Black strings in our world

Hiroyuki Nakano,[1] Ken-ichi Nakao,[1] and Masahide Yamaguchi[2]

[1]*Department of Mathematics and Physics, Graduate School of Science, Osaka City University, Osaka 558-8585, Japan*
[2]*Department of Physics and Mathematics, Aoyama Gakuin University, Sagamihara 229-8558, Japan*

The brane-world scenario is a new approach to resolve the problem on how to compactify the higher dimensional spacetime to our 4-dimensional world. One of the remarkable features of this scenario is the higher dimensional effects in classical gravitational interactions at short distances. Because of this feature, there are black-string solutions in our 4-dimensional world. In this paper, assuming the simplest model of complex minimally coupled scalar field with the local $U(1)$ symmetry, we show a possibility of black-string formation by merging processes of type I long cosmic strings in our 4-dimensional world. No fine tuning for the parameters in the model might be necessary.

I. INTRODUCTION

The brane-world scenario is a model inspired by superstring theory which is one of the most promising theories for the unification of all elementary interactions including gravity. The basic idea of this scenario has been given by Arkani-Hamed, Dimopoulos, and Dvali; the matter and gauge fields except for gravitational field are confined within the brane which is a 4-dimensional timelike submanifold in the higher dimensional spacetime [1,2]. Then putting their basic idea to account, Randall and Sundrum gave very original two models for compactification of higher dimensional spacetime to our 4-dimensional world [3,4]. In this paper, we assume RS model II in which there is a 4-dimensional positive tension brane embedded in 5-dimensional spacetime called bulk [4]. In order to describe the bulk dimension, we adopt Gaussian normal coordinate w which is set so that the brane is located at $w = 0$. Then in the framework of this model, the spacetime geometry is determined by the following 5-dimensional Einstein equations,

$$G_{ab} = \frac{8\pi}{M_{\rm pl}^3}(T_{ab} - \sigma h_{ab})\delta(w) - \Lambda_5 g_{ab}, \quad (1)$$

where g_{ab}, G_{ab}, $M_{\rm pl}$, Λ_5 are the 5-dimensional metric tensor, Einstein tensor, Planck mass and cosmological constant, respectively, σ is the tension of the brane, T_{ab} is the stress-energy tensor of the matter or gauge fields confined within the brane, h_{ab} is the intrinsic metric of the brane, $\delta(w)$ is the Dirac's delta function, and we have adopted the natural unit $c = \hbar = 1$. In accordance with the Randall and Sundrum, we assume the model of $8\pi\sigma/M_{\rm pl}^3 = \sqrt{-6\Lambda_5}$, where Λ_5 is assumed to be negative. This model recovers the 4-dimensional Einstein gravity for long range gravitational interaction within the brane.

The 5-dimensional Planck mass $M_{\rm pl}$ is related to the 4-dimensional one $m_{\rm pl}(= 1.2 \times 10^{19}$ GeV) by $M_{\rm pl}^3 = m_{\rm pl}^2/l$, where $l = \sqrt{-6/\Lambda_5}$ is the AdS length scale. The experimental constraint on this AdS length is $l < 0.1$ mm[5].

Therefore 5-dimensional Planck mass $M_{\rm pl}$ is then written as

$$M_{\rm pl} = 6.7 \times 10^8 l_{0.1}^{-1/3} \text{ GeV}, \quad (2)$$

where we introduce a normalized AdS length defined by

$$l_{0.1} = \frac{l}{0.1 \text{ mm}}. \quad (3)$$

The brane tension σ is written as

$$\sigma = \frac{3M_{\rm pl}^3}{4\pi l} \simeq (3.4 \times 10^3 l_{0.1}^{-1/2} \text{ GeV})^4. \quad (4)$$

It should be noted that the brane tension σ is much smaller than 5-dimensional Planck scale $M_{\rm pl}^4$ if l is of order 0.1 mm.

In the brane-world scenario, quantum effects in gravity will appear at the energy scale over 5-dimensional Planck mass $M_{\rm pl}$ which may be much smaller than 4-dimensional Planck mass $m_{\rm pl}$. On the other hand, effects of the higher dimension appear in the gravity at the distance scale shorter than the AdS length scale l. Since l is much larger than $M_{\rm pl}^{-1}$ in the case of $l \simeq 0.1$ mm, the higher dimensional effects can be important even in the classical gravitational interactions. One of remarkable features of the higher dimensional Einstein gravity is the existence of black-string solutions which do not exist in 4-dimensional Einstein theory of gravity with the dominant energy conditions on matter fields [6]. Therefore if the distribution of mass-energy is infinitely long but sufficiently thin, a black-string forms also in the RS model II [7]. This is consistent with the extended version of hoop conjecture [8].

