

Superadiabatic-type magnetic amplification in conventional cosmologyChristos G. Tsagas^{1,*} and Alejandra Kandus^{2,†}¹*DAMTP, Centre for Mathematical Sciences, University of Cambridge, Wilberforce Road, Cambridge CB3 0WA, United Kingdom*²*LATO-DCET, Universidade Estadual de Santa Cruz,**Rodovia Ilhéus-Itabuna km 16 s/n, Salobrinho CEP-05508-900, Ilhéus - BA, Brazil*

(Received 4 April 2005; published 6 June 2005)

We consider the evolution of cosmological magnetic fields in Friedmann-Robertson-Walker models and outline a geometrical mechanism for their superadiabatic amplification on large scales. The mechanism operates within standard electromagnetic theory and applies to Friedmann-Robertson-Walker universes with open spatial sections. We discuss the general relativistic nature of the effect and show how it modifies the adiabatic magnetic evolution. Assuming a universe that is only marginally open today, we estimate the main features of the superadiabatically amplified residual field.

DOI: 10.1103/PhysRevD.71.123506

PACS numbers: 98.80.Jk, 41.20.-q, 98.80.Cq

I. INTRODUCTION

Magnetic fields appear everywhere in the universe. From Earth and the nearby stars, all the way to the remote galaxy clusters and high-redshift protogalaxies, the existence of magnetic fields has been repeatedly verified [1]. Despite this widespread presence, however, the origin of cosmic magnetism remains a mystery and it is still the subject of debate. Over the years, numerous mechanisms of magnetogenesis have appeared in the literature (see [2] for recent reviews). Broadly speaking, one can classify these scenarios into those arguing for a late (post-recombination) magnetic generation and those advocating a primordial origin for the field [3]. Prior to recombination we have superstring and inflation based models, mechanisms operating during the electroweak and the quark-hadron phase transitions, eddies in the pre-recombination plasma, and effects at electron-proton recombination. These early time scenarios are mainly global amplification mechanisms of primeval magnetic seeds. In the post-recombination era there exist local astrophysical processes that generate the magnetic seeds and operate simultaneously with the amplifying process. For example, weak magnetic fields produced via the Biermann battery [4] can be amplified to galactic size fields during the protogalactic collapse [5]. Alternatively, stronger magnetic seeds can be injected into the intragalactic medium by stellar winds and supernova explosions (e.g. see [6]). In addition, there is still relatively little knowledge on the reionization of our universe and the related magnetohydrodynamics.

An attractive aspect of early magnetogenesis is that it makes the ubiquity of large-scale magnetic fields in the universe, particularly those observed in high-redshift protogalaxies, easier to explain. Inflation seems the most plausible candidate for producing the primordial fields, as it naturally leads to large-scale phenomena from sub-horizon microphysics. The main obstacle in this scenario is

that any early magnetic field that survives an epoch of inflation is so drastically diluted that it can never seed the galactic dynamo for the ordered, large-scale field.¹ The reason is cosmological magnetic flux conservation, namely, the fact that the strength of the large-scale fields drops as a^{-2} (a is the scale factor of the universe). The root of the problem is traced down to the conformal invariance of electromagnetism and to the conformal flatness of the Friedmann-Robertson-Walker (FRW) models. Together, these two are thought to guarantee that $B \propto a^{-2}$ always and irrespective of plasma effects. When the FRW background has nonzero spatial curvature, however, we will show this is not necessarily the case.

The most common way of modifying the “adiabatic” $B \propto a^{-2}$ law is by breaking away from standard electromagnetic theory. There is more than one way of doing that, which explains the large number of relevant scenarios in the literature. Perhaps the first detailed discussion of the issue was the one given in [7]. Among other suggestions, the authors introduced a coupling between the Maxwell field and the curvature of the space in their Lagrangian. As a result, both the conformal invariance and the gauge invariance of Maxwell’s equations were lost. However, when applied to a spatially flat FRW universe, the aforementioned interaction led to an extra magneto-curvature term in the magnetic wave equation. The immediate consequence was that superhorizon-sized magnetic fields, evolving in a poorly conducting inflationary universe, decayed slower than the standard a^{-2} law. This meant an effective superadiabatic amplification of the field on these scales, a concept that was originally introduced in gravitational wave studies [8]. In other words, magnetic fields on large enough scales could go through an epoch of

¹In the galaxy there is also a random magnetic field, which is stirred up by turbulent motions and saturates on scales of $\sim 50 - 100$ pc. This field has magnitude slightly larger than that of the ordered magnetic field but its growth time is only 3×10^7 yrs (i.e. one tenth of the typical growth time scale associated with the ordered field dynamo).

