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Equivalence of covariant and light front QED at one loop level

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We consider the equivalence of covariant and light front quantum electrodynamics at one loop level. We show that the one loop expressions for fermion self-energy, vacuum polarization, and vertex correction in the covariant perturbation theory can be reduced to a sum of propagating and instantaneous diagrams of light front time-ordered perturbation theory by performing k^- integration. We show that the third term in the doubly transverse gauge propagator is necessary to generate the diagrams involving instantaneous photon exchange both in the case of fermion self-energy as well as the vertex correction. We also show that the correspondence between the covariant and light front diagrams cannot be established if one removes the $k^+ = 0$ modes by an infrared cutoff before performing k^- integration.

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I. INTRODUCTION

There has been a considerable amount of work on the equivalence of covariant perturbation theory and light front Hamiltonian perturbation theory in recent years [1–5]. One of the approaches to show equivalence is at the level of Feynman diagrams [1]. In this approach, one starts with the covariant expression for a Feynman diagram and performs the integration over the light-cone energy k^-. This approach has been found to be successful in theories describing spinless particles. Ligterink and Bakker [1] have given a general algorithm for proving equivalence in theories involving scalars as well as spin-$\frac{1}{2}$ particles. Equivalence at the Feynman diagram level in Yukawa theory has been discussed in Refs. [2–4]. Correspondence between the light front Hamiltonian approach and the Lorentz-covariant approach has been discussed for QED $1 + 1$ and also for QCD by bosonization of the model [5].

Eyck and Rohrlich have shown the equivalence of null-plane $(3 + 1)$ QED in the light front gauge and conventional QED in the Coulomb gauge within the framework of Feynman-Dyson-Schwinger theory [6]. In this work, we address the issue of the equivalence of light front quantum electrodynamics (LFQED) and covariant QED at the one loop level using the Feynman diagram approach.

One loop renormalization of LFQED has been discussed by Mustaki *et al.* [7] in the light front gauge and by Ligterink and Bakker [8] in the Feynman gauge. There are two ways to obtain the one loop expressions for electron self-energy, vacuum polarization, and vertex correction graphs in LFQED. One way is to start with the light front Hamiltonian P^- and use the Heitler method of the old fashioned Hamiltonian perturbation theory. This is the method used in Ref. [7] and in most of the earlier work on LFQCD [9]. The other way is to start with the covariant expressions for the one loop diagrams and obtain the light-cone time-ordered diagrams by performing the k^- integration. This is the method used in Ref. [8] to obtain the light front expressions for one loop diagrams in Feynman gauge. The two treatments differ in the choice of gauge and also use different regularization schemes. Mustaki *et al.* use a mixed regularization scheme in which dimensional regularization is used in the transverse direction and a cutoff is used in the longitudinal direction. Ligterink and Bakker, on the other hand, use the "minus regularization" scheme which treats both kinds of divergences on the same footing. It is well known that in light front QED, there are additional one loop diagrams due to the presence of instantaneous interactions. In the minus regularization of Ligterink and Bakker [8], the instantaneous terms are removed by the regularization procedure as they are independent of p^- and therefore drop under differentiation. However, in the work of Mustaki *et al.*, the instantaneous diagrams are calculated by putting a cutoff on small values of k^+. Both the procedures have been used to evaluate the one loop expressions for LFQED.

In this work, we address the issue of showing equivalence of one loop expressions in the light front gauge at the level of Feynman diagrams. We use the method of performing k^- integration on covariant expressions and obtain the same expressions for all the propagating as well as instantaneous one loop diagrams which are obtained by Mustaki *et al.* using the Heitler method. Our treatment differs from that of Ligterink and Bakker in two respects. First, we use the light front gauge, which results in additional terms in the gauge boson propagator. Second, Ligterink and Bakker have shown equivalence using the minus regularization scheme, whereas the expressions that we have obtained are the same as those obtained by Mustaki *et al.* using the Heitler method. Evaluation of the resulting 3-dimensional integrals in the light front gauge using the mixed regularization scheme has already been done in Ref. [7]. We plan to do the corresponding calculation in the light front gauge using the minus regularization scheme in a future work. It has been shown in Ref. [7] that in their mixed regularization scheme the diagrams involving instantaneous interactions are neces-

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sary to obtain the covariant result. It is our aim to show that these diagrams can also be obtained by the method of performing k^- integration. The aim of the present work is to show that the unregularized expressions for regular as well as instantaneous diagrams obtained by using the Heitler method can also be obtained from the covariant expression by performing k^- integration provided the correct form of the photon propagator is used and the zero modes are not ignored.

The major difficulties that one faces while showing equivalence in the case of QED are the following:

(1) The treatment of fermions in light front field theories (LFFT's) is difficult due to the presence of a nonpropagating part in the Feynman propagator. The free propagator for the fermion in the covariant theory can be written as

$$\frac{i(\not{p} + m)}{p^2 - m^2 + i\epsilon} = \frac{i\not{p}_{\rm on} + m}{p^2 - m^2 + i\epsilon} + \frac{\gamma^+}{2p^+}. \qquad (1)$$

The first term is the on-shell propagator used in LFFT's and the second term is the nonpropagating part, which gives rise to integrals that are more singular than those in the time-ordered or the manifestly covariant formulation [1]. In the Yukawa model, it has been shown [2,3] that the nonpropagating part in the Feynman propagator gives, on k^- integration, the instantaneous diagrams involving the instantaneous fermion line of light front perturbation theory. We will show the same result for one loop diagrams in QED.

(2) The second issue and one that has not been addressed so far is the presence of diagrams involving instantaneous photons and how do they arise by performing k^- integration in covariant expressions. To establish equivalence, one must be able to show that the covariant expressions for diagrams involving internal photon lines reduce, on k^- integration, to a sum of propagating and instantaneous diagrams of light-cone-time-ordered field theory.

As an initial step in proving the equivalence of covariant QED and LFQED at the Feynman diagram level, we have considered in this paper one loop renormalization of LFQED in the light front gauge [7] and have shown that one can obtain all the propagating as well as instantaneous diagrams by performing the k^- integration. However, if one imposes an infrared cutoff on k^+ before performing the k^- integration, then the correct expressions for instantaneous diagrams cannot be reproduced.

(3) The main difficulty that one faces in proving equivalence arises due to an ambiguity in literature over the form of the photon propagator in the light-cone gauge [7,10]. Mustaki *et al.*[7] use the following two term photon propagator in the light-cone gauge:

$$D_{\mu\nu} = \frac{1}{k^2 + i\epsilon}\left[-g_{\mu\nu} + \frac{\delta_{\mu+}k_\nu + \delta_{\nu+}k_\mu}{k^+}\right]. \qquad (2)$$

There exists an alternative form of the photon propagator with an extra term [6,10–16] given by

$$D_{\mu\nu} = \frac{1}{k^2 + i\epsilon}\left[-g_{\mu\nu} + \frac{\delta_{\mu+}k_\nu + \delta_{\nu+}k_\mu}{k^+} - \frac{k^2\delta_{\mu+}\delta_{\nu+}}{(k^+)^2}\right]. \qquad (3)$$

The third term in this expression has always been dropped from actual calculations on the grounds that it has no physical significance because it does not propagate any information [15]. Recently, Suzuki *et al.* have presented the gauge fixing conditions which can lead to the three term propagator [16]. However, the question of which of the two propagators is the correct one is still not resolved.

In this work, we consider the question of equivalence of covariant and LFQED at the Feynman diagram level using the three term photon propagator. We derive the one loop expressions of LFQED obtained by Mustaki *et al.* [7] by performing the k^- integrations in the covariant expressions. We show that the nonpropagating terms in the fermion propagator as well as in the photon propagator are necessary to obtain the instantaneous diagrams involving the instantaneous fermion exchange and the instantaneous photon exchange. For the photon propagator, one must include the third term also. We also find that if one puts a cutoff on small k^+ values before performing the k^- integration then the instantaneous diagrams cannot be reproduced.

The plan of the paper is as follows: In Sec. II, we summarize the one loop renormalization of LFQED [7]. Here, we present only those results of Ref. [7] which are needed for our discussion. In Sec. III, we consider the self-energy diagram and show that the covariant expression on k^- integration reduces to a sum of propagating as well as instantaneous graphs of LFQED. In Secs. IV and V, we carry out a similar analysis for vacuum polarization and vertex correction graphs. In Sec. VI, we summarize and discuss our results. The Appendix contains the notations and basics.

II. PERTURBATIVE RENORMALIZATION OF LIGHT FRONT QED

In this section, we briefly review the work done by Mustaki *et al.* [7] on one loop renormalization of light front QED in the Hamiltonian formalism. Here, we will summarize the relevant parts of the calculation of the mass shift for the fermion as well as the photon and the one loop vertex correction in LFQED.

