\aleph_0 hypergravity in three dimensions

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(Received 11 April 2005; published 7 June 2005)

We construct hypergravity theory in three-dimensions with the gravitino $\psi_{\mu m_1 \cdots m_n}{}^A$ with an arbitrary half-integral spin n + 3/2, carrying also the index A for certain real representations of any gauge group G. The possible real representations are restricted by the condition that the matrix representation of all the generators are antisymmetric: $(T^I)^{AB} = -(T^I)^{BA}$. Since such a real representation can be arbitrarily large, this implies \aleph_0 hypergravity with infinitely many (\aleph_0) extended local hypersymmetries.

DOI: 10.1103/PhysRevD.71.125002

PACS numbers: 04.65.+e, 02.40.Pc, 11.15.-q, 11.30.Pb

I. INTRODUCTION

It is well known that a graviton in three-dimensions (3D) has zero physical degree of freedom. This is because the conventional counting for a symmetric traceless tensor for transverse components in 3D gives zero: $(3-2) \times (4-2)/2 - 1 = 0$. Similarly, the gravitino also has no physical degree of freedom: $(3-3) \times 2 = 0$. Therefore, the multiplet of supergravity has 0 + 0 physical degrees of freedom.

This fact leads to the interesting concept of "hypergravity" [1] in 3D with a gravitino with spin 5/2 or higher [2]. In 4D, on the other hand, it is difficult to formulate consistent hypergravity, due to the problem with the free indices of a gravitino field equation whose divergences do not vanish on nontrivial backgrounds (Velo-Zwanziger disease) [3].

In contrast, this situation is drastically improved in 3D due to the zero physical degree of freedom of the gravitino. The first hypergravity theory in 3D was given in [2], where it was shown that consistent hypergravity indeed exists with the gravitino $\psi_{\mu m_1 \cdots m_n}$ carrying spin n + 3/2, where the indices $m_1 \cdots m_n$ are totally symmetric γ -traceless Lorentz indices. Interacting models of higher spin gauge fields with extended hypersymmetry in anti-de Sitter 3D also have been developed [4].

Independent of this, there has been a different development about 3D physics related to Chern-Simons theories [5–8]. It had been known for some time that arbitrarily many (\aleph_0) supersymmetrics [9–12] can be accommodated in supersymmetric Chern-Simons theory. Typical examples are $OSp(p|2; \mathbb{R}) \times OSp(q|2; \mathbb{R})$ [9] or SO(N) [10] which can be arbitrarily large. In our recent paper [13] we have shown that the gravitino, coupling to locally supersymmetric Chern-Simons theory in 3D, can be in the adjoint representation of any gauge group G, with the relationship $N = \dim G$,

instead of limited groups, such as *OSp*- or *SO*-type gauge groups [9,10].

In this paper, we combine these two developments in 3D. Namely, we generalize massless hypergravity [2] further to a system, where the gravitino $\psi_{mm_1\cdots m_n}^{A}$ carries the real representation index A of an arbitrary gauge group G, such that the matrix representations of all the generators are antisymmetric. Since such a representation can be arbitrarily large, this means that we have " \aleph_0 hypergravity" with infinitely many extended hypersymmetries. In the next section, we first will give the action I_0 which is a generalization of our previous system [13] to the new system where the gravitino has the index structures $\psi_{m_1m_2\cdots m_n}^A$. We next consider additional Chern-Simons actions $I_{R\omega}$ and I_{GB} , where $I_{R\omega}$ is a Lorentz Chern-Simons term, while I_{GB} is a Chern-Simons term starting as $G \wedge B + \dots$ with the field strength $G = dB + B \wedge B$ and its vector potential B. Fortunately, the invariance of the total action $I_{\text{tot}} \equiv I_0 + I_{R\omega} + I_{GB}$ under hypersymmetry is confirmed under a slight modification of the hypersymmetry transformation rule.

II. TOTAL LAGRANGIAN AND ×₀ HYPERSYMMETRIES

Our field content of the system is $(e_{\mu}{}^{m}, \psi_{m_{1}\cdots m_{n}}{}^{A}, A_{\mu}{}^{I}, B_{\mu}{}^{I}, C_{\mu}{}^{I}, \lambda_{m_{1}\cdots m_{n}}{}^{A})$ which is similar to [11,13]. The fields $A_{\mu}{}^{I}, B_{\mu}{}^{I}$, and $C_{\mu}{}^{I}$ are all vector fields in the adjoint representation. As in the cases of [11,13], the *B* field is coupling minimally to the gravitino, *A* field forms a Chern-Simons term starting with $F \wedge A$, the *C* field is needed for the vanishing field strength G = dB. For the consistency of coupling, G = 0 is required, necessitating the $C \wedge G$ term in the Lagrangian. The structure of the system is parallel to [11,13], but their meaning will be clearer, when our Lagrangian is fixed by (4) later.

