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Detection of gravity waves by phase modulation of the light from a distant star

G. B. Lesovik,[1] A. V. Lebedev,[1,2] V. Mounutcharyan,[1] and T. Martin[2]

[1]*Landau Institute for Theoretical Physics RAS, 117940 Moscow, Russia*

[2]*Centre de Physique Théorique et Université de la Méditerranée, Case 907, 13288 Marseille, France*

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We propose a novel method for detecting gravitational waves (GW), where a light signal emitted from a distant star interacts with a local (also distant) GW source and travels towards the Earth, where it is detected. While traveling in the field of the GW, the light acquires specific phase modulation (which we account in the eikonal approximation). This phase modulation can be considered as a coherent spreading of the given initial photons energy over a set of satellite lines, spaced at the frequency of GW (from quantum point of view it is multigraviton absorption and emission processes). This coherent state of photons with the energy distributed among the set of equidistant lines, can be analyzed and identified on Earth either by passing the signal through a Fabry-Perot filter or by monitoring the intensity-intensity correlations at different times.

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I. INTRODUCTION

The detection of gravity waves (GW) has stimulated a lot of interest for decades. There are two major GW detection concepts: acoustic and interferometric detection. The acoustic method deals with a resonance response of massive elastic bodies on GW excitations. Historically the acoustic method was proposed first by J. Weber [1] where he suggested to use long and narrow elastic cylinders as GW antennas. Although a significant progress has been achieved in fabrication and increasing sensitivity of such type of detectors [2–4] the interpretation of obtained data is still far to claim undoubtedly the detection of GW. On the other hand a considerable attention has been shifted recently to more promising interferometric detection methods. The interferometric gravitational-wave detector like Laser Interferometric Gravitational-Wave Observatory (LIGO) and VIRGO [5,6] represents a Michelson interferometer with a laser beam split between two perpendicular arms of interferometer. The principles of operation of such type of detectors are reviewed in Refs [7–11]. The action of gravitational waves on an interferometer can be presented as relative deformation of both interferometer arms. A gravitational wave with dimensionless amplitude h induces the opposite length changes $\delta l/l = 1/2h\cos\Omega t$ in each arm of the Michelson interferometer, where l is the length of the arm, Ω is the gravitational-wave frequency. These length changes produce opposite phase shifts between two light beams in interferometer arms, when interference occurs at the beam splitter of Michelson interferometer. The resulting phase shift of a single beam of light spending time τ in the interferometer can be written as [11]

$$\delta\phi = h\frac{\omega}{\Omega}\sin\frac{\Omega\tau}{2}, \tag{1}$$

where ω is the light frequency. This phase shift results an intensity signal change of the light from interferometer beam splitter hitting the photodetector.

The main problem of the acoustic and interferometric methods that they both deal with is gravitational waves with extremely small amplitudes of the order $h \sim 10^{-21}$ [12] reached the Earth from deep space. One can see from Eq. (1) that for gravity wave frequencies in the 1 kHz range, $\Omega \sim 10^3$ Hz, and for the light in visual frequency range, $\omega \sim 10^{14}$ Hz, one has the maximum phase shift of the order $\delta\phi \sim 10^{-10}$ for interferometer arms length of the order 150 kilometers. Such extraordinarily weak effect requires an exceedingly high detector sensitivity both acoustic and interferometric detectors.

Alternatively, GW detection may be based on effects associated with propagation of light or electromagnetic waves in gravitational fields. There are two primary effects for the light in constant gravitational field i) the deflection of light rays near massive bodies [13] and ii) the Shapiro effect accounting for integrated time delay of the signal passing near a strong source of gravitational field [14]. The same effects have to be observed for light propagating in gravity waves: the gravitational waves have to induce a weak time dependent deflection of light ray propagating through these waves and also lead to gravity-wave-induced variation in time delay. The idea to use astrometry to detect periodic variation in apparent angular separation of appropriate light sources was proposed by Fakir [15]. It is shown that for a gravity wave source located between the Earth and the light source (with line of sight close aligned to gravity wave source) a periodical variation of the order $\Delta\phi \sim \pi h(\Lambda)$ in the angular position of the light source has to be observed. Here $h(\Lambda)$ is the dimensionless strength of the gravity wave at distances of the order gravitational wavelength Λ which is many orders of magnitude greater than the strength of the same waves when they reach the Earth. On the other hand one can directly measure the variation of the integrated time delay induced by gravity waves on the light emitted by a distant star passing through space region with strong gravity wave. The idea to use timing observation for detection of the gravitational waves was suggested first by Sazhin [16] and then this problem was studied in details by several authors [17–19]. The estimations carried out in Ref. [17] give the following answer for the rate of change in the gravity-wave-induced time delay, $\dot{\tau}$:

$$|\dot{\tau}| \sim |h(r = D)|, \tag{2}$$

where $h(r = D)$ is the gravity wave strength at distances of the order of impact parameter D for the light beam passing near the gravity wave source. To put some numbers let us consider a dissymmetric rotating neutron star with spin frequency of the order 10^3 Hz and gravity wave amplitude of the order $H \sim 10^{-5}$ cm [8], then Eq. (2) gives $|\dot{\tau}| \sim 10^{-12}$ while the same effect measured near Earth results $|\dot{\tau}| \sim 10^{-26}$ assuming the neutron star to be at a typical distance of a few kiloparsecs from the Earth. One can see that all effects due to gravitational waves near the gravity wave source are several orders of magnitude stronger than the same ones on the Earth.

In the present paper we want to suggest a new gravitational waves detection method based on the interaction of the photon with gravitational waves. Assuming that the photons from a distant star passing near gravitational-wave source, where the photon-gravity wave interaction assumed to be strong, the photon-gravity wave interaction leads to relatively strong modulation in time of photon frequency. The latter allows to analyze this modulation of the photons reaching the Earth by means of standard optical methods including the Fabry-Perot analysis and quantum photon correlations measurements. It is important that while the interaction of photons with gravitational wave is rather weak (proportional to strength of gravitational wave) the frequency modulation can be accumulated over large distances during the photon propagation that could result in an experimentally measurable effect on the Earth. In some aspects our treatment of the photon-gravity wave interaction resembles the effect of photon acceleration by gravitational wave [20], where the photon propagating in plane gravitational wave long enough time acquires a considerable increase of the frequency. However while in Ref. [20] the photon-gravity wave interaction was treated in the frame of reference of the photon, we have considered this interaction in the point of observation frame coordinate which seems to give the widening of the initial monochromatic photon wave packet rather than the increase of the photon frequency.

