PHYSICAL REVIEW D **71,** 123512 (2005)

An emergent universe from a loop

David J. Mulryne,[1] Reza Tavakol,[1] James E. Lidsey,[1] and George F. R. Ellis[2]

[1]*Astronomy Unit, School of Mathematical Sciences, Queen Mary, University of London, London E1 4NS, United Kingdom*

[2]*Department of Applied Mathematics, University of Cape Town, Cape Town, South Africa*

(Received 28 February 2005; published 9 June 2005)

Closed, singularity-free, inflationary cosmological models have recently been studied in the context of general relativity. Despite their appeal, these so called emergent models suffer from a number of limitations. These include the fact that they rely on an initial Einstein static state to describe the past-eternal phase of the universe. Given the instability of such a state within the context of general relativity, this amounts to a very severe fine tuning. Also in order to be able to study the dynamics of the universe within the context of general relativity, they set the initial conditions for the universe in the classical phase. Here we study the existence and stability of such models in the context of Loop Quantum Cosmology and show that both these limitations can be partially remedied, once semiclassical effects are taken into account. An important consequence of these effects is to give rise to a static solution (not present in GR), which dynamically is a center equilibrium point and located in the more natural semiclassical regime. This allows the construction of emergent models in which the universe oscillates indefinitely about such an initial static state. We construct an explicit emergent model of this type, in which a nonsingular past-eternal oscillating universe enters a phase where the symmetry of the oscillations is broken, leading to an emergent inflationary epoch, while satisfying all observational and semiclassical constraints. We also discuss emergent models in which the universe possesses both early- and late-time accelerating phases.

DOI: 10.1103/PhysRevD.71.123512 PACS numbers: 98.80.Cq, 04.60.Pp

I. INTRODUCTION

One of the fundamental questions of modern cosmology is whether the universe had a definite origin or whether it is past eternal. The central paradigm for structure formation in the universe is the inflationary scenario. (For a review, see, e.g., Ref. [1]). Under very general conditions, inflation is future eternal, in the sense that once inflation has started, most of the volume of the universe will remain in an inflating state [2]. Given certain assumptions, however, it has been argued [3] that inflation can not be past null complete—and therefore past eternal—if it is future eternal.

Leading alternatives to inflation that are motivated by recent advances in string/M-theory are the pre-big bang [4,5] and ekpyrotic/cyclic [6] scenarios, respectively. The fundamental postulate of the pre-big bang model is that the set of initial data for the universe lies in the infinite past in the perturbative regime of small string coupling and spacetime curvature [4]. The universe then evolves into a strongly coupled and highly curved regime before exiting into the standard, post–big bang phase.

On the other hand, the big bang singularity is interpreted in the cyclic scenario in terms of the collisions of two codimension one branes propagating in a higher-dimensional spacetime [6]. In this picture, the branes undergo an infinite sequence of oscillations where they move towards and subsequently away from each other. From the perspective of a four-dimensional observer, the collision of the branes is interpreted as the bounce of a collapsing universe into a decelerating, post-big bang expansion.

Despite these attractive properties, however, the process that leads to a nonsingular transition between the pre- and post-big bang phases is unclear. A further intriguing possibility is the ‘‘Cosmological Natural Selection‘‘ scenario based on the idea that a baby universe may form inside a black hole [7]. However, such a model does not completely address the singularity problem either and furthermore it is a different type of scenario as it involves variations in the fundamental constants of physics.

The search for singularity-free inflationary models within the context of classical General Relativity (GR) has recently led to the development of the emergent universe scenario [8,9]. In this model, the universe is positively curved and initially in a past-eternal Einstein static (ES) state that eventually evolves into a subsequent inflationary phase. Such models are appealing since they provide specific examples of nonsingular (geodesically complete) inflationary universes. Furthermore, it has been proposed that entropy considerations favor the ES state as the initial state for our universe [10].

Classical emergent models, however, suffer from a number of important shortcomings. In particular, the instability of the ES state (represented by a saddle equilibrium point in the phase space of the system) makes it extremely difficult to maintain such a state for an infinitely long time in the presence of fluctuations, such as quantum fluctuations, that will inevitably arise. Moreover, the initial ES state must be set in the classical domain in order to study the dynamics within the context of GR. A more natural choice for the initial state of the universe would, however, be in the semiclassical or quantum gravity regimes.

A leading framework for a nonperturbative theory of quantum gravity is loop quantum gravity (LQG). (For reviews, see, e.g., Ref. [11,12]). Recently, there has been considerable interest in loop quantum cosmology (LQC), which is the application of LQG to symmetric states [13]. Within the framework of LQC, there can exist a 'semiclassical' regime, where spacetime is approximated by a continuous manifold, but where nonperturbative quantum corrections modify the form of the classical Einstein field equations [14].

Motivated by the above discussion, the present paper studies the existence and nature of static solutions within the framework of semiclassical LQC in the presence of a self-interacting scalar field. Significant progress has recently been made in understanding the dynamics of this system, and its relevance to inflation and nonsingular behavior [15–22]. In addition to the ES universe of classical cosmology, LQC effects result in a second equilibrium point in the phase space that we refer to as the "loop static" (LS) solution. Crucially, this solution corresponds to a *center* equilibrium point in the phase space and, consequently, the universe may undergo a series of (possibly) infinite, non–singular oscillations about this point. During these oscillations, the scalar field can be *driven up* its potential [18]. This leads us to consider a new picture for the origin of the universe, where the universe is initially oscillating about the LS static solution in the infinite past and eventually emerges into a classical inflationary era. The model shares some of the attractive features of the pre–big bang and cyclic scenarios, in the sense that it is past eternal, although it exhibits a significant difference in that the cycles are broken by inflationary expansion. Additionally, unlike the pre-big bang and cyclic scenarios, the emergent model considered here is genuinely nonsingular.

The paper is organized as follows. The existence and nature of static universes in semiclassical LQC is studied in Sec. II. Section III discusses the dynamics that leads to the emergence of an inflationary universe and a particular inflationary model that is consistent with presentday cosmological observations is outlined in Sec. IV. In Sec. V we discuss a class of models where the field that drives the preinflationary oscillations may also act as the source of dark energy in the presentday universe. We conclude with a discussion in Sec. VI.

II. STATIC UNIVERSES

A. Einstein static universe in classical gravity

Before considering static solutions within the context of LQC, it is instructive to review the corresponding results for classical Einstein gravity.

Throughout this paper, we consider an isotropic and homogeneous Friedmann-Robertson-Walker (FRW) universe sourced by a scalar field with energy density and pressure given by $\rho = \frac{1}{2}\dot{\phi}^2 + V(\phi)$ and $p = \frac{1}{2}\dot{\phi}^2 - V(\phi)$, respectively, where $V(\phi)$ represents the self-interaction potential of the field. The Raychaudhuri equation for such a universe is given by

$$\frac{\ddot{a}}{a} = -\frac{8\pi l_{\rm Pl}^2}{3}[\dot{\phi}^2 - V(\phi)], \tag{1}$$

and local conservation of energy-momentum implies the scalar field satisfies

$$\ddot{\phi} + 3H\dot{\phi} + V'(\phi) = 0, \tag{2}$$

where a prime denotes differentiation with respect to the field. Eqs. (1) and (2) together admit the first integral (the Friedmann equation):

$$H^2 = \frac{8\pi l_{\rm Pl}^2}{3}[\dot{\phi}^2 + V(\phi)] - \frac{K}{a^2}, \tag{3}$$

where as usual $K = 0, \pm 1$ parametrizes the spatial curvature. Combining Eq. (3) with Eq. (1) yields

$$\dot{H} = -4\pi l_{\rm Pl}^2 \dot{\phi}^2 + \frac{K}{a^2}. \tag{4}$$

The Einstein static universe is characterized by the conditions $K = 1$ and $\dot{a} = 0 = \ddot{a}$. In the presence of a scalar field with a constant potential V, it is straightforward to verify that there exists a unique solution satisfying these conditions, where the scale factor is given by $a = a_0$ with $a_0 = (4\pi l_{\rm Pl}^2 \dot{\phi}^2)^{1/2}$. There are two important points to note regarding this solution. Firstly, it represents a saddle equilibrium point in the phase space, which is unstable to linear perturbations. Secondly, in order to enable the analysis to be performed within the context of GR, it is necessary to assume that a_0 lies in the classical domain.