Our most important ingredient here, however, is the representation of the gravitino ψ and the gaugino λ . The indices $m_1 \cdots m_n$ are totally symmetric Lorentz indices carrying spin n + 3/2 as in [2], and the gravitino has

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the additional constraint of γ tracelessness as in [2]:

$$\gamma^{m_1}\psi_{\mu m_1 m_2 \cdots m_n}{}^A = 0. \tag{1}$$

The superscript A is a "collective" index for any real representation of an arbitrary gauge group G, satisfying the condition

$$(T^{I})^{AB} = -(T^{I})^{BA}$$
 $(I = 1, 2, \cdots, \dim G)$ (2)

for the generators T^{I} of G. Typical examples of representations satisfying (2) are

- (i) Vectorial representation of SO(M): $A \equiv a, B \equiv b, (T^I)^{AB} = (T^I)^{ab} = -(T^I)^{ba}$.
- (ii) Adjoint representation of $\forall G: A \equiv J, B \equiv K, (T^I)^{AB} = -f^{IJK}$.
- (iii) Totally symmetric *m* vectorial indices of SO(M): $A \equiv (a_1 \cdots a_m), B \equiv (b_1 \cdots b_m),$

$$(T^{I})^{a_{1}\cdots a_{m},b_{1}\cdots b_{m}} = + m(T^{I})^{(a_{1}|}{}_{(b_{1}|}\delta_{|b_{2}|}^{|a_{2}|}\cdots \delta_{|b_{m}|}^{|a_{m}|}$$
$$= -(T^{I})^{b_{1}\cdots b_{m},a_{1}\cdots a_{m}}.$$
(3)

Note that the indices A, B are collective indices in the case (iii). This case is analogous to the totally symmetric Lorentz indices $m_1 \cdots m_n$ on the gravitino [2].

We propose an action $I_0 \equiv \int d^3x \mathcal{L}_0$ with the Lagrangian

$$\mathcal{L}_{0} = -\frac{1}{4}eR(\omega) + \frac{1}{2}\epsilon^{\mu\nu\rho}(\overline{\psi}_{\mu(n)}{}^{A}D_{\nu}(\omega, B)\psi_{\rho}{}^{(n)A}) + \frac{1}{2}g\epsilon^{\mu\nu\rho}C_{\mu}{}^{I}G_{\nu\rho}{}^{I} + \frac{1}{2}gh\epsilon^{\mu\nu\rho}(F_{\mu\nu}{}^{I}A_{\rho}{}^{I}) - \frac{1}{3}gf^{IJK}A_{\mu}{}^{I}A_{\nu}{}^{J}A_{\rho}{}^{K}) + \frac{1}{2}ghe(\overline{\lambda}_{(n)}{}^{A}\lambda^{(n)A}), \quad (4)$$

where the suffix (n) stands for the totally symmetric nLorentz indices $m_1 \cdots m_n$ in order to save space. The gand h are real coupling constants but are related to each other for certain gauge groups, as will be seen shortly in (25). The third *CG* term is a kind of *BF* term used in topological field theory [14], while the last line is a hypersymmetric Chern-Simons term with the gaugino mass term [10]. In the case when the suffix A needs a nontrivial metric, we have to distinguish the superscript and subscript [15].

The covariant derivative on the gravitino has also a minimal coupling to the *B* field:

$$\mathcal{R}_{\mu\nu r_{1}\cdots r_{n}}{}^{A} \equiv D_{\mu}(\omega, B)\psi_{\nu r_{1}\cdots r_{n}}{}^{A} - D_{\nu}(\omega, B)\psi_{\mu r_{1}\cdots r_{n}}{}^{A}$$
$$\equiv \left[+\partial_{\mu}\psi_{\nu r_{1}\cdots r_{n}}{}^{A} + \frac{1}{4}\omega_{\mu}{}^{tu}\gamma_{tu}\psi_{\nu r_{1}\cdots r_{n}}{}^{A} + n\omega_{\mu(r_{1})}{}^{s}\psi_{s|r_{2}\cdots r_{n}}{}^{A} + gB_{\mu}{}^{I}(T^{I}){}^{AB}\psi_{\nu r_{1}\cdots r_{n}}{}^{B} \right]$$
$$- (\mu \leftrightarrow \nu), \tag{5}$$

