Neutrino mass limit from galaxy cluster number density evolution

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Measurements of the evolution with redshift of the number density of massive galaxy clusters are used to constrain the energy density of massive neutrinos and so the sum of neutrino masses $\sum m_\nu$. We consider a spatially flat cosmological model with cosmological constant, cold dark matter, baryonic matter, and massive neutrinos. Accounting for the uncertainties in the measurements of the relevant cosmological parameters we obtain a limit of $\sum m_\nu < 2.4$ eV (95% C.L.).

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Constraints on neutrino masses are of great interest for particle physics as well as for cosmology, and thus attract a lot of scientific attention (for recent reviews see Refs. [1–4]). Current upper limits on the sum of neutrino masses, $\sum m_\nu$, from cosmological structure formation data [5,6], cosmic microwave background (CMB) fluctuation data [7,8], or combined CMB + large-scale structure data [9–13], are of order an eV [14] (for various limits see Table 1 of Ref. [4]). The number of neutrino species can be constrained from big bang nucleosynthesis or by using CMB and large-scale structure data [11,15].

High energy physics experiments also constrain neutrino masses, and have measured the number of light neutrino species with high precision [3]. Direct searches for neutrino mass effects in beta decays yield limits in the region of several eV, but the sum over all neutrino masses is almost unconstrained by beta decay and other experiments, mainly due to the weak limit on the tau-neutrino mass. The measurement of neutrino oscillations, on the other hand, constrains the differences between the squared masses of the neutrino mass eigenstates Δm^2. With the justified assumption that neutrino masses are non-negative and for mass splittings $\Delta m^2_\odot \approx 7 \times 10^{-5}$ eV^2 and $\Delta m^2_{\rm atm} > 1.3 \times 10^{-3}$ eV^2 [3], we obtain $\sum m_\nu > 0.04$ eV if the solar mass splitting is between the highest and second highest mass eigenstates ($\sum m_\nu > 0.07$ eV if the atmospheric mass splitting is between the two highest states). Results from LSND Collaboration yield a larger lower limit on $\sum m_\nu$, and must be considered if confirmed by the MiniBooNE experiment [16], which is currently taking data.

In this paper we use the dependence of galaxy cluster number density evolution on the massive neutrino energy density parameter Ω_ν to set a limit on $\sum m_\nu$. We consider the standard spatially flat ΛCDM Friedmann–Lemaître-Robertson-Walker cosmological spacetime model with baryons, cold dark matter (CDM), massive neutrinos, and a nonzero cosmological constant Λ (for a recent review see Ref. [17]). To compute the cluster number density as a function of redshift z we use the Press-Schechter approach [18,19] as modified by Sheth and Tormen (ST) [20].[1] This approach makes use of the mass function $N(M > M_0)$ of clusters (cluster number density as a function of cluster mass M greater than a fiducial mass M_0), which depends on cosmological model parameters [21–24]. In particular, it is very sensitive to the matter density parameter Ω_M ($= \Omega_b + \Omega_{\rm cdm} + \Omega_\nu$, where Ω_b and $\Omega_{\rm cdm}$ are the density parameters of baryons and CDM, respectively) and the value of σ_8 [the root mean square (r.m.s.) amplitude of density fluctuations smoothed over a sphere of $8h^{-1}$ Mpc radius, where h is the Hubble constant in units of 100 km s^{-1}Mpc^{-1}] [25]. The observationally viable ranges of these two parameters are related (for recent reviews see Refs. [12,26,27]), and a current version of the relation between Ω_M and σ_8 is given in Table 5 of Ref. [26]. The parameter σ_8 is determined by the matter fluctuation power spectrum which is sensitive to Ω_ν [2,6]. This is because the neutrinos are light particles and have a much larger free-streaming path length[2] than the CDM particles. Gravitational instability is therefore unable to confine the neutrinos on small and intermediate length scales, resulting in a suppression of small- and intermediate-scale power. See Fig. 6 of Ref. [4] for σ_8 as a function of Ω_ν for models normalized to the Wilkinson Microwave Anisotropy Probe (WMAP) data.