In the emergent universe scenario, it is assumed that the potential becomes asymptotically flat in the limit $\phi \rightarrow -\infty$ and that the initial conditions are specified such that the ES configuration represents the past-eternal state of the universe, out of which the universe slowly evolves into an inflationary phase. However, the instability of the ES universe ensures that any perturbations—no matter how small—rapidly force the universe away from the ES state, thereby aborting the scenario.

In the next subsection we shall see that employing a more general LQC setting can partially resolve both these issues.

B. Static solutions in semiclassical LQC

It has recently been shown that in the semiclassical regime the cosmological evolution equations become modified [14]. When restricted to FRW backgrounds, this regime is characterized by the scale factor of the universe lying in the range $a_i < a < a_*$, where $a_i \equiv \sqrt{\gamma} l_{\rm Pl}$ determines the scale for the discrete quantum nature of spacetime to become important and $\gamma \approx 0.274$ is the Barbero-Immirzi parameter [23], and $a_* \equiv \sqrt{\gamma j/3} l_{\rm Pl}$, where j is a parameter that arises due to ambiguities in the quantization procedure [24]. It must take positive, half-integer values but is otherwise arbitrary [1]. The standard classical cosmology is recovered above the scale a_* and the parameter j therefore sets the effective quantum gravity scale. The dynamics in this regime is determined by replacing the inverse volume term a^{-3} that arises in the classical matter Hamiltonian $\mathcal{H}_\phi = \frac{1}{2}a^{-3}p_\phi^2 + a^3V(\phi)$ with a continuous function $d_j(a)$ that approximates the eigenvalues of the geometrical density operator in LQC. This "quantum correction" function is given by $d_j(a) \equiv D(q)a^{-3}$, where

$$D(q) = \left(\frac{8}{77}\right)^6 q^{3/2}\{7[(q+1)^{11/4} - |q-1|^{11/4}] - 11q[(q+1)^{7/4} - \mathrm{sgn}(q-1)|q-1|^{7/4}]\}^6, \quad (5)$$

and $q \equiv (a/a_*)^2$. As the universe evolves through the semiclassical phase, this function varies as $D \propto a^{15}$ for $a \ll a_*$, has a global maximum at $a \approx a_*$, and falls monotonically to $D = 1$ for $a > a_*$.

The effective field equations then follow from the Hamiltonian [14]:

$$\hat{\mathcal{H}} = -\frac{3}{8\pi\ell_{\rm Pl}^2}(\dot{a}^2 + K)a + \frac{1}{2}d_j p_\phi^2 + a^3V = 0, \quad (6)$$

where $p_\phi = d_j^{-1}\dot{\phi}$ is the momentum canonically conjugate to the scalar field. The Raycharduri equation becomes

$$\frac{\ddot{a}}{a} = -\frac{8\pi l_{\rm Pl}^2}{3D}\dot{\phi}^2\left(1 - \frac{1}{4}\frac{d\ln D}{d\ln a}\right) + \frac{8\pi l_{\rm Pl}^2}{3}V(\phi), \quad (7)$$

and the modified scalar field equation takes the form

$$\ddot{\phi} = -3H\left(1 - \frac{1}{3}\frac{d\ln D}{d\ln a}\right)\dot{\phi} - DV'. \quad (8)$$

The first integral of Eqs. (7) and (8) is given by the modified Friedmann equation

$$H^2 = \frac{8\pi l_{\rm Pl}^2}{3}\left[\frac{\dot{\phi}^2}{2D} + V(\phi)\right] - \frac{K}{a^2}, \quad (9)$$

where in the LQC context K can only take values 0, +1. Combining Eq. (7) with Eq. (9) then implies that

$$\dot{H} = -\frac{4\pi l_{\rm Pl}^2\dot{\phi}^2}{D}\left(1 - \frac{1}{6}\frac{d\ln D}{d\ln a}\right) + \frac{K}{a^2}. \quad (10)$$

Equation (7)–(10) can be rewritten in the form of a three-dimensional dynamical system in terms of variables $\{a, H, V\}$ [21]. (The present study extends the results of [21] and, more importantly, provides an analytical account of the dynamics). Employing the Friedmann Eq. (9) to eliminate the scalar field's kinetic term, and assuming that $dV/d\phi$ can be expressed as a function of the potential, allows the complete dynamical system to be described by equations:

$$\dot{H} = (8\pi l_{\rm Pl}^2 V - 3H^2)\left(1 - \frac{A}{6}\right) + \frac{K}{a^2}\left(\frac{A}{2} - 2\right), \quad (11)$$

$$\dot{a} = Ha, \quad (12)$$

$$\dot{V} = V'(V)\left(\frac{6DH^2}{8\pi l_{\rm Pl}^2} - 2DV + \frac{6D}{8\pi l_{\rm Pl}^2 a^2}\right)^{1/2}, \quad (13)$$

where the function $A(a) \equiv d\ln D/d\ln a$. In this Section, with the aim of considering emergent universes we shall take $K = +1$ and initially consider constant potentials. In this case, Eq. (13) is trivially satisfied and the system reduces to the two-dimensional autonomous system (11) and (12).

The static solutions ($\dot{a} = 0 = \ddot{a}$) then correspond to the equilibrium points of this system, which are given by

$$H_{\rm eq} = 0, \qquad a = a_{\rm eq}, \quad (14)$$

where $a_{\rm eq}$ is given by solutions to the constraint equation

$$A(a_{\rm eq}) = B(a_{\rm eq}), \qquad B(a) \equiv \frac{6(8\pi l_{\rm Pl}^2 Va^2 - 2)}{(8\pi l_{\rm Pl}^2 Va^2 - 3)}. \quad (15)$$

It follows from Eq. (10) that the field's kinetic energy at the equilibrium points is given by

$$\dot{\phi}_{\rm eq}^2 = \frac{D_{\rm eq}}{4\pi a_{\rm eq}^2 l_{\rm Pl}^2(1 - A_{\rm eq}/6)}. \quad (16)$$

Equation (15) implies that a necessary and sufficient condition for the existence of the equilibrium points is that the functions $A(a)$ and $B(a)$ should intersect.

Here we are interested in how the properties of the equilibrium points alter as the potential is varied. This is best seen by considering how the functions $A(a)$ and $B(a)$ (and hence their points of intersection) change. The function A has the form shown in the top panel of Fig. 1. It asymptotes to the constant value $A = 15$ at $a = 0$, decreases to a minimum value of $A_{\rm min} = -5/2$ at $a = a_*$, and then asymptotes to zero from below as $a \to \infty$. Increasing the parameter j increases the value of a_* and this results in moving the turning point of the function A to larger values of the scale factor. The important point to note, however, is that the qualitative form of the function A remains unaltered for all values of j, as can be seen from Fig. 1.

