) | 0(t) \rangle_\nu \tag{A6}$$

$$= \left(\prod_{\mathbf{p} \neq \mathbf{k}} \langle 0 | G_{\mathbf{p},\theta}(0) \alpha_{\mathbf{k},\nu_e}^r(0) \alpha_{\mathbf{k},\nu_e}^{r\dagger}(t) \left(\prod_{\mathbf{q}} G_{\mathbf{q},\theta}^{-1}(t) | 0 \right)_{1,2} \right), \tag{A7}$$

and, since for $\mathbf{p} \neq \mathbf{q}$ the mixing generators commute among themselves and with $\alpha_{\mathbf{k},\nu_e}^r$ for $\mathbf{k} \neq \mathbf{p}, \mathbf{q}$, it is

$$\begin{aligned}
\langle \nu_{\mathbf{k},e}^r(0) | \nu_{\mathbf{k},e}^r(t) \rangle &\propto_\nu \langle 0^{\mathbf{k}} | \alpha_{\mathbf{k},\nu_e}^r(0) \alpha_{\mathbf{k},\nu_e}^{r\dagger}(t) | 0^{\mathbf{k}}(t) \rangle_\nu \\
&\times \prod_{\mathbf{p} \neq \mathbf{k}, 1, 2} \langle 0 | G_{\mathbf{p},\theta}(0) G_{\mathbf{p},\theta}^{-1}(t) | 0 \rangle_{1,2} \tag{A8}
\end{aligned}$$

$$= {}_\nu \langle 0^{\mathbf{k}} | \alpha_{\mathbf{k},\nu_e}^r(0) \alpha_{\mathbf{k},\nu_e}^{r\dagger}(t) | 0^{\mathbf{k}}(t) \rangle_{\nu\nu} \langle 0 | 0(t) \rangle_\nu. \tag{A9}$$

By using Eq. (A3), we obtain the orthogonality of flavor states at different times in the infinite volume limit, provided ${}_\nu \langle 0^{\mathbf{k}} | \alpha_{\mathbf{k},\nu_e}^r(0) \alpha_{\mathbf{k},\nu_e}^{r\dagger}(t) | 0^{\mathbf{k}}(t) \rangle_\nu$ is finite or zero, as it is indeed.

APPENDIX B: EXPLICIT EXPRESSIONS OF EQS. (64), (69), AND (70)

The complete Eq. (64)

