Comment on "New Brans-Dicke wormholes"

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(Received 8 June 2004; revised manuscript received 3 January 2005; published 27 June 2005)

It is shown that the recently claimed two new Brans-Dicke wormhole solutions [F. He and S-W. Kim, Phys. Rev. D **65**, 084022 (2002)] are not really new solutions. They are just the well known Brans-Dicke solutions of Class I and II in a different conformal gauge.

DOI: 10.1103/PhysRevD.71.128501 PACS numbers: 04.20.Gz, 04.50.+h

There has been a revival of interest in the Brans-Dicke theory (BDT) in recent times, particularly in the context of traversable Lorentzian wormholes. Now a days, BDT is no longer regarded merely as a Machian competitor to Einstein's General Relativity Theory (GRT) but a little more. There are several reasons. The principal reason is of course that BDT describes weak field tests of gravity reasonably well. Apart from this, it is known that the BD scalar field ϕ plays the role of classical exotic matter required for the construction of traversable Lorentzian wormholes [1].

In a recent paper, He and Kim [2] have found two new classes of solutions of BDT and showed that the solutions represent massive Lorentzian traversable wormholes. The purpose of this short Comment is to demonstrate that these solutions can be alternatively derived by exploiting the conformal invariance of the vacuum BDT action. That is, we show that the claimed solutions are merely the Class I and II solutions of BDT in a different gauge and thus are not essentially new.

We start from the BD action given by (we take units $G = c = 1$):

$$S_{BD} = \frac{1}{16\pi}\int d^4x\sqrt{-g}\left[\phi R - \frac{\omega}{\phi} g^{\mu\nu}\nabla_\mu\phi\nabla_\nu\phi\right] + S_{\rm matter} \tag{1}$$

in which $S_{\rm matter}$ represents the nongravitational part of the action which is independent of the scalar field ϕ. With Ref. [2], we consider only the pure gravitational part of the action and set $S_{\rm matter} = 0$. Under a conformal transformation, with a constant gauge parameter ξ,

$$\tilde{g}_{\mu\nu} = \phi^{2\xi} g \tag{2}$$

the Lagrangian density becomes

$$L_{BD}\sqrt{-g} = \sqrt{-\tilde{g}}\left[\phi^{1-2\xi}\tilde{R} - 6\phi^{1-5\xi}(\phi^{\xi})_{;\mu}^{;\mu} - \frac{\omega}{\phi^{1+2\xi}}\tilde{g}^{\mu\nu}\tilde{\nabla}_\mu\phi\tilde{\nabla}_\nu\phi\right]. \tag{3}$$

Under a further redefinition of the scalar field

$$\tilde{\phi} = \phi^{1-2\xi} \tag{4}$$

the Lagrangian density simply becomes

$$L_{BD}\sqrt{-g} = \sqrt{-\tilde{g}}\left[\tilde{\phi}\tilde{R} - \frac{\tilde{\omega}}{\tilde{\phi}}\tilde{g}^{\mu\nu}\tilde{\nabla}_\mu\tilde{\phi}\tilde{\nabla}_\nu\tilde{\phi}\right] \tag{5}$$

where

$$\tilde{\omega} = \frac{\omega - 6\xi(\xi-1)}{(1-2\xi)^2}. \tag{6}$$

Therefore the vacuum BD action (1)is invariant under the transformations (2) and (4). We briefly mention some relevant implications of this invariance. Faraoni [3] has shown that the transformations (2) and (4) that map $(g_{\mu\nu}, \phi) \to (\tilde{g}_{\mu\nu}, \tilde{\phi})$ constitute a one-parameter Abelian group with a singularity in the parameter dependence at $\xi = 1/2$. He used this invariance to show that the $\omega \to \infty$ limit of the BDT does not lead to vacuum GRT, when the matter stress energy (other than that of ϕ) is traceless. (However, for a critique of his arguments, see Ref. [4]). Cho [5] called the above conformal invariance as indicating an inherent ambiguity of the vacuum BD action and he argued that the only way to resolve this ambiguity is to specify how the physical metric couples to matter field. On the other hand, if one includes matter field in the action (1), one ends up with "abnormal" coupling with it in the conformally rescaled action so that the principle of equivalence is violated by the motion of ordinary matter. This violation is of no concern as there are important gains: The so called ambiguity is removed and that the gravitational interaction is described by spin-two massless graviton.