The qualitative nature of the function B, on the other hand, is sensitive to the sign of the potential, as can be seen from the bottom panel of Fig. 1. We briefly discuss the behavior of B for each case in turn. For $V = 0$, it is given by the (solid) horizontal line $B = 4$. For $V > 0$ it represents a hyperbola (dot-dashed curve) with a single (since the scale factor a is non-negative) vertical asymptote given by

[1] More specifically, any irreducible SU(2) representation with spin j may be chosen in the quantization scheme when rewriting the classical scale factor in terms of holonomies. The fundamental representation corresponds to $j = 1/2$.

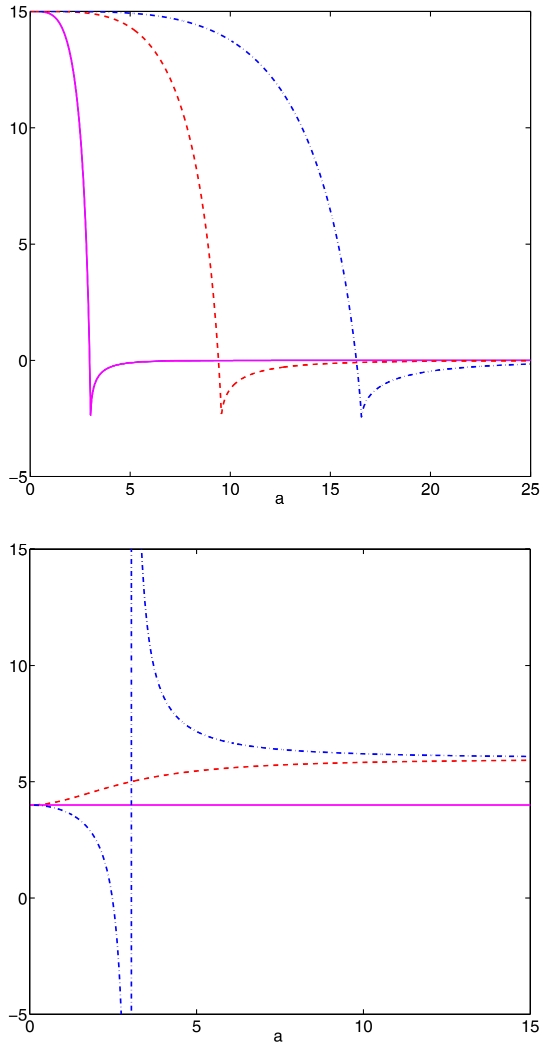

FIG. 1 (color online). The top panel shows the plots for the function A against a for $j = 100$ (solid line), $j = 1000$ (dashed line) and $j = 3000$ (dot-dashed line). The bottom panel shows plots of the function B against a for $V = 0$ (solid line), $V > 0$ (dot-dashed line) and $V < 0$ (dashed line). Note the dramatic change in the shape of the function as the sign of V is altered.

$$a = \sqrt{\frac{3}{8\pi l_{\rm Pl}^2 V}}. \qquad (17)$$

The region to the left of this asymptote defines the region in the $\{A/B, a\}$ plane where the reality condition, $\dot{\phi}^2 > 0$, is satisfied. Thus, the upper-right branch of the hyperbola plays no role in determining the existence of the equilibrium points. In the limit of $a \to 0$, the function $B \to 4$ which coincides with the value of the function in the case $V = 0$. As a is increased, the function takes progressively smaller values as the vertical asymptote is approached. As V tends to zero from above, it causes the asymptote to move to progressively larger values of a, and we may therefore formally view the $V = 0$ case as the limit where the asymptote moves to infinity. Finally, for negative potentials the qualitative behavior of the function B is changed to the dashed line in the bottom panel of Fig. 1 and there is no vertical asymptote. As in the case of positive potentials, $B \to 4$ as $a \to 0$, but the function B now increases monotonically as a increases, ultimately tending to the value 6 as $a \to \infty$. As the potential tends to zero from below, the function B still tends to the asymptotic value $B = 6$, but at larger a. An important point to note is that for *all* values of V, the function B satisfies the condition $B < 6$ (for physically relevant regions of the $\{A/B, a\}$ plane), and hence *equilibrium points can only occur when* $A = B < 6$.

Once the positions of the equilibrium points have been determined, their nature can be found by linearizing about these points. The eigenvalues are given by

$$\lambda^2 = \left[\frac{4 - A}{a^2} + \frac{1}{6a(1 - A/6)}\frac{dA}{da}\right]_{a_{\rm eq}}. \qquad (18)$$

When $\lambda^2 < 0$, this leads to imaginary eigenvalues and implies that the equilibrium point is a center, whereas the point is a saddle when $\lambda^2 > 0$. Although this expression is rather complicated, it can be verified that for $A < 6$ (which is a necessary condition for the existence of equilibrium points), λ^2 is negative for $a < a_*$ and positive for $a > a_*$. Hence we have the important result that, *equilibrium points occurring in the semiclassical regime are centers and those occurring in the classical regime are saddles.*

To determine the existence of equilibrium points, we again consider the different signs of the potential separately.

$V = 0$: In the case of a massless scalar field with $V = 0$, the condition (15) has a single solution given by $A = 4$, implying a single equilibrium point. The position of this equilibrium point can be seen from the intersection point in the top panel of Fig. 2. Since this point occurs at $a < a_*$, it is a center, with its phase portrait consisting of a continuum of concentric orbits (see the bottom panel of Fig. 2).

$V > 0$: As described above, for small positive values of the potential, the vertical asymptote of the function B is far to the right of the origin, which results in two points of intersections between the functions A and B (see the top panel of Fig. 3). There are therefore *two* equilibrium points in this case: the first occurs at $a < a_*$ and hence is a center, while the second occurs at $a > a_*$ and is therefore a saddle. As the potential is increased, the asymptote moves towards the origin causing the equilibrium points to eventually coalesce when $V = V_*$. This occurs at the point of tangency of the functions A and B, i.e. at the minimum (which is a cusp) of the function A located at $a = a_*$. Thus, using B and the fact that $A|_{a_*} = -5/2$, we obtain

$$V_* = \frac{39}{136\pi l_{\rm Pl}^2 a_*^2} = \frac{117}{136\pi\gamma j} m_{\rm Pl}^4. \qquad (19)$$

For values of $V > V_*$, the asymptote moves further towards the origin. Consequently, the functions A and B

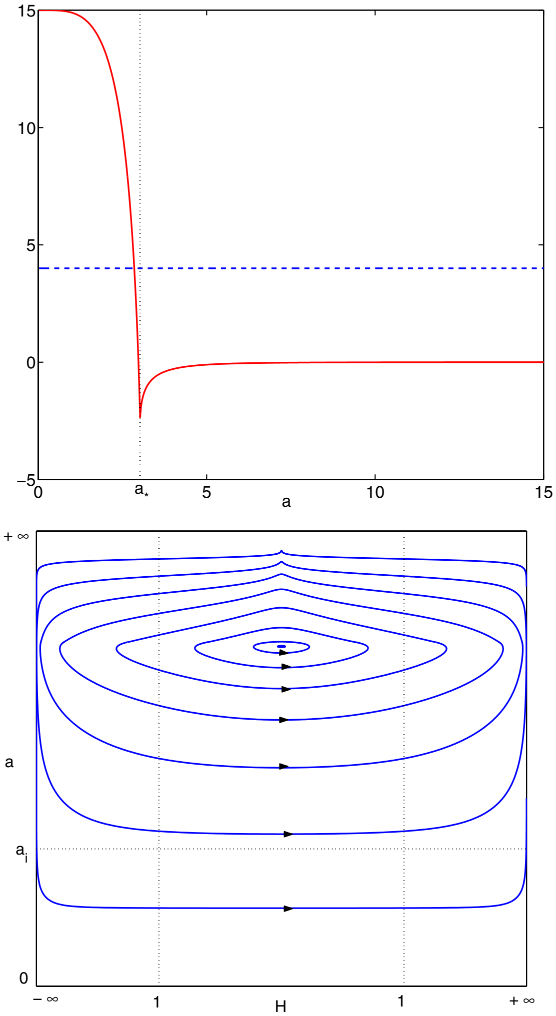

FIG. 2 (color online). The top panel shows the plots of functions A (solid line) and B (dashed line) against a for $V = 0$. The vertical dotted line denotes the position of $a = a_*$. The bottom panel is the corresponding phase portrait demonstrating the center equilibrium point that occurs in this case. The plot is compactified using $x = \arctan(H)$ and $y = \arctan(\ln a)$, in order to present the entire phase space. The vertical dotted lines in the bottom panel demarcate the region in which the Hubble parameter is less than the Planck scale. The horizontal dotted line marks a_i, where the quantum regime begins. All axes are in Planck units, and $j = 100$.