$$\begin{aligned}
\langle :: Q_\nu(x^0) :: \rangle &= -|g_\pi|^2 \int_{x_I^0}^{x^0} \int_{x_I^0}^{x^0} d^4z d^4y \langle \pi(\langle \mathbf{k} \rangle, \mathbf{x}, x_I^0) | \partial_\lambda \pi^\dagger(z) \partial_\sigma \pi(y) | \pi(\langle \mathbf{k} \rangle, \mathbf{x}, x_I^0) \rangle \sum_{r,t,u,v} \int \frac{d^3\mathbf{p} d^3\mathbf{q} d^3\mathbf{q}'}{(2\pi)^6} \\
&\times (e^{i(\mathbf{p}+\mathbf{q})\cdot\mathbf{z}} e^{-i(\mathbf{p}+\mathbf{q}')\cdot\mathbf{y}} e^{-i(\omega_p+\omega_{q,2})z^0} e^{i(\omega_p+\omega_{q',2})y^0} \bar{v}_{\mathbf{p},\mu}^r \gamma^\lambda (1 + \gamma_5) u_{\mathbf{q},2}^t \bar{u}_{\mathbf{q}',2}^u \gamma^\sigma (1 + \gamma_5) v_{\mathbf{p},\mu\nu}^r \langle 0(x_I^0) | \alpha_{\mathbf{q},\nu_\mu}^t(z^0) \\
&\times [\alpha_{\mathbf{q}',\nu_\mu}^{v\dagger}(x^0) \alpha_{\mathbf{q},\nu_\mu}^v(x^0) - \beta_{\mathbf{q}',\nu_\mu}^{v\dagger}(x^0) \beta_{\mathbf{q},\nu_\mu}^v(x^0)] \alpha_{\mathbf{q}',\nu_\mu}^{u\dagger}(y^0) | 0(x_I^0) \rangle_\nu \\
&+ e^{i(\mathbf{p}-\mathbf{q})\cdot\mathbf{z}} e^{-i(\mathbf{p}-\mathbf{q}')\cdot\mathbf{y}} e^{-i(\omega_p-\omega_{q,2})z^0} e^{i(\omega_p-\omega_{q',2})y^0} \bar{v}_{\mathbf{p},\mu}^r \gamma^\lambda (1 + \gamma_5) v_{\mathbf{q},2}^t \bar{v}_{\mathbf{q}',2}^u \gamma^\sigma (1 + \gamma_5) v_{\mathbf{p},\mu\nu}^r \langle 0(x_I^0) | \beta_{\mathbf{q},\nu_\mu}^{t\dagger}(z^0) \\
&\times [\alpha_{\mathbf{q}',\nu_\mu}^{v\dagger}(x^0) \alpha_{\mathbf{q},\nu_\mu}^v(x^0) - \beta_{\mathbf{q}',\nu_\mu}^{v\dagger}(x^0) \beta_{\mathbf{q},\nu_\mu}^v(x^0)] \beta_{\mathbf{q}',\nu_\mu}^{u\dagger}(y^0) | 0(x_I^0) \rangle_\nu \\
&+ e^{i(\mathbf{p}+\mathbf{q})\cdot\mathbf{z}} e^{-i(\mathbf{p}-\mathbf{q}')\cdot\mathbf{y}} e^{-i(\omega_p+\omega_{q,2})z^0} e^{i(\omega_p-\omega_{q',2})y^0} \bar{v}_{\mathbf{p},\mu}^r \gamma^\lambda (1 + \gamma_5) u_{\mathbf{q},2}^t \bar{v}_{\mathbf{q}',2}^u \gamma^\sigma (1 + \gamma_5) v_{\mathbf{p},\mu\nu}^r \langle 0(x_I^0) | \alpha_{\mathbf{q},\nu_\mu}^t(z^0) \\
&\times [\alpha_{\mathbf{q}',\nu_\mu}^{v\dagger}(x^0) \alpha_{\mathbf{q},\nu_\mu}^v(x^0) - \beta_{\mathbf{q}',\nu_\mu}^{v\dagger}(x^0) \beta_{\mathbf{q},\nu_\mu}^v(x^0)] \beta_{\mathbf{q}',\nu_\mu}^{u\dagger}(y^0) | 0(x_I^0) \rangle_\nu \\
&+ e^{i(\mathbf{p}-\mathbf{q})\cdot\mathbf{z}} e^{-i(\mathbf{p}+\mathbf{q}')\cdot\mathbf{y}} e^{-i(\omega_p-\omega_{q,2})z^0} e^{i(\omega_p+\omega_{q',2})y^0} \bar{v}_{\mathbf{p},\mu}^r \gamma^\lambda (1 + \gamma_5) v_{\mathbf{q},2}^t \bar{u}_{\mathbf{q}',2}^u \gamma^\sigma (1 + \gamma_5) v_{\mathbf{p},\mu\nu}^r \langle 0(x_I^0) | \beta_{\mathbf{q},\nu_\mu}^{t\dagger}(z^0) \\
&\times [\alpha_{\mathbf{q}',\nu_\mu}^{v\dagger}(x^0) \alpha_{\mathbf{q},\nu_\mu}^v(x^0) - \beta_{\mathbf{q}',\nu_\mu}^{v\dagger}(x^0) \beta_{\mathbf{q},\nu_\mu}^v(x^0)] \alpha_{\mathbf{q}',\nu_\mu}^{u\dagger}(y^0) | 0(x_I^0) \rangle_\nu \tag{B1}
\end{aligned}$$

The complete Eq. (69) is:

¹Note that it may be that for some values of the continuous index k , the $C_k(t)$ are periodic functions of time. However, the set of values of k for which this happens is a zero-measure set in the above integration Eq. (A3).