Neutrinos are weakly interacting and this characterizes how they affect cosmology. When $m_\nu > 10^{-3}$ eV neutrinos are nonrelativistic today [11] and thus behave like a hot component of dark matter. The presence of even a

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[1]In the following we use PS to refer to the unmodified Press-Schechter approach.

[2]For ultrarelativistic particles the free-streaming path length is equal to the Hubble radius. After they become nonrelativistic, particle velocities are redshifted away adiabatically and so the free-streaming path length grows only slowly or decreases [11].

small fraction of massive neutrinos (hot dark matter) $f_\nu \equiv \Omega_\nu/\Omega_M$, of order 10%–20%, requires a smaller value for the cosmological constant in comparison to a pure ΛCDM model, while other cosmological parameters are largely unaffected [6]. This is because a smaller Ω_Λ results in a larger Ω_M and hence a faster fluctuation growth rate, which compensates for the reduction of small- and intermediate-scale power caused by the neutrinos. Neutrino free-streaming suppression of the linear[3] growth of density perturbations on small and intermediate scales results in only a fraction of matter of order $1 - f_\nu$ being involved in gravitational clustering [29]. Being an integral over the power spectrum, σ_8 depends sensitively on Ω_ν. This makes the cluster number density evolution with redshift very sensitive to the value of Ω_ν [30,31], and so to the value of $\sum m_\nu$ (since $\sum m_\nu = 94\Omega_\nu h^2$ eV [3]).

As with other cosmological tests, the cluster number density evolution test for neutrino masses requires fixing the range of some cosmological parameters. This may be viewed as a choice of priors; see Ref. [6] for a detailed discussion of priors in the context of deriving neutrino mass limits from cosmological data. The parameter ranges we consider in this computation are picked as follows. Based on Hubble Space Telescope (HST) measurements of the Hubble constant [32], we use $h = 0.71 \pm 0.07$ (1 standard deviation limit).[4] We assume adiabatic density perturbations with a primordial power spectral index close to scale-invariant $n = 0.98 \pm 0.02$ (1-σ range) [12,26]. Concerning the value of the matter density parameter Ω_M, there is evidence for $\Omega_M \in (0.2, 0.35)$ from different data such as Type Ia supernovae [34], WMAP [35], Sloan Digital Sky Survey (SDSS) and 2 degree Field Galaxy Redshift Survey (2dFGRS) galaxy clustering [26,27], and galaxy cluster gas mass fraction evolution [36]. For a summary see Ref. [37], who find $0.2 \lesssim \Omega_M \lesssim 0.35$ at 2 standard deviations; in our computation we use this as a 1 standard deviation range. We choose $\sigma_8 \in (0.77, 1.11)$ as the 2 standard deviation range; for a discussion see Sec. 3.1 of Ref. [38]. For the baryon density parameter we use from big bang nucleosynthesis studies $\Omega_b h^2 \in (0.018, 0.022)$.[5] We work in a spatially flat model which requires $1 - \Omega_\Lambda = \Omega_b + \Omega_{\rm cdm} + \Omega_\nu = \Omega_M$. The prior on Ω_b is less important than the priors on σ_8, Ω_M, h, or n [6].

The paucity of galaxy cluster evolution data makes it inappropriate to use data analysis techniques based on χ^2 fits. Reviewing such data-poor situations in the literature one finds either modified χ^2 fits assuming gaussian errors on the logarithm of the observed cluster number density or likelihood approaches with Poisson errors based on the number of observed clusters. In our analysis we use a likelihood approach and we define our likelihood function by $\mathcal{L} = \prod_i e^{-\mu_i}\mu_i^{k_i}/k_i!$. Here k_i is the number of observed clusters with mass greater than M_0, in the ith redshift bin (centered at redshift z_i and of width Δz) and $\mu_i(M > M_0, z_i, \Delta z) = \int_{z_i}^{z_i+\Delta z} N_{\rm pred}(M > M_0, z)dz/(\Delta z \alpha_i)$ is the predicted cluster number in this bin. $N_{\rm pred}(M > M_0, z)$ is the predicted cluster number density. The normalization factor α_i, which has dimensions of inverse volume, is the detection efficiency defined by $N_{\rm obs}(M > M_0, z_i) = \alpha k_i$ where $N_{\rm obs}(M > M_0, z_i)$ is the observed cluster number density in the ith redshift bin.