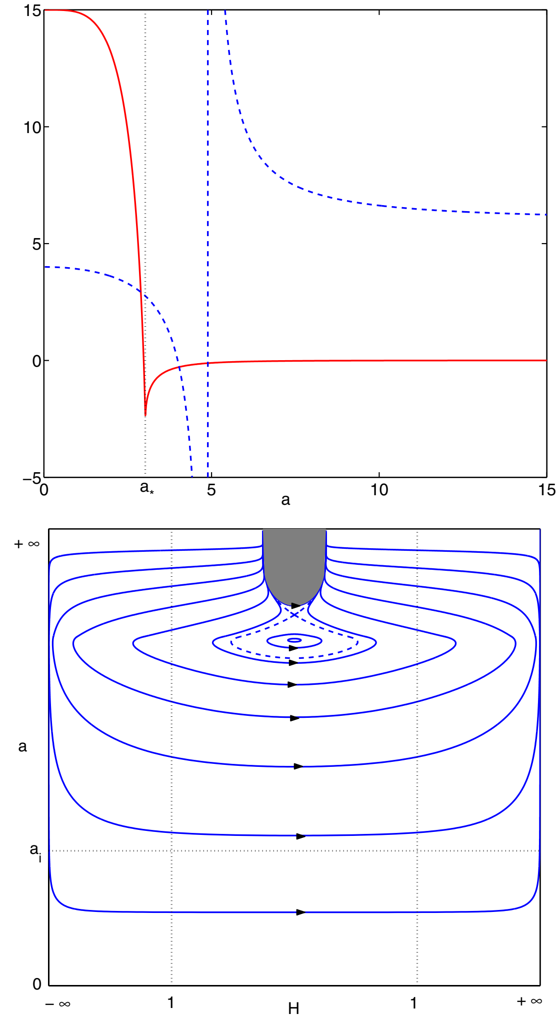

FIG. 3 (color online). The top panel shows the plots of functions A (solid line) and B (dashed line) against a for $V = 0.005$. For this value of V, the equilibrium points are close to coalescing. The vertical dotted line denotes the position of $a = a_*$. The bottom panel is the corresponding phase portrait demonstrating the center equilibrium point and saddle point which occurs in this case with the dashed line indicating the separatrix. The shaded area is the unphysical region where $\dot{\phi}^2$ would be negative. The plot is compactified as in Fig. 2. The dotted lines in the bottom panel are as described in Fig. 2. All axes are in Planck units, and $j = 100$.

will no longer intersect in the part of the plane which satisfies the reality condition, and therefore there will be no equilibrium points in this case. It is also easy to understand the effect of changing the parameter j on V_*. Increasing j increases a_*, which means that the point of tangency occurs for smaller values of the potential. This implies that larger values of the quantization parameter, and hence a_*, lead to smaller values of the potential beyond which no equilibrium points can exist.

$V < 0$: Negative potentials are known to arise in string/M-theory compactifications (e.g., [25]) and are of interest in connection with the ekpyriotic/cyclic models considered recently [6]. In such cases the qualitative change in the form of B results in only one equilibrium point, which occurs inside the semiclassical region and hence is a center (see the bottom panel of Fig. 4). Furthermore, the center equilibrium point will persist for *any* negative value of V.

To summarize, the important consequence of considering LQC effects in this context is that they admit two possible static solutions, rather than the single solution in the case of GR. The first static solution corresponds to a saddle point (as in GR) and is referred to as the Einstein Static (ES) solution. The second static solution is a direct

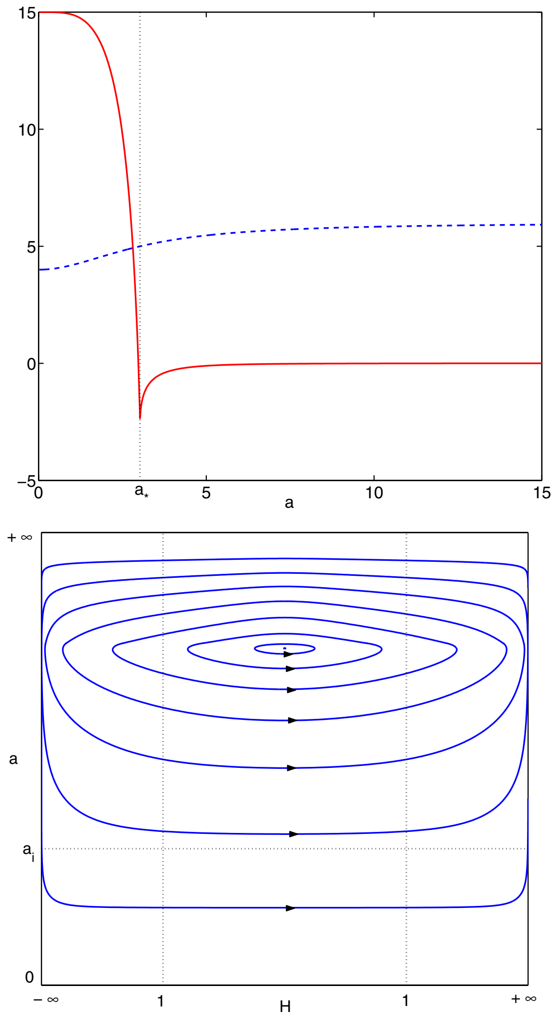

FIG. 4 (color online). The top panel shows the plots of functions A (solid line) and B (dashed line) against a for $V = -0.013$. The vertical dotted line denotes the position of $a = a_*$. The bottom panel is the corresponding phase portrait demonstrating the center equilibrium point that occurs in this case. The plot is compactified as in Fig. 2. The dotted lines in the bottom panel are as described in Fig. 2. All axes are in Planck units and $j = 100$.

consequence of LQC effects. It is a center and we shall refer to it as the loop static (LS) solution. We have shown analytically that the LS solution arises for a wide range of values for the potential given by $V \in (-V_{lb}, V^*)$, where the lower bound $-V_{lb}$ is determined by the need to satisfy the Planck bounds. The importance of the LS solution is that, in contrast to the ES solution present in GR, slight perturbations do not result in an exponential divergence from the static universe, but lead instead to oscillations about it. Moreover, since the LS solution always lies within the semiclassical region $a < a_*$, it is ideally suited to act as the past asymptotic initial state for an emerging universe. Indeed, it is the only equilibrium point in models with a vanishing or negative potential.

In the following Section, we employ these novel features of LQC dynamics within the context of the emergent inflationary universe.