A. Electron mass renormalization

In light-cone time-ordered perturbation theory, the transition matrix is given by the perturbative expansion,

$$T = V + V\frac{1}{p^- - H_0}V + \cdots, \tag{4}$$

where

$$V = V_1 + V_2 + V_3, \tag{5}$$

with V_1, V_2, and V_3 being the standard three-point interaction, an $O(e^2)$-nonlocal effective four-point vertex corresponding to instantaneous fermion exchange and an $O(e^2)$-nonlocal effective four-point vertex corresponding to an instantaneous photon exchange, respectively. The forms of these are given in the Appendix.

The one loop correction to fermion self-energy is obtained by calculating the matrix element of the first two terms of the above series between the initial and final one electron states $|p, s\rangle$ and $|p', s'\rangle$:

$$2m\delta m\delta_{ss'} = 2p^+ T_{pp} \tag{6}$$

which implies

$$\delta m\delta_{ss'} = \frac{p^+}{m}T_{pp}. \tag{7}$$

Here, the electron states are chosen as

$$|\bar{p}, s\rangle = \sqrt{2p^+}|p, s\rangle \tag{8}$$

for Lorentz-invariant normalization. One can also identify a matrix element $\Sigma(p)$ through the following expression:

$$\delta m\delta_{ss'} = \overline{u}(p, s')\Sigma(p)u(p, s). \tag{9}$$

At $O(e^2)$, there are three contributions to this amplitude. The second term in the expansion of T yields a contribution which is second order in V_1 and is given by

$$\bar{u}(p, s')\Sigma_1(p)u(p, s) = \langle p, s'|V_1\frac{1}{p^- - H_0}V_1|p, s\rangle. \tag{10}$$

This corresponds to the diagram in Fig. 1(a). Unlike conventional QED, in LFQED, there are additional diagrams due to first order contributions from the instantaneous interactions V_2 and V_3, which are given by

$$\bar{u}(p, s')\Sigma_2(p)u(p, s) = \langle p, s'|V_2|p, s\rangle \tag{11}$$

corresponding to the diagram in Fig. 1(b) and

$$\bar{u}(p, s')\Sigma_3(p)u(p, s) = \langle p, s'|V_3|p, s\rangle \tag{12}$$

corresponding to the sum of diagrams in Figs. 1(c) and 1(d). Here all the particles are on shell:

$$p = \left(p^+, \frac{p_\perp^2 + m^2}{2p^+}, p_\perp\right), \tag{13}$$

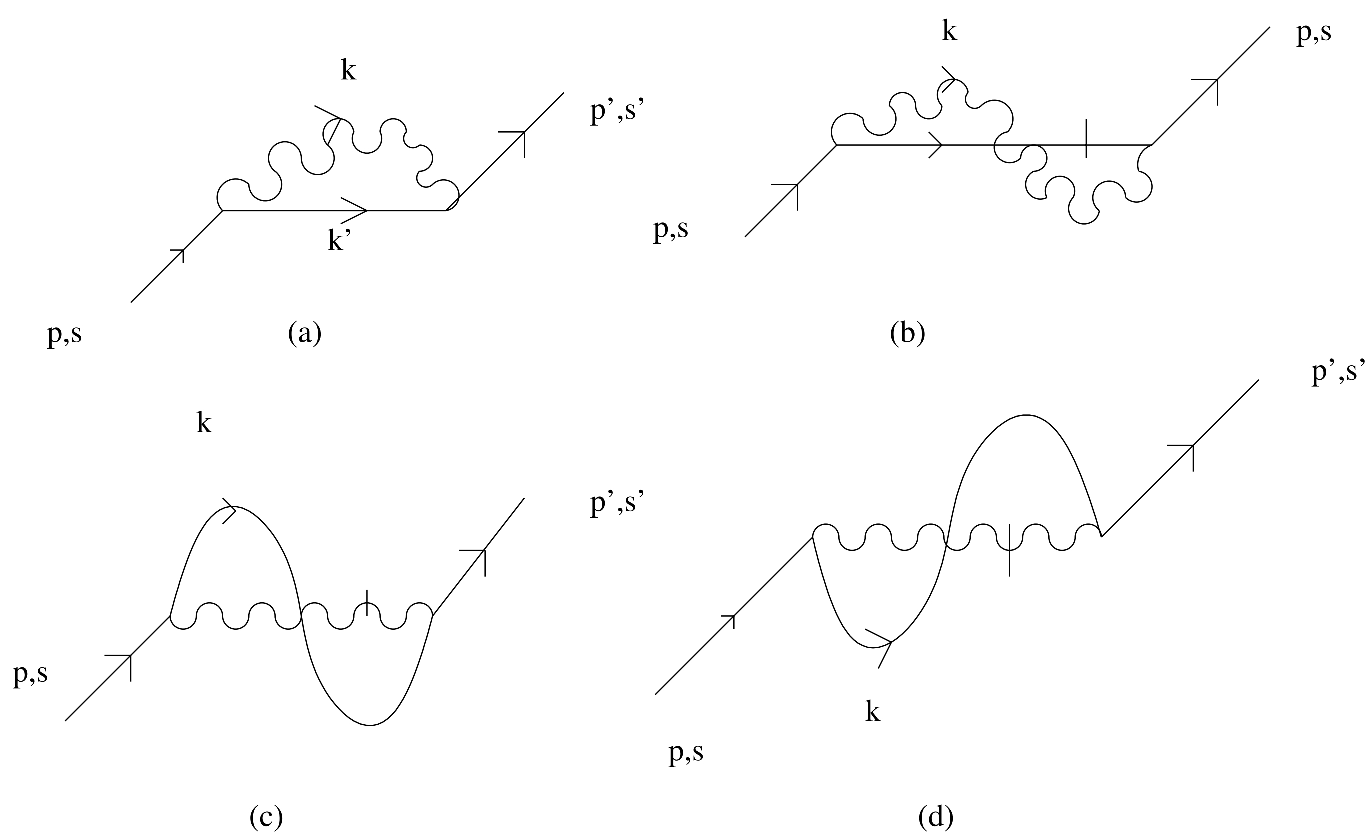

FIG. 1. Diagrams for electron mass shift in LFQED.

$$k = \left(k^+, \frac{k_\perp^2}{2k^+}, k_\perp\right), \tag{14}$$

and

$$k' = \left(p^+ - k^+, \frac{(p_\perp - k_\perp)^2 + m^2}{2(p^+ - k^+)}, p_\perp - k_\perp\right). \tag{15}$$

The contribution of Fig. 1(a) to δm is given by Eq. (10) and leads to the light-cone expression

$$\delta m_a \delta_{s\sigma} = \frac{e^2}{m} \int \frac{d^2 k_\perp}{(4\pi)^3} \int_0^{p^+} \frac{dk^+}{k^+(p^+ - k^+)} \times \frac{\bar{u}(p, \sigma)\gamma^\mu (\not{k}' + m)\gamma^\nu u(p, s) d_{\mu\nu}(k)}{p^- - k^- - k'^-}. \tag{16}$$

Similarly, the contribution of Fig. 1(b) is

$$\delta m_b \delta_{ss'} = \frac{e^2 p^+ \delta_{ss'}}{2m} \int \frac{d^2 k_\perp}{(2\pi)^3} \int_0^{+\infty} \frac{dk^+}{k^+(p^+ - k^+)} \tag{17}$$

and the sum of contributions of Figs. 1(c) and 1(d) is

$$\delta m_{c+d} \delta_{ss'} = \frac{e^2 p^+ \delta_{ss'}}{2m} \int \frac{d^2 k_\perp}{(2\pi)^3} \left[\int_0^{+\infty} \frac{dk^+}{(p^+ - k^+)^2} - \int_0^{+\infty} \frac{dk^+}{(p^+ + k^+)^2} \right]. \tag{18}$$

These integrals have singularities at $k^+ = 0$ and $k^+ = p^+$ which are regularized by introducing small cutoffs α and β,

$$\alpha < k^+ < p^+ - \beta, \tag{19}$$

and the poles at $k^+ = p^+$ in δm_b and δm_c are removed by the principal-value prescription. These integrals have been evaluated in Ref. [7] and finally add up to yield

$$\delta m = \frac{e^2 m}{8\pi^2 \epsilon}. \tag{20}$$

B. Photon mass renormalization

Consider now the amplitude T_{pp} of the transition matrix T between free photon states (p, λ) and (p, λ') at order e^2. $\delta\mu^2$ is given by

$$\delta\mu^2 \delta_{\lambda\lambda'} = 2p^+ T_{pp}, \tag{21}$$

where now

$$p = \left(p^+, \frac{p_\perp^2}{2p^+}, p_\perp\right) \tag{22}$$

is the initial (or final) photon momentum. One can also identify a "tensor" $\Pi^{\mu\nu}(p)$ through

$$\delta\mu^2 \delta_{\lambda\lambda'} = \epsilon_\mu^\lambda(p) \Pi^{\mu\nu}(p) \epsilon_\nu^{\lambda'}(p). \tag{23}$$