The predicted cluster number density $N_{\rm pred}(M > M_0, z)$ depends on the cosmological model considered. An important characteristic of a cosmological model is the linear energy density perturbation power spectrum $P(k, z)$. This is sensitive to the values of the cosmological parameters h, Ω_M, and Ω_b, as well as to the density parameter of each dark matter component, i.e., $\Omega_{\rm cdm}$ and Ω_ν (although the requirement of flat spatial hyperspaces can be used to eliminate the dependence on $\Omega_{\rm cdm}$), and to additional parameters discussed below. This wavenumber-space two-point correlation function of density perturbations, $P(k, z)$, at given redshift z, is determined by the power spectrum of initial perturbations $P_0(k)$, the energy density perturbation transfer function $T(k, z)$, and the perturbation growth rate $D(z)$. The functions $T(k, z)$ and $D(z)$ depend on model parameters and describe the evolution of density inhomogeneities [39]. To compute $P(k, z)$ we assume a close to scale-invariant (Harrison-Peebles-Yu-Zeldovich) post-inflation energy density perturbation power spectrum $P_0(k) \propto k^n$, with $n \sim 1$, and we use a semianalytical approximation for the transfer function $T(k, z)$ in the ΛCDM model with three species of equal-mass neutrinos [40].[6] For the growth rate $D(z)$ we use Eqs. (2)–(4) of Ref. [31], which are based on results from Refs. [41]. In summary, in our model the free parameters are n, h, Ω_M, $\sum m_\nu$ (or Ω_ν), and σ_8 (which fixes the normalization of the power spectrum, as discussed below).

The cluster mass function at redshift z is $N(M > M_0, z) = \int_{M_0}^{\infty} dM n(M, z)$, where $n(M, z)dM$ is the comoving number density of collapsed objects with mass lying in the interval $(M, M + dM)$. In the PS approach the cluster

[3]The effect of massive neutrino infall into CDM halos is studied in Ref. [28]. They found that having three degenerate-mass neutrinos with $\sum m_\nu \sim 2.7$ eV alters the nonlinear matter power spectrum by about 1%.

[4]An analysis of all available measurements of the Hubble constant results in the more restrictive, but HST consistent, estimate $h = 0.68 \pm 0.07$ (2 standard deviation range) [33].

[5]This is more consistent with estimates from WMAP data and from big bang nucleosynthesis using the mean of the primordial deuterium abundance measurements, but it is significantly larger than an estimate based on helium and lithium abundance measurements; see, e.g., Ref. [17].

[6]The effects of neutrino mass differences are irrelevant for our considerations, since if $\sum m_\nu > 0.4$ eV the mass eigenvalues are essentially degenerate [4], while if $\sum m_\nu < 0.4$ eV the mass differences do not much affect cluster number density evolution [11].

mass function is determined by $\sigma(R, z)$, the r.m.s. amplitude of density fluctuations smoothed over a sphere of radius $R = (3M/4\pi\rho_M)^{1/3}$, where ρ_M is the mean matter density [18]. The function $n(M, z)$ is a universal function of the peak height $\delta_C/\sigma(R)$, where $\delta_C = 1.686$. For gaussian fluctuations,

$$n(M, z) \propto \frac{\delta_C}{\sigma^2(R, z)} \left| \frac{d\sigma(R, z)}{dM} \right| \exp\left[-\frac{\delta_C^2}{2\sigma^2(R, z)} \right]; \quad (1)$$

see, e.g., Eq. (1) of Ref. [19]. The evolution of the cluster mass function is determined by the z dependence of $\sigma(R, z)$. Now $\sigma^2(R, z)$ is related to the power spectrum $P(k, z)$ through

$$\sigma^2(R, z) = \frac{1}{2\pi^2} \int_0^\infty P(k, z)|W(kR)|^2 k^2 dk, \quad (2)$$

where $W(kR)$ is the Fourier transform of the top-hat window function, $W(x) = 3(\sin x - x\cos x)/x^3$. Numerical computation results for $n(M, z)$ are not accurately fit by the PS expression of Eq. (1); see Refs. [20,42,43]. Several more accurate modifications of $n(M, z)$ have been proposed; see Refs. [20,42,44]. Here we use the ST modification [20], as defined in Eq. (5) of Ref. [24],