II. PROPAGATION OF LIGHT NEAR THE LOCALIZED SOURCE OF GRAVITATIONAL WAVES

In this section we consider how the plane electromagnetic waves interact with gravitational-wave field emitted by some localized source. The situation we have in mind is depicted in Fig. 1. The light signal originates from a distant star S and travels towards the Earth to be detected by an observer O, but along the way it interacts with a gravitational body M, which also emits gravity waves. We consider the trajectory of the light ray in the two-dimensional plane formed by the star S, the body which emits the GW M, and the Earth. Provided that the wavelength of the GW is large compared to the wavelength of the light signal, we will use the eikonal or geometrical optics approximation to describe the interaction of the light in gravitational waves. The propagation of the electromagnetic waves in the field of gravitational waves was considered many times and we can refer the reader to several papers dealing with these questions in the geometrical optics approximation [7,21–25]. Our subsequent analysis of this problem will be carried out in close analogy with the paper of Sazhin and Maslova [25], where they have considered the structure of electromagnetic field in Fabry-Perot resonator in the field of gravitational waves. However in the present paper we will be interested in a different aspect of the interaction of the light beam passing in immediate proximity to the gravitational-wave source, where the amplitude of the gravitational waves is assumed to be strong in comparison with the amplitude of the same waves reaching the Earth. It is important that since the light interacts with gravitational waves in the region where they are strong, the resulting photons frequency modulation seems to be appreciable to detect it on the Earth.

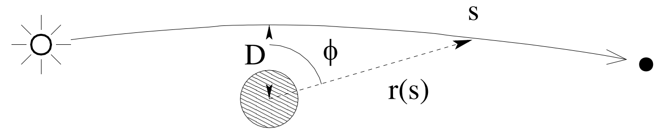

FIG. 1. A star emits light which is deformed by the gravity field, but its phase is modulated, an effect which can be detected on the Earth.

A. The eikonal equation

In the geometrical optics approximation the propagation of the light ray is described by the eikonal equation:

$$g^{ik} \frac{\partial \psi}{\partial x^i} \frac{\partial \psi}{\partial x^k} = 0, \tag{3}$$

where $g^{ik} = g_0^{ik} - h^{ik}$ the metric tensor associated with the Schwarzschildian static metric g_0^{ik} of the object M, which is perturbed by the tensor h^{ik} associated with the GW emitted from M [13]. Here we will neglect the true form of the static metric g_0^{ik} assuming that light ray propagates in the flat Minkovsky space $g_0^{ik} = \eta^{ik} = \mathrm{diag}\{1, -1, -1, -1\}$. It is known that the static metric leads both to the deflection of the light ray near the massive body and time delay of the light signal passing along the ray (so called Shapiro effect). It can be rigorously shown in the subsequent analysis that the former effect of the light deviation leads to the negligible correction to the additional phase accumulated by photons due to the interaction with GW and one can assume the light trajectory to be a straight line. As for static Shapiro time delay it can be easily incorporated to the final answer for photon phase and actually does not affect on the subsequent detection method of modulated photons.

Let $\psi_0 = -\omega t + ky$ be the eikonal in the absence of gravitational waves perturbation with metric η^{ik} (we assumed the light ray to be propagating along the y axis in the plane formed by three bodies). Assuming that the perturbation is small, in the presence of gravitational waves the eikonal becomes $\psi = \psi_0 + \psi_1$, where ψ_1 a small addition to the eikonal ψ_0 computed to first order in h^{ik}—it satisfies the equation:

$$\frac{\partial \psi_1}{\partial t} + c\frac{\partial \psi_1}{\partial y} = -\frac{\omega}{2}F(t, r), \tag{4}$$

where we have introduced the notation $F(t, r) = h^{00} + h^{yy} - 2h^{0y}$. Using the Green's function formalism one can find the general solution of this equation. Then to first order in the gravitational-wave perturbation h^{ik} the additional phase acquired by the photon due to the interaction with the GW, can be written for arbitrary time dependence of the perturbed metric:

$$\psi_1(t, y) = -\frac{1}{2}\frac{\omega}{c}\int_{-\infty}^{y} F\left(t - \frac{y - y'}{c}, y'\right)dy'. \tag{5}$$

B. The emission of GW

Assuming that the GW tensor h^{ik} is a small perturbation to the static flat metric in the quadrupole approximation one can write the coordinate components of the GW tensor as [13]:

$$\tilde{h}_{\alpha\beta} = \frac{2G}{c^4}\frac{\ddot{Q}_{\alpha\beta}(t - r/c)}{r}, \tag{6}$$

where G is the Newtonian gravitation constant, $\tilde{h}_{ik} = h_{ik} - \frac{1}{2}\eta_{ik}h$ (where $h \equiv h_i^i$), $Q_{\alpha\beta}$ is the quadrupole moment:

$$Q_{\alpha\beta}(t) = \int \tilde{T}_{00}(t, r)x^\alpha x^\beta d^3x, \tag{7}$$

where $\tilde{T}_{ik} = T_{ik} - \frac{1}{2}\eta_{ik}T$, T_{ik} is the energy-momentum tensor of the GW source, $T \equiv T_i^i$. Equation (6) implies that the so-called harmonic gauge condition, $\partial_k\tilde{h}_i^k = 0$ has been chosen for the components of GW tensor h_{ik}.

Using the gauge condition one can find the $\tilde{h}_{00}$ and $\tilde{h}_{0\alpha}$ components of the GW tensor. To do this let us write the equation $\partial_k\tilde{h}_i^k = 0$ separately for coordinate and time components:

$$\frac{\partial \tilde{h}_{\alpha\beta}}{\partial x^\beta} = \frac{\partial \tilde{h}_{\alpha 0}}{\partial x^0}, \qquad \frac{\partial \tilde{h}_{0\alpha}}{\partial x^\alpha} = \frac{\partial \tilde{h}_{00}}{\partial x^0}. \tag{8}$$

Combining the Eq. (6) with the first Eq. (8) the $\tilde{h}^{\alpha 0}$ components of GW tensor can be written as:

$$\tilde{h}_{\alpha 0} = \frac{2G}{c^3}\frac{\partial}{\partial x^\beta}\left[\frac{\dot{Q}_{\alpha\beta}(t - r/c)}{r}\right], \tag{9}$$

while from this equation and the second Eq. (8) one has:

$$\tilde{h}_{00} = \frac{2G}{c^2}\frac{\partial^2}{\partial x^\alpha \partial x^\beta}\left[\frac{Q_{\alpha\beta}(t - r/c)}{r}\right]. \tag{10}$$