One of the present authors, Nakamura and Mishima showed that a black-string forms in the RS model II if the circumferential radius, i.e., "thickness" of the cylindrically distributed mass within the brane is smaller than its gravitational thickness $r_{\rm g}$ defined by

$$r_{\rm g} := \frac{\mu}{M_{\rm pl}^3}, \quad (5)$$

as long as $\mu/m_{\rm pl}^2 \ll 1$, where μ is the line energy density, i.e., the energy per unit length scale; the inequality $\mu/m_{\rm pl}^2 \ll 1$ is equivalent to the condition that the gravity produced by the brane tension σ is much smaller than that of the black string at the horizon. However, at first glance, it is not likely that such configurations are realized in our universe, even if the brane-world scenario describes our universe. An exception might be a cosmic string and thus in this paper, we focus on it and study the possibility of its gravitational collapse to form a black string.

This paper is organized as follows. In Sec. II, we briefly review the local $U(1)$ gauged cosmic string and give a crude criterion for the formation of a black string by the gravitational collapse of a cosmic string. In Sec. III, we show the scenario of the black-string formation caused by mergers of type I cosmic strings in the expanding universe. Finally, Sec. IV is devoted to summary and discussion about several issues of black strings in the expanding universe.

II. GRAVITATIONAL COLLAPSE OF TYPE I COSMIC STRING

We assume the $U(1)$ gauged cosmic string [9] whose motion is confined within the brane. The Lagrangian density of the complex scalar field ϕ and $U(1)$ gauge field A^μ is

$$\mathcal{L} = -h^{\mu\nu}(\partial_\mu + ieA_\mu)\bar{\phi}(\partial_\nu - ieA_\nu)\phi$$
$$+ \frac{1}{4}F^{\mu\nu}F_{\mu\nu} + V(|\phi|), \qquad (6)$$

where e is a positive real constant, ∂_μ is the partial derivative operator within the brane, $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$ is the field strength tensor, and the potential $V(|\phi|)$ is assumed to be

$$V(|\phi|) = \frac{1}{4}\lambda(|\phi|^2 - \eta^2)^2, \qquad (7)$$

where λ and η are positive real parameters.

In this model, there are two length scales; one is Compton length of the scalar field $r_{\rm s}$ and the other is that of the gauge field $r_{\rm v}$. These are, respectively, given as

$$r_{\rm s} = \frac{1}{\sqrt{\lambda}\eta} \quad \text{and} \quad r_{\rm v} = \frac{1}{\sqrt{2}e\eta}. \qquad (8)$$

Here we introduce a parameter q defined by

$$q: = \frac{2e^2}{\lambda}. \qquad (9)$$

The model of $q > 1$ $(r_{\rm s} > r_{\rm v})$ is called type I, that of $q = 1$ $(r_{\rm s} = r_{\rm v})$ is called Bogomol'nyi limit and the case of $0 < q < 1$ $(r_{\rm s} < r_{\rm v})$ is called type II. The line energy density $\mu_{\rm sl}$ of a single winding cosmic string has been numerically computed and its fitting formula is given as [10]

$$\mu_{\rm sl} = \frac{1.04\pi}{(2q^2)^{0.195}}\eta^2 = :\alpha(q)\eta^2. \qquad (10)$$

The above formula is valid to about 5% accuracy in the range $0.01 < 2q^2 < 100$. Although this formula is derived by assuming no self-gravity, this might be sufficient for our purpose to get a crude criterion for black-string formation.