$$\begin{aligned}
\langle\langle \mathcal{Q}_{\nu_\mu}(x^0) \rangle\rangle = & -|g_\pi|^2 \int_{x_I^0}^{x^0} \int_{x_I^0}^{x^0} d^4z d^4y \sum_{r,t,u,v} \int \frac{d^3\mathbf{k} d^3\mathbf{k}' d^3\mathbf{p} d^3\mathbf{q} d^3\mathbf{q}'}{4(2\pi)^9 \Omega_{\mathbf{k}} \Omega_{\mathbf{k}'}} A_\pi^*(\mathbf{k}, \langle \mathbf{k} \rangle) A_\pi(\mathbf{k}', \langle \mathbf{k}' \rangle) k_\lambda k'_\sigma e^{i(\mathbf{k}-\mathbf{k}') \cdot \mathbf{x}} \\
& \times e^{-i(\Omega_{\mathbf{k}} - \Omega_{\mathbf{k}'}) x_I^0} (e^{-i(\mathbf{k}-\mathbf{p}-\mathbf{q}) \cdot \mathbf{z}} e^{i(\mathbf{k}'-\mathbf{p}-\mathbf{q}') \cdot \mathbf{y}} e^{i(\Omega_{\mathbf{k}} - \omega_p - \omega_{q,2})(z^0 - x_I^0)} e^{-i(\Omega_{\mathbf{k}'} - \omega_p - \omega_{q',2})(y^0 - x_I^0)} \bar{v}_{\mathbf{p},\mu}^r \gamma^\lambda (1 + \gamma_5) \\
& \times u_{\mathbf{q},2}^t \bar{u}_{\mathbf{q}',2}^u \gamma^\sigma (1 + \gamma_5) v_{\mathbf{p},\mu\nu}^r \langle 0(x_I^0) | \alpha_{\mathbf{q},\nu_\mu}^{t\dagger}(z^0) [\alpha_{\mathbf{q},\nu_\mu}^{v\dagger}(x^0) \alpha_{\mathbf{q},\nu_\mu}^v(x^0) - \beta_{\mathbf{q},\nu_\mu}^{v\dagger}(x^0) \beta_{\mathbf{q},\nu_\mu}^v(x^0)] \alpha_{\mathbf{q},\nu_\mu}^{u\dagger}(y^0) | 0(x_I^0) \rangle_\nu \\
& + e^{-i(\mathbf{k}-\mathbf{p}+\mathbf{q}) \cdot \mathbf{z}} e^{i(\mathbf{k}'-\mathbf{p}+\mathbf{q}') \cdot \mathbf{y}} e^{i(\Omega_{\mathbf{k}} - \omega_p + \omega_{q,2})(z^0 - x_I^0)} e^{-i(\Omega_{\mathbf{k}'} - \omega_p + \omega_{q',2})(y^0 - x_I^0)} \bar{v}_{\mathbf{p},\mu}^r \gamma^\lambda (1 + \gamma_5) v_{\mathbf{q},2}^t \bar{u}_{\mathbf{q}',2}^u \gamma^\sigma (1 + \gamma_5) \\