in addition to the nontrivial Lorentz connection term as in [2]. The field strengths F, G and H are defined by

$$F_{\mu\nu}{}^{I} \equiv \partial_{\mu}A_{\nu}{}^{I} - \partial_{\nu}A_{\mu}{}^{I} + gf^{IJK}A_{\mu}{}^{J}A_{\nu}{}^{K},$$

$$G_{\mu\nu}{}^{I} \equiv \partial_{\mu}B_{\nu}{}^{I} - \partial_{\nu}B_{\mu}{}^{I} + gf^{IJK}B_{\mu}{}^{J}B_{\nu}{}^{K},$$

$$H_{\mu\nu}{}^{I} \equiv \partial_{\mu}C_{\nu}{}^{I} - \partial_{\nu}C_{\mu}{}^{I} + 2gf^{IJK}B_{[\mu}{}^{J}C_{\nu]}{}^{K}$$

$$\equiv D_{\mu}C_{\nu}{}^{I} - D_{\nu}C_{\mu}{}^{I}.$$
(6)

As is clear from the Lagrangian (4) and field strengths (5) and (6) we need the B field for a nontrivial minimal gauge coupling to the gravitino. On the other hand, the $C \wedge$ G term in (4) is necessary, in order to yield the field equation $G \doteq 0$ [16] for consistency for gravitino field equation, avoiding Velo-Zwanziger disease [3]. The presence of the constant h in the $F \wedge A$ Chern-Simons term shows that this Chern-Simons term can be set to zero by taking the $h \rightarrow 0$ limit. For these reasons, the two vectors B and C are indispensable, while A field can be truncated. Even though all of these three vectors carry the adjoint index I, only A and B transform as a gauge field for the group G, while the C field is not a gauge field but is simply in the adjoint representation, as is reflected in their field strengths (6). These points are completely parallel to the earlier works in [11,13].

We adopt the 1.5-order formalism, so that the Lorentz connection ω in the Lagrangian is regarded as an independent variable, yielding its field equation

$$\omega_{mrs} \doteq \hat{\omega}_{mrs} \equiv \frac{1}{2} (\hat{C}_{mrs} - \hat{C}_{msr} + \hat{C}_{srm}),$$

$$\hat{C}_{\mu\nu}{}^{m} \equiv \partial_{\mu} e_{\nu}{}^{m} - \partial_{\nu} e_{\mu}{}^{m} - (2n+1) (\overline{\psi}_{\mu(n)}{}^{A} \gamma^{m} \psi_{\nu}{}^{(n)A}).$$
(7)

Or equivalently, the torsion tensor is

$$T_{\mu\nu}{}^{m} \doteq + (2n+1)(\overline{\psi}_{\mu(n)}{}^{A}\gamma^{m}\psi_{\nu}{}^{(n)A}), \tag{8}$$

with the dependence on n, in agreement with [2]. Our action I_0 is invariant under hypersymmetry א₀ HYPERGRAVITY IN THREE DIMENSIONS

 $\delta_{\Omega} e_{\mu}{}^{m} = +(2n+1)(\overline{\epsilon}_{(n)}{}^{A}\gamma^{m}\psi^{(n)A}),$

$$\delta_{Q}\psi_{\mu m_{1}\cdots m_{n}}{}^{A} = +\partial_{\mu}\epsilon_{m_{1}\cdots m_{n}}{}^{A} + \frac{1}{4}\hat{\omega}_{\mu}{}^{rs}\gamma_{rs}\epsilon_{m_{1}\cdots m_{n}}{}^{A} + n\hat{\omega}_{\mu(m_{1}]}{}^{t}\epsilon_{l|m_{2}\cdots m_{n}}{}^{A} + g(T^{I}){}^{AB}B_{\mu}{}^{I}\epsilon_{m_{1}\cdots m_{n}}{}^{B}$$

$$- g(T^{I}){}^{AB}\gamma^{\nu}\epsilon_{m_{1}\cdots m_{n}}{}^{B}\hat{H}_{\mu\nu}{}^{I} \equiv +D_{\mu}(\hat{\omega}, B)\epsilon_{m_{1}\cdots m_{n}}{}^{A} - g(T^{I}){}^{AB}\gamma^{\nu}\epsilon_{m_{1}\cdots m_{n}}{}^{B}\hat{H}_{\mu\nu}{}^{I},$$