Using the expressions (6), (9),(10) for the components of the GW tensor write the explicit expression for $F(t, r) = h^{00} + h^{yy} - 2h^{0y}$ in the right-hand side of Eq. (5). Taking coordinate derivatives and keeping in mind that along the ray trajectory $\partial_x r = D/r$, $\partial_y r = y/r$, $\partial_z r = 0$ (where D is the impact parameter) the $F(t, r)$ takes the form

$$\begin{aligned} F(t, r) = {}& \frac{2G}{c^4}\left[\frac{\ddot{Q}_{xx}}{r}\frac{D^2}{r^2} + 2\frac{\ddot{Q}_{xy}}{r}\frac{D}{r}\left(\frac{y}{r} - 1\right) + \frac{\ddot{Q}_{yy}}{r}\left(\frac{y}{r} - 1\right)^2\right] \\ & + \frac{2G}{c^3}\left[3\frac{\dot{Q}_{xx}}{r^2}\frac{D^2}{r^2} + 2\frac{\dot{Q}_{xy}}{r^2}\frac{D}{r}\left(3\frac{y}{r} - 1\right)\right. \\ & \left. + \frac{\dot{Q}_{yy}}{r^2}\left(3\frac{y^2}{r^2} - 2\frac{y}{r} - 1\right)\right] \\ & + \frac{2G}{c^2}\left[3\frac{Q_{xx}}{r^3}\frac{D^2}{r^2} + 6\frac{Q_{xy}}{r^3}\frac{yD}{r^2} + \frac{Q_{yy}}{r^3}\left(3\frac{y^2}{r^2} - 1\right)\right], \end{aligned} \tag{11}$$

where all quadrupole moments $Q_{\alpha\beta}$ are assumed to be the functions of the retarded variable $t - r/c$.

C. Solution of the eikonal equation

For simplicity the source of GW is assumed to emit only one frequency Ω. We consider the two following situations: either the GW source has been emitting since $t = -\infty$, or the signal is bounded in time, with an exponential decay. In the former case one can write $\ddot{Q}_{\alpha\beta}(t) = q_{\alpha\beta}\sin\Omega t$. Then substituting the expression for $F(t, r)$, see Eq. (11), into Eq. (5) one could write the solution for ψ_1 as an integral over the variable $z = y'/D - \sqrt{1 + (y'/D)^2}$:

$$\begin{aligned} \psi_1(\xi) = {}& -2\frac{\omega}{c}\int_{D/(2y)}^{\infty}\frac{z^3H_{yy} - 2z^2H_{xy} + zH_{xx}}{(z^2 + 1)^2}\sin\left[\Omega\xi - 2\pi\frac{D}{\Lambda}z\right]dz + 4\frac{\omega}{c}\left(\frac{\Lambda}{2\pi D}\right) \\ & \times \int_{D/(2y)}^{\infty}\frac{z^2(z^2 - 2)H_{yy} - 2(2z^3 - z)H_{xy} + 3z^2H_{xx}}{(z^2 + 1)^3}\cos\left[\Omega\xi - 2\pi\frac{D}{\Lambda}z\right]dz + 4\frac{\omega}{c}\left(\frac{\Lambda}{2\pi D}\right)^2 \\ & \times \int_{D/(2y)}^{\infty}\frac{z(z^4 - 4z^2 + 1)H_{yy} - 6z^2(z^2 - 1)H_{xy} + 6z^3H_{xx}}{(z^2 + 1)^4}\sin\left[\Omega\xi - 2\pi\frac{D}{\Lambda}z\right]dz, \end{aligned} \tag{12}$$

where the variable $\xi = t - y/c$ accounts for the effect of retardation of the light signal during its propagation to the point of observation, $H_{\alpha\beta} = 2Gq_{\alpha\beta}/c^4$, Λ is the wavelength of gravitational waves. Assuming that at the point of observation $y \gg D$, one can safely set the lower limit for all integrals in this expression equal to zero. Then for large values of the impact parameter $2\pi D/\Lambda \gg 1$ one can asymptotically estimate these integrals and the main contribution to ψ_1 is given by the following expression:

$$\psi_1(\xi) \approx \frac{1}{2\pi^2}\left(\frac{\Lambda}{D}\right)^2 \frac{\omega}{c} H_{xx} \sin\Omega\xi \tag{13}$$

$$\approx \frac{h_{xx}}{\pi}\frac{\Lambda}{\lambda}\left(\frac{\Lambda}{D}\right)^2 \sin\Omega\xi, \tag{14}$$

where in the last line the answer is rewritten in terms of the dimensionless strength of gravitational waves: $h_{xx} = H_{xx}\Lambda^{-1}$. One could see that the main contribution to the photon phase ψ comes from the h_{xx} component of gravitational radiation confirming the well-known fact [18,19] that only gravitational waves propagating perpendicular to the photon wave vector (directed along y axis in our geometry) have a considerable effect on the electromagnetic radiation.

Let us now consider the case where gravity waves are emitted starting at $t = 0$ decaying exponentially with characteristic time τ_0. In this case let us choose $\ddot{Q}_{\alpha\beta}(t)$ to be equal to:

$$\ddot{Q}_{\alpha\beta}(t) = q_{\alpha\beta}\theta(t)\exp\left(-\frac{t}{\tau_0}\right)\sin\Omega t, \tag{15}$$

where $\theta(x) = 0$ for $x < 0$ and 1 for $x > 0$. Then for large values of the impact parameter, $2\pi D/\Lambda \gg 1$, the solution for ψ_1 is given by the following formula:

$$\psi_1(\xi) \approx \frac{h_{xx}}{\pi}\left(\frac{\Lambda}{D}\right)^2 \frac{\Lambda}{\lambda}\theta(\xi)e^{-\xi/\tau_0}\sin\Omega\xi. \tag{16}$$

One can see that in this case at the point of observation the modulated contribution to the eikonal also decaying exponentially with the same characteristic time τ_0.

D. The preexponential factor

Let us now go beyond the geometrical optics or eikonal approximation and calculate the preexponential factor for the photon wave function. The photon wave function $\varphi(t, y)$ with momentum k satisfies the wave equation:

$$\frac{1}{\sqrt{-g}}\frac{\partial}{\partial x^i}\left(\sqrt{-g}g^{ik}\frac{\partial\varphi}{\partial x^k}\right) = 0, \tag{17}$$

where $g = \det g_{ik}$. Up to the first order contribution in h^{ik} one could find $g = -(1 + h)$ with $h = h_i^i$. Taking the derivative over x^i and keeping only the first order terms in h^{ik} one could arrive to expression:

$$g^{ik}\frac{\partial^2\varphi}{\partial x^i\partial x^k} - \frac{\partial\varphi}{\partial x^k}\frac{\partial}{\partial x^i}\left(h^{ik} - \frac{1}{2}\eta^{ik}h\right) = 0. \tag{18}$$