The corresponding diagrams are displayed in Fig. 2. The sea gulls, which are displayed in Figs. 2(b) and 2(c), yield

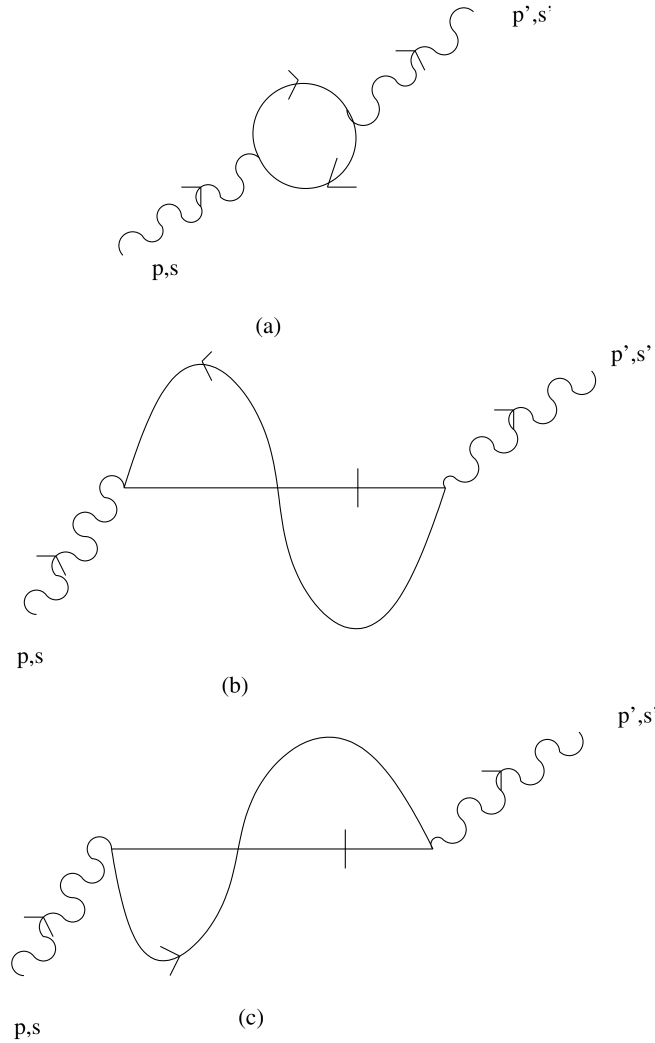

FIG. 2. Diagrams for vacuum polarization in LFQED.

$$\delta\mu_{b+c}^2 = \langle p, \lambda | V_2 | p, \lambda \rangle \tag{24}$$

and have been evaluated to be [7]

$$\delta\mu_{b+c}^2 = e^2 \int \frac{d^2 k_\perp}{(2\pi)^3} \int_0^\infty dk^+ \left[\frac{1}{p^+ - k^+} - \frac{1}{p^+ + k^+} \right]. \tag{25}$$

Here a principal-value prescription is implied. $\delta\mu_a^2$ is given by

$$\delta\mu_a^2 \delta_{\lambda\lambda'} = \langle p, \lambda | V_1 \frac{1}{p^- - H_0} V_1 | p, \lambda \rangle. \tag{26}$$

Inserting appropriate sets of intermediate states and following the standard procedure, one obtains

$$\delta\mu_a^2 \delta_{\lambda\lambda'} = 2e^2 \int \frac{d^2 k_\perp}{(4\pi)^3} \int_\alpha^{p^+ - \beta} \frac{dk^+}{k^+(p^+ - k^+)} \times \frac{\mathrm{Tr}[\not{\epsilon}^{(\lambda)}(p)(\not{k} + m)\not{\epsilon}^{(\lambda')}(p)(\not{k}' - m)]}{p^- - k^- - k'^-}, \tag{27}$$

where

$$k = \left(k^+, \frac{k_\perp^2 + m^2}{2k^+}, k_\perp\right) \tag{28}$$

and

$$k' = \left(p^+ - k^+, \frac{(p_\perp - k_\perp)^2 + m^2}{2(p^+ - k^+)}, p_\perp - k_\perp\right). \tag{29}$$

The sum of $\delta\mu_a^2$ and $\delta\mu_{b+c}^2$ is

$$\delta\mu^2 = e^2 \int \frac{d^2k_\perp}{(2\pi)^3} \int_\alpha^{p^+-\beta} dk^+ \left[\frac{p^+}{k^+(k^+ - p^+)} + \frac{2k_\perp^2}{p^+(k_\perp^2 + m^2)}\right]. \tag{30}$$

Evaluating this integral using dimensional regularization, one obtains

$$\delta\mu^2 = -\frac{e^2 m^2}{4\pi^2 \epsilon}. \tag{31}$$

C. Vertex correction

The vertex corrections $\Lambda^\mu(p, p')$ are of order e^3 as they get contributions from V_1^3, $V_2\ V_1$ and $V_3\ V_1$. It will be adequate for the purpose of finding vertex corrections to calculate Λ^+. Figure 3 contains the diagrams which contribute to Λ^+ excluding the diagrams corresponding to incoming or outgoing fermion line renormalization. The full set of diagrams is given in Ref. [7]. However, the diagrams that we have omitted do not contribute to Λ^+ due to their tensor structure.

In Hamiltonian perturbation theory, corrections to Λ^μ are obtained by calculating the matrix elements of the series on the right-hand side of Eq. (4) between $|p, \sigma\rangle$ and $|p', \sigma', q, \lambda\rangle$ states. The one loop correction to Λ^μ is given by the diagram in Fig. 3. Contributions to Λ^μ from Figs. 3(a) and 3(b) arise from

$$\Lambda^\mu_{(3a)} + \Lambda^\mu_{(3b)} = \langle p', s', k\lambda | V_1 \frac{1}{p^- - H_0} V_1 \frac{1}{p^- - H_0} V_1 | p, s\rangle \tag{32}$$

and the contribution of the instantaneous diagram in Fig. 3(c) comes from

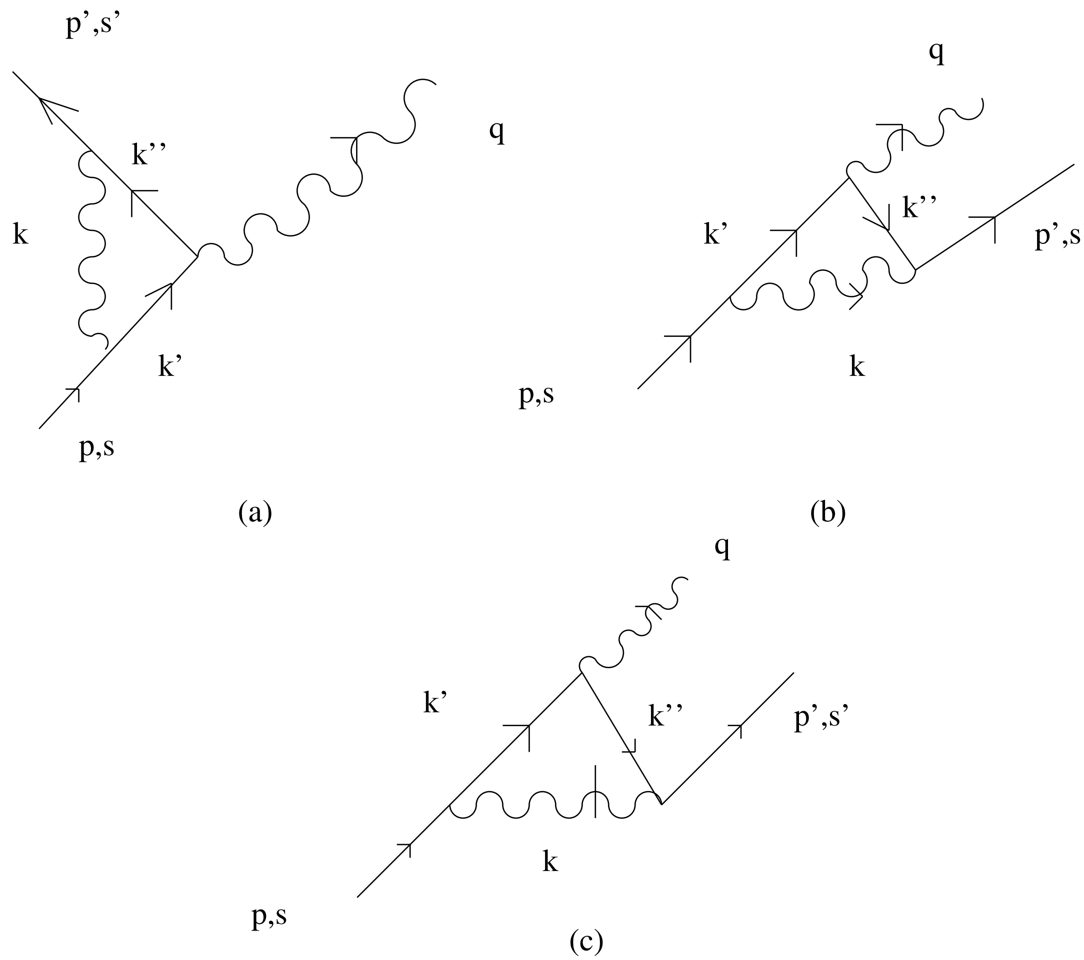

FIG. 3. Diagrams for vertex correction Λ^+.

