Braneworld puzzle about entropy bounds and a maximal temperature

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Entropy bounds applied to a system of $\mathcal{N}$ species of light quantum fields in thermal equilibrium at temperature T are saturated in four dimensions at a maximal temperature $T_{\max} = M_{\rm Planck}/\sqrt{\mathcal{N}}$. We show that the correct setup for understanding the reason for the saturation is a cosmological one, and that a possible explanation is the copious production of black holes at this maximal temperature which prevents any further rise in temperature. The proposed explanation implies, if correct, that $\mathcal{N}$ light fields cannot be in thermal equilibrium in an ideal gas phase at temperatures T above $T_{\max}$. However, we have been unable to identify a concrete mechanism that is efficient and quick enough to prevent the universe from exceeding this limiting temperature. The same issues can be studied in the framework of AdS/CFT by using a brane moving in a five dimensional AdS-Schwarzschild space to model a radiation dominated universe. In this case we show that $T_{\max}$ is the temperature at which the brane just reaches the horizon of the black hole, and that entropy bounds and the generalized second law of thermodynamics seem to be violated when the brane continues to fall into the black hole. We find, again, that the known physical mechanisms, including black hole production, are not efficient enough to prevent the brane from falling into the black hole. We propose several possible explanations for the apparent violation of entropy bounds, but none is a conclusive one.

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I. INTRODUCTION

Entropy bounds seem to imply that $\mathcal{N}$ light quantum fields cannot be in thermal equilibrium in an ideal gas phase at an arbitrarily high temperature. In four dimensions they are saturated at a temperature equal to $T_{\max} = M_P/\mathcal{N}^{1/2}$ (here M_P is the Planck mass). When entropy bounds are saturated it is possible, in many cases, to identify a physical mechanism that enforces them. The prime candidate for such a mechanism is black hole (BH) production. If many BH's are produced, the system goes into a kind of phase transition. In the new phase the previous energy and entropy estimates are no longer valid. Since BH's are more efficient in storing entropy, the ratio of entropy to energy saturates and the bounds are not violated.

We seek a physical mechanism that places an upper bound on the temperature, if such an upper bound indeed exists. Since we wish to use semiclassical methods and avoid the quantum regime, we focus on the limit of large $\mathcal{N}$ since then $T_{\max} \ll M_P$. As we will show, the correct context for studying this issue is a cosmological context.

Previously, Bekenstein [1] argued that if the entropy of a visible part of the universe obeys the usual entropy bound from nearly flat space situations [2], then the temperature is bounded and therefore certain cosmological singularities are avoided. More recently, there have been several discussions following a similar logic. Veneziano [3] suggested that since a BH larger than a cosmological horizon cannot form [4], the entropy of the universe is always bounded. This suggestion is related, although not always equivalent, to the application of the holographic principle [5] in cosmology [6–15]. In [16–18] it was argued that the Hubble parameter H is bounded by entropy considerations, $H \leq H_{\max} \equiv \frac{M_P}{\sqrt{\mathcal{N}}}$. In a cosmological context this is equivalent to $T \leq T_{\max}$.

The AdS/CFT correspondence [19,20] offers an alternative route and a new perspective for the study of a system of a large number $\mathcal{N}$ of light fields in thermal equilibrium in a cosmological setup by studying brane propagation in an AdS-Schwarzschild background [21–25]. Branes moving in AdS-Schwarzschild space are expected to be dual to finite temperature CFT's in a cosmological background [26,27]. However, the status of the conjecture is somewhat weaker than the one relating to an AdS space without a brane (see, for example, [28–30]). In this particular case the branes in AdS-Schwarzschild are conjectured to be dual to a radiation dominated FRW universe, which is exactly the setup that we are interested in. As we will show, the maximal temperature $T_{\max}$ has a geometric 5D interpretation: it corresponds to the brane "just" reaching the BH horizon.

The conjectured duality between branes propagating in AdS-Schwarzschild space and a radiation dominated FRW universe offers a novel perspective for studying the saturation of the entropy bounds at $T_{\max}$. The issue becomes whether the brane can continue to fall into the BH and continues to be dual to a CFT in a cosmological background at temperatures above $T_{\max}$.