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inflation and still remain strong enough to sustain the galactic dynamo.

Following [7], several mechanisms producing magnetic fields during inflation and reheating have appeared in the literature [9]. Most of these scenarios break the conformal invariance of the Maxwell field by introducing extra couplings with the spacetime curvature, or nonconformally invariant sources to Maxwell's equations. In this paper we consider a conventional interaction between the electromagnetic and the gravitational field, which so far has been sparsely studied in cosmology. This is the natural general relativistic coupling between electromagnetism and spacetime geometry that emerges from the vector nature of the Maxwell field and from the geometrical approach of Einstein's theory. The best known effect of the aforementioned interaction, which emerges from the Ricci identities, is probably the "scattering" of electromagnetic radiation by the gravitational field [10]. In what follows we will show that, under certain circumstances, the same coupling also can lead to the superadiabatic amplification of cosmological magnetic fields without violating or modifying standard electromagnetism. Our mechanism operates primarily on magnetic fields coherent on the largest subcurvature scales of a spatially open FRW universe, which asymptotically approaches flatness as it undergoes a period of inflationary expansion. The result is that these fields decay as a^{-1} , a rate considerably slower than the adiabatic a^2 law. Therefore, primordial magnetic fields that survive an epoch of inflation could be considerably stronger than previously anticipated due to curvature effects alone. In practice this means that magnetic fields which are coherent on very large scales could have appreciable strengths. Assuming that $1 - \Omega \sim 10^{-2}$ today, in particular, we find a residual field of 10^{-35} G spanning a comoving length of $\sim 10^4$ Mpc. This is much stronger than any other large-scale field obtained by conventional methods. Moreover, in a universe currently dominated by a dark-energy component, a seed field of 10^{-35} G lies within the broad galactic dynamo requirements [11].

The attractive aspects of the mechanism presented below are its simplicity and the fact that it operates within standard electromagnetic theory. If the universe is marginally open today, this scenario could provide a viable method for a superadiabatic type of early magnetic amplification and lead to fields with astrophysically interesting strengths on very large scales. Even if the universe is not open, however, our mechanism still offers a simple general relativistic counterexample to the widespread perception that the superadiabatic amplification of magnetic fields in FRW cosmologies is not possible within conventional electromagnetism. In either case, we believe that this study will further facilitate our theoretical understanding of the subject, while it also may prove a valuable step in the ongoing quest for an answer to the origin of cosmic magnetism.

II. MAGNETIC FIELDS IN CURVED FRW UNIVERSES

We begin by reminding the reader that, with the exception of completely random radiation or a fully tangled magnetic field, electromagnetic fields are not compatible with the highly symmetric FRW spacetimes. The isotropy of the latter means that these models cannot naturally accommodate inherently anisotropic sources like the Maxwell field. The implication is that, strictly speaking, even weak cosmological electromagnetic fields should be studied in perturbed Friedmann models. Here we will show also that the standard magnetic evolution of $B \propto a^{-2}$ does not always hold in Friedmann models with nontrivial spatial curvature. All these mean that studying cosmological magnetic fields in flat Minkowski space is a good approximation only when the fields are weak and only on small scales in models with nontrivial spatial curvature. In the latter case the approximation becomes progressively less accurate as one moves to larger scales and the 3-curvature effects start kicking in. Technically speaking, this means that certain linear couplings between the field and the geometry of the 3-space, which vanish only when the background is identically flat, are bypassed. It is the purpose of this paper to examine the implications of these magneto-geometrical couplings for the evolution of cosmological magnetic fields.

Our analysis uses the covariant approach to general relativity, which introduces a family of timelike fundamental observers moving with 4-velocity u_a (i.e. $u_a u^a = -1$). We assume that relative to u_a the cosmic medium has a perfect fluid form with a barotropic equation of state, and that the fundamental observers experience an electromagnetic field with components E_a and B_a . Both E_a and the pseudovector B_a live on the observers' local rest space (i.e. $E_a u^a = 0 = B_a u^a$). In the absence of vorticity, the projection tensor $h_{ab} = g_{ab} + u_a u_b$, where g_{ab} is the spacetime metric, is also the metric of the spatial hypersurfaces. The electromagnetic field obeys the standard Maxwell's formulas, consisting of two propagation equations²

$$\begin{aligned} \dot{B}_{\langle a} &= -\frac{2}{3}\Theta B_a + (\sigma_{ab} + \varepsilon_{abc}\omega^c)B^b - \varepsilon_{abc}\dot{u}^b E^c \\ &\quad - \text{curl}E_a, \end{aligned} \quad (1)$$

$$\begin{aligned} \dot{E}_{\langle a} &= -\frac{2}{3}\Theta E_a + (\sigma_{ab} + \varepsilon_{abc}\omega^c)E^b + \varepsilon_{abc}\dot{u}^b B^c \\ &\quad + \text{curl}B_a - J_a, \end{aligned} \quad (2)$$

with Θ representing the volume expansion, σ_{ab} the shear, ω_a the vorticity, \dot{u}_a the 4-acceleration, and $J_a = J_{\langle a}$ the