First we consider black-string formation by a cosmic string of single winding number. The thickness of a type I cosmic string is given by $r_{\rm s}$, while that of Bogomol'nyi limit or of type II cosmic string is equal to $r_{\rm v}$. Hence the type I cosmic string forms into a black string if $r_{\rm s} \lesssim r_{\rm g}$, while in the case of Bogomol'nyi limit and type II cosmic string, the condition on black-string formation is given by $r_{\rm v} \lesssim r_{\rm g}$. By Eqs. (5), (8) and (10), these conditions lead to

$$\left(\frac{\eta}{M_{\rm pl}}\right) \gtrsim \frac{1}{\omega^{1/6}\alpha^{1/3}} \qquad (11)$$

with

$$\omega = \begin{cases} \lambda & \text{for type I,} \\ 2e^2 & \text{for Bogomol'nyi limit or type II.} \end{cases} \qquad (12)$$

Therefore in order that a single winding cosmic string collapses to form into a black string, the phase transition should occur at the energy scale almost equal to or larger than 5-dimensional Planck scale $M_{\rm pl}$ if λ and $2e^2$ are of order unity. The phase transition over Planck scale seems to be unnatural. The parameters λ for type I and $2e^2$ for Bogomol'nyi limit or type II should be larger than 10^6 so that the breaking scale η is sufficiently lower than 5-dimensional Planck scale $M_{\rm pl}$. However, on the physical ground, it is preferable that dimensionless parameters in the physical model are of order unity. Therefore in the following discussion, we assume moderate values for λ and $2e^2$ and do not consider black-string formation by a single winding cosmic string.

Large line energy density and small thickness of a cosmic string is necessary for black-string formation by its gravitational collapse. Hence multiple winding cosmic strings will be important. In fact, the line energy density of a multiple winding cosmic string is larger than that of a single winding one while its thickness is almost the same as that of a single winding one [9,11]. The multiple winding cosmic strings in type I model are stable while those of Bogomol'nyi limit or type II models are unstable and will decay into single winding ones. Therefore in the following discussion, we focus on type I cosmic strings. The line energy density $\mu_{\rm ml}(n)$ of a cosmic string with the winding number n might be written in the form,

$$\mu_{\rm ml}(n) = \beta(n)n\mu_{\rm sl} = \alpha(q)\beta(n)n\eta^2, \qquad (13)$$

where $\beta(n)$ represents the effect of the interaction energy and should be less than unity due to the stable nature of the type I multiple winding cosmic string. Then, in the case of a multiple winding cosmic string, the condition for black-

string formation is given by

$$n > n_{\rm c} \equiv \frac{1}{\alpha\beta\sqrt{\lambda}} \left(\frac{M_{\rm pl}}{\eta}\right)^3. \tag{14}$$

If the winding number n of a cosmic string is larger than the critical value $n_{\rm c}$ at its formation, it would collapse and a black string would be formed. However, the production probability of multiple winding cosmic strings by Kibble mechanism is very small [12]. Hence we investigate the possibility of mergers of type I long cosmic strings in the expanding universe as a formation process of a highly multiple winding cosmic string.

III. FORMATION OF BLACK STRINGS BY MERGER OF COSMIC STRINGS

Bettencourt and Kibble discussed the merger of type I long strings and analytically estimated escape velocities for various intersection angles and the model parameter q [13]. The numerical simulations were performed by Bettencourt, Laguna and Matzner [14] and their results are basically consistent to the previous analytic estimation. For example, in the case of $q > 2$, the escape velocity between the cosmic strings of intersection angle $30° - 60°$ is about 0.1. Since each collision will be highly inelastic, the merger can occur even if the colliding velocity is larger than 0.1. According to the numerical simulations of cosmological evolution of type II cosmic strings, typical velocity of cosmic string is $0.63 - 0.67$ during radiation dominated era and $0.57 - 0.65$ during matter dominated era [15–17]. However, in these simulations, peculiar nature of type I cosmic strings is not considered adequately. Thus, the above results are not necessarily applicable to type I cosmic strings. Furthermore, it is pointed out that the estimated velocity includes not only coherent velocity but also that due to small scale motion so that coherent velocity is much smaller and could be 0.15 [18]. This means that the merging probability is not so small. Therefore, the winding number might grow as a result of merger, though the string network basically grows according to the scaling solution because loops are also produced through intercommutation. Then, we will investigate the development of a typical winding number in the context of scaling evolution of the cosmic string network, in which the typical network of cosmic strings grows with the cosmic time.[1]

The homogeneous and isotropic universe model in the framework of the RS-model II is considered here. Assuming the bulk is the exact 5-dimensional *AdS* spacetime, the equation for the scale factor a of the Friedmann brane is given by