$$\delta_{Q}A_{\mu}{}^{I} = -(T^{I}){}^{AB}(\overline{\epsilon}_{(n)}{}^{A}\gamma_{\mu}\lambda^{(n)B}), \qquad \delta_{Q}B_{\mu}{}^{I} = -(T^{I}){}^{AB}(\overline{\epsilon}_{(n)}{}^{A}\gamma^{\nu}\mathcal{R}_{\mu\nu}^{(n)B}) - h(T^{I}){}^{AB}(\overline{\epsilon}_{(n)}{}^{A}\gamma_{\mu}\lambda^{(n)B}),$$

$$\delta_{Q}C_{\mu}{}^{I} = +(T^{I}){}^{AB}(\overline{\epsilon}_{(n)}{}^{A}\psi_{\mu}^{(n)B}) - h(T^{I}){}^{AB}(\overline{\epsilon}_{(n)}{}^{A}\gamma_{\mu}\lambda^{(n)B}),$$

$$\delta_{Q}\lambda_{(n)}{}^{A} = +\frac{1}{2}(T^{I}){}^{AB}\gamma^{\mu\nu}\epsilon_{(n)}{}^{B}(2F_{\mu\nu}{}^{I} + G_{\mu\nu}{}^{I} + \hat{H}_{\mu\nu}{}^{I}) - \frac{1}{2}(2n+1)(\overline{\epsilon}_{(n)}{}^{B}\gamma^{\mu}\psi_{\mu}^{(n)'B})\lambda_{(n)}{}^{A}$$

$$+ 2(T^{I}){}^{AB}(T^{I}){}^{CD}(\gamma_{[\mu}\psi_{\nu](n)}{}^{B})(\overline{\epsilon}_{(n)'}{}^{C}\gamma^{\mu}\tilde{\mathcal{R}}{}^{\nu(n)'D}),$$
(9)

where $\hat{H}_{\mu\nu}{}^{I}$ is the "hypercovariantization" of $H_{\mu\nu}{}^{I}$:

$$\hat{H}^{I}_{\mu\nu} \equiv H_{\mu\nu}{}^{I} - (T^{I})^{AB} (\overline{\psi}_{\mu(n)}{}^{A} \psi_{\nu}^{(n)B}) + 2h(T^{I})^{AB} \\ \times (\overline{\psi}_{[\mu|(n)}{}^{A} \gamma_{|\nu]} \lambda^{(n)B}),$$
(10)

similar to supercovariantization [17–19]. The $\tilde{\mathcal{R}}$ is the Hodge dual of the gravitino field strength (5):

$$\tilde{\mathcal{R}}_{\mu(n)}{}^{A} \equiv +\frac{1}{2}e^{-1}\epsilon_{\mu}{}^{\rho\sigma}\mathcal{R}_{\rho\sigma(n)}{}^{A}.$$
 (11)

In accordance with (1), the parameter of hypersymmetry ϵ

should have an extra constraint:

$$\gamma^{m_1} \boldsymbol{\epsilon}_{m_1 m_2 \cdots m_n}^A = 0. \tag{12}$$

The confirmation of action invariance under hypersymmetry is very similar to usual supergravity [17–19] or the case for n = 0 in [13]. There are in total ten different categories of g or h-dependent terms (sectors): (i) $g\psi G$, (ii) $g\mathcal{R}H$, (iii) $g\psi^2\mathcal{R}$, (iv) $ghF\lambda$, (v) $ghG\lambda$, (vi) $ghH\lambda$, (vii) $gh\psi\lambda^2$, (viii) $gh\psi^2\lambda$, (ix) $h\psi^2\mathcal{R}$, (x) $g\psi\mathcal{R}\lambda$. Among these, the antisymmetry of T^I is frequently used in the sectors (ii), (iii), (iv), (v), (vi), (viii), and (ix). Other cancellation patters are more or less parallel to [13].