²Angled brackets denote spatially projected vectors and the projected, symmetric, and trace-free part of spacelike second-rank tensors (e.g. $\dot{B}_{\langle a} = h_a^b \dot{B}_b$). Also, round brackets indicate symmetrization and square ones antisymmetrization.

projected 4-current [12]. The above are supplemented by the constraints

$$D^a B_a = 2\omega^a E_a, \quad (3)$$

$$D^a E_a = \rho_e - 2\omega^a B_a, \quad (4)$$

where ρ_e is the charge density. Note that overdots indicate proper time derivatives and $D_a = h_a^b \nabla_b$ is the covariant derivative operator on the observer's local 3-space. Also, $\text{curl} v_a = \epsilon_{abc} D^b v^c$ for any orthogonally projected vector v_a (i.e. with $v_a u^a = 0$) and ϵ_{abc} is the projected permutation tensor. By differentiating Eq. (1) with respect to time and then using (2) to eliminate E_a , one arrives at the covariant wave equation of B_a in a general spacetime [13]. Linearized about a FRW background the latter reads

$$\begin{aligned} \ddot{B}_a - D^2 B_a = & -5H\dot{B}_a - 4H^2 B_a + \frac{1}{3}\rho(1+3w)B_a \\ & - \mathcal{R}_{ab} B^b + \text{curl} J_a, \end{aligned} \quad (5)$$

where $H = \dot{\Theta}/3 = \dot{a}/a$ is the background Hubble parameter, ρ is the energy density of the matter, and $w = p/\rho$, where $p = p(\rho)$ is the barotropic pressure. When linearizing the full equations we assume that the magnetic field vanishes in the unperturbed FRW background. This guarantees the gauge-invariance of the analysis and frees our results from any gauge related ambiguities (see [13] for further discussion and technical details). Note the second last term in the right-hand side of the above, where $\mathcal{R}_{ab} = (2k/a^2)h_{ab}$ is the zero-order spatial Ricci tensor and $k = 0, \pm 1$ is the associated curvature index. This term is manifestly linear and vanishes only when the background model is spatially flat. Cosmological magnetic field studies in flat spaces will clearly bypass such magneto-geometrical terms. The latter result from the general relativistic coupling between the electromagnetic and the gravitational field and are an unavoidable consequence of the geometrical nature of Einstein's theory. Technically speaking this magneto-geometrical interaction is manifested in the 3-Ricci identity. In the absence of rotation, the latter reads $2D_{[c} D_{b]} B_a = \mathcal{R}_{dabc} B^d$, where \mathcal{R}_{abcd} represents the spatial Riemann tensor and $\mathcal{R}_{ab} = \mathcal{R}^c{}_{acb}$ [13,14]. In what follows we will consider the implications of the magneto-curvature term in the right-hand side of Eq. (5) for the evolution of cosmological magnetic fields.

The effect of the current term in the right-hand side of (5) depends crucially on the conducting properties of the medium in which the magnetic field evolves. If Ohm's law holds, then the electrical conductivity is the quantity that describes these properties. In general there are additional terms in what is known as the generalized Ohm's law. For example, when building a magneto-hydrodynamical model of three separate fluids, namely, electrons, protons and neutrals, the interaction of the first two gives rise to the Hall effect and the last two lead to ambipolar diffusion (e.g.

see [15]). However, only the resistive term is responsible for the dissipation of the magnetic energy into heat, while the other effects do not cause dissipation. The Hall term might have influenced the evolution of the field during the radiation era [16] and the ambipolar diffusion is effective in the intragalactic medium [17,18]. During inflation the universe is normally treated as a very poor conductor. Thus, Ohm's law guarantees that all spatial currents vanish, despite the presence of nonzero electric fields (e.g. see [13]). Given that we are primarily interested in the evolution of a large-scale primordial magnetic field during a early period of inflation, we will from now on ignore the current contribution to Eq. (5).³

After inflation, the reheating process reinstates the high electrical conductivity of the cosmic medium. Of course, the resistivity of the plasma is not identically zero. Nevertheless, the amount of magnetic dissipation on large scales is negligible. We can estimate the decay time of a magnetic field coherent over a scale L (with L smaller than the horizon scale) as $t_d \sim L^2/\lambda$, where λ is the magnetic diffusivity (e.g. see [19]). Assuming Spitzer conductivity at an epoch when the temperature of the universe is $T \sim 10^4$ K, we have $\lambda \sim 10^7$ cm²/sec and obtain $t_d \sim 10^{26}$ yrs for fields coherent on approximately 100 pc. This time scale is many orders of magnitude larger than the current age of the universe. Therefore, the magnetic flux on astrophysically interesting scales is effectively frozen into the cosmic plasma.