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[1]Such scaling property is confirmed not only for local strings [15,16,19,20] but also for global strings [21] and global monopoles [22].

$$\left(\frac{\dot{a}}{a}\right)^2 = \left(\frac{4\pi}{3M_{\rm pl}^3}\right)^2 \rho(2\sigma + \rho) - \frac{k}{a^2} + \frac{\Lambda_4}{3}, \tag{15}$$

where the dot means the time derivative, ρ is the energy density of the matter and radiation, k is the sign of the spatial curvature and Λ_4 is the 4-dimensional cosmological constant [23]. The ρ-square term represents the higher dimensional effect and thus if ρ is larger than the brane tension σ but smaller than the 5-dimensional Planck scale $M_{\rm pl}^4$, the higher dimensional classical gravity is realized. In the present setting, σ is given by Eq. (4) and thus, when the temperature of the universe T is much higher than $\sigma^{1/4}$, the ρ-square term becomes dominant. On the other hand, it is negligible when $T \ll \sigma^{1/4}$.

Before the cosmic string network obeys the scaling solution, the friction coming from interactions with surrounding plasma dominates its dynamics. The transition temperature $T_{\rm tr}$ from the friction dominated era to the scaling regime is determined by the balance between the Hubble parameter and the friction coefficient $T_{\rm tr}^3/\nu\mu$, where ν is a numerical factor related to the coupling between the cosmic string and surrounding plasma and takes a value of $1 - 10^3$. Assuming that the ρ-square term is negligible and single winding strings are dominant, the transition temperature is given and constrained from above, as follows.

$$T_{\rm tr} \simeq \frac{\nu\mu_{\rm sl}}{m_{\rm pl}} \ll \frac{\nu M_{\rm pl}^2}{m_{\rm pl}} \simeq 3.6 \times 10^{-2} \frac{\nu}{l_{0.1}^{2/3}} \text{ GeV}. \tag{16}$$

Thus, unless l is too small, $T_{\rm tr}$ is much lower than $\sigma^{1/4}$. Then, we assume that the ρ-square term is subdominant at the transition time.

In the scaling regime, the energy density of the long cosmic strings is given by

$$\rho_\infty = \frac{\mu_{\rm ml}(n)}{L^2}, \tag{17}$$

where L is a scale characterizing the cosmic string network and is proportional to the cosmic time in the manner $L = \gamma t$ with a positive constant γ. In this regime, mean coherent velocity v of long cosmic strings is constant. Then denoting the merging probability per collision by p, the evolution equation for the number N of mergers per the characteristic length scale L is given by

$$\frac{dN}{dt} = \frac{pv}{L}(N + 1) + \frac{Z}{L}\frac{dL}{dt}N, \tag{18}$$

where Z is non-negative parameter smaller than or equal to unity. The first term of R.H.S. in the above equation comes from the increase of merger number due to collisions, while the second term describes the evolution of configurations of cosmic strings already interconnected to each other. A merger of two cosmic strings at $t = t_{\rm m}$ might produce a segment interconnected to each other over the

characteristic length scale γt_{m} at the moment of this merger. There is large uncertainty on the evolution of the interconnected segment. If the length of interconnection does not change from γt_{m} even for $t > t_{\mathrm{m}}$, the parameter Z should vanish. In this case, the length scale over which N cosmic strings are interconnected to each other is the characteristic length γt_1, where $t = t_1$ is the moment of the first merger. By contrast, if the interconnected segment evolves in the manner of a zipper [13,14], all of the cosmic strings attached to the interval of the characteristic length γt of some cosmic string might completely merge into a cosmic string over the length scale γt. In this case, the parameter Z should be taken unity.

We assume that the probability p is constant. Then the solution of Eq. (18) is given by

$$N = \frac{P}{P+Z}\left[\left(1 + \frac{P+Z}{P}N_{\mathrm{tr}}\right)\left(\frac{t}{t_{\mathrm{tr}}}\right)^{P+Z} - 1\right], \quad (19)$$

where $P := pv/\gamma$ and N_{tr} is an integration constant which corresponds to the number of mergers at the transition time $t = t_{\mathrm{tr}}$. N_{tr} may take a large value because a typical coherent velocity is smaller in the friction dominated epoch so that the merger may happen more often. However, the evolution of the cosmic string network in the friction dominated era strongly depends on its distribution at its formation. So, we do not mention this any more. Instead, we simply assume that N_{tr} vanishes to give a conservative estimate because the main purpose of the present paper is to show that black strings can be actually produced in our universe.