III. EMERGENT INFLATIONARY UNIVERSE IN LQC

A. The dynamics of emergence

The phase plane analysis of the previous Section determined the qualitative LQC dynamics for the case of a constant potential. However, any realistic inflationary model clearly requires the potential to vary as the scalar field evolves. Nevertheless, the constant potential dynamics (see Figs. 2–4) provides a good approximation to more general dynamics if variations in the potential are negligible over a few oscillations. This implies that cosmic dynamics with a variable potential may be studied by treating the potential as a sequence of separate, constant potentials. Here, with the emergent inflationary models in mind (see below), we shall consider a general class of potentials that asymptote to a constant $V_{-\infty} < V_*$ as $\phi \rightarrow -\infty$ and rise monotonically once the value of the scalar field exceeds a certain value.

As we saw in the previous Section, for any constant potential $V < V_*$, there exists a region of parameter space where the universe undergoes non–singular oscillations about the point LS. For $V > V_*$, the equilibrium points LS and ES merge and the phase plane then resembles that of classical GR: a collapsing universe bounces and asymptotically evolves into a de Sitter (exponential) expansion in the infinite future. This leads us to propose the following picture for the origin of the universe. The universe is initially at, or in the neighborhood of, the static point LS, with the field located on the plateau region of the potential with a positive kinetic energy, $\dot{\phi}_{\rm init} > 0$. The universe undergoes a series of nonsingular oscillations in a (possibly) past-eternal phase with the field evolving monotonically along the potential. As the magnitude of the potential increases, the cycles are eventually broken by the emergence of the universe into an inflationary epoch.

We proceed to discus the dynamics of this scenario in more detail. The universe will exhibit cyclic behavior around the LS point for a very wide range of initial values $\dot{\phi}_{\rm init}$ when $V_{-\infty} < 0$. If $0 < V_{-\infty} < V_*$, the range becomes more limited as $V_{-\infty}$ is increased. An important feature to note is that in all cases the field's kinetic energy never vanishes during a given cycle. In the case of a positive potential, the unphysical region of phase space where the field violates the null energy condition lies outside the cyclic region that is bounded by the separatrix. Indeed, for a constant potential, the evolution of the scalar field is determined by integrating Eq. (8):

$$\dot{\phi} = \dot{\phi}_{\rm init}\left(\frac{a_{\rm init}}{a}\right)^3 \frac{D}{D_{\rm init}}. \qquad (20)$$

Thus, the field will vary monotonically along the potential and eventually reach the region where the potential begins to rise.

Increasing the magnitude of the potential over a series of cycles has the effect of moving the location of the LS point to progressively higher values of the scale factor, although the shift is moderate. In the case of a positive potential, for example, a necessary condition for the existence of LS is that $4 < A(a_{\rm eq}) < -2.5$, where the function A is defined in Eq. (15), and this corresponds to the range $0.94 a_* < a < a_*$. Equation (16) then implies that the field's kinetic energy does not alter significantly, since the universe remains in the vicinity of LS. On the other hand, the saddle point ES occurs at progressively smaller values of the scale factor as the magnitude of the potential increases. As discussed in Sec. II B, this equilibrium point occurs in the range $a_* \le a \le \infty$, where the limits are approached as $V \to V_*$ and $V \to 0$, respectively.

The overall effect of increasing the potential, therefore, is to distort the separatrix in the phase plane, making it narrower in the vertical direction but introducing little change to the position of the LS point. This implies that the dynamics varies only slightly from cycle to cycle for orbits that are close to the LS point. If the magnitude of the potential continues to increase as the field evolves, however, a cycle is eventually reached where the trajectory that represents the universe's evolution now lies *outside* the finite region bounded by the separatrix and this effectively breaks the oscillatory cycles. From a physical point of view, the magnitude of the field's potential energy, relative to its kinetic energy, is now sufficiently large that a recollapse of the universe is prevented, i.e., the strong energy condition of GR is violated, thereby leading to accelerated expansion. The field decelerates as it moves further up the potential, subsequently reaching a point of maximal displacement and then rolling back down. If the potential has a suitable form in this region, slow-roll inflation will occur.

B. The energy scale of inflation

An important question to address is the energy scale at the onset of inflation, $V_{\rm inf}$. In general, inflation begins (in the classical regime) when the strong energy condition is violated:

$$V(\phi_{\rm inf}) \approx \dot{\phi}^2_{\rm inf}, \tag{21}$$

and, moreover, the structure of the phase space indicates that the potential energy of the field remains dynamically negligible until the onset of inflation[2]. In this case, the evolution of the scalar field prior to inflation is determined by Eq. (20).

The inflationary energy scale may then be estimated by considering the penultimate cycle before the onset of inflation. The turnaround in the expansion occurs when the field's energy density satisfies $\rho = 3/(8\pi l_{\rm Pl}^2 a^2)$. Since the energy density of the field will not have changed significantly at the equivalent stage of the following cycle, the scale factor at the onset of inflation is determined approximately by $a_{\rm inf} \approx (4\pi l_{\rm Pl}^2 V_{\rm inf})^{-1/2}$. Substituting this condition into Eq. (20) then yields an estimate for the magnitude of the potential in terms of initial conditions:

$$V_{\rm inf} \approx \frac{1}{(4\pi l_{\rm Pl}^2)^{3/2}} \frac{D_{\rm init}}{a^3_{\rm init}\dot{\phi}_{\rm init}}. \tag{22}$$

For a universe located near to the equilibrium point LS, the scale factor is given by $a_{\rm init} = f a_*$, where $0.94 \le f \le 1$ and it may be further verified that $D_{\rm init} = \mathcal{O}(1)$ in this range. The field's initial kinetic energy is then determined by Eq. (16): $\dot{\phi}^2_{\rm init} \approx 3/(4\pi l_{\rm Pl}^2 a_{\rm eq}^2)$. It follows from Eq. (22), therefore, that a universe "emerging" from the semiclassical LQC phase near to the LS point will begin to inflate when

$$V_{\rm inf} \approx \frac{1}{2jf^2} m^4_{\rm Pl}. \tag{23}$$

As expected, this is in good agreement with the necessary condition (19) for the coalescence of the equilibrium points LS and ES, since the scale factor can not evolve until $V > V_*$ if the universe is located on or near the LS point.

A precise measure on the set of initial data is presently unknown, and we should therefore consider other possible initial conditions. Having investigated the regime $a_{\rm init} \approx a_*$, a second possibility is to consider initial conditions where $a_i \approx a_{\rm init} \ll a_*$. In this regime, the quantum correction factor (5) asymptotes to a power law, $D \approx (12/7)^6 (a/a_*)^{15}$, and Eq. (22) is then equivalent to the condition

$$\dot{\phi}_{\rm init} \approx 20 \frac{\beta^{12}}{j^{3/2}} \left(\frac{V_{\rm inf}}{m^4_{\rm Pl}}\right)^{-1} m^2_{\rm Pl}, \tag{24}$$

where we have defined the ratio $\beta \equiv a_{\rm init}/a_*$. This ratio may be constrained by imposing two necessary conditions for the semi-classical framework to be valid. Firstly, the initial conditions should be set in the regime where spacetime is approximated by a smooth manifold, $a_i/a_{\rm init} < 1$, and this leads to the *lower* limit:

$$\beta > \sqrt{\frac{3}{j}}. \tag{25}$$

Secondly, the scalar field's kinetic energy must not exceed the Planck scale at the onset of the classical regime. Since

[2] For a quadratic potential, numerical integration of the field equations indicates that this estimate yields a very good measure of the energy scale at the onset of inflation when the universe undergoes a large number of pre-inflationary oscillations [19].

the anti–frictional effects in the modified field Eq. (8) accelerate the field when $a < a_*$, the field's kinetic energy must initially be sub-Planckian, $|\dot{\phi}_{\rm init}|/m_{\rm Pl}^2 \leq 1$. Equation (24) then leads to an *upper* limit on β:

$$\beta < \left(\frac{V_{\rm inf}}{m_{\rm Pl}^4}\right)^{1/12} j^{1/8}, \tag{26}$$

and combining the constraints (25) and (26) results in a *necessary* condition for the onset of inflation when $a_{\rm init} \ll a_*$:

$$\frac{V_{\rm inf}}{m_{\rm Pl}^4} > \left(\frac{2}{j}\right)^{15/2}. \tag{27}$$

In other words, for a given value of the parameter j, inflation must begin above the scale $(2/j)^{15/2} m_{\rm Pl}^4$.