We adopt the standard decomposition $B_a = B_{(n)} Q_a^{(n)}$, where $Q_a^{(n)}$ is the n th vector harmonic, $D_a B_{(n)} = 0 = \dot{Q}_a^{(n)}$, $D^a Q_a^{(n)} = 0$, and $D^2 Q_a^{(n)} = -(n^2/a^2)Q_a^{(n)}$. Then, substituting the background expression of \mathcal{R}_{ab} into Eq. (5) we obtain

$$\begin{aligned} \ddot{B}_{(n)} + 5H\dot{B}_{(n)} + 4H^2 B_{(n)} - \frac{1}{3}\rho(1+3w)B_{(n)} \\ + \frac{2k}{a^2} B_{(n)} + \frac{n^2}{a^2} B_{(n)} = 0, \end{aligned} \quad (6)$$

for the evolution of the n th magnetic mode. The Laplacian eigenvalues take continuous values, with $n^2 \geq 0$, when $k = 0, -1$ and discrete ones, with $n^2 \geq 3$, for $k = +1$. In this notation supercurvature modes in spatially open models have $0 \leq n^2 < 1$, which guarantees that the physical wavelength of the perturbation is larger than the curvature scale (i.e. $\lambda_n = a/n > a$). On the other hand, modes with $n^2 > 1$ span lengths smaller than the curvature scale and will be therefore termed subcurvature. Note that the supercurvature modes are always larger than the Hubble length

³On sufficiently large scales the current term in Eq. (5) is negligible even during the standard big bang evolution. Indeed, by definition $\text{curl} J_a = \epsilon_{abc} D^b J^c = \epsilon_{abc} \partial^b J^c$, given the symmetry of the Christoffel symbols. Moreover, $\partial_a J_b \sim J/L$, where $J^2 = J_a J^a$ and L is the scale in question. Clearly, as we move to progressively larger wavelengths $\partial_a J_b \rightarrow 0$.

and consequently never in causal contact. On the other hand, perturbations on subcurvature scales can be causally connected (see [20] for further discussion). For our purposes, the causality of magnetic modes with $n^2 > 1$ is crucial. Finally, we remind the reader that $n^2 = 0$ denotes the so-called homogeneous mode.

To proceed further we recall that the zero-order Raychaudhuri equation does not explicitly depend on the background curvature and takes the form $\ddot{a}/a = -\rho(1+w)/6$. On using this expression and introducing η , the conformal time variable with $\dot{\eta} = 1/a > 0$, Eq. (6) becomes

$$B''_{(n)} + 4\left(\frac{a'}{a}\right)B'_{(n)} + 2\left(\frac{a'}{a}\right)^2 B_{(n)} + 2\left(\frac{a''}{a}\right)B_{(n)} + 2kB_{(n)} + n^2 B_{(n)} = 0, \quad (7)$$

where primes indicate differentiation with respect to η . Finally, employing the ‘‘magnetic flux’’ variable $\mathcal{B}_{(n)} = a^2 B_{(n)}$ the above reduces to

$$\mathcal{B}''_{(n)} + n^2 \mathcal{B}_{(n)} = -2k\mathcal{B}_{(n)}. \quad (8)$$

This wave equation shares a very close resemblance with the one obtained in [7] [see Eq. (2.15) there]. The similarity is in the presence of a curvature-related source term in both expressions. The difference is that here the magneto-curvature term is a natural and unavoidable consequence of the vector nature of the magnetic field and of the geometrical approach of general relativity. No new physics has been introduced and standard electromagnetism still holds.

III. THE SUPERADIABATICALLY AMPLIFIED MAGNETIC FIELD

For a spatially flat background, the magneto-curvature term in Eq. (8) vanishes and one recovers the standard wavelike evolution of the field, with an amplitude decreasing according to the familiar a^{-2} law. The adiabatic depletion rate is preserved also when the background is spatially closed, despite the presence of a nonzero magneto-curvature term in (8). Indeed, for $k = +1$ the latter exhibits an oscillatory solution of the form [13]

$$B_{(n)} = \frac{1}{a^2} \left\{ C_1 \cos \left[\sqrt{n^2 + 2} \eta \right] + C_2 \sin \left[\sqrt{n^2 + 2} \eta \right] \right\}, \quad (9)$$

for the n th magnetic mode (with C_1, C_2 constants). Apart from modifying the oscillation frequency, the magneto-curvature term in Eq. (8) has no significant effect on the evolution of the field when $k = +1$. Note that in this case the oscillatory behavior of the magnetic field is ensured on all scales by the closed geometry (i.e. by the compactness) of the space.