Let us introduce the parameter ξ into the Class I solution of BDT [6] described by the action (1). The general solution then looks like, using (2) and (4),

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$$ds^2 = -e^{2\alpha(r)}dt^2 + e^{2\beta(r)}[dr^2 + r^2 d\theta^2 + r^2 \sin^2\theta d\varphi^2] \tag{7}$$

where

$$e^{\alpha(r)} = e^{\alpha_0}\left(\frac{1-\frac{B}{r}}{1+\frac{B}{r}}\right)^{1+\xi C/\lambda} \tag{8}$$

$$e^{\beta(r)} = e^{\beta_0}\left(1+\frac{B}{r}\right)^2\left(\frac{1-\frac{B}{r}}{1+\frac{B}{r}}\right)^{\lambda - C - 1 + \xi C/\lambda} \tag{9}$$

$$\phi(r) = \phi_0\left(\frac{1-\frac{B}{r}}{1+\frac{B}{r}}\right)^{C(1-2\xi)/\lambda} \tag{10}$$

where the constants λ, C are still connected via the BD field equations by

$$\lambda^2 \equiv (C+1)^2 - C\left(1 - \frac{\omega C}{2}\right). \tag{11}$$

Evidently, the parameter ξ has cancelled out and that is why we called it a gauge: It can be fixed to any value. The arbitrary constants $\alpha_0 = 0, \beta_0 = 0$ are determined by asymptotic flatness. Now choose the gauge and redefine the constant λ such that

$$\xi = -\frac{1}{C}, \qquad \frac{C+2}{\lambda} = C'. \tag{12}$$

Then the metric and the scalar function (8)–(10) immediately become

$$e^{\alpha(r)} = 1 \tag{13}$$

$$e^{\beta(r)} = \left(1+\frac{B}{r}\right)^2\left(\frac{1-\frac{B}{r}}{1+\frac{B}{r}}\right)^{1-C'} \tag{14}$$

$$\phi(r) = \phi_0\left(\frac{1-\frac{B}{r}}{1+\frac{B}{r}}\right)^{C'}. \tag{15}$$

Using the Eqs. (12) and (6), the identity (11) can be rewritten as

$$C'^2 = \left(\frac{\tilde{\omega}+2}{2}\right)^{-1}. \tag{16}$$

This is exactly the solution obtained in Ref. [2] on solving the complete set of BD field equations *ab initio*. Similarly, consider the Class II BD solution with the gauge ξ, viz.,

$$\alpha(r) = \alpha_0 + \frac{2(1+\xi C)}{\Lambda}\arctan\left(\frac{r}{B}\right) \tag{17}$$

$$\beta(r) = \beta_0 - \frac{2(C+1-\xi C)}{\Lambda}\arctan\left(\frac{r}{B}\right) - \ln\left(\frac{r^2}{r^2+B^2}\right) \tag{18}$$

$$\phi = \phi_0 e^{[2C(1-2\xi)/\Lambda]\arctan(r/B)} \tag{19}$$

$$\lambda^2 \equiv C\left(1 - \frac{\omega C}{2}\right) - (C+1)^2. \tag{20}$$

One can choose the gauge, again of the same form, $\xi = -\frac{1}{C}$, $\frac{C+2}{\Lambda} = C'$ to find that the resulting solution precisely coincides with the other solution in Ref. [2]. The key point is that, it is possible to generate an infinite set of formal solutions simply by assigning arbitrary values to the gauge parameter ξ and the resulting sets of solutions form equivalence classes having the same physical content as the original 1962 solutions.

It should be noted in passing that, under the identifications

$$\tilde{\lambda} = \frac{\lambda}{1+\xi C}, \qquad \tilde{C} = \frac{C(1-2\xi)}{1+\xi C}, \tag{21}$$

the solution set (8)–(11) has exactly the same form as the original BD Class I solutions [6] with $\xi = 0$. It is possible to obtain the solutions (13)–(16) from the solution with $\xi = 0$ simply by letting $C, \lambda \to \infty$ such that $(C/\lambda) \to C'$. This is obviously equivalent to choosing the gauge $\xi = -1/C$ in Eqs. (21) so that $\tilde{C}, \tilde{\lambda} \to \infty$, but $(\tilde{C}/\tilde{\lambda}) = C'$, in virtue of Eq. (12). Similar considerations apply for the solution set (17)–(20) in respect of BD Class II solutions. The above analyses make it clear that conformal invariance of the vacuum BD action can be used as a tool to separate genuinely new solutions from the existing ones. This concludes what we wished to demonstrate.

A. B. wishes to thank IRC-NBU for providing its journal facility. K. K. N. wishes to thank Professor Ou-Yang Zhong-Can for providing hospitality and excellent working facilities at ITP, CAS. Unstinted assistance from Sun Liqun is also gratefully acknowledged. This work is supported in part by the TWAS-UNESCO program of ICTP, Italy, in part by the financial assistance from the Chinese Academy of Sciences as well as in part by the National Basic Research Program of China under Grant No. 2003CB716300 and by the NNSFC under Grant No. 90403032.

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