$$n(M, z) \propto \frac{\delta_C}{\sigma^2(R, z)} \left| \frac{d\sigma(R, z)}{dM} \right| \left[1 + \left(\frac{a\delta_C^2}{\sigma^2(R, z)} \right)^{-p} \right] \times \left(\frac{a\delta_C^2}{\sigma^2(R, z)} \right)^{-1/2} \exp\left[-\frac{a\delta_C^2}{2\sigma^2(R, z)} \right], \quad (3)$$

where the parameters $a = 0.303$ and $p = 0.707$ are fixed by fitting to the numerical results (for the PS case $a = 1$ and $p = 0$) [20,24]. With this choice of parameter values the mass of collapsed objects in Eq. (3) must be defined using a fixed over-density contrast with respect to the background density ρ_M [23–25,42], and this requires accounting for the mass conversion between M_{180b} and M_{200c} [23,24].[7] Such a conversion depends on cosmological parameters (see Fig. 1 of Ref. [23]); we use an analytical extrapolation of this figure to do the conversion for $\Omega_M \in (0.2, 0.35)$.

Our analysis is based on data from a compilation of massive clusters (with $M > M_0 = 8 \times 10^{14} h^{-1} M_\odot$, where

[7]The mass of a collapsed object is defined with respect to the Einstein-de Sitter critical density $\rho_{\rm cr} = 3H^2/(8\pi G)$, as the mass within a radius $R_{\Delta c}$, inside of which the mean interior density is Δ times the critical density $\rho_{\rm cr}$. Assuming a $\rho(R)$ density profile, $\int_0^{R_{\Delta c}} \rho(R) R^2 dR = \Delta\rho_{cr} R_{\Delta c}^3/3$. M_{200c}, which corresponds to an over-density $\Delta = 200$ with respect to the critical density, is a common definition of the virial mass of the cluster [21,24], while M_{180b} corresponds to the mass within a sphere of radius R_{180b} inside which the mean density is 180 times the background density ρ_M ($R_{180b} = R_{54c}$ for $\Omega_M = 0.3$) [24]. For details of mass conversion assuming a Navarro-Frenk-White density profile [21] see [24].

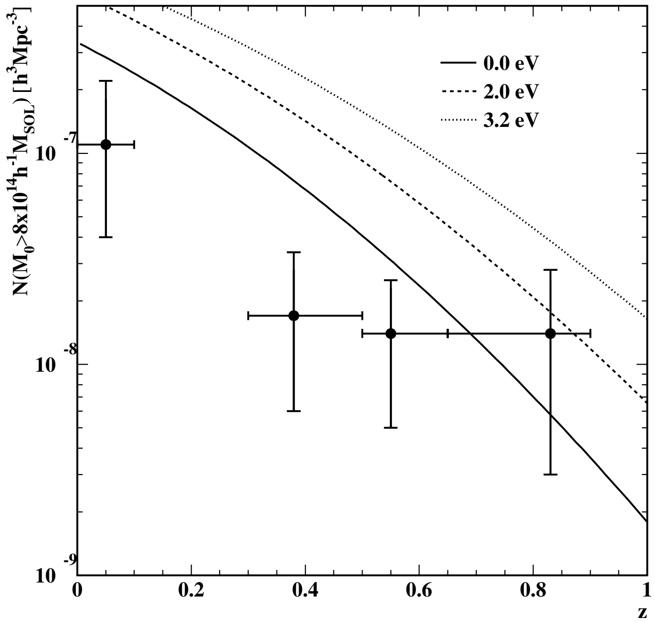

FIG. 1. The curves show the number density evolution of massive clusters ($M > 8 \times 10^{14} h^{-1} M_\odot$) for models in which the sum of neutrino masses $\sum m_\nu = 0$, 2, 3.2 eV (from bottom to top). The other five parameters Ω_M, Ω_b, σ_8, n, and h are set at the center of the scan interval. The crosses show the observational data of Eq. (4) in four redshifts bins with 1-σ Poisson error bars.