Conditions (23) and (27) both imply that the inflationary energy scale is *higher* for *lower* values of the parameter j. Indeed, it is comparable to the Planck scale for $j \leq \mathcal{O}(10)$ and this generic behavior is only weakly dependent on the initial conditions. This has implications for the asymptotic form of the potential as the field reaches progressively higher values. Unless the parameter j is sufficiently large, it is unlikely that the oscillatory dynamics will end if the potential asymptotes to a constant value, or reaches a local maximum, that is significantly below the Planck scale. In this sense, therefore, the scenario outlined above favors potentials that increase monotonically once the value of the scalar field has exceeded some critical value.

In the following Section, we consider a concrete example of a potential that exhibits the appropriate asymptotic behavior.

IV. A SPECIFIC MODEL OF AN EMERGING UNIVERSE

From a dynamical point of view, the emergent universe scenario can be realized within the context of semiclassical LQC if the potential satisfies a number of rather weak constraints. Asymptotically, it should have a horizontal branch as $\phi \to -\infty$ such that $dV/d\phi \to 0$ and increase monotonically in the region $\phi > \phi_{\rm grow}$, where without loss of generality we may choose $\phi_{\rm grow} = 0$. The reheating of the universe imposes a further constraint that there should be a global minimum in the potential at $V_{\rm min} = 0$ if reheating is to proceed through coherent oscillations of the inflaton. Since inflation will end after the field has rolled back down the potential, this should occur for $\phi \leq \phi_{\rm grow}$. Finally, the region of the potential that drove the last 60 e-foldings of inflationary expansion is then constrained by cosmological observations, as in the standard scenario.

Examples of potentials that exhibit these generic properties are illustrated in Fig. 5. It is interesting that potentials of this form have been considered previously in a number of different settings, including cases where higher-order curvature invariants of the form

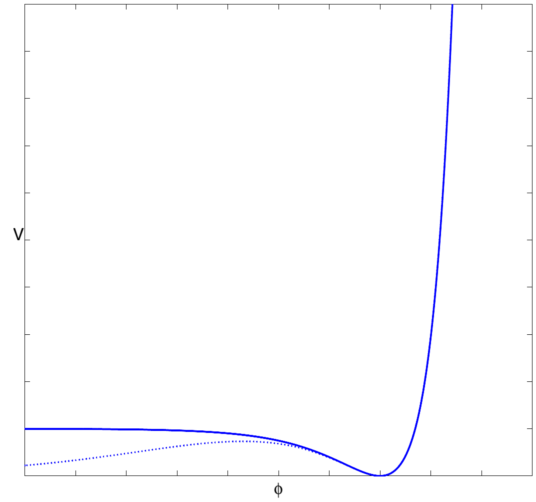

FIG. 5 (color online). The figure depicts two possible forms of emergent potentials that allow for conventional reheating. The solid line is the form of the potential motivated by the inclusion of a R^2 term in the Einstein-Hilbert action, and the dotted line the form motivated by the inclusion of higher-order terms.

$$L_N = \frac{1}{16\pi l_{\rm Pl}^2} \sum_{i=1}^{N} \epsilon_i R^i \tag{28}$$

are introduced into the Einstein-Hilbert action, where R is the Ricci scalar, ϵ_i are coupling constants and $\epsilon_1 = 1$. Such corrections are required when attempting to renormalize theories of quantum gravity [26] and also arise in low-energy limits of superstring theories [27]. In general, such theories are conformally equivalent to Einstein gravity sourced by a minimally coupled, self-interacting scalar field. For example, potentials with a nonzero asymptote at $\phi \to -\infty$ (as shown by the solid line in Fig. 5) can be obtained from theories that include a quadratic term in the action, whereas those with a zero asymptotic value and a local maximum (shown by a dotted line) arise when cubic and higher–order terms in the Ricci scalar are introduced [28,29]. In general, all these potentials possess a global minimum at $V = 0$. Finally, potentials of this form have also been considered within the context of the classical emergent universe [9] (see also [30]).

Motivated by the above discussion, we consider, as an example, the potential

$$V = \alpha[(\exp(\beta\phi/\sqrt{3}) - 1]^2, \tag{29}$$

where α and β are constants. This potential is qualitatively similar to that illustrated by the solid line in Fig. 5. The parameters in the potential are constrained primarily by the Cosmic Microwave Background (CMB) anisotropy power spectrum. Assuming inflation proceeds in the region $\phi \gg 0$, the parameter β determines the spectral index of the

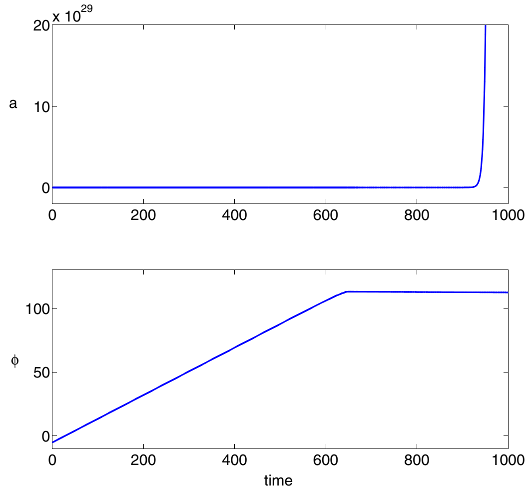

FIG. 6 (color online). Time evolution of the scale factor (top panel) and scalar field (bottom panel) with the field initially on the asymptotically flat region of the potential (29) with $\alpha = 10^{-12}$ and $\beta = 0.1$. The field increases in value from initial conditions close to the LS (center) solution defined by Eq. (14).

scalar perturbation spectrum together with the relative amplitude of the gravitational wave perturbations. The parameter α is then constrained by the COBE normalization of the power spectrum on large scales [31].

We chose $\alpha = 10^{-12}$ and $\beta = 0.1$ as representing typical values satisfying the constraints imposed recently by the WMAP satellite [32] and numerically integrated the field Eqs. (7)–(9) for a universe starting from an initial state close to the LS static state. The results of the integration are illustrated in Figs. 6 and 7. The field starts in the asymptotically flat region of the potential and gradually increases in value, as the scale factor oscillates about the LS point. The field moves past the minimum and climbs up the potential. The scale factor continues to oscillate until the field reaches the point where it slows down significantly, thereby bringing the oscillations to an end and initiating the inflationary expansion. The behavior at this stage is qualitatively similar to that discussed previously for a quadratic potential [18].