One could see that under the harmonic gauge condition $\partial_i\tilde{h}_k^i = 0$ with $\tilde{h}_k^i = h_k^i - \frac{1}{2}\eta_k^i h$ the second term in the above equation vanishes and the photon wave equation reduces to

$$g^{ik}\frac{\partial^2\varphi}{\partial x^i\partial x^k} = 0. \tag{19}$$

Let us write the photon wave function as $\varphi(t, y) = c(t, y)e^{i\psi}$, where ψ is the eikonal of the photon interacting with gravitational waves. Then assuming that $c = 1 + a$, where a is the small contribution of the first order in h^{ik}, one could rewrite the above equation as

$$g^{ik}\left(\frac{\partial^2 a}{\partial x^i\partial x^k} + 2i\frac{\partial\psi}{\partial x^i}\frac{\partial a}{\partial x^k} + i(1 + a)\frac{\partial^2\psi}{\partial x^i\partial x^k} - (1 + a)\frac{\partial\psi}{\partial x^i}\frac{\partial\psi}{\partial x^k}\right) = 0. \tag{20}$$

The last term in this equation vanishes due to the eikonal Eq. (3). Since the free eikonal $\psi_0 = ky - \omega t$ is the linear function of x^i the all second derivatives of ψ_0 vanish and the remaining wave equation up to the first order terms in h^{ik} takes the form:

$$\eta^{ik}\frac{\partial^2 a}{\partial x^i\partial x^k} + 2i\eta^{ik}\frac{\partial\psi_0}{\partial x^i}\frac{\partial a}{\partial x^k} + i\eta^{ik}\frac{\partial^2\psi_1}{\partial x^i\partial x^k} = 0. \tag{21}$$

Assuming that the $a(t, y) = a(t - y/c)$ thus the first term above vanishes one could finally find

$$\frac{\partial a}{\partial t} + c\frac{\partial a}{\partial y} = \frac{1}{2}\frac{c^2}{\omega}\left(\frac{1}{c^2}\frac{\partial^2\psi_1}{\partial t^2} - \frac{\partial^2\psi_1}{\partial y^2}\right). \tag{22}$$

Substituting here the general solution for ψ_1 (see Eq. (5)) one can express the right-hand side of Eq. (22) through the function $F(t, y)$:

$$\frac{\partial a}{\partial t} + c\frac{\partial a}{\partial y} = \frac{c}{4}\left(\frac{\partial F}{\partial y} - \frac{1}{c}\frac{\partial F}{\partial t}\right). \tag{23}$$

According to our previous analysis the main contribution to $a(t, y)$ is given by the H_{xx} component of gravitational radiation:

$$F(t, r) \approx H_{xx}\frac{\sin\Omega(t - r/c)}{r}\frac{D^2}{r^2}. \tag{24}$$

Substituting this expression to the Eq. (23) the solution for $a(\xi)$ for large values of impact parameter $2\pi D/\Lambda \gg 1$ has the form:

$$a(\xi) \approx -\frac{h_{xx}}{\pi}\left(\frac{\Lambda}{D}\right)^2\cos\Omega\xi. \tag{25}$$

Combining together Eq. (14) with Eq. (25) one can finally write the photon wave function at the point of observation in the form:

$$\varphi_k(\xi) = \frac{e^{-i\omega(\xi+\tau_g \sin\Omega\xi)}}{1 + \tau_g\Omega\cos\Omega\xi}, \tag{26}$$

where we have introduced the time τ_g associated with integrated Shapiro time delay (see below):

$$\tau_g = -\frac{h_{xx}}{\pi}\left(\frac{\Lambda}{D}\right)^2\Omega^{-1}. \tag{27}$$

The same solution Eq. (26) for the photon wave function is valid for the case where gravitational radiation is bounded exponentially in time (see Eq. (15)) however in this case τ_g depends on time and equals to

$$\tau_g(\xi) = -\frac{h_{xx}}{\pi}\left(\frac{\Lambda}{D}\right)^2\Omega^{-1}\theta(\xi)e^{-\xi/\tau_0}. \tag{28}$$

E. Modulation due to alternating Newtonian gravitational field

The observation and detection of the gravitational waves undoubtedly provides us a new astrophysical tool which can give us a deeper insight on the processes like the neutron star formation, the processes happening in close binaries at the final stage of its evolution, etc. However the observation of gravitational waves gives us also an additional proof of the General Theory of Relativity itself and it seems reasonable to consider the effect of photon phase modulation in Newtonian theory of gravitation where the gravitational potential instantly adjusts to the current configuration of moving bodies.

Consider, for example, an oscillating neutron star or a rotating double star system which in Newtonian theory of gravity causes an alternating Newtonian gravitational potential $\phi(\mathbf{r}, t)$. This alternating field can also lead to photon phase modulation and results in principle in the same effect like for the case of photon in the field of gravitational waves. The aim of this section is to compare the effect of photon phase modulation between these two theories.

As the rotation or the oscillation frequency of the stars is much less than the frequency of the photon we can work in the same eikonal approximation as in the previous sections. The Newtonian gravitational field contributes only to the diagonal metric elements: $h^{00} = h^{ii} = 2\phi/c^2$, where ϕ is a gravitational potential at the point of observation. At large distances compared to the size of the star system we can leave only the quadrupole term in the potential

$$\phi(\mathbf{r}, t) = GQ_{\alpha\beta}\frac{n_\alpha n_\beta}{2r^3}. \tag{29}$$

Here r is the distance from the star to the photon, the coordinate axis are chosen to be the same as before, so $r = \sqrt{y^2 + D^2}$, n_i—the direction vectors of $\mathbf{r}$: $n_x = D/\sqrt{y^2 + D^2}$, $n_y = y/\sqrt{D^2 + y^2}$, $n_z = 0$, $Q_{\alpha\beta}(t)$ is the alternating tensor of the mass quadrupole moment of the star. The values of $Q_{\alpha\beta}$ are determined by the parameters of the star, like orientation of the angular velocity, eccentricity, etc. We will further discuss the general case, assuming only, that due to rotation or oscillation $\ddot{Q}_{\alpha\beta}(t) = q_{\alpha\beta}\sin\Omega t$.

The correction to the eikonal due to rotation is given by the same Eq. (5) as for the GW case solution of the eikonal equation, where in our case $F(t, y) = h^{00} + h^{yy} = 2h^{00}$

$$\Psi_n = -\frac{\omega}{c}\int_{-\infty}^{y} h^{00}\left(t - \frac{y - y'}{c}, y'\right)dy'. \tag{30}$$

Substituting the expression for ϕ into the formula above and making the substitution $z = y'/D$ one obtains:

$$\Psi_n(\xi) = \frac{1}{2}\frac{\omega}{c}\left(\frac{\Lambda}{2\pi D}\right)^2\int_{-\infty}^{y/D}\frac{H_{xx} + 2H_{xy}z + H_{yy}z^2}{(1 + z^2)^{5/2}} \times \sin\left[\Omega\xi + 2\pi\frac{D}{\Lambda}z\right]dz, \tag{31}$$

where we have introduced $H_{\alpha\beta} = 2Gq_{\alpha\beta}/c^4$.