A possible way of viewing the propagation of branes in AdS-Schwarzschild is the following: a thermal system with a known form of entropy is thrown into a BH, a process

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analogous to the Geroch process (See [31] for a review). The original Geroch process is a thought experiment in which a small thermodynamic system is moved from infinity into a BH. The small system is lowered slowly until it is just outside the black hole horizon, and then falls in. Here a 4D universe is thrown whole into a 5D BH, and so issues concerning the generalized second law (GSL) and its relation to entropy bounds can be addressed. As in the standard case, it is then possible to compare the total entropy of the system before and after and to discuss cases in which a decrease in the total entropy is suspected. We do indeed find that the GSL is violated as the brane falls into the BH.

In Sec. II we explain the saturation of entropy bounds at $T_{\max}$, and discuss possible physical mechanisms that may lead to this saturation. In Sec. III we discuss the issue from a 5D perspective, and discuss possible physical mechanisms that may alter the propagation of branes with respect to naive expectations. In Sec. IV we offer several possible resolutions of the puzzle that we have posed in the previous sections.

II. BLACK HOLE CREATION AND A MAXIMAL TEMPERATURE IN FOUR DIMENSIONS

This section is concerned with studying under which conditions a relativistic gas ceases to be in thermal equilibrium due to gravitational effects.

In the first part of this section we will show by means of simple scaling arguments, that this problem cannot be addressed in a flat space-time-independent setup, rather it has to be addressed in a cosmological setup. We apply simple scaling arguments in flat space, and find that the assumption that we can ignore the back reaction of the gas on the geometry is incorrect. We then continue to study the problem in a time-dependent setup in Sec. II B, where some known results about entropy bounds will be interpreted in a way relevant to the problem at hand. We explain how entropy bounds lead to the notion of a maximal temperature, which, as we show, can be much lower than the Planck energy.

We then, in the last subsection, look for an explanation of the appearance of a maximal temperature as resulting from some semiclassical physics that so to speak enforces the entropy bounds. We perform a more quantitative analysis of the candidate physical effects that could in principle invalidate our picture, and find that they do not alter the previous, more qualitative analysis. Our conclusion follows: that in the semiclassical domain it is not possible to determine the principle that enforces the entropy bounds.

A. Thermal equilibrium of a relativistic gas in a rigid box

Consider a relativistic gas in thermal equilibrium at a temperature T. We assume that the gas consists of $\mathcal{N}$ independent degrees of freedom in a box of macroscopic linear size R, we further assume that R is larger than any fundamental length scale in the system, and in particular R is much larger than the Planck length $R \gg l_P$. The volume of the box is $V = R^3$. Since the gas is in thermal equilibrium its energy density is $\rho = \mathcal{N} T^4$ and its entropy density is $s = \mathcal{N} T^3$ (here and in the following we systematically neglect numerical factors). As explained previously, we are interested in the limit of large $\mathcal{N}$.

Under what conditions is this relativistic gas unstable to the creation of BH's? The simplest criterion which may be used to determine whether an instability is present is a comparison of the total energy in the box $E_{\mathrm{Th}} = \mathcal{N} T^4 R^3$ to the energy of a BH of the same size $E_{\mathrm{BH}} = M_P^2 R$. The two energies are equal when $T^4 = 1/\mathcal{N} M_P^2/R^2$. So thermal radiation in a box has a lower energy than a BH of the same size if

$$(TR)^4 < \frac{1}{\mathcal{N}} M_P^2 R^2. \tag{1}$$

Another criterion that may help us to determine the presence of an instability to BH's creation is to compare the thermal entropy $S_{\mathrm{Th}} = \mathcal{N} T^3 R^3$ to the entropy of the BH $S_{\mathrm{BH}} = M_P^2 R^2$. They are equal when $T^3 = 1/\mathcal{N} M_P^2/R$. So thermal radiation in a box has a lower entropy than and a BH of the same size if

$$(TR)^3 < 1/\mathcal{N} M_P^2 R^2. \tag{2}$$

From Eqs. (1) and (2) it is possible to conclude the well known fact that for fixed R and $\mathcal{N}$, if the temperature is low enough the average thermal free energy is not sufficient to form BH's. For low temperatures the thermal fluctuations are weak and they do not alter the conclusion qualitatively.