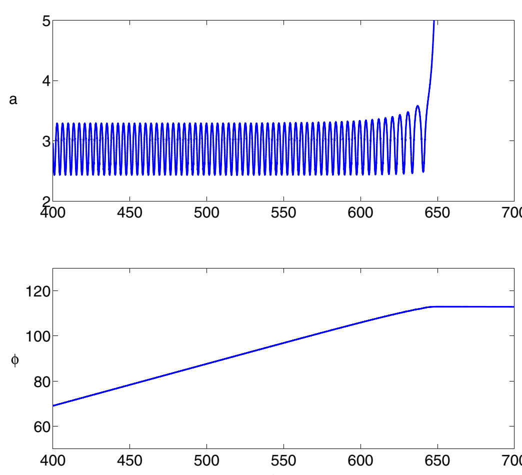

FIG. 7 (color online). Plots illustrating the magnification of Fig. 6 around the region where emergence commences, the oscillations cease and inflation begins.

V. EMERGING QUINTESSENTIAL INFLATION

One drawback of reheating through inflaton decay in the emergent scenario is that the coupling of the scalar field to the standard model degrees of freedom must be strongly suppressed if the field is to survive for a (possibly) infinite time as it emerges from the oscillatory semiclassical phase. It is more natural, therefore, to invoke a ‘‘sterile‘‘ inflaton that is not coupled directly to standard model fields, and where reheating proceeds through an alternative mechanism such as gravitational particle production [33]. In this case, the potential need not exhibit a minimum and the field could continue to roll back down the potential towards $\phi \rightarrow -\infty$ at late times.

It is notable that the general requirements for a sterile inflaton with a potential exhibiting a decaying tail as $\phi \rightarrow -\infty$ are *precisely* those features that are characteristic of the quintessential inflationary scenario, where the field that drove early universe inflation is also identified as the

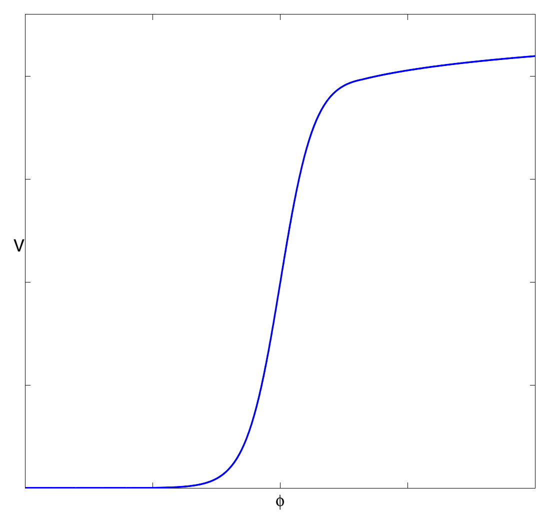

FIG. 8 (color online). Plots illustrating the generic form of the potential that leads to early- and late-time accelerating phases. The potential exhibits a decaying tail as $\phi \rightarrow -\infty$. As the field moves up this tail and increases in value, the universe can oscillate about the LS point. The inflationary regime rises (possibly) towards the Planck scale for large values of ϕ. As the field turns round, it can drive a phase of inflation and, if the potential exhibits a sufficiently steep middle section around $\phi \approx 0$, reheating may proceed through gravitational particle production. Consequently, the field may survive until the present epoch, where it can act as the source of dark energy by slowly rolling along the tail towards $\phi \rightarrow -\infty$.

source of dark energy today [34]. In the present context, this suggests that the scalar field could play a three-fold role in the history of the universe, acting as the mechanism that enables the universe to emerge into the classical domain, and then subsequently driving both the early- and late-time accelerated expansion.

The specific constraints that must be satisfied by the potential in standard quintessential inflation have been considered in detail in Ref. [35]. In particular, one of the simplest asymptotic forms for the low-energy tail that simultaneously leads to tracking behavior and satisfies primordial nucleosynthesis bounds is given by $V \propto (m/\phi)^k \exp(\lambda\phi/m_{\rm Pl})$, where m, k and λ are constants. Cyclic behavior in LQC will arise for such a potential.

A further requirement is that the potential must be sufficiently steep immediately after the end of inflation if the field's energy density is to redshift more rapidly than the subdominant radiation component. This requires a second point of inflexion in the potential, as illustrated qualitatively in Fig. 8. Beyond this region, the potential must continue to rise in order for the oscillatory dynamics to come to end and, as discussed above, this is expected to occur near to the Planck scale. From a dynamical point of view, there are no further constraints on how rapidly the potential energy need increase in this region. The only remaining consideration is that a phase of successful slow-roll inflation should arise as the field rolls back down the potential. Given the ease with which the inflaton is able to move up the potential due to LQC effects, we anticipate that any potential satisfying the existing constraints for successful quintessential inflation will also lead to a successful emergence of the classical universe.

VI. DISCUSSION

In this paper, we have investigated the occurrence of static solutions in loop quantum cosmology settings sourced by a scalar field with a constant potential. We have shown that there are in principle two such solutions, depending upon the sign and magnitude of the potential. The point ES is always in the classical domain and corresponds to the unstable saddle point that is also present in GR. The important characteristic of this solution is that any perturbations, no matter how small, necessarily force the universe to deviate exponentially from the static state. The second solution LS is always in the semiclassical domain and corresponds to a center. This is a solution made possible by LQC effects. We have shown that it exists for a wide range of values of the potential, including positive, zero and negative values. Its importance lies in the fact that it allows a universe that is slightly displaced from the static state to oscillate in the neighborhood of the static solution for an arbitrarily long time.

We have exploited these characteristics to develop a working scenario of the emerging inflationary universe, in which a past-eternal, cyclic cosmology eventually enters a phase where the symmetry of the oscillations is broken by the scalar field potential, thereby leading in principle to a phase of successful slow-roll inflation. The mechanism that enables the universe to emerge depends very weakly on the form of the potential, and requires only that it asymptotes to a constant value at $\phi \rightarrow -\infty$ and grows in magnitude at larger ϕ in order to break the cycles. The asymptotic value of the potential can be either positive, zero or negative.

The above emergent scenario has a further advantage that the initial state of the universe is set in the more natural semiclassical regime, rather than the classical arena of GR. Nevertheless, an important question that arises is the likelihood of these initial conditions within a more general framework. In particular, there is the issue of why the scalar field should initially be located in the asymptotic low-energy region of the potential. Although a detailed study of such questions is beyond the scope of the present paper, it has been argued [36] that for the case of a constant potential, the wavefunction in LQC most closely resembles the Hartle-Hawking no-boundary wavefunction [37]. More specifically, the difference equation in LQC requires the wavefunction to tend to zero near to the classical singularity [38], and in this sense resembles DeWitt's initial condition [39]. However, within the context of solutions to the Wheeler-DeWitt equation, requiring the wavefunction to be bounded as $a \rightarrow 0$ selects the exponentially increasing WKB mode [36] and this corresponds to the no-boundary wavefunction.

This is of interest since the square of the wavefunction in quantum cosmology is interpreted as the probability distribution for initial values of the scalar field in an ensemble of universes. For the no-boundary boundary condition, the probability, $P \propto \exp(3/[8\pi l_P^4 V(\phi)])$, is peaked at $V(\phi) = 0$, thereby suggesting that the no-boundary proposal disfavors inflation. In the present context, however, this implies that the most natural initial condition for the field is to be located either at the minimum of the potential, or in the case where the potential has no minimum, at $\phi = -\infty$. It would be interesting to explore this possibility further. In particular, previous analyses have so far neglected kinetic terms in the matter Hamiltonian and these may be important in the emerging universe scenario. There is also the related question of whether the field moves from left to right initially.