Then for large values of the impact parameter $2\pi D/\Lambda \gg 1$ and large distances $y/D \gg 1$ one can obtain the following approximate expression for Ψ_n:

$$\Psi_n(\xi) = \frac{1}{6}\frac{\omega}{c}\sqrt{\frac{\Lambda}{D}}e^{-2\pi D/\Lambda}[(H_{xx} - H_{yy})\sin\Omega\xi + 2H_{xy}\cos\Omega\xi]. \tag{32}$$

One can see that the photon phase modulation due to the Newtonian alternating potential vanishes exponentially for large impact parameters in contrast to the GW modulation which vanishes as $1/D^2$ (see Eq. (14)). Technically the difference between Newtonian and GW cases follows from the different y' dependence of the $F(t, y')$ function. For the properly retarded GW radiation the solution for ψ_1, see Eq. (14), involves the oscillating integral over $\sin\Omega[\xi + (y' - \sqrt{(y')^2 + D^2})/c]$, where the oscillating dependence on y' near the source $|y'| \sim D$ almost vanishes making the light-GW interaction more effective. Oppositely for the instantly adjusted Newtonian radiation one has in Eq. (30) fast oscillating integral over y' with $\sin\Omega(\xi + y'/c)$ near vicinity of the gravitational-wave source, which leads to a very small value of modulation.

F. The Shapiro effect in GW

Using the results of the previous sections let us calculate the integrated time delay of the light signal propagating in the field of gravitational waves. Consider an arbitrary wave packet emitted by some distant light source traveling toward Earth near a source of GW:

$$\varphi(\xi) = \int \varphi_\omega e^{-i\omega\xi}\frac{d\omega}{2\pi}, \tag{33}$$

where $\xi = t - y/c$. Using the results for monochromatic photons propagating in the field of gravitational radiation one can find the signal at the point of observation:

$$\varphi_{ob}(\xi) = \int \varphi_\omega e^{-i\omega[\xi+\tau_g \sin\Omega\xi]} \frac{d\omega}{2\pi} = \varphi(\xi + \tau_g \sin\Omega\xi). \tag{34}$$

One can see that the interaction with the gravitational radiation leads to alternating time delay of the signal $\delta t = \tau_g \sin\Omega t$, (note that the τ_g is negative, see Eq. (27)). This effect was suggested independently first by Sazhin and Detweiler [16] for detection of gravitational waves using the timing observation of a pulsar the line of sight to which passes near the source of GW.

III. DETECTION OF THE GW

Let us now turn to the discussion about how the photons modulated by the interaction with the gravitational waves can be measured in some realistic setup.

Consider first the simple case where the modulated photons (26) simply hit a photodetector which can react at all photon's frequencies. However, any real photodetector reacts not on the vector potential $\mathbf{A}(t)$ of the photons in the photodetector directly, but rather on the electrical field $\mathbf{E}(t)$ induced by the photons in this photodetector. Then the one photon photo-detection probability, $P(t)\Delta t$, to observe a photon during time interval Δt can be expressed as [26]:

$$P(t)\Delta t = \alpha \frac{Sc\Delta t}{E_{\rm det}} \langle \hat{\mathbf{E}}^{(-)}(t, \mathbf{r}) \cdot \hat{\mathbf{E}}^{(+)}(t, \mathbf{r})\rangle, \tag{35}$$

where S is the area of the photodetector, $E_{\rm det}$ some characteristic energy describing the interaction of the photodetector with incoming photons, α is the quantum efficiency of the photo-detection, $\hat{\mathbf{E}}^{(+)}(t, \mathbf{r})$ is the positive frequency part of the electrical field operator, $\hat{\mathbf{E}}^{(-)}(t, \mathbf{r})$ is the Hermitian conjugate of $\hat{\mathbf{E}}^{(+)}(t, \mathbf{r})$:

$$\hat{\mathbf{E}}^{(+)}(t, \mathbf{r}) = -\frac{\sqrt{4\pi}}{L^{3/2}} \sum_{\mathbf{k}s} \sqrt{\frac{\hbar}{2\omega_k}} \dot{\varphi}_k(t, \mathbf{r}) \mathbf{e}_{ks} \hat{a}_{ks}. \tag{36}$$

At the point of observation we set $\mathbf{r} = 0$ to shorten the notations. Taking the time derivative of the photon wave function (26) one gets the photo-detection probability equal to

$$\begin{aligned} P(t)\Delta t &= \frac{4\pi\alpha Sc\Delta t}{E_{\rm det}L^3} \sum_{\mathbf{k}s} \frac{\hbar\omega_k}{2} n_{ks} \\ &= 2\alpha \frac{S}{r^2} \Delta t \int d\omega \frac{\hbar\omega}{E_{\rm det}} n(\omega), \end{aligned} \tag{37}$$

where r is the distance between the light source and the Earth, $n(\omega)d\omega$ is the number of the photons with frequency ω emitted by the distant light source per unit time into a unit angular domain.

One can see that the photo-detection probability does not feel any photon phase modulation due to GW and this probability is the same as it would be for the monochromatic photons emitted by a distant light source. Thus for such a simple setup it is not possible to retrieve any information about the gravity wave from the photodetector signal.

Consider however a more realistic setup where the photodetector has a sensitivity edge ω_s which means that the photodetector does not react on the photons with frequency less than ω_s. One can treat this situation as if the photons passed through a filter with transparency $t(\omega) = \theta(\omega - \omega_s)$ before hitting the photodetector. It is convenient to write this transparency as an operator acting on the photon wave function: $\hat{t} = \theta[i\partial_t - \omega_s]$. Then after passing such a filter the photon has a wave function equal to $\hat{t}\varphi_k(t) \approx \theta[\omega_k(t) - \omega_s]\varphi_k(t)$, where $\omega_k(t) = \omega_k[1 + \Omega\tau_g \cos\Omega t] = \omega_k f(t)$. Then for the photo-detection probability one has:

$$P(t)\Delta t \approx 2\alpha \frac{S\Delta t}{r^2} \int d\omega \frac{\hbar\omega}{E_{det}} \theta[\omega f(t) - \omega_s] n(\omega). \tag{38}$$

Consider now the case where photons coming to the photodetector from a sharp Lorentzian spectral line with maximum ω_0 near the photodetector sensitivity edge $\omega_0 \approx \omega_s$ and width Γ_0:

$$n(\omega) = n(\omega_0) \frac{\Gamma_0^2}{(\omega - \omega_0)^2 + \Gamma_0^2}, \tag{39}$$

then according to Eq. (38) the photo-detection probability for the modulated photons (26) has an appreciable contribution which is periodic in time at the GW frequency, and which is proportional to the strength of gravitational waves:

$$P(t)\Delta t \approx P_0 \Delta t \left(1 + \Omega\tau_g \frac{2\omega_0}{\pi\Gamma_0} \cos\Omega t\right), \tag{40}$$

where $P_0\Delta t$ is the photo-detection probability for non-modulated photons.