$$\Lambda^{\mu}_{(3c)} = \langle p's', k\lambda | V_1 \frac{1}{p^- - H_0} V_3 | ps \rangle. \tag{33}$$

These expressions can be evaluated by inserting appropriate sets of intermediate states. Following the standard procedure one obtains

$$\Lambda^{+}_{(3a)} = e^3 \int \frac{d^2k_\perp}{(4\pi)^3} \int_0^{p'^+} \frac{dk^+}{k^+ k'^+ k''^+} \times \frac{\gamma^+ (\not{k}'' + m)\gamma^\mu (\not{k}' + m)\gamma^\nu d_{\mu\nu}}{(p^- - k^- - k'^-)(p^- - k^- - k''^- - q^-)}, \tag{34}$$

$$\Lambda^{+}_{(3b)} = -e^3 \int \frac{d^2k_\perp}{(4\pi)^3} \int_{p'^+}^{p^+} \frac{dk^+}{k^+ k'^+ k''^+} \times \frac{\gamma^\rho (\not{k}'' + m)\gamma^+ (\not{k}' + m)\gamma^\nu d_{\rho\nu}}{(p^- - k^- - k')(p^- - k'^- + k''^- - p'^-)}, \tag{35}$$

$$\Lambda^{+}_{(3c)} = 2e^3 \int \frac{d^2k_\perp}{(4\pi)^3} \int_{p^+ - q^+}^{p^+} \frac{dk^+}{(k^+)^2 k'^+ k''^+} \times \frac{\gamma^+ (\not{k}'' + m)\gamma^+ (\not{k}' + m)\gamma^+}{[p^- - p'^- + k''^- - k'^-]}. \tag{36}$$

These expressions will be used in Sec. V for demonstrating the equivalence of covariant and light front expressions. These integrals have been evaluated in Appendix D of Ref. [7] using dimensional regularization. However, we will not need the details of this calculation. The sum of $\Lambda^+_{(3a)}$, $\Lambda^+_{(3b)}$, and $\Lambda^+_{(3c)}$ yields the vertex correction

$$\Lambda^+ = \Lambda_0^+ \frac{e^2}{8\pi^2 \epsilon}\left[-\frac{3}{2} + \ln p^+ \bar{p}^+ \alpha^2 \right]. \tag{37}$$

III. EQUIVALENCE OF COVARIANT AND LIGHT FRONT FERMION SELF-ENERGY GRAPHS

In this section, we will demonstrate the equivalence of self-energy graphs in covariant QED and light front QED at $O(e^2)$ at the level of Feynman diagrams. We start with the one loop expressions for electron self-energy and show that by performing the k^- integration, these covariant expressions can be reduced to the light-cone time-ordered expressions given in Sec. II.

The covariant expression for electron self-energy in the light front gauge is given by

$$\sum(p) = \frac{(ie)^2}{2mi} \int \frac{d^4k}{(2\pi)^4} \times \frac{\gamma^\mu (\not{p} - \not{k} + m)\gamma^\nu d'_{\mu\nu}(k)}{[(p-k)^2 - m^2 + i\epsilon][k^2 - \mu^2 + i\epsilon]}, \tag{38}$$

where $d'_{\mu\nu}/k^2$ is the photon propagator in the light-cone gauge in covariant perturbation theory with $d'_{\mu\nu}(k)$ given by

$$d'_{\mu\nu}(k) = d_{\mu\nu}(k) - \frac{\delta_{\mu+}\delta_{\nu+}k^2}{(k^+)^2}. \tag{39}$$

Here, $d_{\mu\nu}(k)/k^2$ is the photon propagator in the light-cone gauge used in light front QED [7] where $d_{\mu\nu}(k)$ is given by

$$d_{\mu\nu} = -g_{\mu\nu}(k) + \frac{\delta_{\mu+}k_\nu + \delta_{\nu+}k_\mu}{k^+}. \tag{40}$$

To prove equivalence, we note that the momenta in the light front expressions in Sec. II are on-shell momenta. Therefore, we substitute in Eq. (38):

$$\not{p} - \not{k} + m = \gamma^+\left[\left(\frac{(p_\perp - k_\perp)^2 + m^2}{2(p^+ - k^+)}\right)\right] + \gamma^-(p^+ - k^+) - \gamma_\perp(p_\perp - k_\perp) + \gamma^+\left[p^- - k^- - \frac{(p_\perp - k_\perp)^2 + m^2}{2(p^+ - k^+)}\right] \tag{41}$$

and write

$$-i\Sigma(p) = -i\Sigma_1(p) - i\Sigma_2(p), \tag{42}$$

where

$$-i\Sigma_1(p) = +\frac{e^2}{2m}\int \frac{d^4k}{(2\pi)^4} \times \frac{\gamma^\mu(\not{k}' + m)\gamma^\nu d'_{\mu\nu}(k)}{[(p-k)^2 - m^2 + i\epsilon][k^2 - \mu^2 + i\epsilon]} \tag{43}$$

and

$$-i\Sigma_2(p) = \frac{e^2}{2m}\int \frac{d^4k}{(2\pi)^4} \frac{\gamma^\mu\gamma^+\gamma^\nu d'_{\mu\nu}(k)}{2(p^+ - k^+)(k^2 - \mu^2 + i\epsilon)}. \tag{44}$$

$\Sigma_2(p)$ can be rewritten as

$$\Sigma_2(p) = \frac{ie^2}{2m}\int \frac{dk^+}{2\pi}\int \frac{d^2k_\perp}{(2\pi)}\int \frac{dk^-}{2\pi} \times \frac{\gamma^\mu\gamma^+\gamma^\nu[-g_{\mu\nu} + \frac{k_\mu\delta_{\nu+} + k_\nu\delta_{\mu+}}{k^+}]}{2(p^+ - k^+)2k^+[k^- - \frac{k_\perp^2 + \mu^2 - i\epsilon}{2k^+}]}. \tag{45}$$

The k^- integral here has a pole at $k^- = (k_\perp^2 + \mu^2)/2k^+$ and another pole at $k^- = \infty$. To deal with the pole at infinity, we change the variable from k^- to $u = \frac{1}{k^-}$, so that the pole at infinity is shifted to the origin and the k^- integral reduces to

$$D = i\int \frac{du}{u[2k^+ - (k_\perp^2 + m^2 - i\epsilon)u]}. \tag{46}$$

Next, we regularize the pole at the origin by the replacement [1]

$$\frac{1}{u} \to \frac{1}{2}\left(\frac{1}{u+i\delta} + \frac{1}{u-i\delta}\right) \tag{47}$$

which gives

$$D^{\rm reg} = i\int du \frac{1}{2}\left[\frac{1}{u+i\delta} + \frac{1}{u-i\delta}\right] \times \frac{1}{2k^+ - (k_\perp^2 + \mu^2 - i\epsilon)u} \tag{48}$$

$$= i\int \frac{du}{2}\frac{1}{(u+i\delta)[2k^+ - (k_\perp^2 + \mu^2 - i\epsilon)u]} + i\int \frac{du}{2}\frac{1}{(u-i\delta)[2k^+ - (k_\perp^2 + \mu^2 - i\epsilon)u]}. \tag{49}$$

The first integral has a pole at $u = -i\delta$ which is below the real axis and another pole at $u = 2k^+/(k_\perp^2 + \mu^2 - i\epsilon)$. For $k^+ < 0$, both the poles lie below the real axis. Therefore, the integral vanishes. For $k^+ > 0$, the second pole is above the real axis. Therefore, we perform contour integration along a semicircle which goes along the real line and is closed at infinity in the lower half complex plane. The second integral has poles at $u = +i\delta$ and at $u = 2k^+/(k_\perp^2 + \mu^2 - i\epsilon)$. For $k^+ > 0$ both the poles are above the real axis, and therefore $I = 0$ for $k^+ > 0$. For $k^+ < 0$, we close the contour in the upper half plane. Performing complex integration with this choice of contours, we obtain

$$D^{\rm reg} = \frac{\pi\theta(k^+)}{2k^+ + i(k_\perp^2 + \mu^2 - i\epsilon)\delta} + \frac{-\pi\theta(-k^+)}{2k^+ - i(k_\perp^2 + \mu^2 - i\epsilon)\delta} \tag{50}$$

which reduces, in the limit $\delta \to 0$, to

$$D = \frac{2\pi\theta(k^+)}{2k^+}. \tag{51}$$

Substituting this into $\Sigma_2(p)$ we get

$$\Sigma_2(p) = \frac{e^2}{2m}\int_0^\infty \frac{dk^+}{2k^+}\int \frac{d^2k_\perp}{(2\pi)^3}\frac{\gamma^\mu\gamma^+\gamma^\nu d'_{\mu\nu}(k)}{2(p^+ - k^+)}. \tag{52}$$