In general, we have found that the onset of slow-roll inflation will occur at a relatively high energy scale, unless the quantization parameter j is extremely large. This indicates that a large amount of slow-roll inflation should arise, at least for a wide class of smoothly varying potentials, and it is expected, therefore, that the density of the presentday universe should be exponentially close to the critical density, $\Omega_0 = 1 + \epsilon$, where $\epsilon \ll 1$. In principle, therefore, the emergent scenario we have proposed could be ruled out if a significant detection of spatial curvature is ultimately reported by future cosmological observations.

It is also worth remarking that if the field is pushed sufficiently far up the potential for the condition $3V'^2 < 128\pi l_{\rm Pl}^6 V^3$ to be met, the inflating universe will enter a phase of eternal self-reproduction, where quantum fluctuations in the inflaton become more important than its classical dynamics [2]. In our model, inflation can commence at energy scales $\sim \frac{1}{j} m_{pl}^4$, which is of the order of the Planck scale for small enough values of j. Now since the self-reproduction condition is met well below this scale for many inflationary potentials, the eternal self-reproducing universe is a likely outcome for such potentials. When self-reproduction occurs, causally disconnected regions evolve independently, and most of the universe inflates indefinitely. In some regions, however, the potential energy of the inflaton will fall sufficiently that the self-reproduction condition is no longer satisfied, and a classical evolution follows. In these regions, which give rise to viable late-time cosmologies, the scenarios we have presented above remain unaltered.

Finally, there is an interesting symmetry in the emerging quintessential scenario between the initial and final states of the universe. Although the size of the universe differs by many orders of magnitude, the field evolves along the tail of the potential at both early and late times, $\phi(t \to -\infty) = \phi(t \to +\infty)$, but with its kinetic energy having changed sign. This implies that a reconstruction of the dark energy equation of state at the present epoch could yield direct observational insight into the nature of the *preinflationary* potential in this scenario.

ACKNOWLEDGMENTS

D. J. M. is supported by PPARC.

[1] A. R. Liddle and D. H. Lyth, *Cosmological Inflation and Large-Scale Structure* (Cambridge University Press, Cambridge, 2000).
[2] A. D. Linde, Phys. Lett. B **175**, 395 (1986).
[3] A. Borde and A. Vilenkin, Phys. Rev. Lett. **72**, 3305 (1994); Phys. Rev. D **56**, 717 (1997); A. Borde, A. H. Guth, and A. Vilenkin, Phys. Rev. Lett. **90**, 151301 (2003); A. H. Guth, astro-ph/0101507; A. Vilenkin, gr-qc/0204061.
[4] M. Gasperini and G. Veneziano, Phys. Rep. **373**, 1 (2003).
[5] J. E. Lidsey, D. Wands, and E. J. Copeland, Phys. Rep. **337**, 343 (2000).
[6] J. Khoury, B. A. Ovrut, P. J. Steinhardt, and N. Turok, Phys. Rev. D **64**, 123522 (2001); P. J. Steinhardt and N. Turok, Science **296**, 1436 (2002); P. J. Steinhardt and N. Turok, Phys. Rev. D **65**, 126003 (2002); J. Khoury, P. J. Steinhardt, and N. Turok, Phys. Rev. Lett. **92**, 031302 (2004).
[7] L. Smolin, Class. Quant. Grav. **9**, 173 1992.
[8] G. F. R. Ellis and R. Maartens, Class. Quant. Grav. **21**, 223 (2004).
[9] G. F. R. Ellis, J. Murugan, and C. G. Tsagas, Class. Quant. Grav. **21**, 233 (2004).
[10] G. W. Gibbons, Nucl. Phys. **B292**, 784 (1987); **B310**, 636 (1988).
[11] C. Rovelli, Living Rev. Relativity **1**, 1 (1998).
[12] T. Thiemann, Lect. Notes Phys. **631**, 41 (2003).
[13] M. Bojowald, Class. Quant. Grav. **19**, 2717 (2002).
[14] M. Bojowald, Phys. Rev. Lett. **89**, 261301 (2002).
[15] P. Singh and A. Toporensky, Phys. Rev. D **69**, 104008 (2004).
[16] S. Tsujikawa, P. Singh, and R. Maartens, Class. Quant. Grav. **21**, 5767 (2004).
[17] M. Bojowald, J. E. Lidsey, D. J. Mulryne, P. Singh, and R. Tavakol, Phys. Rev. D **70**, 043530 (2004).
[18] J. E. Lidsey, D. J. Mulryne, N. J. Nunes, and R. Tavakol, Phys. Rev. D **70**, 063521 (2004).
[19] D. J. Mulryne, N. J. Nunes, R. Tavakol, and J. E. Lidsey, gr-qc/0411125.
[20] J. E. Lidsey, J. Cosmol. Astropart. Phys. 12 (2004) 007.
[21] G. V. Vereshchaign, J. Cosmol. Astropart. Phys. 07 (2004) 013.
[22] M. Bojowald, R. Maartens, and P. Singh, Phys. Rev. D **70**, 083517 (2004).
[23] M. Domagala and J. Lewandowski, Class. Quant. Grav. **21**, 5233 (2004); K. A. Meissner, Class. Quant. Grav. **21**, 5245 (2004).
[24] M. Gaul and C. Rovelli, Class. Quant. Grav. **18**, 1593 (2001); M. Bojowald, Class. Quant. Grav. **19**, 5113 (2002).
[25] P. K. Townsend and M. N. R. Wohlfarth, Phys. Rev. Lett. **91**, 061302 (2003); N. Ohta, Phys. Rev. Lett. **91**, 061303 (2003); R. Emparan and J. Garriga, J. High Energy Phys. 05 (2003) 028; N. Ohta, Prog. Theor. Phys. **110**, 269 (2003); C.-M. Chen, P.-M. Ho, I. P. Neupane, N. Ohta, and J. E. Wang, J. High Energy Phys. 10 (2003) 058; S. Roy, Phys. Lett. B **567**, 322 (2003).
[26] I. Antoniadis and E. T. Tomboulis, Phys. Rev. D **33**, 2756 (1986).
[27] P. Candelas, G. T. Horowitz, A. Strominger, and E. Witten, Nucl. Phys. **B258**, 46 (1985).
[28] K. Maeda, Phys. Rev. D **39**, 3159 (1989).
[29] J. D. Barrow and S. Cotsakis, Phys. Lett. B **258**, 305 (1991).
[30] E. I. Guendelman, Mod. Phys. Lett. A **14**, 1043 (1999).
[31] E. F. Bunn, A. R. Liddle, and M. White, Phys. Rev. D **54**, R5917 (1996); E. F. Bunn and M. White, Astrophys. J. **480**, 6 (1997).
[32] D. N. Spergel *et al.*, Astrophys. J. Suppl. Ser. **148**, 175 (2003); H. V. Peiris *et al.*, Astrophys. J. Suppl. Ser. **148**, 213 (2003).

[33] L. H. Ford, Phys. Rev. D **35**, 2955 (1987); L. P. Grishchuk and Y. V. Sidorov, Phys. Rev. D **42**, 3413 (1990); B. Spokoiny, Phys. Lett. B **315**, 40 (1993).
[34] P. J. E. Peebles and A. Vilenkin, Phys. Rev. D **59**, 063505 (1999).
[35] K. Dimopoulos, Astropart. Phys. **18**, 287 (2002).
[36] M. Bojowald and K. Vandersloot, Phys. Rev. D **67**, 124023 (2003).
[37] J. B. Hartle and S. W. Hawking, Phys. Rev. D **28**, 2960 (1983).
[38] M. Bojowald, Phys. Rev. Lett. **87**, 121301 (2001).
[39] B. S. DeWitt, Phys. Rev. **160**, 1113 (1967).