The closure of gauge algebra is similar to [13], as

$$\begin{bmatrix} \delta_{Q}(\boldsymbol{\epsilon}_{1}), \delta_{Q}(\boldsymbol{\epsilon}_{2}) \end{bmatrix} = \delta_{P}(\boldsymbol{\xi}) + \delta_{G}(\boldsymbol{\xi}) + \delta_{Q}(\boldsymbol{\epsilon}_{3}) + \delta_{L}(\boldsymbol{\lambda}^{rs}) + \delta_{\Lambda} + \delta_{\tilde{\Lambda}} + \delta_{\tilde{\Lambda}}, \\ \boldsymbol{\xi}^{m} \equiv +(2n+1)(\overline{\boldsymbol{\epsilon}}_{2(n)}{}^{A}\boldsymbol{\gamma}^{m}\boldsymbol{\epsilon}_{1}{}^{(n)A}), \\ \boldsymbol{\epsilon}_{3(n)}{}^{A} \equiv -\boldsymbol{\xi}^{\mu}\boldsymbol{\psi}_{\mu(n)}{}^{A}, \quad \boldsymbol{\lambda}^{rs} \equiv +\boldsymbol{\xi}^{\mu}\hat{\boldsymbol{\omega}}_{\mu}{}^{rs} - 2(2n+1)\boldsymbol{g}(T^{I}){}^{AB}(\overline{\boldsymbol{\epsilon}}_{1(n)}{}^{A}\boldsymbol{\epsilon}_{2}{}^{(n)B})\hat{H}^{rsI}, \\ \boldsymbol{\Lambda}^{I} \equiv -\boldsymbol{\xi}^{\mu}\boldsymbol{A}_{\mu}{}^{I}, \quad \tilde{\boldsymbol{\Lambda}}^{I} \equiv -\boldsymbol{\xi}^{\mu}\boldsymbol{B}_{\mu}{}^{I}, \quad \tilde{\boldsymbol{\Lambda}}^{\tilde{I}} \equiv -\boldsymbol{\xi}^{\mu}\boldsymbol{C}_{\mu}{}^{I} - (T^{I}){}^{AB}(\overline{\boldsymbol{\epsilon}}_{1(n)}{}^{A}\boldsymbol{\epsilon}_{2}{}^{(n)B}), \end{aligned}$$
(13)

where δ_P , δ_G , δ_Q , δ_L are the translation, general coordinate, hypersymmetry, and local Lorentz transformations, while δ_Λ , $\delta_{\overline{\Lambda}}$, and $\delta_{\overline{\Lambda}}$ are the *G* gauge transformations

$$\begin{split} \delta_{\Lambda}A_{\mu}{}^{I} &\equiv \partial_{\mu}\Lambda^{I} + gf^{IJK}A_{\mu}{}^{J}\Lambda^{K}, \\ \delta_{\tilde{\Lambda}}B_{\mu}{}^{I} &\equiv \partial_{\mu}\tilde{\Lambda}^{I} + gf^{IJK}B_{\mu}{}^{J}\tilde{\Lambda}^{K}, \\ \delta_{\tilde{\Lambda}}C_{\mu}{}^{I} &\equiv \partial_{\mu}\tilde{\Lambda}^{I} + gf^{IJK}B_{\mu}{}^{J}\tilde{\Lambda}^{K}, \end{split}$$
(14)

As usual, the closure of on shell hypersymmetry can be confirmed by the use of the field equations:

$$F_{\mu\nu}{}^{I} \doteq 0, \qquad G_{\mu\nu}{}^{I} \doteq 0, \qquad \hat{H}_{\mu\nu}{}^{I} \doteq 0, \qquad (15)$$

$$\mathcal{R}_{\mu\nu}{}^{A} \doteq 0. \tag{16}$$

The special case when the gravitino carries no group index recovers the result by Aragone-Deser [2]. Also, the case n = 0 with the adjoint group index maintained is the \aleph_0 supergravity with the spin 3/2 gravitino with the adjoint index A = I [13]. There is a technical subtlety associated with the field equation of ω and the torsion (8), which was not stressed well in [2]. The ω field equation from our Lagrangian is

$$\frac{\delta I_0}{\delta \omega_{\mu}{}^{rs}} = -\frac{1}{4} e T_{rs}{}^{\mu} - \frac{1}{2} e e_{[r}{}^{\mu} T_{s]}$$
$$-\frac{1}{8} \epsilon^{\mu\rho\sigma} (\overline{\psi}_{\rho(n)}{}^{A} \gamma_{rs} \psi_{\sigma}{}^{(n)A})$$
$$-\frac{1}{2} n \epsilon^{\mu\rho\sigma} (\overline{\psi}_{\rho r(n-1)}{}^{A} \psi_{\sigma s}{}^{(n-1)A}) \doteq 0, \quad (17)$$

where $T_{\mu} \equiv T_{\mu\nu}^{\nu}$. By multiplying this by e_s^{μ} , we get the expression for T_{μ} which in turn is substituted into (17) to eliminate the T_{μ} term, as

$$T_{\rho\sigma}{}^{m} = + (\overline{\psi}_{\rho(n)}{}^{A} \gamma_{m} \psi_{\sigma}{}^{(n)A}) - 6n e^{-1} \epsilon^{\tau\lambda\omega} e_{[\rho]}{}^{m} (\overline{\psi}_{\tau|\sigma|}{}^{(n-1)A} \psi_{\lambda|\omega]}{}^{(n-1)A}).$$
(18)