Using Eqs. (39) and (A11), we finally obtain

$$\delta m_2 \delta_{ss'} = \bar{u}(p,s')\Sigma_2(p)u(p,s) = \frac{e^2 p^+ \delta_{s\sigma}}{2m}\int \frac{d^2k_\perp}{(2\pi)^3}\int_0^\infty \frac{dk^+}{k^+(p^+ - k^+)}. \tag{53}$$

$\Sigma_1(p)$ can be written as

$$\Sigma_1(p) = i\frac{e^2}{2m}\int \frac{d^3k}{(2\pi)^3}\int \frac{dk^-}{2\pi}\frac{\gamma^\mu(\not{k}' + m)\gamma^\nu[d_{\mu\nu} - \frac{\delta_{\mu+}\delta_{\nu+}k^2}{(k^+)^2}]}{2k^+2(p^+ - k^+)[k^- - \frac{k_\perp^2 + \mu^2 - i\epsilon}{2k^+}][p^- - k^- - \frac{(p_\perp - k_\perp)^2 + m^2 - i\epsilon}{2(p^+ - k^)}]}. \tag{54}$$

We define

$$\Sigma_1(p) = \Sigma_1^{(a)}(p) + \Sigma_1^{(b)}(p), \tag{55}$$

where

$$\Sigma_1^{(a)}(p) = -i\frac{e^2}{2m}\int \frac{d^3k}{(2\pi)^3}\int \frac{dk^-}{2\pi}\frac{\gamma^\mu(\not{k}' + m)\gamma^\nu d_{\mu\nu}(k)}{2k^+2(p^+ - k^+)[k^- - \frac{k_\perp^2 + \mu^2 - i\epsilon}{2k^+}][k^- - p^- + \frac{(p_\perp - k_\perp)^2 + m^2 - i\epsilon}{2(p^+ - k^+)}]} \tag{56}$$

and

$$\Sigma_1^{(b)}(p) = i\frac{e^2}{2m}\int \frac{d^3k}{(2\pi)^3}\int \frac{dk^-}{2\pi}\frac{\gamma^\mu(\not{k}' + m)\gamma^\nu[\delta_{\mu+}\delta_{\nu+}k^2]}{2k^+(2p^+ - k^+)[k^- - \frac{k_\perp^2 + \mu^2 - i\epsilon}{2k^+}][k^- - p^- + \frac{(p_\perp - k_\perp)^2 + m^2 - i\epsilon}{2(p^+ - k^+)}](k^+)^2}. \tag{57}$$

The integrand in $\Sigma_1^{(a)}(p)$ has poles at $k^- = (k_\perp^2 + \mu^2 - i\epsilon)/2k^+$ and $k^- = p^- - \{[(p_\perp - k_\perp)^2 + m^2 - i\epsilon]/(p^+ - k^+)\}$. One can notice that

(1) For $k^+ < 0$, both the poles are above the real axis.
(2) For $k^+ > p^+$, both poles are below the real axis.
(3) For $0 < k^+ < p^+$, the pole at $k^- = (k_\perp^2 + m^2 - i\epsilon)/2k^+$ is below the real axis and the pole at $k^- = p^- - \{[(p_\perp - k_\perp)^2 + m^2 - i\epsilon]/2(p^+ - k^+)\}$ is above the real axis.

Thus the integral vanishes for $k^+ < 0$ and $k^+ > p^+$ and for $0 < k^+ < p^+$, we can close the contour in the lower half plane . The integral is then equal to

$$\Sigma_1^{(a)}(p) = \frac{ie^2}{2m}\int_0^{p^+} \frac{d^3k}{(2\pi)^3} \times \frac{\gamma^\mu(\not{k}'_{\rm on} + m)\gamma^\nu d_{\mu\nu}(k)}{2k^+2(p^+ - k^+)[p^- - k^-_{\rm on} - k'^-]}, \tag{58}$$

where

$$k'^{-} = \frac{(p_\perp - k_\perp)^2 + m^2}{2(p^+ - k^+)}. \tag{59}$$

$\Sigma_1^{(b)}$ is given by

$$\Sigma_1^{(b)} = i\frac{e^2}{2m}\int\frac{d^2k_\perp}{(2\pi)^3}\int dk^+\int\frac{dk^-}{(2\pi)}\frac{k^2\gamma^\mu(\not{k}' + m)\gamma^\nu\delta_{\mu+}\delta_{\nu+}}{2k^+(2p^+ - 2k^+)[k^- - \frac{k_\perp^2+\mu^2-i\epsilon}{2k^+}][p^- - k^- - \frac{(p_\perp-k_\perp)^2+m^2-i\epsilon}{2(p^+-k^+)}](k^+)^2}. \tag{60}$$

Using the properties of gamma matrices Eqs. (A4)–(A9), this reduces to

$$\Sigma_1^{(b)} = i\frac{e^2}{2m}\int\frac{d^2k_\perp}{(2\pi)^3}\frac{dk^+}{(k^+)^2}\int\frac{dk^-}{2\pi} \times\frac{\gamma^+\gamma^-\gamma^+}{2[p^- - k^- - \frac{(p_\perp-k_\perp)^2+m^2-i\epsilon}{2(p^+-k^+)}]}. \tag{61}$$

This integral has a pole at $k^- = \infty$ at $k^+ = p^+$. To deal with the pole at infinity, we change the variable from k^- to $u = \frac{1}{k^-}$, so that the k^- integral changes to

$$I = -\frac{1}{2}\int\frac{du}{2\pi}\frac{1}{u}\frac{1}{[1 - u[p^- - \frac{(p_\perp-k_\perp)^2+m^2-i\epsilon}{2(p^+-k^+)}]]} \tag{62}$$

and the pole at infinity is shifted to the origin. I has poles at $u = 0$ and

$$u = \frac{1}{p^- - \frac{(p_\perp-k_\perp)^2+m^2-i\epsilon}{2(p^+-k^+)}}.$$

We regularize the integral by the replacement

$$\frac{1}{u} = \frac{1}{2}\left(\frac{1}{u+i\delta} + \frac{1}{u-i\delta}\right) \tag{63}$$

and write

$$I = I_1 + I_2, \tag{64}$$

where

$$I_1 = -\frac{1}{4}\int\frac{du}{2\pi}\frac{1}{u+i\delta}\frac{1}{1 - u[p^- - \frac{(p_\perp-k_\perp)^2+m^2-i\epsilon}{2(p^+-k^+)}]} \tag{65}$$

and

$$I_2 = -\frac{1}{4}\int\frac{du}{2\pi}\frac{1}{u-i\delta}\frac{1}{1 - u[p^- - \frac{(p_\perp-k_\perp)^2+m^2-i\epsilon}{2(p^+-k^+)}]}. \tag{66}$$

The first integral has poles at $u = -i\delta$ which is below the real axis and at

$$u = \frac{1}{p^- - \frac{(p_\perp-k_\perp)^2+m^2-i\epsilon}{2(p^+-k^+)}}$$

which is above the real axis only when $p^+ < k^+$. I_2 has poles at $u = +i\delta$ which is above the real axis and at

$$u = \frac{1}{p^- - \frac{(p_\perp-k_\perp)^2+m^2-i\epsilon}{2(p^+-k^+)}}$$

which is below the real axis only for $p^+ > k^+$. Therefore, we close the contour by a semicircle at infinity in the lower half plane for I_1 and in the upper half plane for I_2. Thus we obtain

$$I^{\text{reg}} = \frac{i}{4}\frac{\theta(k^+ - p^+)}{1 + i\delta[p^- - \frac{(p_\perp-k_\perp)^2+m^2-i\epsilon}{2(p^+-k^+)}]} - \frac{i}{4} \times\frac{\theta(p^+ - k^+)}{1 - i\delta[p^- - \frac{(p_\perp-k_\perp)^2+m^2-i\epsilon}{2(p^+-k^+)}]}. \tag{67}$$

Taking the limit $\delta \to 0$ and substituting in $\Sigma_1^{(b)}$, we obtain

$$\Sigma_1^{(b)} = \frac{ie^2}{2m}\int\frac{d^2k_\perp}{(2\pi)^2}\int\frac{dk^+}{(2\pi)(k^+)^2} \times\frac{i}{4}\gamma^+\gamma^-\gamma^+[\theta(k^+ - p^+) - \theta(p^+ - k^+)]. \tag{68}$$