Since the winding number n randomly increases by each merger because a string encounters a string or an antistring with equal probability, it is estimated as $n \simeq N^{1/2}$. When a winding number exceeds the critical value n_c, black strings are formed. Assuming the formation time t_{f} is much larger than the transition time t_{tr}, t_{f} is estimated from Eqs. (14) and (19) as

$$t_{\mathrm{f}} = t_{\mathrm{tr}}\left[\frac{P+Z}{P\alpha^2\beta^2\lambda}\left(\frac{M_{\mathrm{pl}}}{\eta}\right)^6\right]^{1/(P+Z)}. \quad (20)$$

At the formation, the ratio of the energy density of black strings ρ_{BS} to the total energy density of the universe ρ_{tot} is estimated as

$$\frac{\rho_{\mathrm{BS}}}{\rho_{\mathrm{tot}}}(t_{\mathrm{f}}) = \left(\frac{n_c\alpha\beta\eta^2}{\gamma^2 t_{\mathrm{f}}^2}\right)\left(\frac{3m_{\mathrm{pl}}^2\epsilon^2}{8\pi t_{\mathrm{f}}^2}\right)^{-1}$$

$$= 2.5 \times 10^{-20}\frac{M_{\mathrm{pl}}}{\epsilon^2\gamma^2\lambda^{1/2}l_{0.1}^{2/3}\eta} \quad (21)$$

with $\epsilon = 1/2$ for the case of the formation in the radiation dominant era and $\epsilon = 2/3$ for the formation in the matter dominant era. In the following estimations, we assume the cosmic concordance model of density parameter of non-relativistic matter $\Omega_{\mathrm{M}} \simeq 0.3$ and that of the cosmological constant $\Omega_{\Lambda} \simeq 0.7$ since recent observational results are

consistent with this model. Therefore the formation of black strings in the cosmological constant dominant era is possible but we do not consider it here.

In the present scenario, the winding number is not constant along a cosmic string. In the case of nonzipper evolution, the winding number varies on the length scale γt_1, while in the case of complete zipper, its variation scale evolves as γt. Then segments with sufficiently large winding number will be enclosed by horizons which are compact and disconnected to each other. Therefore, exactly speaking, the formed black objects are not strings but rather thin worms of finite lengths. However, once a black worm is formed on a segment of large winding number, it will grow to indefinitely long configuration by swallowing up the cosmic strings interconnected to this black worm and hence it might as well be called a black string.

Black strings are formed in the radiation dominated era when $\eta \geq \eta_{\mathrm{RD}}$, where

$$\frac{\eta_{\mathrm{RD}}}{M_{\mathrm{pl}}} \equiv (8.8 \times 10^{-16})^{P+Z/4(P+Z)+6}\mathcal{F} \quad (22)$$

with

$$\mathcal{F} \equiv \left[\left(\frac{l_{0.1}^{4/3}}{\mathcal{N}_{\mathrm{tr}}^{1/2}\nu^2}\right)^{P+Z}\frac{P+Z}{\alpha^{2(P+Z+1)}\beta^2\lambda P}\right]^{1/[4(P+Z)+6]}. \quad (23)$$

Here, $\mathcal{N}_{\mathrm{tr}}$ is a factor coming from the effective number of distinct helicity states at the transition time. The above condition has been obtained from the condition that t_{f} is smaller than or equal to the matter-radiation equality time $t_{\mathrm{eq}} \simeq (16\pi^3 T_{\mathrm{eq}}^4/(45m_{\mathrm{pl}}^2))^{-1/2}/2$ and we have used $T_{\mathrm{eq}} \simeq 0.76$ eV. On the other hand, the condition that black strings are formed by now is obtained from the condition $t_{\mathrm{f}} \leq t_0$ which leads to $\eta \geq \eta_{\mathrm{MD}}$, where

$$\frac{\eta_{\mathrm{MD}}}{M_{\mathrm{pl}}} \equiv \left(4.2 \times 10^{-21} \times \frac{14 \text{ Gyr}}{t_0}\right)^{P+Z/[4(P+Z)+6]}\mathcal{F} \quad (24)$$

and t_0 is the present cosmic time. Note that η_{RD} and η_{MD} are smaller as l is smaller. Here we have used the relation $t_{\mathrm{tr}} = (4\pi^3 \mathcal{N}_{\mathrm{tr}}T_{\mathrm{tr}}^4/(45m_{\mathrm{pl}}^2))^{-1/2}/2$.