From the above analysis one can conclude that in order to detect the modulation of the photons by GW one should place before the photodetector some filter which has a finite frequency band. Then the formula for photo-detection probability (38) with a sensitivity edge has a rather general sense. In fact let us consider an arbitrary filter placed before the photodetector with transparency $T(\omega)$. Then provided that the time spent by the photon in this filter (that is an inverse frequency band of the filter) is much smaller than frequency of the GW, Ω, one can treat the modulated photons as monochromatic photons with slowly varying frequency $\omega_k(t) = \omega_k[1 + \Omega\tau_g \cos\Omega t] = \omega_k f(t)$. In this case the photo-detection probability can be written simply as

$$P(t)\Delta t \approx 2\alpha \frac{S\Delta t}{r^2} \int d\omega \frac{\hbar\omega}{E_{det}} T(\omega f(t)) n(\omega). \tag{41}$$

From this expression one can see that there are two different regimes of GW detection in this proposed setup. Consider the first regime where the distribution function of the light source $n(\omega)$ has a sharp peak of the width Γ_0 (or in more general case some sufficient irregularity like the edge of the spectrum) centered near the transmission window of the filter. Then, provided that the amplitude of the photon's frequency modulation, $\omega_k\Omega\tau_g$, is much bigger than the width of the spectrum irregularity, $\omega_k\Omega\tau_g \gg \Gamma_0$, the resulting photo-detection signal will be a periodic sequence of peaks.

In the opposite case where the light spectrum $n(\omega)$ is a slowly varying function of the frequency within the transparency window of the filter one can consider $n(\omega)$ as a constant in Eq. (41). Then the time dependence of the photo-detection probability can be written as:

$$P(t)\Delta t \approx 2\alpha \frac{S\Delta t}{r^2}\frac{1}{f(t)}\int d\omega \frac{\hbar\omega}{E_{det}}T(\omega)n(\omega). \qquad (42)$$

This result can be understood as the time modulation of the number of incoming photons per unit time within a finite frequency interval.

From another point of view one can note that the modulated photon wave function (26) can be written as a superposition of waves with energies shifted by $n\Omega$ (n integer):

$$\varphi_k(t) = \frac{1}{1+\Omega\tau_g\cos\Omega t}\sum_{-\infty}^{+\infty} J_n(\omega\tau_g)e^{-i(\omega+n\Omega)t}, \qquad (43)$$

where J_n is the Bessel function. In order to detect a gravity wave signal it is therefore necessary to provoke an interference between these Fourier components. To do it one can both investigate the modulated light signal passing through the Fabry-Perot interferometer or on the other hand study the intensity-intensity correlation of the photons coming on the photodetector.

A. Analysis of the light signal with an interferometer

In this section we consider in detail the setup where at the point of observation—the Earth—the modulated light signal before hitting the photodetector passes through an interferometer, which is characterized by a complex transmission amplitude $t(\omega)$. We will assume that the light passing through a Fabry-Perot filter comes from a single Lorentzian spectral line of the type (39). For simplicity, let us first assume that there is only one Fabry-Perot (FP) resonance within this spectral line. The transparency of FP is written in the usual way:

$$t(\omega) = \frac{i\Gamma/2}{\omega-\omega_0+i\Gamma/2}. \qquad (44)$$

Here a derivation of the photo-detection probability is provided. It relies on the assumption $\Omega\tau_g \ll 1$, which is relevant for experimental situations (see below).

Consider the propagation of a photon wave packet $\varphi_k(t)$ (26) through the FP interferometer. The resulting wave packet after the FP can be written $\phi_k(t) = \hat{t}\varphi_k(t)$, where $\hat{t}$ is the transparency operator of the FP in real time representation:

$$\hat{t} = \frac{i\Gamma}{2}[i\partial_t - \omega_0 + i\Gamma/2]^{-1}, \qquad (45)$$

where Γ is the width of FP resonance. However the real FP filter does not act directly on the vector potential of the incoming photons but rather on the electromagnetic field induced by these photons. Using similar arguments the positive frequency part of electrical field operator $\mathbf{E}^{(+)}(t, r)$, after passing the Fabry-Perot filter, can be written in terms of the transmission operator of the filter:

$$\hat{\mathbf{E}}^{(+)}(t,\mathbf{r}) = -\frac{\sqrt{4\pi}}{L^{3/2}}\sum_{\mathbf{k}s}\sqrt{\frac{\hbar}{2\omega_k}}[\hat{t}\dot{\varphi}_k(t,\mathbf{r})]\mathbf{e}_{ks}\hat{a}_{ks}. \qquad (46)$$

With these notations the photo-detection probability can be written as

$$P(t) = 2\alpha\frac{S}{r^2}\int\frac{\hbar d\omega}{\omega E_{\rm det}}[\hat{t}\dot{\varphi}_k(t)]^*[\hat{t}\dot{\varphi}_k(t)]n(\omega)d\omega. \qquad (47)$$

The action of the operator $\hat{t}$ on the function $\dot{\varphi}_k(t)$ can be found using the Green's function formalism:

$$\hat{t}\dot{\varphi}_k(t) = \frac{\Gamma}{2}\int_{-\infty}^{t} e^{-i\omega_0(t-\tau)-\frac{\Gamma}{2}(t-\tau)}\dot{\varphi}_k(\tau)d\tau. \qquad (48)$$

Substituting this expression into Eq. (47) in the limit $\omega_0 \gg \Gamma$ one finally obtains the following expression for the photo-detection probability:

$$P(t)=2\alpha\frac{S}{r^2}\frac{\hbar\omega_0}{E_{\rm det}}\frac{\Gamma^2}{4}\int n(\omega)e^{-\Gamma t}d\omega \times\int_{-\infty}^{t}\int_{-\infty}^{t} e^{-i\omega_0(\tau-s)+\frac{\Gamma}{2}(\tau+s)}\dot{\varphi}_k^*(\tau)\dot{\varphi}_k(s)d\tau ds. \qquad (49)$$

Performing the integral over ω and introducing the relative time $\xi = \tau - s$, and the total time $\eta = (\tau + s)/2$, in the working limit $\Omega\tau_g \ll 1$ one can write the photo-detection probability as:

$$P(t) = \alpha\pi\frac{S}{r^2}\frac{\hbar\omega_0}{E_{\rm det}}n(\omega_0)\Gamma_0\frac{\Gamma^2}{4}\int_{-\infty}^{t} d\eta e^{-\Gamma(t-\eta)} \times\int_{-2(t-\eta)}^{2(t-\eta)} e^{i\xi[\omega_0\Omega\tau_g\cos\Omega\eta]}e^{-\frac{\Gamma_0(\eta)}{2}|\xi|}d\xi. \qquad (50)$$