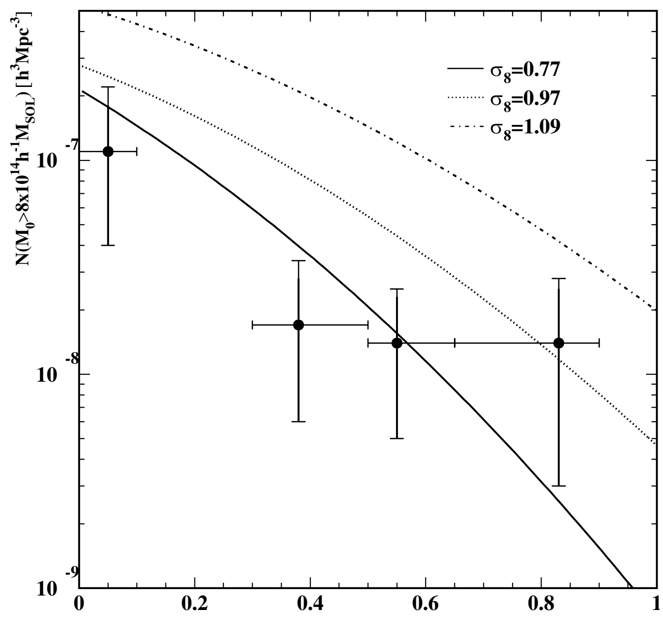

FIG. 2. Variation of cluster number density evolution $N(M > 8 \times 10^{14} h^{-1} M_\odot, z)$ as a function of σ_8 with parameters kept at the same values as in Fig. 1 except for σ_8 which is varied and $\sum m_\nu$ which is kept at zero. The observational data are the same as those shown in Fig. 1.

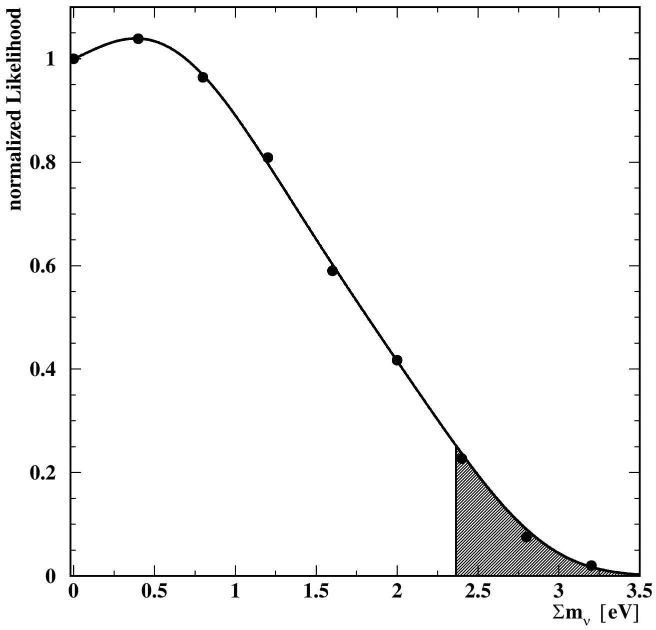

FIG. 3. The marginalized (integrated) likelihood $\mathcal{L}/\mathcal{L}_{\max}$ as a function of $\sum m_\nu$, accounting for the observational uncertainties in Ω_M, Ω_b, σ_8, n, and h. The shaded region is excluded at 95% confidence level. The likelihood function has been scaled for display purposes to a value of one at $\sum m_\nu = 0$.

M_0 here corresponds to the mass within a comoving radius of $1.5h^{-1}$ Mpc and $M_\odot$ is the solar mass) observed at redshifts up to $z \approx 0.8$, [19,25],

$$N(M > 8 \times 10^{14} h^{-1} M_\odot) h^{-3}\ \mathrm{Mpc}^3 = \begin{cases} 1.1^{+1.1}_{-0.7} \times 10^{-7}, & \text{at} \quad z = 0.0 - 0.1, \\ 1.7^{+1.7}_{-1.1} \times 10^{-8}, & \text{at} \quad z = 0.3 - 0.5, \\ 1.4^{+1.1}_{-0.9} \times 10^{-8}, & \text{at} \quad z = 0.5 - 0.65, \\ 1.4^{+1.4}_{-1.1} \times 10^{-8}, & \text{at} \quad z = 0.65 - 0.9. \end{cases} \tag{4}$$

Here the errors in N are from counting uncertainty and are automatically included in our Poisson likelihood analysis. The bin width in z is chosen to reflect the uncertainty in the redshift measurement.