Interestingly enough, the last term antisymmetric in $[\rho\sigma\omega]$ can be shown to be proportional to the first term in (18) by the peculiar algebra

$$3e^{-1}\epsilon^{\tau\lambda\omega}e_{[\rho]}{}^{m}(\overline{\psi}_{\tau|\sigma|(n-1)}{}^{A}\psi_{\lambda|\omega]}{}^{(n-1)A})$$

$$= -\frac{1}{2}e^{-3}\epsilon^{\tau\lambda\omega}\epsilon_{\rho\sigma\omega}\epsilon^{\mu\nu\psi}e_{\mu}{}^{m}(\overline{\psi}_{\tau\nu(n-1)}{}^{A}\psi_{\lambda\psi}{}^{(n-1)A})$$

$$= +e^{-1}\epsilon^{m\tau\lambda}(\overline{\psi}_{\rho\tau(n-1)}{}^{A}\psi_{\sigma\lambda}{}^{(n-1)A}) = +(\overline{\psi}_{\rho\tau(n-1)}{}^{A}\gamma^{m\tau\lambda}\psi_{\sigma\lambda}{}^{(n-1)A})$$

$$= +(\overline{\psi}_{\rho\tau(n-1)}{}^{A}\gamma^{m\tau}\gamma^{\lambda}\psi_{\sigma\lambda}{}^{(n-1)A}) - (\overline{\psi}_{\rho\tau(n-1)}{}^{A}\gamma^{m}\psi_{\sigma}{}^{\tau(n-1)A}) + (\overline{\psi}_{\rho\tau(n-1)}{}^{A}\gamma^{\tau}\psi_{\sigma}{}^{m(n-1)A})$$

$$= -(\overline{\psi}_{\rho(n)}{}^{A}\gamma^{m}\psi_{\sigma}{}^{(n)A}).$$
(19)

Here use is made of the important constraint (1) for dropping the first and last terms in the penultimate line in (19). This combines the last term in (18) with its first term, with its coefficient changed from unity to (2n + 1).

It is sometimes convenient to note the difference in the transformation of the Lorentz connection between the firstand second-order formalisms:

(i) First-order:

$$\delta_Q \omega_{\mu rs} = 0, \qquad (20)$$

(ii) Second-order:

$$\delta_{Q}\hat{\omega}_{\mu rs} = \frac{1}{2}(n+1)[2(\overline{\epsilon}^{(n)A}\gamma_{[r}\mathcal{R}_{s]\mu(n)}^{A}) - (\overline{\epsilon}^{(n)A}\gamma_{\mu}\mathcal{R}_{rs(n)}^{A})].$$
(21)

These two are equivalent to each other on shell under (16), as in the 4D case [17–19].

We mention the consistency of gravitino field equation (18) in our system, associated with so-called "Velo-Zwanziger disease" [3]. This is about the problem with the divergence of the gravitino field equation on curved backgrounds or nontrivial field strengths frequently encountered in 4D or higher. Fortunately in 3D, the covariant

$$\pi_3(G) = \begin{cases} \mathbb{Z} & \text{(for } G = A_n, B_n, C_n, D_n (n \ge 2, G \neq D_2), \\ \mathbb{Z} \oplus \mathbb{Z} & \text{(for } G = SO(4)), \\ 0 & \text{(for } G = U(1)). \end{cases}$$

Especially for a compact gauge group with $\pi_3(G) = \mathbb{Z}$, the quantization condition is [5] [21]

$$gk = 8\pi h$$
 $(k = 0, \pm 1, \pm 2, \cdots).$ (26)

Under this condition, the two initially independent constant g and h are now proportional to each other.

III. ADDITIONAL CHERN-SIMONS TERMS

Because of the peculiar property of 3D, we can further add some Chern-Simons terms to our initial Lagrangian \mathcal{L}_0 . The first example is the Lorentz Chern-Simons term $I_{R\omega} \equiv \int d^3x \mathcal{L}_{R\omega}$ [5], where

divergence of gravitino field equation (16) vanishes, as desired:

$$0 \stackrel{?}{=} D_{\mu} \left(\frac{1}{2} \epsilon^{\mu \rho \sigma} \mathcal{R}_{\rho \sigma m_{1} \cdots m_{n}}^{A} \right)$$

$$= + \frac{1}{8} \epsilon^{\mu \nu \rho} (\gamma_{rs} \psi_{\rho m_{1} \cdots m_{n}}^{A}) \mathcal{R}_{\mu \nu}^{rs}(\omega)$$

$$+ \frac{1}{2} n \epsilon^{\mu \nu \rho} \psi_{\rho t(m_{2} \cdots m_{n}]}^{A} \mathcal{R}_{\mu \nu | m_{1}}^{I}(\omega)$$

$$+ \frac{1}{2} g \epsilon^{\mu \nu \rho} (T^{I})^{AB} \psi_{\rho m_{1} \cdots m_{n}}^{B} \mathcal{G}_{\mu \nu}^{I} \doteq 0.$$
(22)