& \times v_{\mathbf{p},\mu\nu}^r \langle 0(x_I^0) | \beta_{\mathbf{q},\nu_\mu}^{t\dagger}(z^0) [\alpha_{\mathbf{q},\nu_\mu}^{v\dagger}(x^0) \alpha_{\mathbf{q},\nu_\mu}^v(x^0) - \beta_{\mathbf{q},\nu_\mu}^{v\dagger}(x^0) \beta_{\mathbf{q},\nu_\mu}^v(x^0)] \beta_{\mathbf{q},\nu_\mu}^{u\dagger}(y^0) | 0(x_I^0) \rangle_\nu \\
& + e^{-i(\mathbf{k}-\mathbf{p}-\mathbf{q}) \cdot \mathbf{z}} e^{i(\mathbf{k}'-\mathbf{p}+\mathbf{q}') \cdot \mathbf{y}} e^{i(\Omega_{\mathbf{k}} - \omega_p - \omega_{q,2})(z^0 - x_I^0)} e^{-i(\Omega_{\mathbf{k}'} - \omega_p + \omega_{q',2})(y^0 - x_I^0)} \bar{v}_{\mathbf{p},\mu}^r \gamma^\lambda (1 + \gamma_5) u_{\mathbf{q},2}^t \bar{u}_{\mathbf{q}',2}^u \gamma^\sigma (1 + \gamma_5) \\
& \times v_{\mathbf{p},\mu\nu}^r \langle 0(x_I^0) | \alpha_{\mathbf{q},\nu_\mu}^{t\dagger}(z^0) [\alpha_{\mathbf{q},\nu_\mu}^{v\dagger}(x^0) \alpha_{\mathbf{q},\nu_\mu}^v(x^0) - \beta_{\mathbf{q},\nu_\mu}^{v\dagger}(x^0) \beta_{\mathbf{q},\nu_\mu}^v(x^0)] \beta_{\mathbf{q},\nu_\mu}^{u\dagger}(y^0) | 0(x_I^0) \rangle_\nu \\
& + e^{-i(\mathbf{k}-\mathbf{p}+\mathbf{q}) \cdot \mathbf{z}} e^{i(\mathbf{k}'-\mathbf{p}-\mathbf{q}') \cdot \mathbf{y}} e^{i(\Omega_{\mathbf{k}} - \omega_p + \omega_{q,2})(z^0 - x_I^0)} e^{-i(\Omega_{\mathbf{k}'} - \omega_p - \omega_{q',2})(y^0 - x_I^0)} \bar{v}_{\mathbf{p},\mu}^r \gamma^\lambda (1 + \gamma_5) v_{\mathbf{q},2}^t \bar{u}_{\mathbf{q}',2}^u \gamma^\sigma (1 + \gamma_5) \\
& \times v_{\mathbf{p},\mu\nu}^r \langle 0(x_I^0) | \beta_{\mathbf{q},\nu_\mu}^{t\dagger}(z^0) [\alpha_{\mathbf{q},\nu_\mu}^{v\dagger}(x^0) \alpha_{\mathbf{q},\nu_\mu}^v(x^0) - \beta_{\mathbf{q},\nu_\mu}^{v\dagger}(x^0) \beta_{\mathbf{q},\nu_\mu}^v(x^0)] \alpha_{\mathbf{q},\nu_\mu}^{u\dagger}(y^0) | 0(x_I^0) \rangle_\nu. \tag{B2}
\end{aligned}$$