A. Non-Zipper Evolution

First we consider the case of nonzipper evolution $Z = 0$. Though there are various unknown physics, we assume the merger probability p, typical coherent velocity v and the parameter γ determining characteristic length L so that

$$P = 0.4\left(\frac{p}{0.8}\right)\left(\frac{v}{0.15}\right)\left(\frac{0.3}{\gamma}\right). \quad (25)$$

Substituting this value into Eqs. (22) and (24), we obtain

$$\frac{\eta_{\mathrm{RD}}}{M_{\mathrm{pl}}} = 0.16\mathcal{F}_{\mathrm{nz}}, \quad (26)$$

$$\frac{\eta_{\mathrm{MD}}}{M_{\mathrm{pl}}} = 8.5 \times 10^{-2} \left(\frac{14 \text{ Gyr}}{t_0}\right)^{1/19} \mathcal{F}_{\mathrm{nz}}, \tag{27}$$

where

$$\mathcal{F}_{\mathrm{nz}} \equiv l_{0.1}^{4/57} \mathcal{N}_{\mathrm{tr}}^{-1/38} \nu^{-2/19} \alpha^{-7/19} \beta^{-5/19} \lambda^{-5/38}. \tag{28}$$

Note that although η_{RD} and η_{MD} are smaller as l is smaller, their dependence on l is very weak. It is easily seen from the above equation that the dependence of η_{RD} and η_{MD} on α is the strongest while their dependence on the other unknown parameters $\mathcal{N}_{\mathrm{tr}}$, ν, β and λ is very weak. Therefore if α is order of unity, we may say that if the merger probability p is sufficiently large, the formation of black strings is possible even though the breaking scale η is much smaller than the 5-dimensional Planck scale.

B. Zipper Evolution

In the case of the zipper evolution, we assume that P is much smaller than unity for conservative estimation. Then we find

$$\frac{\eta_{\mathrm{RD}}}{M_{\mathrm{pl}}} = 3.1 \times 10^{-2} \mathcal{F}_{\mathrm{zp}}, \tag{29}$$

$$\frac{\eta_{\mathrm{MD}}}{M_{\mathrm{pl}}} = 9.2 \times 10^{-3} \left(\frac{14 \text{ Gyr}}{t_0}\right)^{1/10} \mathcal{F}_{\mathrm{zp}}, \tag{30}$$

where

$$\mathcal{F}_{\mathrm{zp}} \equiv l_{0.1}^{2/15} \mathcal{N}_{\mathrm{tr}}^{-1/20} \nu^{-1/5} \alpha^{-2/5} \beta^{-1/5} \lambda^{-1/10} P^{-1/10}. \tag{31}$$

Also in this case, the dependence of η_{RD} and η_{MD} on α is the strongest. Here it should be noted that the dependence of η_{RD} and η_{MD} on P is very weak. This means that even though the probability p of a merger is very small, large winding number of a cosmic string might be realized and as a result, the black string might form even for rather small breaking scale η if the interconnected segments sufficiently rapidly evolve in the manner of zipper.

C. Present Energy Density of Black Strings

Once black strings are formed, reconnection and formation of loops cease. Therefore, the energy density of black strings is not suppressed in the same manner as cosmic strings, instead its energy density might decrease in proportional to the inverse squared of the scale factor. Then, the present energy density of black strings is estimated as

$$\frac{\rho_{\mathrm{BS}}}{\rho_{\mathrm{tot}}}(t_0) = \frac{\rho_{\mathrm{BS}}}{\rho_{\mathrm{tot}}}(t_f) \left(\frac{a(t_{\mathrm{eq}})}{a(t_f)}\right)^{(2-2\epsilon)/\epsilon} (1 + z_{\mathrm{eq}})(1 + z_\Lambda)^{-3}, \tag{32}$$

where z_{eq} is the redshift of the matter-radiation equality time, which is estimated to be 3233 [24], while z_Λ is the redshift of matter-Λ equality time estimated to be 0.33. Therefore in the case of the black-string formation in the radiation dominant era, the present energy density of the black string is given by