Thus, $\bar{u}\Sigma_1^{(b)}u$ is given by

$$\bar{u}\Sigma_1^{(b)}u = -\frac{e^2p^+}{2m}\int\frac{d^2k_\perp}{(2\pi)^3}\left[-\int_{-\infty}^{p^+}\frac{dk^+}{(k^+)^2} + \int_{p^+}^{\infty}\frac{dk^+}{(k^+)^2}\right] = \frac{e^2p^+}{2m}\int\frac{d^2k_\perp}{(2\pi)^3}2\int_0^{p^+}\frac{dk^+}{(k^+)^2}, \tag{69}$$

where we have used Eqs. (A8) and (A13). Using [10]

$$2\int_0^{p^+}\frac{dk^+}{(k^+)^2} = \int_0^\infty\frac{dk^+}{(p^+ - k^+)^2} - \int_0^\infty\frac{dk^+}{(p^+ + k^+)^2} \tag{70}$$

we finally obtain

$$\bar{u}\Sigma_1^{(b)}(p)u = \frac{e^2p^+}{2m}\int d^2k_\perp\int_0^\infty\frac{dk^+}{(2\pi)^3} \times\left[\frac{1}{(p^+ - k^+)^2} - \frac{1}{(p^+ + k^+)^2}\right]. \tag{71}$$

Adding Eqs. (53), (58), and (71), one obtains the self-energy expression obtained using light-cone time-ordered Hamiltonian perturbation theory, i.e., the sum of Eqs. (16)–(18). One may notice that the contribution corresponding to the instantaneous diagrams arises from the consideration of the pole at infinity, i.e., if we regularize the k^+ integrals by putting a cutoff on small values, then we get only δm_a. The pole at infinity is essential to generate the instantaneous diagrams. Also we get δm_{c+d} to be zero if we drop the third term in the photon propagator.

IV. EQUIVALENCE OF COVARIANT AND LIGHT FRONT VACUUM POLARIZATION GRAPHS

Now we will show the equivalence of covariant and light-cone expressions for vacuum polarization graphs using the same procedure. Photon self-energy is defined by

$$\delta\mu^2\delta_{\lambda\lambda'} = \epsilon_\mu^\lambda(p)\Pi^{\mu\nu}(p)\epsilon_\nu^{\lambda'}(p), \tag{72}$$

where

$$i\Pi^{\mu\nu}(p) = -(-ie)^2 \times \int \frac{d^4k}{(2\pi)^4}\mathrm{Tr}\Big(\gamma^\mu \frac{i}{\not k - m}\gamma^\nu \frac{i}{\not p - \not k + m}\Big). \tag{73}$$

One can rewrite Eq. (73) as

$$i\Pi^{\mu\nu}(p) = -e^2\int\frac{d^3k}{(2\pi)^3}\int\frac{dk^-}{2\pi}\frac{\mathrm{Tr}[\gamma^\nu(\not k + m)\gamma^\nu(\not p - \not k - m)]}{2k^+2(p^+ - k^+)[k^- - \frac{k_\perp^2+m^2-i\epsilon}{2k^+}][p^- - k^- - \frac{(p_\perp-k_\perp)^2+m^2-i\epsilon}{2(p^+-k^+)}]}. \tag{74}$$

To reduce this expression to a sum of light front diagrams, we first change the off-shell momenta to on-shell momenta by using

$$\not p - \not k = \gamma^+\Big[p^- - k^- - \frac{(p_\perp - k_\perp)^2 + m^2}{2(p^+ - k^+)}\Big] + \Big[\gamma^+\Big(\frac{(p_\perp - k_\perp)^2 + m^2}{2(p^+ - k^+)}\Big) + \gamma^-(p^+ - k^+) - \gamma_\perp(p_\perp - k_\perp)\Big] \tag{75}$$

obtaining

$$\Pi^{\mu\nu}(p) = \Pi_1^{\mu\nu}(p) + \Pi_2^{\mu\nu}(p), \tag{76}$$

where

$$i\Pi_1^{\mu\nu}(p) = e^2\int\frac{d^3k}{(2\pi)^3}\int\frac{dk^-}{(2\pi)}\frac{\mathrm{Tr}[\gamma^\mu(\not k + m)\gamma^\nu(\not k' - m)]}{2k^+2(p^+ - k^+)[k^- - \frac{k_\perp^2+m^2}{2k^+}][p^- - k^- - \frac{(p_\perp-k_\perp)^2+m^2}{2(p^+-k^+)}]} \tag{77}$$

and

$$i\Pi_2^{\mu\nu}(p) = -e^2\int\frac{d^3k}{(2\pi)^3}\int\frac{dk^-}{(2\pi)}\frac{\mathrm{Tr}[\gamma^\mu(\not k + m)\gamma^\nu\gamma^+]}{2k^+2(p^+ - k^+)[k^- - \frac{k_\perp^2+m^2-i\epsilon}{2k^+}]}. \tag{78}$$

Here k'^- is the on-shell momentum:

$$k'^- = \frac{(p_\perp - k_\perp)^2 + m^2}{2(p^+ - k^+)}. \tag{79}$$

$\Pi_1^{\mu\nu}(p)$ has poles at $k^- = (k_\perp^2 + m^2 - i\epsilon)/2k^+$ and $k^- = p^- - \{[(p_\perp - k_\perp)^2 + m^2 - i\epsilon]/2(p^+ - k^+)\}$. We note that

(1) For $k^+ < 0$, both poles are above the real line.
(2) For $k^+ > p^+$, both poles are below the real line.
(3) For $0 < k^+ < p^+$ the first pole is below the real line whereas the second pole is above the real line.

As a result, the k^- integration yields zero for $k^+ < 0$ and for $k^+ > p^+$. For $0 < k^+ < p^+$, we perform the k^- integration by closing the contour in the lower half plane and obtain

$$\Pi_1^{\mu\nu} = 2e^2\int\frac{d^3k}{(4\pi)^3}\frac{\mathrm{Tr}[\gamma^\mu(\not k_{\rm on} + m)\gamma^\nu(\not k'_{\rm on} - m)]}{k^+(p^+ - k^+)[p^- - k^-_{\rm on} - k'^-_{\rm on}]}, \tag{80}$$

where

$$k^-_{\rm on} = \frac{k_\perp^2 + m^2}{2k^+} \tag{81}$$

and

$$k'^-_{\rm on} = \frac{(p_\perp - k_\perp)^2 + m^2}{2(p^+ - k^+)}. \tag{82}$$

Thus, the contribution of $\Pi_1^{\mu\nu}$ to the photon self-energy expression is given by

$$\epsilon_\mu^\lambda\Pi_1^{\mu\nu}\epsilon_\nu^{\lambda'} = 2e^2\int\frac{d^3k}{(4\pi)^3}\frac{\mathrm{Tr}[\not\epsilon^\lambda(\not k + m)\not\epsilon^{\lambda'}(\not k' - m)]}{k^+(p^+ - k^+)[p^- - k^- - k'^-]} \tag{83}$$

which is the same as Eq. (27), i.e., the propagating part of the photon self-energy in the light front calculation (Here, the subscript ''on'' has been dropped but the momenta k and k' are on-shell momenta.) $\Pi_2^{\mu\nu}$ can be simplified by using

$$\gamma^\mu \not{k} \gamma^\nu \gamma^+ = \gamma^\mu \gamma^+ \gamma^\nu \gamma^+ k^- + \gamma^\mu \gamma^- \gamma^\nu \gamma^+ k^+ - \gamma^\mu \gamma^i \gamma^\nu \gamma^+ k^i \tag{84}$$

and dropping the terms linear in k^i to obtain

$$\Pi_2^{\mu\nu} = \Pi_{(2a)}^{\mu\nu} + \Pi_{(2b)}^{\mu\nu}, \tag{85}$$

where

$$\Pi_{(2a)}^{\mu\nu} = ie^2 \int \frac{d^3k}{(2\pi)^3} \int \frac{dk^-}{(2\pi)} \times \frac{\mathrm{Tr}[\gamma^\mu \gamma^+ \gamma^\nu \gamma^+] k^-}{4k^+(p^+ - k^+)[k^- - \frac{k_\perp^2 + m^2 - i\epsilon}{2k^+}]} \tag{86}$$

and

$$\Pi_{(2b)}^{\mu\nu} = ie^2 \int \frac{d^3k}{(2\pi)^3} \int \frac{dk^-}{(2\pi)} \times \frac{\mathrm{Tr}[\gamma^\mu \gamma^- \gamma^\nu \gamma^+] k^+}{4k^+(p^+ - k^+)[k^- - \frac{k_\perp^2 + m^2 - i\epsilon}{2k^+}]}. \tag{87}$$