In particular, $G_{\mu\nu}{}^{I} \doteq 0$ (15) and the dreibein field equation

$$R_{\mu\nu}(\omega) \doteq 0 \iff R_{\mu\nu}^{rs}(\omega) \doteq 0$$
 (23)

are used. The latter is associated with the peculiar identity in 3D:

$$R_{\mu\nu}{}^{\rho\sigma}(\omega) \equiv +4\delta_{[\mu}{}^{[\rho}R_{\nu]}{}^{\sigma]}(\omega) - \delta_{[\mu}{}^{[\rho}\delta_{\nu]}{}^{\sigma]}R(\omega), \quad (24)$$

due to the vanishing of the conformally invariant Weyl tensor $C_{\mu\nu\rho}^{\ \sigma} \equiv 0$ [5].

As usual in 3D, the coefficient of the Chern-Simons term should be quantized [5] for a compact gauge group whose π_3 -homotopy mapping is nontrivial, e.g.,

$$=\begin{cases} \mathbb{Z} & \text{(for } G = A_n, B_n, C_n, D_n (n \ge 2, G \ne D_2), G_2, F_4, E_6, E_7, E_8), \\ \mathbb{Z} \oplus \mathbb{Z} & \text{(for } G = SO(4)), \\ 0 & \text{(for } G = U(1)) \end{cases}$$
(25)

$$\mathcal{L}_{R\omega} \equiv +\frac{1}{2} \alpha \epsilon^{\mu\nu\rho} [R_{\mu\nu}{}^{rs}(\omega)\omega_{\rho rs} + \frac{1}{3} \omega_{\mu r}{}^{s}\omega_{\nu s}{}^{t}\omega_{\rho t}{}^{r}].$$
(27)

Here α is a real constant. This Lagrangian also was presented in the context of topological massive gravity [5].

The ω field equation is now modified by $\mathcal{L}_{R\omega}$ with α , as

$$\frac{\delta I_{\text{tot}}}{\delta \omega_{\mu}^{rs}} = -\frac{1}{4} e T_{rs}^{\mu} - \frac{1}{2} e e_{[r}^{\mu} T_{s]}$$
$$-\frac{1}{8} \epsilon^{\mu\rho\sigma} (\overline{\psi}_{\rho(n)}^{A} \gamma_{rs} \psi_{\sigma}^{(n)A})$$
$$-\frac{1}{2} n \epsilon^{\mu\rho\sigma} (\overline{\psi}_{\rho r(n-1)}^{A} \psi_{\sigma s}^{(n-1)A})$$
$$+ \alpha \epsilon^{\mu\rho\sigma} R_{\rho\sigma rs}(\omega) \doteq 0.$$
(28)

However, the new term is proportional to the Riemann tensor $R_{\mu\nu}{}^{rs}(\omega)$, and it vanishes because of (24) and the dreibein field equation (23) which is not modified. In other words, the last α -dependent term in (28) vanishes, leaving the original algebraic field equation for ω . Accordingly, the torsion (8) stays intact, and we can still use the first or 1.5-order formalism for ω . Consequently, the invariance of the total action $I_{\text{tot}} \equiv I_0 + I_{R\omega}$ is still valid despite the presence of $I_{R\omega}$.

This is based on the 1.5-order formalism, but the invariance $\delta_Q I_{\text{tot}} = 0$ is much more transparent in the first-order formalism. This is because in the first-order formalism, the Lorentz connection is invariant under hypersymmetry as in (20), and therefore the invariance $\delta_Q I_{R\omega} = 0$ is manifest.

Note that the fact $R_{\mu\nu}^{rs}(\omega) \doteq 0$ does *not* make the Lorentz-Chern-Simons term (27) trivial, because of its last term $\omega \wedge \omega \wedge \omega$, as in the usual Chern-Simons term in 3D [5]. Additionally, since the Lorentz group SO(2, 1) in 3D is noncompact, there is no quantization for the coefficient α [5].