Here we are interested in the case $RT > 1$ which means that the size of the box is larger than the thermal wavelength $1/T$. The case $RT < 1$ has been considered previously in [32]. In this case the temperature is not relevant. Instead, the field theory cutoff Λ was shown to be the relevant scale. In [32] we found a relationship between Λ, M_P and the number of fields $\mathcal{N}$ which is somewhat different than what we find here between T, M_P, and $\mathcal{N}$.

Imagine raising the temperature of the radiation from some low value for which condition (1) is not satisfied to higher and higher values such that eventually condition (1) is saturated. Note that since $TR > 1$ Eq. (1) is saturated before Eq. (2). We assume that the size of the box R is fixed during this process (the number of species $\mathcal{N}$ is also fixed), and estimate the backreaction of the radiation energy density on the geometry of the box to determine whether the assumption that the geometry of box is fixed is consistent. To obtain a simple estimate we assume that the box is spherical, homogeneous and isotropic. Then its expansion or contraction rate is given by the Hubble parameter $H = \dot{R}/R$, which is determined by the 00 Einstein equation $H^2M_P^2 = \mathcal{N}T^4$. However, if Eq. (1) is satisfied then $\frac{1}{R^2}M_P^2 = \mathcal{N}T^4$, and therefore $HR \sim 1$. The conclusion is that if Eq. (1) is saturated then the gravitational time scale is comparable to the light crossing time of the box, and therefore it is inconsistent to assume that the box has a fixed size which is independent of the energy density inside it.

Thus we have shown that the assumption that it is possible to ignore the back reaction of the gas on the geometry is incorrect. The back reaction has to be taken into account.

B. Thermal equilibrium of a relativistic gas in a cosmological setup

The conclusion from the previous subsection is that the hypothesis that copious BH production is responsible for the appearance of a maximal temperature for a gas of relativistic particles needs to be studied in a time-dependent setting, namely, in a cosmological one. Different asymptotic boundary conditions have to be used where

$$H^2M_P^2 = \mathcal{N}T^4. \tag{3}$$

Entropy bounds such as the Hubble entropy bound and others are saturated if $S_{\rm Th} = \mathcal{N}T^3H^{-3} = S_{\rm BH} = M_P^2H^{-2}$. From Eq. (3) we understand that this happens for

$$H = T, \qquad T = T_{\max} = \frac{M_P}{\sqrt{\mathcal{N}}}. \tag{4}$$

Let us examine in more detail the physics of a radiation dominated (RD) universe at temperatures near $T_{\max}$. If $H = T$, the cosmological horizon size H^{-1} becomes comparable to the wavelength of a typical particle of the relativistic gas, $\lambda \sim T^{-1}$. If we go beyond this temperature, the classical description of the particles that compose the gas in terms of a homogeneous and isotropic fluid is no longer appropriate, and thus neither is Eq. (3). Alternatively, one can think of $T_{\max}$ as the temperature at which the Jeans length of a typical thermal fluctuation becomes comparable to the thermal wavelength, thus suggesting, again, that the approximation of the gas by a homogeneous and isotropic fluid becomes inappropriate. Yet another way to think about $T_{\max}$ is as the temperature at which the entropy within a thermal wavelength becomes comparable to the entropy of a BH of the same size, thus making BH entropically favored over single particle excitations. Similarly, at $T = T_{\max}$ the thermal energy inside a "box" of size H^{-1}, $E = \mathcal{N}T^4H^{-3}$ is equal to the energy of a BH of the same size, and also the free energies of both states become comparable.

Our simple scaling arguments and qualitative considerations indicate that a gas of particles cannot be in thermal equilibrium in an ideal gas phase at temperatures above $T_{\max} = M_p/\sqrt{\mathcal{N}}$. It is also clear that at $T_{\max}$ the assumption that we can treat gravity as semiclassical, only providing matter with a geometric background, is incorrect. All these considerations are well known in the case $\mathcal{N} = 1$; but if $\mathcal{N}$ is a large number the relevant scale can be much smaller than the Planck scale. Our conclusion will remain the same also after a more quantitative examination presented below.