When dealing with the hyperbolic geometry of a spatially open FRW model, however, the oscillatory behavior

of $B_{(n)}$ is not always guaranteed. Indeed, for $k = -1$ Eq. (8) reads

$$\mathcal{B}''_{(n)} + (n^2 - 2)\mathcal{B}_{(n)} = 0, \quad (10)$$

which clearly does not accept an oscillatory solution on sufficiently long wavelengths (i.e. for $n^2 < 2$). These wavelengths extend from large subcurvature scales, with $1 \leq n^2 < 2$, to supercurvature lengths with $0 \leq n^2 < 1$. Let us consider the largest subcurvature scales first, since on these wavelengths the associated magnetic modes can be causally connected. It is convenient to introduce the parameter $k^2 = 2 - n^2$, so that the range $0 < k^2 \leq 1$ corresponds to the largest subcurvature scales. Then, Eq. (10) assumes the form

$$\mathcal{B}''_{(k)} - k^2 \mathcal{B}_{(k)} = 0, \quad (11)$$

yielding the following solution for large-scale magnetic fields

$$B_{(k)} = \frac{1}{a^2} [C_1 \cosh(|k|\eta) + C_2 \sinh(|k|\eta)]. \quad (12)$$

On these scales the standard $B \propto a^{-2}$ law is not *a priori* guaranteed. Indeed, consider a FRW universe with open spatial sections. Then, the Friedmann and the Raychaudhuri equations combine to provide the expression

$$aH = \coth \left[\frac{1}{2} (1 + 3w)\eta + C \right], \quad (13)$$

where C depends on the normalization. The above governs the expansion dynamics during the various epochs in the lifetime of this universe, provided that the barotropic index w remains constant throughout each period. For our purposes the key period is that of an inflationary expansion with $p/\rho = w = -1$. The reason is that then the conductivity of the cosmic medium is effectively zero and the magnetic evolution is monitored by Eqs. (6)–(8). Also, the most dramatic suppression of the field occurs during inflation and therefore any change in the magnetic depletion rate during that period could prove crucial. Note that inflation does not change the geometry of the 3-space, but simply makes it look flatter by pushing the curvature scale well beyond the observer’s horizon. Setting $C = 0$, which means that $\eta < 0$, reduces (13) to $aH = -\coth \eta$. The latter integrates to give

$$a = \frac{A_0 e^\eta}{1 - e^{2\eta}}, \quad (14)$$

with $A_0 = a_0(1 - e^{2\eta_0})/e^{\eta_0}$ a positive constant (see [13] for details).⁴ Substituting this result into the right-hand side of Eq. (12) we can express the evolution of the magnetic field in terms of the cosmological scale factor. For simplicity consider the case of $|k| \rightarrow 1^-$, which corresponds to

⁴The adopted normalization scheme, where $C = 0$ and $\eta < 0$, has allowed us to streamline the key equations considerably without loss of generality. Within these conventions, $a \rightarrow 0$ for $\eta \rightarrow -\infty$ and $a \rightarrow +\infty$ as $\eta \rightarrow 0^-$.

the largest subcurvature scales with $n^2 \rightarrow 1^+$. Then, from (12) and (14) we arrive at

$$B = C_3(1 - e^{2\eta})a^{-1} + C_4e^{-\eta}a^{-2}, \quad (15)$$

where C_3 and C_4 are constants. Therefore, on the largest subcurvature scales, the dominant magnetic mode never depletes faster than a^{-1} . This decay rate is considerably slower than the typical a^{-2} law and holds throughout the inflationary era. Note that the magnetic depletion switches to the adiabatic a^{-2} rate at the $\eta \rightarrow 0^-$ limit only.⁵ Result (15) immediately implies that, beyond a certain scale, the cosmological magnetic flux increases with time instead of being preserved. Hence, in spatially open almost-FRW universes, large-scale magnetic fields that survive inflation are significantly stronger than anticipated because of curvature effects alone.

IV. THE RESIDUAL MAGNETIC FIELD

In the previous sections we have studied the evolution of large-scale primordial magnetic fields, emphasizing on their behavior during the inflationary regime of a spatially open FRW cosmology. So far we have provided a qualitative analysis that identified a superadiabatic-type amplification for magnetic fields spanning the largest subcurvature scales of the universe. Next we will attempt to estimate the key properties of these superadiabatically amplified fields, namely, their strength and coherence length.