Using Eqs. (A4)–(A9) one obtains

$$\epsilon_\mu^{(\lambda)} \Pi_{(2a)}^{\mu\nu} \epsilon_\nu^{(\lambda)} = 0 \tag{88}$$

and

$$\epsilon_\mu^{(\lambda)} \Pi_{(2b)}^{\mu\nu} \epsilon_\nu^{(\lambda)} = -8ie^2 \int \frac{d^3k}{(2\pi)^3} \int \frac{dk^-}{(2\pi)} \times \frac{1}{4(p^+ - k^+)[k^- - \frac{k_\perp^2 + m^2 - i\epsilon}{2k^+}]}. \tag{89}$$

To deal with the pole at $k^- = \infty$, we make a change of variable, $u = \frac{1}{k^-}$ leading to

$$\epsilon_\mu^{(\lambda)} \Pi_{(2b)}^{\mu\nu} \epsilon_\nu^{(\lambda)} = 8ie^2 \int \frac{d^3k}{(2\pi)^3} \int \frac{du}{2\pi u^2} \times \frac{1}{4(p^+ - k^+)[\frac{1}{u} - \frac{k_\perp^2 + m^2 - i\epsilon}{2k^+}]} \tag{90}$$

$$= 2ie^2 \int \frac{d^3k}{(2\pi)^3} \frac{1}{p^+ - k^+} \int \frac{du}{2\pi u} \frac{1}{[1 - u(\frac{k_\perp^2 + m^2 - i\epsilon}{2k^+})]}. \tag{91}$$

This expression is regularized to

$$\epsilon_\mu^{(\lambda)} \Pi_{(2b)}^{\mu\nu} \epsilon_\nu^{(\lambda)} = ie^2 \int \frac{d^3k}{(2\pi)^3} \frac{1}{p^+ - k^+} \times \int \frac{du}{2\pi} \left[\frac{1}{u + i\delta} \frac{1}{1 - u(\frac{k_\perp^2 + m^2 - i\epsilon}{2k^+})} + \frac{1}{u - i\delta} \frac{1}{1 - u(\frac{k_\perp^2 + m^2 - i\epsilon}{2k^+})} \right]. \tag{92}$$

The first term in this expression has a pole at $u = -i\delta$ which is below the real line and another pole at $u = 2k^+/(k_\perp^2 + m^2 - i\epsilon)$ which is below the real line for $k^+ < 0$ and above the real line for $k^+ > 0$. Similarly, the second term has a pole at $u = +i\delta$ and another at $u = 2k^+/(k_\perp^2 + m^2 - i\epsilon)$. Therefore, we close the contour in the lower half plane in the former case and in the upper half plane in the latter. Performing the k^- integration one obtains

$$\epsilon_\mu^{(\lambda)} \Pi_{(2b)}^{\mu\nu} \epsilon_\nu^{(\lambda)} = e^2 \int \frac{d^2k}{(2\pi)^2} \times \int \frac{dk^+}{2\pi(p^+ - k^+)} \left[\frac{\theta(k^+)}{1 + i\delta(\frac{k_\perp^2 + m^2 - i\epsilon}{2k^+})} - \frac{\theta(-k^+)}{1 - i\delta(\frac{k_\perp^2 + m^2 - i\epsilon}{2k^+})} \right]. \tag{93}$$

Taking the limit $\delta \to 0$, one gets

$$\epsilon_\mu^{(\lambda)} \Pi_2^{\mu\nu} \epsilon_\nu^{(\lambda)} = e^2 \int \frac{d^2k}{(2\pi)^3} \int_0^\infty \left[\frac{dk^+}{(p^+ - k^+)} - \frac{dk^+}{p^+ + k^+} \right] \tag{94}$$

which is the same as the contribution of the instantaneous diagrams to $\delta\mu^2$ in the light front expression Eq. (25).

V. EQUIVALENCE OF COVARIANT AND LIGHT FRONT VERTEX CORRECTION GRAPHS

We will now consider the covariant expression for one loop vertex correction $\Lambda^\mu(p, p', q)$ and show that the + component Λ^+ can be reduced to the sum of corresponding light front diagrams evaluated in the light front time-ordered perturbation theory. The covariant expression for the one loop vertex correction is

$$\Lambda^\mu(p, p', q) = ie^3 \int \frac{d^4k}{(2\pi)^4} \frac{\gamma_\rho(\not{p}' - \not{k} + m)\gamma^\mu(\not{p} - \not{k} + m)\gamma_\nu d'^{\rho\nu}(k)}{[(p-k)^2 - m^2 + i\epsilon][(p'-k)^2 - m^2 + i\epsilon](k^2 + i\epsilon)} \tag{95}$$

which can be rewritten as

$$\Lambda^\mu(p, p', q) = \Lambda_{(a)}^\mu(p, p', q) + \Lambda_{(b)}^\mu(p, p', q) + \Lambda_{(c)}^\mu(p, p', q). \tag{96}$$

Here

$$\Lambda^\mu_{(a)}(p,p',q)+\Lambda^\mu_{(b)}(p,p',q)=ie^3\int\frac{dk^+d^2k_\perp}{(2\pi)^4}\int dk^-\frac{\gamma^\rho(\not p'-\not k+m)\gamma^\mu(\not p-\not k+m)\gamma^\nu d_{\rho\nu}(k)}{2k^+2(p^+-k^+)2(p'^+-k^+)}\times\frac{1}{[k^--\frac{k_\perp^2+\mu^2-i\epsilon}{2k^+}][(p^--k^-)-\frac{(p_\perp-k_\perp)^2+m^2-i\epsilon}{2(p^+-k^+)}]}\frac{1}{[p'^--k^--\frac{(p'_\perp-k_\perp)^2+m^2-i\epsilon}{2(p'^+-k^+)}]}\tag{97}$$

is the contribution of the conventional two term photon propagator and

$$\Lambda^\mu_{(c)}(p,p',q)=-ie^3\int\frac{dk^+d^2k_\perp}{(2\pi)^4}\int dk^-\frac{\gamma^+(\not p'-\not k+m)\gamma^\mu(\not p-\not k+m)\gamma^+}{2(p^+-k^+)2(p'^+-k^+)}\times\frac{1}{(k^+)^2[(p^--k^-)-\frac{(p_\perp-k_\perp)^2+m^2-i\epsilon}{2(p^+-k^+)}][p'^--k^--\frac{(p'_\perp-k_\perp)^2+m^2-i\epsilon}{2(p'^+-k^+)}]}\tag{98}$$

arises from the third term in the modified propagator in the light front gauge. The integrand in $\Lambda^\mu_{(c)}(p,p',q)$ has poles at

(1) $k^-=p^--\{[(p_\perp-k_\perp)^2+m^2-i\epsilon]/2(p^+-k^+)\}$.

(2) $k^-=p'^--\{[(p'^-_\perp-k_\perp)^2+m^2-i\epsilon]/2(p'^+-k^+)\}$.

For $k^+<p'^+$, both the poles are above the real axis and hence the integral is zero for $k^+<p'^+$. Similarly for $k^+>p^+$, both the poles are below the real axis, and hence $\Lambda^\mu_{(c)}(p,p',q)$ is zero in this region also. In the region defined by $p'^+<k^+<p^+$, the pole at $k^-=p^--\{[(p_\perp-k_\perp)^2+m^2-i\epsilon]/2(p^+-k^+)\}$ is above the real axis and the pole at $k^-=p'^--\{[(p'_\perp-k_\perp)^2+m^2-i\epsilon]/2(p'^--k^+)\}$ is below the real axis. Closing the contour in the lower half plane, $\Lambda^\mu_{(c)}(p,p',q)$ reduces to

$$\Lambda^\mu_{(c)}(p,p',q)=e^3\int\frac{d^2k_\perp}{(2\pi)^3}\int_{p'^+}^{p^+}dk^+\frac{\gamma^+(\not p'-\not k+m)\gamma^\mu(\not p-\not k+m)\gamma^+}{(k^+)^2 2(p^+-k^+)2(p'^+-k^+)[p^--p'^-+k''^--k'^-]},\tag{99}$$

where

$$k'^-=\frac{(p_\perp-k_\perp)^2+m^2-i\epsilon}{2(p^+-k^+)}\tag{100}$$

and

$$k''^-=\frac{(p'^-_\perp-k_\perp)^2+m^2-i\epsilon}{2(p'^+-k^+)}.\tag{101}$$

Equation (99) is the same as the expression for $\Lambda^+_{(3c)}$ given by Eq. (36).

Similarly, $\Lambda^\mu_{(a)}(p,p',q)+\Lambda^\mu_{(b)}(p,p',q)$ has poles at

(1) $k^-=(k_\perp^2+m^2-i\epsilon)/2k^+$.

(2) $k^-=p^--\{[(p_\perp-k_\perp)^2+m^2-i\epsilon]/2(p^+-k^+)\}$.

(3) $k^-=p'^--\{[(p'^-_\perp-k_\perp)^2+m^2-i\epsilon]/2(p'^+-k^+)\}$.