C. Additional quantitative tests

Let us first close some possible loopholes in our analysis. One possible loophole could have been if thermal fluctuations were too large, invalidating our simple scaling arguments that implicitly assume that the fluctuations are small. This is not the case. The ratio of the energy in thermal fluctuations,

$$\frac{\Delta E^2}{E^2} = \frac{1}{\mathcal{N}}\frac{1}{T^3R^3} \tag{5}$$

is small compared to the average value of the energy in this regime and is much smaller than unity for $RT > 1$, and $\mathcal{N} \gg 1$. Another possible loophole could have been, as in [32], a clash with the assumption that the semiclassical treatment is valid. Since, in the case at hand, the energy is dominated by the mean value of ρ, and not by the fluctuations, we do not have problems with black hole evaporation: in fact it turns out that for

$$T \leq T_C = \sqrt{\frac{640\pi}{\mathcal{N}}}M_P, \tag{6}$$

BH's can be treated classically and, as can be seen by inserting the correct numerical factors into the definition of $T_{\max}$, $T_{\max} < T_C$.

We would like to estimate the time scale for the collapse of perturbations which, if frequent and strong enough, will lead to production of black holes. The perturbation equations which govern their evolution are well known [33]; we present here the equation governing the dynamics of the Bardeen potential Φ, in longitudinal gauge

$$6\ddot{\Phi} + 24\mathcal{H}\dot{\Phi} + 12(\dot{\mathcal{H}} + \mathcal{H}^2)\Phi - 2\Delta\Phi = 0, \tag{7}$$

with $\mathcal{H} \equiv \dot{R}/R$, the dot denoting the derivative with respect to conformal time η, and Δ the spatial Laplacian operator. The solution of the perturbation equations is quite standard. First, by means of the spatial Fourier transform the Laplacian operator is expressed in terms of the comoving wavenumber k as $\Delta \to -k^2$. Then one notices that, since the background evolves as a power law in conformal time, and, in particular, for a radiation dominated contracting universe one has $R(\eta) \sim -\eta$, with $-\infty < \eta < 0$, the solution for the mode Φ_k can be expressed in terms of the variable $x \equiv k\eta$ as

$$\Phi_k(\eta) = A_kF(x), \tag{8}$$

where $F(x)$ (whose explicit form is not needed here) scales as x^{-2} for $x \to -\infty$, diverges as x^{-3} for $x \to 0$, and is of order one for $x \sim -1$.

The factor A_k can be determined through the perturbed Friedmann equation which gives a relation between the Bardeen potential and the density perturbation:

$$\Phi_k(\eta) = \frac{3}{2x^2}\frac{\delta\rho_k(x)}{\rho}. \tag{9}$$

We now observe that the thermal energy fluctuations are dominated by the comoving wavenumber $k_T \equiv RT$ since the higher modes are exponentially suppressed in the Boltzmann distribution, and that we can estimate them via Eq. (5). We further observe that at the time $\eta_{\max}$ when the critical temperature $T_{\max}$ is attained, one has $x = -1$ for the mode that dominates the fluctuations. By combining all these elements we may express the Bardeen potential at $\eta_{\max}$ in terms of an initial thermal fluctuation at some early time η_i:

$$\Phi_{k_T}(\eta_{\max}) = \frac{3}{2}\frac{1}{\sqrt{\mathcal{N}}(RT)^{3/2}}\frac{1}{x_i^2 F(x_i)}F(-1). \tag{10}$$

The factor $x_i^2 F(x_i)$ is of order one, and so is $F(-1)$; this leads us to the conclusion that the Bardeen potential is still small at the critical time, due to the large factor $\sqrt{\mathcal{N}}(RT)^{3/2}$ in the denominator.

To summarize, we have found that if the initial perturbations are provided by thermal fluctuations, then their initial amplitude is very small, and since they grow only as a power law, they do not have enough time to become large before the critical temperature is reached. We conclude that BH production from thermal perturbations is not quick enough, so entropy bounds do seem to be violated.

At this point we cannot proceed further with semiclassical methods and get a better idea on the state of a system when the temperature is increased beyond $T_{\max}$, or even whether this is possible at all.

III. FEEDING A 4D BRANEWORLD TO A 5D BLACK HOLE

We can gain some insight about the meaning of $T_{\max}$, and perhaps some further technical control by modelling a 4D RD universe as a brane moving in an AdS_5-Schwarzschild space-time.