Following [7], the energy density stored in the n th magnetic mode as it crosses outside the horizon is $\rho_B = (M/m_{\text{Pl}})^4 \rho$, where $\rho \approx M^4$ is the total energy density of the universe and m_{Pl} is the Planck mass. Then, assuming that $B^2 \propto a^{-4}$, the energy density in the mode at the end of the inflationary regime is given by [7]

$$\rho_B = \frac{B^2}{8\pi} \sim 10^{-104} \tilde{\lambda}_{\text{Mpc}}^{-4} \rho_\gamma. \quad (17)$$

Here ρ_γ is the radiation energy density and $\tilde{\lambda}$ is the comoving scale of the field. The latter is measured in Mpc and it is normalized so that $\tilde{\lambda}$ coincides with the physical scale today. Note that the magnetic mode crossed outside the horizon $N = N(\tilde{\lambda})$ e-folds before the end of

⁵According to Eq. (10), the curvature effects modify the magnetic evolution on large scales with $n^2 < 2$. Expression (15) shows that as $|k| \rightarrow 1^-$, which corresponds to $n^2 \rightarrow 1^+$ and the largest subcurvature scales, the magnetic field decays as a^{-1} . When $n^2 \rightarrow 2^-$, on the other hand, we have $|k| \rightarrow 0^+$ and $B \propto a^{-2}$. In particular, expressions (12) and (14) combine to provide the general solution

$$B_{(k)} = C_3(1 - e^{2\eta})^{|k|} a^{|k|-2} + C_4(1 - e^{2\eta})^{-|k|} a^{-|k|-2}, \quad (16)$$

with $|k| \leq 1$. Clearly, when $|k|$ takes its values in the open interval (0,1) the decay rate of the dominant magnetic mode varies between a^{-2} and a^{-1} , which is always slower than the adiabatic a^{-2} law.

inflation (see [7] for details). The underlying assumption leading to the above result is that any given mode is excited quantum mechanically while inside the horizon and “freezes in” as a classical perturbation once it crosses through the Hubble radius. The dramatic weakness of the residual field demonstrated in Eq. (17), reflects the drastic suppression of the magnetic energy density relative to the vacuum energy, which remains constant throughout the inflationary regime. After inflation ρ_γ also decays as a^{-4} and the ratio $r = \rho_B/\rho_\gamma$ does not change.

If the dynamo amplification of large-scale fields is efficient, the strength of the required magnetic seed, as measured at the time of completed galaxy formation, ranges from $\sim 10^{-19}$ G down to $\sim 10^{-23}$ G. In addition, the coherence length of the initial field should be at least as large as the size of the largest turbulent eddy, namely, no less than ~ 100 pc. The aforementioned magnetic strengths, which correspond to $r \sim 10^{-27}$ and $r \sim 10^{-35}$, respectively, have been obtained in a spatially flat universe with zero cosmological constant. However, if the universe is open or if it is dominated by a dark-energy component, the above quoted requirements are considerably relaxed. In particular, the standard dynamo can produce the currently observed galactic magnetic fields from a seed of the order of 10^{-30} G, or even less, at the end of galaxy formation [11]. Note that a “collapsed” magnetic field of $\sim 10^{-30}$ G coherent on approximately 100 pc corresponds to a comoving field of the order of 10^{-34} G spanning a scale of ~ 10 kpc. Nevertheless, even seeds as weak as 10^{-34} G have been very difficult to produce in a conventional way on the required scales. For example, assuming a field with a coherence length of 10 kpc and using Eq. (17), we find a residual strength of approximately 10^{-53} G. Clearly, such fields cannot seed the galactic dynamo and are therefore astrophysically irrelevant.

The situation changes considerably if during inflation the magnetic energy density decays as a^{-2} instead of following the adiabatic a^{-4} law. As we have already seen, this happens on the largest subcurvature scales (and beyond) when the inflationary patch has negative spatial curvature. Therefore, the universe can be permeated by substantially strong large-scale magnetic fields even if it is only marginally open today. For a direct comparison with the spatially flat case scenario, it helps to follow the analysis of [7] [see also Eq. (17) above]. Consider a typical grand unified theory-scale inflationary scenario with $M \sim 10^{17}$ GeV and reheating temperature $T_{\text{RH}} \sim 10^9$ GeV. Then, for $B^2 \propto a^{-2}$, the energy density stored in a given magnetic mode at the end of inflation is given by

$$\rho_B \sim 10^{-90} M^{8/3} T_{\text{RH}}^{-2/3} \tilde{\lambda}_{\text{Mpc}}^{-2} \rho_\gamma \sim 10^{-51} \tilde{\lambda}_{\text{Mpc}}^{-2} \rho_\gamma, \quad (18)$$