Performing the integration over ξ in the limit $\Gamma_0 \gg \Gamma$ the leading contribution to the photo-detection probability is given by the following expression:

$$P(t) = \alpha\pi \frac{S}{r^2} \frac{\hbar\omega_0}{E_{\rm det}} n(\omega_0)\Gamma_0 \frac{\Gamma^2}{4} \times \int_0^\infty \frac{\Gamma_0(t-z)e^{-\Gamma z}dz}{(\omega_0\Omega\tau_g)^2\cos^2\Omega(t-z) + \Gamma_0^2(t-z)/4}, \quad (51)$$

where we have made the substitution $z = (t - \eta)$ and we introduced the notation $\Gamma_0(t) = \Gamma_0|1 + \Omega\tau_g \cos\Omega t|$.

1. "Strong" GW or narrow spectrum

Let us consider the limit where the frequency broadening of the initially monochromatic photon wave packet due to the interaction with gravitational waves, $\delta\Gamma = \omega_0\Omega\tau_g$, is much bigger than the width of the spectral line Γ_0. The condition $\delta\Gamma \gg \Gamma_0$ can be achieved both in the case where the amplitude of GW is strong enough or when one has a very narrow spectral line. In this limit one can safely neglect the dependence on $|1 + \Omega\tau_g \cos\Omega t|$ and put $\Gamma_0(t) = \Gamma_0$ in Eq. (51) assuming $\Omega\tau_g \ll 1$. Then the leading contribution to the photo-detection probability can be written in terms of the occupation number $n(\omega)$ of the spectral line:

$$P(t) = \alpha\pi \frac{S}{r^2} \frac{\hbar\omega_0}{E_{\rm det}} \Gamma^2 \int_0^\infty n[\omega_0(t-z)]e^{-\Gamma z}dz, \quad (52)$$

where $\omega_0(t) = \omega_0(1 + \Omega\tau_g \cos\Omega t)$. This result has a very simple qualitative explanation which is consistent with our previous simple arguments for the photodetector with sensitivity edge (see Eq. (41)). Consider a photon wave packet coming to FP filter: $\varphi_k = C(t)e^{-i\omega_k(t+\tau_g \sin\Omega t)}$. Such a wave packet in the limit $\Omega \ll \omega_k$ can be interpreted as a photon with a frequency which is slowly varying in time $\omega_k(t) = \omega_k(1 + \Omega\tau_g \cos\Omega t)$. If however the time spent by this photon in the FP filter, Γ^{-1}, is much smaller than the characteristic time of the frequency change Ω^{-1} one can assume the photon at any instant of time to be monochromatic with frequency $\omega_k(t)$. Then one can take advantage of our Eq. (41). Performing the integration over ω in Eq. (41) with Lorentzian spectral line $n(\omega)$ (39) one has

$$P(t) = \alpha\pi \frac{S}{r^2} \frac{\hbar\omega_0}{E_{\rm det}} n[\omega_0(t)]\Gamma. \quad (53)$$

On the other hand this result can be derived directly from our elaborate analysis by putting $n(\omega_0(t-z)) = n(\omega_0(t))$ in Eq. (52). The exponential factor in Eq. (52) describes a time delay of the photon wave packet propagating through the FP filter.

The result Eq. (53) describes the periodic sequence of symmetric peaks with a half period of gravitational waves π/Ω and widths equal to

$$\tau = \frac{\Gamma_0}{\delta\Gamma}\Omega^{-1}. \quad (54)$$

If however the width of these peaks becomes smaller than the time needed for the photon to penetrate through the FP filter: $\tau \ll \Gamma^{-1}$ then the form of peaks becomes asymmetric and the more accurate expression (52) has to be used.

It should be noted that the expression (52) is valid not only for a single spectral line (39) but also for arbitrary distribution function for the light source, where there is a sufficient irregularity in $n(\omega)$ like, for example, at the edge of the spectrum. Indeed if one considers the arbitrary distribution function $n(\omega)$ which changes much at frequency scale $\Delta\omega$ around ω_0 then it follows from Eq. (52) that one again has a periodical sequence of peaks for photo-detection signal in regime $\delta\Gamma \gg \Delta\omega$. In this sense the described effects (52), (53) have nothing to do with the interference of the different components of the photon wave packet (43) and deal rather with the effect of the photon frequency modulation $\omega_k(t) = \omega_k(1 + \Omega\tau_g \cos\Omega t)$ due to the interaction with the GW. If however the amplitude of the frequency modulation $\delta\Gamma$ becomes smaller than the characteristic scale of the spectral function $n(\omega)$ (the case of a broad spectrum) the effects associated with frequency modulation completely disappear.

B. "Weak" GW or broad spectrum

Consider now the opposite limit to the previous case when the amplitude of GW is small or when the width of the spectral line is large enough so that the amplitude of the frequency modulation $\delta\Gamma$ is much smaller than the width of the spectral line Γ_0. It should be noted that this situation is the most common situation in practice since the amplitude of GW is extremely small even in the vicinity of GW source. In the limit $\delta\Gamma \ll \Gamma_0$ one can safely expand the integral expression in Eq. (51) up to the lowest order in the small ratio $\delta\Gamma/\Gamma_0 \ll 1$ and perform the integration over z. Keeping all terms of the type $1 + \Omega\tau_g \cos\Omega t$ in the working assumption $\Gamma \gg \Omega$ the photo-detection probability can be now written as:

$$P(t) \approx \alpha\pi \frac{S}{r^2} \frac{\hbar\omega_0}{E_{\rm det}} n(\omega_0)\Gamma\left(1 - \Omega\tau_g \cos\Omega t \times -2\frac{\delta\Gamma^2}{\Gamma_0^2}\cos 2\Omega t\right). \quad (55)$$

One can see that in the case of a broad spectrum one obtains small oscillations against a huge constant background with frequencies Ω and 2Ω suppressed by small factors $\Omega\tau_g \ll 1$ and $\delta\Gamma^2/\Gamma_0^2 \ll 1$, correspondingly. However as one can see from Eq. (55) even for the extreme case of infinitely broad spectral line there is still a time dependent contribution to the $P(t)$:

$$P(t) \approx \alpha\pi \frac{S}{r^2} \frac{\hbar\omega_0}{E_{\rm det}} n(\omega_0)\Gamma[1 - \Omega\tau_g \cos\Omega t]. \quad (56)$$

It should be noted that this expression does not depend absolutely on the form of the spectra of the distant light source and the time modulation of the photo-detection signal defined only by the strength of the gravitational waves. The amplitude of the photo-detection probability in this case is proportional to the number of photons $n(\omega_0)\Gamma$ coming to the photodetector per unit time and one can practically use the FP filter with appropriate width Γ to collect enough number of photons.