$$\frac{\rho_{\mathrm{BS}}}{\rho_{\mathrm{tot}}}(t_0) = 0.16 \mathcal{N}_{\mathrm{tr}}^{\frac{1}{2}} l_{0.1}^{-2} \nu^2 \gamma^{-2} \left[\alpha^{2(P+Z+1)} \beta^2 \lambda^{2-(P+Z)/2} \left(\frac{\eta}{M_{\mathrm{pl}}}\right)^{3(P+Z+2)} \left(\frac{P}{P+Z}\right)\right]^{1/(P+Z)}, \tag{33}$$

while in the case of the black-string formation in the matter dominant era, it is given by

$$\frac{\rho_{\mathrm{BS}}}{\rho_{\mathrm{tot}}}(t_0) = 8.4 \times 10^{-7} \mathcal{N}_{\mathrm{tr}}^{1/3} l_{0.1}^{-14/9} \nu^{4/3} \gamma^{-2} \left[\alpha^{4(P+Z+1)} \beta^4 \lambda^{4-3(P+Z/2)} \left(\frac{\eta}{M_{\mathrm{pl}}}\right)^{5(P+Z)+12} \left(\frac{P}{P+Z}\right)^2\right]^{1/3(P+Z)}. \tag{34}$$

Thus, since $\eta < M_{\mathrm{pl}}$, and since $P + Z$ is at most order of unity, the present energy density of black strings is subdominant as long as the parameters α, β, and λ are also order of unity and $l_{0.1}$ is not too small. However we should note that in contrast to η_{RD} and η_{MD}, the energy density ρ_{BS} strongly depends on these unknown parameters. Thus it is possible that the black strings are a dominant component in the present universe.

IV. SUMMARY AND DISCUSSION

In this paper, we discuss the black-string formation by considering multiple winding strings. Though the formation probability of such strings is small at the formation of strings, they can be formed through merging processes of

type I cosmic strings. Based on several numerical simulations of type I cosmic strings, we discuss the formation of multiple winding cosmic strings and subsequent collapse into black strings. We show that black strings can be formed in our 4-dimensional world for reasonable sets of parameters. However, in the numerical simulations of type I cosmic strings, the setting of collisions of two strings is given by hand and far from realistic. Collisions of type I cosmic strings will be investigated for the realistic evolution in the expanding universe.

A black string is a source of gravitational radiation. The shortest wavelength of the gravitational waves might be determined by Gregory-Laflamme instability [25]. The Schwarzschild black string solution is unstable against linear perturbations of the wavelength longer than several

times of r_g. In the present scenario, the gravitational thickness r_g cannot exceed the *AdS* length l. If $r_g > l$, the gravity near the horizon of the black string is that for the 4-dimensional spacetime. This fact leads to an inconsistency since there is no black-string solution in 4-dimensional gravity. Thus black strings might emit gravitational waves of various wavelength larger than some very short lower cutoff which must be less than l.

Like as a cosmic string, self-intersections of a black string will occur. In this case, black loops are produced but those might be kept attaching to their parent black string and then the loop structures may disappear by "shrinking". If the black loops leave their parent black string, naked singularities appear. At present, it is not clear whether isolated loops are released from their parent black string. However, we may say that such a process will produce much gravitational waves. The intersections between individual black strings may also produce very complicated network and much gravitational radiation. The spectrum and amplitude of such a gravitational radiation should be studied in detail and this is a future work.

It is worthwhile to note that the condition of black-string formation is identical to that for the topological inflation [26]. However, in the present case, the deflation rather than the inflation may be realized. Farhi and Guth presented a proof for the impossibility of inflation in the laboratory [27]. Although the present situation is not identical to the assumption in their proof, the same result might be obtained. Therefore, inside the black-string horizon, the multiple winding cosmic string might form spacetime singularities rather than the child universes [28].

The evaporation of the black string due to Hawking effect is also an important problem. Here we should note that the black string produced by the multiple winding cosmic string also has a winding number which is observable for outside observers and is a topological invariant. Thus in order that the black string with the nonvanishing winding number evaporates completely, the winding number should be released. This means that cosmic strings with some winding numbers should be created outside the horizon through the quantum processes in order for the complete evaporation. However the creation probability of infinite cosmic strings seems to be very small and therefore the black string considered in this paper might not evaporate completely, although detailed study is necessary.

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