instead of (17). According to the above, on a given scale, the earlier inflation starts and the lower the reheating temperature, the stronger the superadiabatically amplified residual field. After inflation the high conductivity of the

plasma is restored. This ensures that $B^2 \propto a^{-4}$ and consequently that the ratio $r = \rho_B/\rho_\gamma \sim 10^{-51} \tilde{\lambda}_{\text{Mpc}}^{-2}$ remains fixed. To proceed we note that $\tilde{\lambda}$ is nearly the curvature scale at the end of inflation. Also, in a universe with non-trivial spatial geometry the effect of curvature in a comoving region remains unchanged, since the curvature scale simply redshifts with the expansion (e.g. see [20]). This means that if $1 - \Omega_0$ is of the order of 10^{-2} , as it appears to be today [21], the current curvature scale is

$$(\lambda_k)_0 = \frac{(\lambda_H)_0}{\sqrt{1 - \Omega_0}} \sim 10^4 \text{ Mpc}, \quad (19)$$

where $(\lambda_H)_0 = H_0^{-1}$, $H_0 \simeq 2h \times 10^{-42} \text{ GeV}$, and $0.5 \leq h \leq 1$. The above is also the approximate scale of the superadiabatically amplified primordial magnetic field, redshifted to the present. Then, by substituting this comoving scale into expression (18) we find that

$$r = \frac{\rho_B}{\rho_\gamma} \sim 10^{-59}, \quad (20)$$

which corresponds to a magnetic field with current strength around 10^{-35} G . Note that the above quoted strength depends on the current values of the Hubble and the density parameters, although this dependence is weak. Also, in order to satisfy the conventional causality requirements we have implicitly assumed that the universe was sufficiently open at the onset of inflation. In particular, a relatively mild initial value of $\Omega_i < 0.1$ will suffice for all practical purposes. Such a value ensures that effectively all the largest subcurvature modes are initially inside the horizon and therefore in causal contact when inflation starts.

The first point to underline is that, to the best of our knowledge, magnetic fields with $B_0 \sim 10^{-35} \text{ G}$ and coherence lengths of $\sim 10^4 \text{ Mpc}$ are greatly stronger than any field obtained within standard electromagnetic theory on such scales. Moreover, fields with this strength are of astrophysical interest because they can successfully seed the galactic dynamo, as long as the current energy density of the universe is dominated by a dark component; a scenario favored by recent observations [21]. For a nearly flat universe with the dark energy making up to 70% of the present density parameter, in particular, a seed field of $\sim 10^{-35} \text{ G}$ is within the lower strength required for the galactic dynamo to operate [11]. Note that the above given magnetic strengths do not account for the effects of the physically more realistic scenario of anisotropic protogalactic collapse. The latter is expected to add a few more orders of magnitude to any field obtained through the highly idealized spherical collapse models [22].

For completeness, let us also consider the magnetic evolution on supercurvature scales. During inflation supercurvature modes also obey Eqs. (11) and (15). On these scales the eigenvalue (n) lies in the interval $[0, 1)$, which implies that $1 < k^2 \leq 2$. Then, near the $k^2 = 2$ limit that

corresponds to the homogeneous mode, the magnetic decay rate becomes $B \propto a^{\sqrt{2}-2}$. The latter is considerably slower than the a^{-1} law associated with the largest subcurvature scales. One should keep in mind, however, that supercurvature scales in spatially open FRW cosmologies lie always outside the Hubble radius and therefore are not causally connected. Nevertheless, any magnetic field that happens to span over these scales at the onset of inflation will decay much slower than its subcurvature counterparts.

Finally, we should note that the linear amplification mechanism outlined here, which is purely geometrical in nature, is in some respects analogous to the one discussed in [23]. There, the electromagnetic field is coupled to the inhomogeneous metric of a perturbed FRW model. Given the *a priori* weakness of the field, however, the magnetic amplification achieved in [23] is presumably a nonlinear effect. The same also can be said about the scenario discussed in [24], where a weak primordial magnetic field was amplified through its coupling to gravity wave perturbations soon after the end of inflation.