We compute the likelihood from the data using Poisson errors and the predicted number of clusters in each bin, and perform a maximum likelihood fit over a discretized parameter space. We compare the observed cluster number density evolution of massive clusters with $M > 8 \times 10^{14} h^{-1} M_\odot$ to model predictions for different values of $\sum m_\nu$ (see Fig. 1 for some examples) and for each value of $\sum m_\nu$ we marginalize the likelihood by integrating over the parameter space (Ω_M, Ω_b, h, n, σ_8) with Gaussian weighting. Figure 2 shows the dependence of cluster number density evolution on σ_8. Here $\sum m_\nu = 0$ and the other four parameters (Ω_M, Ω_b, h, n) are set at the center of the scan interval.

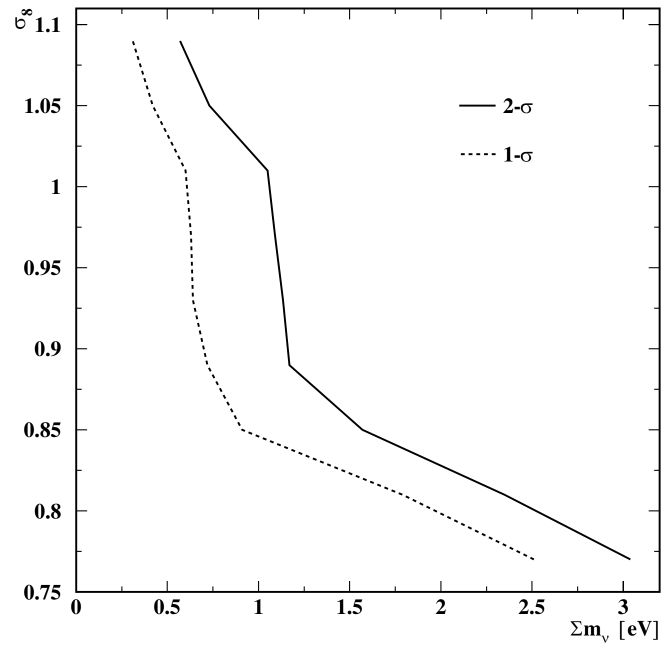

FIG. 4. The curves show the two-dimensional likelihood in $(\sigma_8, \sum m_\nu)$. The continuous line is the 2-σ contour and the dashed line is the 1-σ contour. Lower values of σ_8 favor a higher neutrino mass; for example, at $\sigma_8 = 0.77$ the most likely neutrino mass value is $\sum m_\nu = 1.3$ eV (also see Ref. [45,46]) whereas at high σ_8 values only upper limits on $\sum m_\nu$ apply.

The likelihood as a function of $\sum m_\nu$ is shown in Fig. 3. Low values of $\sum m_\nu$ are favored but we find no preference for a nonzero value. We obtain an upper limit of $\sum m_\nu < 2.37$ eV (95% C.L.), accounting for the uncertainties in all five cosmological parameters Ω_M, n, h, σ_8, and Ω_b. The cluster number density evolution depends significantly on σ_8 (see Fig. 2). The two-dimensional likelihood in $(\sigma_8, \sum m_\nu)$ is shown in Fig. 4. Lower values of σ_8 favor a higher neutrino mass.

Our result of $\sum m_\nu < 2.4$ eV, which is based on the cluster number density evolution, is in good agreement with bounds on neutrino masses from CMB (and other) measurements [4,8] and corroborates that evidence. Our limit indicates that effects from the neutrinos on the evolution of galaxy clusters cannot be excluded and also indicates that these effects should be taken into account in the determination of cosmological parameters. Based on our result and on the particle physics limit of $\sum m_\nu > 0.04$ eV, we find Ω_ν is in the range of 0.1% to 5%.

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