For precision, we will take the following representation for the bulk space-time

$$\mathrm{d}s^2 = -H(R)\mathrm{d}t^2 + \frac{1}{H(R)}\mathrm{d}R^2 + R^2\mathrm{d}\Omega_3^2, \tag{11}$$

where $H(R) = 1 + \frac{R^2}{L^2} - \frac{b^4L^2}{R^2}$ vanishes at the black hole horizon R_H and $b = (\frac{8G_N^{(5)}}{3\pi}\frac{M}{L^2})^{1/4}$, M being the black hole mass. L is related to the cosmological constant of the AdS and also to the brane tension λ, which is tuned in such a way as to make a vanishing effective cosmological constant on the brane. Note that the line element in Eq. (11) describes only the part of space-time outside the BH horizon; this will become important and relevant shortly when we discuss the fate of a brane that is about to fall into the BH.

For the BH in AdS to be the dominant configuration over an AdS space with some thermal radiation as required for our analysis to be relevant, b must be large $b \gg 1$ [26], that is, the black hole must be large and hot compared to the surrounding AdS_5. In this limit the closed 4D universe can be treated as flat, and we can write $R_H \simeq bL$, and $b \simeq \pi L T_0$, where T_0 is the Hawking temperature of the black hole.

The motion of the brane through the bulk space-time is viewed by a brane observer as a cosmological evolution. According to the prescription of the RS II model [34], the 4D brane is placed at the Z_2 symmetric point of the orbifold. On the other hand, in the so called mirage cosmology [22], the brane is treated as a test object following a geodesic motion. In both cases the evolution of the brane in the AdS_5-Schwarzschild bulk mimics a FRW radiation dominated cosmology. Thus, both prescriptions are useful for our purposes. We will keep them in mind in the following discussion.

The brane can be described by its radial position as a function of the proper time of the brane $R_b(\tau)$. The brane proper time is analogous to the proper time of a freely falling point particle in a 4D BH space-time. The evolution of $R_b(\tau)$ is determined by an effective Friedmann equation:

$$\left(\frac{\dot{R}_b}{R_b}\right)^2 = \frac{b^4L^2}{R_b^4} - \frac{1}{R_b^2}, \tag{12}$$

where the dot here stands for a derivative with respect to τ. Since, as we recall, $b \gg 1$, the curvature term is always negligible in the range that we are interested, we will therefore ignore this term in the following. Equation (12) expresses the dynamics of the brane in terms of 5D quantities; we now focus on the case of a *contracting* brane and translate those quantities into 4D ones in order to be able to compare Eq. (12) with Eq. (3).

The AdS/CFT correspondence tells us that the number of species in the CFT is given by $\mathcal{N} = L^3/G_N^{(5)}$, while the 4D and the 5D Newton's constants are related by $LG_N^{(4)} = G_N^{(5)}$ (again, we consistently ignore numerical factors). As can be seen from the line element in Eq. (11), the boundary of space is a 3 sphere, so the CFT "lives" on S^3. Now we have enough information to make a comparison between Eq. (12) and Eq. (3) and to obtain the temperature measured on the brane as $T = b/R_b$, which is also in accordance with the AdS/CFT correspondence. In passing, we notice that one should not confuse the temperature of the boundary CFT that is dual to the AdS bulk theory with the Hawking temperature of the AdS-BH as measured by a bulk observer located at the coordinate R. The latter is given by $T_0/\sqrt{H(R)}$ and scales with R in a similar way to the CFT temperature only in the asymptotic limit $R \to \infty$.

We now wish to see what happens in the 5D picture when the limiting temperature is approached on the brane. By expressing $M_P \equiv \sqrt{G_N^{(4)}}$ and $\mathcal{N}$ in terms of 5D quantities we can see that $T_{\max} \simeq 1/L$ and, since the corresponding value for R is $b/T_{\max}$, we find that

$$T \to T_{\max} \quad \longrightarrow \quad R_b \to R_H. \tag{13}$$

$T_{\max}$ is reached exactly when the brane reaches the BH horizon and is about to enter into the black hole.