V. DISCUSSION

The origin and the evolution of the magnetic fields that we observe almost everywhere in the universe today remains an open issue and a matter of debate. The structure of the galactic large-scale field strongly suggests a dynamo-type amplification mechanism, but the latter requires a seed field to operate. Depending on the efficiency of the large-scale dynamo, the strength of the required seed varies between 10^{-12} and 10^{-23} G at the time of completed galaxy formation, while its coherence length is approximately 10 kpc on comoving scales. However, the questions regarding the origin of cosmic magnetism involve not only the initial seed fields but the dynamo mechanism itself. As yet, there is no final dynamo theory and the whole subject is still under intense research [25]. Therefore, there is no certainty on what the properties of the initial seed magnetic field should be. For instance, the fact that astrophysical plasmas are gas mixtures (neutrals, ions and electrons) can substantially modify the standard single fluid approach (e.g., see [26]) and the dynamo action [27]. Besides, turbulent effects during the radiation era can change the features of a primordial field by enlarging, say, its coherent length [28]. Magnetic helicity also is expected to play a pivotal role in these phenomena. Hence, the requirements necessary for the subsequent MHD process that will amplify the primordial seed could be substantially relaxed.

The geometry of our universe, whether it is open or closed, and whether its energy density is close to the critical one is also an open question of contemporary cosmology [29]. Current observations strongly suggest that the universe is nearly flat, though they stop short from establishing whether it is marginally open or marginally closed. It appears also that at present the expansion dynamics is dictated by a dark-energy component, in the

form of a positive cosmological constant or quintessence. If so, the standard constraints on the magnetic seed strength required for the galactic dynamo to operate efficiently can be relaxed down to 10^{-34} G, or even less. However, even fields as weak as 10^{-34} G, on comoving scales of approximately 10 kpc, are very difficult to produce unless standard electromagnetism is violated. The latter effectively means breaking the conformal invariance of Maxwell's equations and in most of the cases this is achieved by appealing to less well understood phenomenology. The underlying reason is that in spatially flat FRW models the magnetic fields decays as $B \propto a^{-2}$ always and irrespective of plasma effects.

On these grounds, we have studied the evolution of cosmological magnetic fields in perturbed FRW with nontrivial background geometry. By allowing for curved spatial sections, we showed that the adiabatic $B \propto a^{-2}$ law is not always guaranteed because of the linear coupling between the field and the background 3-geometry [13]. When dealing with spatially open FRW models, in particular, the extra curvature-related source term in the magnetic wave equation meant that large-scale fields decay as a^{-1} instead of the standard adiabatic a^{-2} law. This is possible for fields evolving through a period of inflationary expansion, due to the very low electrical conductivity of the latter. As a result, primordial magnetic fields coherent on the largest subcurvature scales could survive an epoch of inflation and still be strong enough to sustain the dynamo process. Our linear mechanism operates near the curvature scale and, in particular, at the largest subcurvature scales. This in turn ensures that the superadiabatically amplified magnetic field has rather specific properties. Assuming that $1 - \Omega \approx 10^{-2}$ today and that $H_0 = 100h$ km/sec·Mpc, with $0.5 \leq h \leq 1$, we find a residual field of the order of 10^{-35} G spanning over a region of approximately 10^4 Mpc. Magnetic fields like these are by far stronger than any other large-scale field obtained within standard electromagnetic theory. Moreover, magnetic fields with the aforementioned properties are of astrophysical interest provided the energy density of our universe is currently dominated by a dark component. If so, a comoving field of strength of

the order of 10^{-35} G can seed the large-scale galactic dynamo when its coherence scale is at least as large as 10 kpc. The latter is much less than the coherence length of our superadiabatically amplified field, though we expect fragmentation of the original seed field during the protogalactic collapse and the subsequent nonlinear era.

If the universe is marginally open today, our mechanism allows for a simple, viable, and rather efficient amplification of large-scale primordial seed magnetic fields to strengths that can seed the galactic dynamo. Even if the universe is not open, this study still brings about a rather important issue. This is the unique nature and nontrivial properties of magnetic fields and their potential implications in the context of general relativity. Magnetic fields, in particular, are the only vector source that we know that exist in the universe today and in the geometrical framework of Einstein's theory vectors have different status than scalars. The special status of the former, which is manifested in the Ricci identities, couples the Maxwell field directly to the geometry of the space in a natural way. This coupling has been largely bypassed in the literature, though its implications are generally nontrivial and in many cases quite counter-intuitive [30]. The best known example is probably the scattering of electromagnetic waves by the gravitational field, which leads to the violation of Huygens principle [10]. Here, we have considered the implications of this relativistic magneto-geometrical interaction for the evolution of large-scale magnetic fields in FRW universes. We found that, contrary to the widespread perception, a superadiabatic-type amplification of cosmological magnetic fields is possible in conventional cosmological models and within standard electromagnetic theory. Therefore, in this case, the magneto-geometrical coupling mimics effects that have been traditionally attributed to new physics.

ACKNOWLEDGMENTS

We would like to thank Anthony Challinor, Anne Davis, Carlos Martins, Kandu Subramanian, Shinji Tsujikava, and Larry Widrow for helpful discussions and comments.

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