Another interesting Chern-Simons term is of the *GB* type $I_{GB} \equiv \int d^3x \mathcal{L}_{GB}$, where

$$\mathcal{L}_{GB} = \frac{1}{2} \beta \epsilon^{\mu\nu\rho} \left(G_{\mu\nu}{}^{I} B_{\rho}{}^{I} - \frac{1}{3} g f^{IJK} B_{\mu}{}^{I} B_{\nu}{}^{J} B_{\rho}{}^{K} \right).$$
(29)

In order to maintain the invariance of the total action $I_{\text{tot}} \equiv I_0 + I_{R\omega} + I_{GB}$ under hypersymmetry, we need to shift the *C* field transformation rule by $\delta_O B$, as

$$\delta_{\mathcal{Q}} C_{\mu}{}^{I} = + (T^{I})^{AB} (\overline{\epsilon}_{(n)}{}^{A} \psi_{\mu}{}^{(n)B}) - h(T^{I})^{AB} (\overline{\epsilon}_{(n)}{}^{A} \gamma_{\mu} \lambda^{(n)B}) - g^{-1} \beta (\delta_{\mathcal{Q}} B_{\mu}{}^{I}).$$
(30)

Since the *C* field appears nowhere in \mathcal{L}_{tot} other than the *CG* term, and the variation of I_{GB} has only $\delta_Q B$, the modification above is sufficient for the invariance of I_{tot} . The *B* field equation seems to get modified by the β term, but actually not, because of the *C* field equation $G_{\mu\nu}^{I} \doteq 0$.

The quantization of the β coefficient for any gauge group with $\pi_3(G) = Z$ is

$$g^2 \ell = 8\pi\beta$$
 $(\ell = 0, \pm 1, \pm 2, \cdots).$ (31)

This restricts the originally independent constant β to be proportional to g.

IV. CONCLUDING REMARKS

In this paper, we have presented $\forall N$ -extended hypergravity with the gravitino of spin n + 3/2 in an arbitrary real representation satisfying the condition $(T^I)^{AB} = -(T^I)^{BA}$ of any gauge group G. Since such representation can be limitlessly large, we have arbitrarily large extended local hypersymmetries, which we call \aleph_0 hypergravity. The number N of hypersymmetries is specified by the dimensionality of the representation of the index A. For example, the adjoint representation A = I yields $N = \dim G$, or the vectorial representation A = a of G = SO(M) yields N = M.

Our result is a generalization of the work by Aragone-Deser for gravitino with arbitrarily large half-integral spins [2] to the system where the gravitino is in an arbitrary real representation such that the generators are antisymmetric. Compared with [2], our system has also additional structures, such as the topological *CG* term and the hypersymmetric Chern-Simons term. When the gauge group is trivial as G = I, our graviton-gravitino sector is reduced to [2], while the *CG* and the Chern-Simons term still remains as an Abelian case. On the other hand, if we keep *G* to be nontrivial, while putting n = 0 with the spin 3/2 gravitino, we get the generalization of \aleph_0 supergravity [13] for more general representations than the adjoint representation.

We have further generalized our system, by adding two more Chern-Simons terms $I_{R\omega}$ and I_{GB} . We have seen that the hypersymmetric invariance of the total action $I_{tot} \equiv$ $I_0 + I_{R\omega} + I_{GB}$ is restored, by a slight modification of the *C* field transformation rule. Even though the field equations imply that $R_{\mu\nu}{}^{mn}(\omega) \doteq 0$ and $G_{\mu\nu}{}^I \doteq 0$ on shell, the $\omega \wedge$ $\omega \wedge \omega$ for the former or $B \wedge B \wedge B$ for the latter has nontrivial contribution as the surface term, as usual in Chern-Simons theory. We also have seen interesting relationships among the coupling constants *g* and *h* by the quantization of the coefficients of the relevant Chern-Simons terms for any gauge group with $\pi_3(G) = \mathbb{Z}$.

In this paper, we have given Chern-Simons formulations with no more fundamental bases. However, it may well be the case that our theory has foundations, such as superparticle, superstring, or supermembrane theory. As a matter of fact, it has been known for some time that a Chern-Simons theory in 10D can be derived from the second quantization of superparticle theory [22]. Therefore, it is not too far fetched to expect that our \aleph_0 hypergravity has also such extended objects as its foundation.

Even though the field strengths in our system vanish upon their field equations, they might have more significance than expected. In fact, a similar situation is found in so-called loop quantum gravity theory [23] or more generally, in topological field theories [14], where vanishing field strengths play nontrivial roles due to topological effects on the boundary. In our system, it is very peculiar that the local gauge symmetry can be arbitrarily large, and accordingly local hypersymmetry also can be arbitrarily large (\aleph_0) as well.

ACKNOWLEDGMENTS

We are grateful to S. Deser for important discussions. This work is supported in part by NSF Grant No. 0308246.

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