From the point of view of the brane nothing special seems to happen when it reaches the horizon (more on this later), just as from the point of view of a freely falling observer in a Schwarzschild geometry nothing special happens when it reaches the horizon. However, the setup that we have used for the AdS/CFT is only capable of describing the brane outside the BH horizon. To find out what happens to it as it crosses the horizon, the setup needs to be changed, for example, in the way proposed by Maldacena [35]. One needs to include the whole AdS-Kruskal geometry that has two boundaries. The dual field theory is therefore in a product space that consists of two CFTs which have to be put in a specific entangled state. The AdS/CFT correspondence is less well developed in this different setup. It would be very interesting to study the correct prescription of applying Maldacena's proposal to our problem. Our arguments suggest that some interesting physics needs to take place as the brane approaches the horizon.

One could perhaps avoid the need to go to the more elaborate setup of the AdS/CFT correspondence to be able to describe what happens when the brane reaches the BH horizon by interpreting the brane motion according to the "mirage" prescription. In this case, the brane is a probe brane moving through the fixed bulk background. A reasonable interpretation of what transpires at horizon crossing is that the 4D universe simply ends its existence and disappears into the BH. The BH "eats" the 4D universe, its mass increases and so does its size, and entropy. Therefore the final state from a 5D point of view is simply an AdS_5-Schwarzschild space with a larger BH. Similarly, from the original point of view of the AdS/CFT correspondence this can also interpreted as a single probe brane joining the N branes on which the CFT lives and are the source of the AdS-BH space-time in the bulk. This interpretation leads to the same conclusion: that the final state is simply a larger BH in AdS.

We are thus in a situation similar to the one envisaged in the Geroch process: the thought experiment in which a thermodynamic system is absorbed by a black hole. The aim is to design the process such that the energy absorbed by the BH is minimal, and in such a way also the entropy that the BH gains will be minimal, as both the energy and the entropy of the BH depend only on its mass after the absorption. By carefully analyzing the entropy balance in the Geroch process, and by requiring the validity of the generalized second law of thermodynamics, Bekenstein was able to state his universal entropy bound for compact weakly gravitating systems, $S \leq 2\pi ER$ [2]. In the case at hand, the significant difference is that an entire universe is thrown into the BH. Therefore we can look at the entropy balance during the process and see under which conditions the GSL is respected or not.

In order to have a vanishing effective cosmological constant on the brane, one has $G_N^{(5)}\lambda \simeq L^{-1}$; this means that at horizon crossing the total energy of the brane is

$$E|_{R=R_H} \simeq \frac{b^3 L^2}{G_N^{(5)}}. \tag{14}$$

Comparing E to $M \simeq \frac{b^4 L^2}{G_N^{(5)}}$ we see that for $b \gg 1$ the total energy of the brane is much smaller than the BH mass $E \ll M$.

The entropy of the 5D black hole is $S = \mathcal{A}(R_H)/4G_N^{(5)}$, with the area of the horizon given by $\mathcal{A}(R_H) = 2\pi^2 R_H^3$. When the brane falls into the BH, the entropy of the BH is increased by the following amount:

$$\delta S \simeq \frac{1}{4G_N^{(5)}} E \frac{\delta \mathcal{A}(R_H(M))}{\delta M} \simeq \frac{EL^2}{R_H} \simeq \frac{EL}{b}. \tag{15}$$

For the GSL to hold, the total entropy of the system should increase in the process

$$\delta S > S_b. \tag{16}$$

Since $S_b = 2\pi^2 R_H^3 \mathcal{N} T^3 \simeq EL$ is the total entropy on the brane when it is about to fall into the BH, we find that for the total entropy to increase

$$b < 1. \tag{17}$$

However, for the BH to be the dominant configuration b has to be much larger than unity $b \gg 1$. If indeed $b \gg 1$, then apparently the GSL is violated in this process. We have thus found that a violation of the GSL in the 5D bulk corresponds to a violation of the entropy bounds in the 4D brane. The situation is completely analogous to the one discussed in connection with the ordinary Geroch process where the GSL is apparently violated if the falling object does not satisfy the Bekenstein bound. This issue has a long history (see, for example, [2], [36–41]) and is controversial to some extent. We do not attempt to take sides in the debate, but rather to simply point out the similarities.