For $k^+<0$, all three poles are above the real axis and for $k^+>p^+$, all three are below the real axis. Therefore, the k^- integration yields zero in these two regions. In the region, $p'^+<k^+<p^+$, the second pole is above the real axis and the other two are below; therefore we perform the k^- integration by closing the contour in the upper half plane. In the region, $0<k^+<p'^+$, the first pole is below the real axis and the other two are above. Therefore, we close the contour of integration in the lower half plane . Thus, the two terms in $\Lambda^\mu_{(a)}(p,p',q)+\Lambda^\mu_{(b)}(p,p',q)$ yield after performing the k^- integration,

$$\Lambda^\mu_{(a)}(p,p',q)=-e^3\int\frac{d^2k_\perp}{(2\pi)^3}\int_0^{p'^+}\frac{dk^+}{2k^+2k'^+2k''^+}\frac{\gamma^\rho(\not k''+m)\gamma^\mu(\not k'+m)\gamma^\nu d_{\rho\nu}}{[p^--\frac{(p_\perp-k_\perp)^2+m^2}{2(p^+-k^+)}-\frac{k_\perp^2+\mu^2}{2k^+}]}\frac{1}{[p'^--\frac{k_\perp^2+\mu^2}{2k^+}-\frac{(p'_\perp-k_\perp)^2+m^2}{2(p'^+-k^+)}]}\tag{102}$$

and

$$\Lambda^\mu_{(b)}(p,p',q)=-e^3\int\frac{d^2k_\perp}{(2\pi)^3}\int_{p'^+}^{p^+}\frac{dk^+}{2k^+2k'^+2k''^+}\frac{\gamma^\rho(\not k''+m)\gamma^\mu(\not k'+m)\gamma^\nu d_{\rho\nu}}{[p'^--\frac{(p_\perp-k_\perp)^2+m^2}{2(p^+-k^+)}-\frac{k_\perp^2+\mu^2}{2k^+}]}\frac{1}{[p^--\frac{(p_\perp-k_\perp)^2+m^2}{2(p^+-k^+)}-p'^-+\frac{(p'_\perp-k_\perp)^2+m^2}{2(p'^+-k^+)}]}\tag{103}$$

which are the same as the expressions for Figs. 3(a) and 3(b) given by Eqs. (34) and (35) respectively. Thus, the covariant expression for Λ^+, after performing the k^- integration reduces to a sum of light front expressions for diagrams in Fig. 3, which are obtained within the framework of light front Hamiltonian perturbation theory. The point to be noticed is that the diagram in Fig. 3(c) cannot be generated if we ignore the third term in the modified photon propagator.

VI. SUMMARY AND DISCUSSION

We have considered the three primitively divergent diagrams in QED, i.e., fermion self-energy, vacuum polarization, and vertex correction and have established the equivalence between these and the corresponding diagrams in light front QED. We have shown that by performing the k^- integration the covariant expressions can be reduced to a sum of propagating and instantaneous diagrams of LFQED. The earlier work on the equivalence at the Feynman diagram level deals with theories involving scalars and spin-$\frac{1}{2}$ fields and (3 + 1) QED in the Feynman gauge. The new feature that comes in when one performs the same analysis in the light front gauge is the noncovariant terms in the photon propagator. We find that the photon propagator in the light front gauge that is often used in the light front literature [7], i.e.,

$$D_{\mu\nu} = \frac{1}{k^2 + i\epsilon}\left[-g_{\mu\nu} + \frac{\delta_{\mu+}k_{\nu+} + \delta_{\nu+}k_{\mu}}{k^+}\right] \quad (104)$$

is not enough to prove equivalence. On the other hand, if one uses the modified three term propagator [10,16]

$$D'_{\mu\nu} = \frac{1}{k^2 + i\epsilon}\left[d_{\mu\nu} - \frac{\delta_{\mu+}\delta_{\nu+}}{(k^+)^2}k^2\right] \quad (105)$$

in the covariant expression then the extra term leads to the correct expressions for instantaneous diagrams. However, these diagrams cannot be generated if one naively uses Eq. (104) for the photon propagator. The reason for the ambiguity in the expression for the photon propagator may lie in the fact that in light front field theories all particles are on mass shell and therefore if one directly writes the expression for diagrams using light front Hamiltonian perturbation theory Feynman rules, then it is sufficient to use Eq. (104). However, for the purpose of proving equivalence one has to start from the covariant expression and therefore one has to take into account the third term in the modified propagator. It has been shown in Ref. [6] that in the light front formulation of QED in the light front gauge, the third term in the photon propagator is canceled by a pseudophoton propagator coming from the instantaneous part of the light front Hamiltonian. Our calculation also shows that the third term in the photon propagator is the one that leads to the one loop diagrams involving instantaneous photon exchange, which are necessary to obtain a covariant result in the mixed regularization scheme of Mustaki *et al.* In Ref. [14], it is shown in the context of a tree level calculation of electron-muon scattering in QED that the instantaneous terms in the light front interaction Hamiltonian restore the manifest covariance of the matrix element which is broken by the noncovariant gauge and the noncovariant terms in the gauge propagator. Our calculation thus illustrates further the correspondence between the third term in the propagator and the instantaneous part of the light front Hamiltonian.

Another difficulty that we encountered in the proof of equivalence was the presence of a pole at infinity which can be avoided by putting a cutoff on small values of k^+. However, it is apparent from our analysis that a proper handling of these poles is essential for proving equivalence. In other words, if we try to avoid the pole at infinity by introducing a cutoff $k^+ > \alpha$ we cannot prove the equivalence of covariant and light front expressions because the light front diagrams involving instantaneous interactions cannot be generated if we ignore the zero modes. Therefore, in order to obtain the light front expressions by performing k^- integration on covariant expressions, one should put a cutoff on small k^+ *only* after performing the k^- integration. In earlier work on equivalence in the context of the Yukawa theory [2] the form of infrared regulator $\alpha(k^+)$ is determined by requiring the equivalence to the covariant calculation, Here we show that one does not need to make a particular choice of α to prove equivalence.

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APPENDIX

1. Basics

We define the light front coordinates by

$$x^+ = \frac{x^0 + x^3}{\sqrt{2}}, \quad (A1)$$

$$x^- = \frac{x^0 - x^3}{\sqrt{2}}, \quad (A2)$$

$$x_\perp = (x^1, x^2). \quad (A3)$$

The metric tensor is given by

$$g^{\mu\nu} = \begin{pmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}.$$

Dirac matrices satisfy the following properties:

$$(\gamma^+)^2 = (\gamma^-)^2 = 0, \quad (A4)$$

$$\{\gamma^\mu, \gamma^\nu\} = 2g^{\mu\nu}, \quad (A5)$$

$$(\gamma^0)^+ = \gamma^0, \quad (A6)$$

$$(\gamma^k)^\dagger = -\gamma^k (k = 1, 2, 3), \quad (A7)$$

$$\gamma^+\gamma^-\gamma^+ = 2\gamma^+, \quad \text{(A8)}$$

$$\gamma^-\gamma^+\gamma^- = 2\gamma^-, \quad \text{(A9)}$$

also,

$$\gamma^\alpha\gamma^\beta d_{\alpha\beta}(p) = -2, \quad \text{(A10)}$$

$$\gamma^\alpha\gamma^\nu\gamma^\beta d_{\alpha\beta}(p) = \frac{2}{p^+}(\gamma^+\gamma^\nu + g^{+\nu}\not{p}). \quad \text{(A11)}$$

Dirac spinors satisfy

$$\bar{u}(p,s)u(p,s') = -\bar{v}(p,s)v(p,s) = 2m\delta_{s,s'}, \quad \text{(A12)}$$

$$\bar{u}(p,s)\gamma^\mu u(p,s') = \bar{v}(p,s)\gamma^\mu v(p,s) = 2p^\mu\delta_{s,s'}. \quad \text{(A13)}$$

2. Light front Hamiltonian

P^-, the light front Hamiltonian, is the operator conjugate to the "time" evolution variable x^+ and is given by

$$P^- = H_0 + V_1 + V_2 + V_3, \quad \text{(A14)}$$

where H_0 is the free Hamiltonian, V_1 is the standard, order-e three-point interaction,

$$V_1 = e\int d^2x_\perp dx^- \bar{\xi}\gamma^\mu\xi a_\mu. \quad \text{(A15)}$$

V_2 is an order-e^2 nonlocal effective four-point vertex corresponding to an instantaneous fermion exchange,

$$V_2 = -\frac{i}{4}e^2\int d^2x_\perp dx^- dy^- \epsilon(x^- - y^-)(\bar{\xi}a_k\gamma^k) \times (x)\gamma^+(a_j\gamma^j\xi)(y) \quad \text{(A16)}$$

and V_3 is an order-e^2 nonlocal effective four-point vertex corresponding to an instantaneous photon exchange,

$$V_3 = -\frac{e^2}{4}\int d^2x_\perp dx^- dy^- (\xi\gamma^+\xi)(x)|x^- - y^-|(\xi\gamma^+\xi)(y). \quad \text{(A17)}$$

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