The complete Eq. (70) is:

$$\begin{aligned}
\langle\langle \mathcal{Q}_{\nu_\mu}(x^0) \rangle\rangle = & -|g_\pi|^2 \int_{x_I^0}^{x^0} \int_{x_I^0}^{x^0} dz^0 dy^0 \sum_{r,t,u,v} \int \frac{d^3\mathbf{k} d^3\mathbf{k}' d^3\mathbf{p} d^3\mathbf{q}}{4(2\pi)^3 \Omega_{\mathbf{k}} \Omega_{\mathbf{k}'}} A_\pi^*(\mathbf{k}, \langle \mathbf{k} \rangle) A_\pi(\mathbf{k}', \langle \mathbf{k}' \rangle) \\
& \times e^{i(\mathbf{k}-\mathbf{k}') \cdot \mathbf{x}} e^{-i(\Omega_{\mathbf{k}} - \Omega_{\mathbf{k}'}) x_I^0} (e^{i(\Omega_{\mathbf{k}} - \omega_p - \omega_{k-p,2})(z^0 - x_I^0)} e^{-i(\Omega_{\mathbf{k}'} - \omega_p - \omega_{k'-p,2})(y^0 - x_I^0)} \bar{v}_{\mathbf{p},\mu}^r k_\lambda \gamma^\lambda (1 + \gamma_5) u_{\mathbf{k}-\mathbf{p},2}^t \bar{u}_{\mathbf{k}'-\mathbf{p},2}^u k'_\sigma \\
& \times \gamma^\sigma (1 + \gamma_5) v_{\mathbf{p},\mu\nu}^r \langle 0(x_I^0) | \alpha_{\mathbf{k}-\mathbf{p},\nu_\mu}^{t\dagger}(z^0) [\alpha_{\mathbf{q},\nu_\mu}^{v\dagger}(x^0) \alpha_{\mathbf{q},\nu_\mu}^v(x^0) - \beta_{\mathbf{q},\nu_\mu}^{v\dagger}(x^0) \beta_{\mathbf{q},\nu_\mu}^v(x^0)] \alpha_{\mathbf{k}'-\mathbf{p},\nu_\mu}^{u\dagger}(y^0) | 0(x_I^0) \rangle_\nu \\
& + e^{i(\Omega_{\mathbf{k}} - \omega_p + \omega_{p-k,2})(z^0 - x_I^0)} e^{-i(\Omega_{\mathbf{k}'} - \omega_p + \omega_{p-k',2})(y^0 - x_I^0)} \bar{v}_{\mathbf{p},\mu}^r k_\lambda \gamma^\lambda (1 + \gamma_5) v_{\mathbf{p}-\mathbf{k},2}^t \bar{u}_{\mathbf{p}-\mathbf{k}',2}^u k'_\sigma \gamma^\sigma (1 + \gamma_5) \\
& \times v_{\mathbf{p},\mu\nu}^r \langle 0(x_I^0) | \beta_{\mathbf{p}-\mathbf{k},\nu_\mu}^{t\dagger}(z^0) [\alpha_{\mathbf{q},\nu_\mu}^{v\dagger}(x^0) \alpha_{\mathbf{q},\nu_\mu}^v(x^0) - \beta_{\mathbf{q},\nu_\mu}^{v\dagger}(x^0) \beta_{\mathbf{q},\nu_\mu}^v(x^0)] \beta_{\mathbf{p}-\mathbf{k}',\nu_\mu}^{u\dagger}(y^0) | 0(x_I^0) \rangle_\nu \\
& + e^{i(\Omega_{\mathbf{k}} - \omega_p - \omega_{k-p,2})(z^0 - x_I^0)} e^{-i(\Omega_{\mathbf{k}'} - \omega_p + \omega_{p-k',2})(y^0 - x_I^0)} \bar{v}_{\mathbf{p},\mu}^r k_\lambda \gamma^\lambda (1 + \gamma_5) u_{\mathbf{k}-\mathbf{p},2}^t \bar{u}_{\mathbf{p}-\mathbf{k}',2}^u k'_\sigma \gamma^\sigma (1 + \gamma_5) \\
& \times v_{\mathbf{p},\mu\nu}^r \langle 0(x_I^0) | \alpha_{\mathbf{k}-\mathbf{p},\nu_\mu}^{t\dagger}(z^0) [\alpha_{\mathbf{q},\nu_\mu}^{v\dagger}(x^0) \alpha_{\mathbf{q},\nu_\mu}^v(x^0) - \beta_{\mathbf{q},\nu_\mu}^{v\dagger}(x^0) \beta_{\mathbf{q},\nu_\mu}^v(x^0)] \beta_{\mathbf{p}-\mathbf{k}',\nu_\mu}^{u\dagger}(y^0) | 0(x_I^0) \rangle_\nu \\
& + e^{i(\Omega_{\mathbf{k}} - \omega_p + \omega_{p-k,2})(z^0 - x_I^0)} e^{-i(\Omega_{\mathbf{k}'} - \omega_p - \omega_{k'+p,2})(y^0 - x_I^0)} \bar{v}_{\mathbf{p},\mu}^r k_\lambda \gamma^\lambda (1 + \gamma_5) v_{\mathbf{p}-\mathbf{k},2}^t \bar{u}_{\mathbf{k}'+\mathbf{p},2}^u k'_\sigma \gamma^\sigma (1 + \gamma_5) \\
& \times v_{\mathbf{p},\mu\nu}^r \langle 0(x_I^0) | \beta_{\mathbf{p}-\mathbf{k},\nu_\mu}^{t\dagger}(z^0) [\alpha_{\mathbf{q},\nu_\mu}^{v\dagger}(x^0) \alpha_{\mathbf{q},\nu_\mu}^v(x^0) - \beta_{\mathbf{q},\nu_\mu}^{v\dagger}(x^0) \beta_{\mathbf{q},\nu_\mu}^v(x^0)] \alpha_{\mathbf{k}'+\mathbf{p},\nu_\mu}^{u\dagger}(y^0) | 0(x_I^0) \rangle_\nu. \tag{B3}
\end{aligned}$$

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