We may try to use the 5D picture to understand in a more quantitative way what is the physical mechanism that renders $T_{\max}$ a limiting temperature. Black hole creation and the subsequent "breaking" of the brane seemed to be one of the possibilities in the 4D picture. From the brane world point of view this would correspond to the formation of "blisters" on the brane. In fact, since the temperature of the brane scales as $1/R_b$, if a piece of brane is closer to the

BH with respect to the rest of the brane, then the local temperature on that piece will be higher, as will its energy density. A piece of the brane that has higher energy density has a higher local magnitude of the Hubble parameter. Therefore the speed at which it falls towards the BH is increased, and we expect a ''blister'' to form on the brane. Thus a local oscillation of the brane position would be seen by a brane observer as a local density perturbation which is further amplified as the brane falls towards the BH. This mechanism can be studied by looking at perturbation equations for the position of the brane. Since these are coupled to the bulk metric perturbations of the AdS_5, it seems that the full set of perturbation equations must be studied.

However, as it turns out, in our case one can study the perturbations directly from the 4D brane point of view: it is sufficient to write down the projected Einstein equations on the brane as

$$G_{\mu\nu} = -E_{\mu\nu}, \tag{18}$$

$E_{\mu\nu}$ being the projected bulk Weyl tensor on the brane (see [42]), and to perturb them. Equation (18) looks so simple because there is no matter on the brane, just the tension which is fine-tuned in order to cancel the bulk cosmological constant, so that both disappear from the dynamics. The only effective source term is then the projection of the bulk Weyl tensor, which we parameterize as a fluid with energy density ρ_ϵ and pressure $p_\epsilon = -\rho_\epsilon/3$ ($E_{\mu\nu}$ is traceless). Thus the system of perturbation equations is closed and can be solved without reference to the 5D picture. Notice that this happens because of the simplicity of the model at hand: if we had some matter on the brane this would no longer have been true.

In the end, the perturbation equations look exactly the same as in the pure 4D scenario discussed in the previous section and the same physical considerations about the growth of perturbations are valid. So it seems that the standard picture is confirmed: as $R \to R_H$, $H \to T$, and at horizon crossing the typical modes in the thermal bath become unstable. However their growth follows a power law only, and thus there is not enough time for the instability to invalidate the whole picture.

Another possible 5D mechanism that could modify our discussion and its conclusion about the saturation of the entropy bounds is the interaction of the bulk Hawking radiation with the brane. Since, as we have seen, the temperature of the Hawking radiation diverges at the horizon, one might have expected that at some point the Hawking radiation pressure becomes so high that it prevents the brane from falling into the BH. Perhaps the Hawking radiation pressure could cause the brane to bounce back and change its contraction into expansion or cause it to float just above the horizon. We think that this is unlikely. However, clearly the issue deserves further study, especially in light of the fact that for $b \gg 1$ the temperature of the Hawking radiation is also very large. For boxes falling into BH's the issue was debated extensively in the context of the relationship between the GSL and entropy bounds [36–41].

We would like to make a few observations about the possible influence of the Hawking radiation on the motion of the brane.

First, in AdS space the geometry provides a confining environment for the radiation which is then in equilibrium with the BH. Notice also that, unlike the pure Schwarzschild case, here the equilibrium is stable. The pressure on the brane results from the difference in the force exerted on the two sides of the brane. If the system is in thermal equilibrium and the brane is moving through the radiation fluid, then the pressure on it depends on the interaction of the brane with the radiation. If it is transparent, then the radiation does not exert any pressure on the brane, and if it is opaque, then the radiation pressure can be estimated by the pressure of a fluid at the Hawking temperature.

The Hawking temperature at the position of the brane is given by $T_H = T_0/\sqrt{H(R_b)} \simeq \frac{b}{L\sqrt{H(R_b)}}$. Substituting $H(R_b) = 1 + \frac{R_b^2}{L^2}(1 - \frac{b^4L^4}{R_b^4})$, we see that as long as the distance of the brane from the horizon $R_b - bL$ remains finite, then $T_H \sim b/R_b \sim T_{\rm brane}$. We then observe that the Hawking radiation pressure is smaller by a factor of $\mathcal{N}$ compared to the pressure on the brane, which in turn determines the acceleration of the brane towards the BH. We conclude that as long as the distance of the brane from the horizon is not particularly small, the Hawking radiation pressure is not likely to alter its motion significantly.

When the brane does get close to the BH it seems that the Hawking radiation pressure can affect the motion of the brane. However, it is not clear whether the fluid description of the Hawking radiation is valid in the vicinity of the horizon. The wavelength of a typical particle in thermal bath at temperature T is $\lambda \sim T^{-1}$, and the typical wavelength of the Hawking radiation in our AdS-Schwarzschild space-time is

$$\lambda_H(R) \sim \frac{\pi L}{b}\sqrt{\frac{R^2}{L^2} - \frac{b^4L^2}{R^2}}, \tag{19}$$

where we have taken into account the behavior of the local Hawking temperature as discussed above. On the other hand the physical distance of a space-time point with radial coordinate R from the horizon is

$$\begin{aligned} d(R) &= \int_{bL}^{R} \sqrt{g_{RR}(x)}\mathrm{d}x \\ &= \frac{L}{2}\log\left[\left(\frac{R}{bL}\right)^2 + \sqrt{\left(\frac{R}{bL}\right)^4 - 1}\right]. \end{aligned} \tag{20}$$

Notice that although g_{RR} diverges near the horizon, $d(R)$ is always finite at finite R.

Now observe that as one gets close to the horizon (i.e. for $R - bL \ll bL$), the following relation holds

$$\lambda_H(R) \rightarrow 2\pi d(R), \tag{21}$$

meaning that the typical wavelength λ_H becomes larger than (or in any case, of the same order of magnitude as) the physical distance from the horizon, thus implying that the description of the Hawking radiation as a fluid becomes inappropriate at this point. One could then argue that the Hawking radiation forms mostly at distances $d \sim bL$ from the black hole and larger, and that for smaller distances there is no significant radiation pressure. This means that Hawking radiation pressure cannot stop the brane from falling into the BH as it approaches the horizon.

These issues were discussed in the context of falling boxes most recently by Marolf and Sorkin [41], and previously by others. We conclude that the answer depends on the detailed dynamics of the system.

IV. DISCUSSION AND POSSIBLE RESOLUTION

We have seen that a special value of the temperature $T_{\max} = M_P/\sqrt{\mathcal{N}}$ emerges in various contexts. We have seen that such a value arises in four dimensional models as the temperature at which entropy bounds are saturated, and in five dimensional models as the effective induced temperature on a brane propagating in AdS-Schwarzschild space-time as it reaches the horizon of the bulk BH and is about to disappear into it. We have also shown that in the five dimensional picture the GSL is violated as the brane falls into the BH.

We have presented some examples for the appearance of this special value of the temperature, and have provided arguments supporting its existence or that a change in the description of equilibrium physics at this temperature is required. We have not provided conclusive evidence as to whether a specific physical mechanism is responsible for enforcing such a maximal temperature, or whether one exists at all. We have investigated several candidate effects, but not been able to identify a single mechanism that is efficient and quick enough to prevent the universe from exceeding the limiting temperature nor to identify the required changes in the description of physics at this temperature.

We list a few possibilities which we leave as unsolved puzzles and interesting problems for future research:

(1) Entropy bounds are wrong, and need to be modified such that the limit on temperature disappears.

(2) The number of light fields $\mathcal{N}$ is fundamentally limited, a fact which is well represented by entropy bounds, and therefore considering the large $\mathcal{N}$ limit $\mathcal{N} \rightarrow \infty$, as is done in the AdS/CFT correspondence, is incorrect.

(3) Entropy bounds give the correct limiting temperature in their currently known form. Some enforcing mechanism exists which is still unknown.

(4) The application of AdS/CFT correspondence in this particular context is more complicated when the brane approaches the horizon and falls into the BH. When modified appropriately, for example, by correctly taking into account the influence of the Hawking radiation pressure or the growth of perturbations or the effects of additional induced matter on the brane, entropy bounds remain valid in their currently known forms.

(5) Both the application of AdS/CFT correspondence in this specific context and the currently known entropy bounds need to be modified for temperatures of about $T_{\max}$.

At this point in time we do not have a clear preference or a clear indication from our calculations as to which of these possibilities is correct. We hope that future research will help to resolve the issues that we have discussed.

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