The second important point which has to be emphasized here is that the time dependent term in Eq. (56) has nothing to do with the effect of the frequency modulation mentioned above (see Eq. (52), (53)) and presents an absolutely different effect associated with interference of the different components of the photon's wave packet (43) after passing the FP filter. To clarify the nature of this interference effect we will consider in the next section an example where there are two extremely narrow FP resonances, $\Gamma \ll \Omega$, within the broad spectrum of the distant light source.

C. Two resonances case

Let us consider a more general case where two narrow resonances at frequencies $\omega_0 \pm \Delta/2$ are present within the spectral line (Δ is the separation between resonances). In this case the transmission amplitude of the FP filter $t(\omega)$ again can be written as an operator acting on the electromagnetic field (e.m.f.) induced by modulated photons: $\hat{t} = \hat{t}_1 + \hat{t}_2$, where $\hat{t}_{1(2)}$ corresponds to the first (second) resonance:

$$\hat{t}_{1(2)} = e^{\pm i\phi}\frac{i\Gamma}{2}\left[i\partial_t - \omega_0 \pm \frac{\Delta}{2} + i\frac{\Gamma}{2}\right]^{-1}, \tag{57}$$

where 2ϕ is the relative phase difference between resonances.

Using similar types of arguments and the same method of calculation as for the case of FP filter with a single resonance, in the limit $\Gamma_0 \gg \Omega$, $\Gamma_0 \gg \Gamma$ one arrives at the following expression for the photo-detection probability:

$$P(t) = \alpha\frac{\pi S}{r^2}\frac{\hbar\omega_0}{E_{\text{det}}}n(\omega_0)\Gamma_0\frac{\Gamma^2}{4}[P_1(t) + 2P_{12}(t) + P_2(t)], \tag{58}$$

where

$$P_{1(2)}(t) = \int_0^\infty \frac{\Gamma_0(t-z)e^{-\Gamma z}dz}{(\delta\Gamma\cos\Omega(t-z) \pm \frac{\Delta}{2})^2 + \frac{\Gamma_0^2(t-z)}{4}}, \tag{59}$$

$$P_{12}(t) = \int_0^\infty \frac{\Gamma_0(t-z)\cos(\Delta z - \phi)e^{-\Gamma z}dz}{\delta\Gamma^2\cos^2\Omega(t-z) + \frac{\Gamma_0^2(t-z)}{4}}. \tag{60}$$

Consider now the extreme limit of infinitely broad spectrum: $\Gamma_0 \to \infty$, then in the limit $\Gamma \gg \Omega$, $\Gamma \ll \Delta$ one has:

$$P(t) = \alpha\pi\frac{S}{r^2}\frac{\hbar\omega_0}{E_{\text{det}}}n(\omega_0)(2\Gamma)[1 - \Omega\tau_g\cos\Omega t]. \tag{61}$$

Comparing this result with the expression for the photo-detection probability for the single resonance case (56) one can see that in the regime of a broad spectrum $\Gamma_0 \to \infty$ the photo-detection probability does not depend on the number of resonances in FP filter and depends rather on the total transparency of the filter (2Γ in present case).

Consider however the opposite limit of extremely narrow resonances $\Gamma \ll \Omega$ (this is hardly possible in practice). Then the resonance factor with respect to separation between resonances, Δ, appears in the expression for photo-detection probability:

$$P(t) \approx \alpha\pi\frac{S}{r^2}\frac{\hbar\omega_0}{E_{\text{det}}}n(\omega_0)(2\Gamma)\left[1 - \frac{\Omega\tau_g\Gamma}{2} \times \frac{\Gamma\cos(\Omega t - \phi) + (\Omega - \Delta)\sin(\Omega t - \phi)}{\Gamma^2 + (\Omega - \Delta)^2}\right]. \tag{62}$$

One can see that there is a parametric resonance for the photo-detection signal at $\Delta = \Omega$. The photo-detection probability at the resonance equals to:

$$P_{\Delta=\Omega}(t) \approx \alpha\frac{\pi S}{r^2}\frac{\hbar\omega_0}{E_{\text{det}}}n(\omega_0)(2\Gamma)\left[1 - \frac{\Omega\tau_g}{2}\cos(\Omega t - \phi)\right], \tag{63}$$

while for $|\Delta - \Omega| \gg \Gamma$ (out of resonance) the term which depends on time in the photo-detection probability is saturated by the small factor $\Gamma/|\Delta - \Omega|$.

A similar resonance appears in regime $\Gamma \ll \Omega$ for the case where the width of the spectrum $n(\omega)$ is broad but $(\omega_0/\Gamma_0)^2\Omega\tau_g \gg 1$ so one can neglect all factors $|1 + \Omega\tau_g\cos\Omega t|$ in Eq. (58):

$$P(t) \approx \alpha\frac{\pi S}{r^2}\frac{\hbar\omega_0}{E_{\text{det}}}n(\omega_0)(2\Gamma)\left[1 - \frac{\delta\Gamma^2}{\Gamma_0^2}\Gamma\frac{\Gamma\cos(2\Omega t - \phi) + (2\Omega - \Delta)\sin(2\Omega t - \phi)}{\Gamma^2 + (2\Omega - \Delta)^2}\right]. \tag{64}$$

One can see that the parametric resonance at $\Delta = 2\Omega$ appears in this case. Now we argue that the resonances at $\Delta = \Omega$ and $\Delta = 2\Omega$ (62), (64) arise because of the interference of different components of the photon's wave packet (26). To show it let us simply write the transparency of the FP filter as $t(\omega) = \delta_\Gamma(\omega - \omega_0) + \delta(\omega - \omega_0 - \Delta)$ where $\delta_\Gamma(\omega)$ is a narrow peaked function with finite width $\Gamma \ll \Omega$. Then using the expansion of the photon wave packet (43) one can write the photo-detection probability in terms of the sums over different components of the photon's wave packet:

$$P(t) = 2\alpha\frac{S}{r^2}\sum_{n,m=-\infty}^{\infty}\int\frac{\hbar\omega}{E_{\text{det}}}n(\omega)d\omega e^{i(n-m)\Omega t}t^*(\omega + n\Omega) \times t(\omega + m\Omega)J_n(\omega\tau_g)J_m(\omega\tau_g). \tag{65}$$

Let us assume that the distance between resonances is exactly equal to $N\Omega = \Delta$ where N is an integer. Then substituting the transmission amplitude $t(\omega)$ into this expression and making the integration over ω one gets:

$$P(t) \approx 4\alpha \frac{S}{r^2} \sum_{n=-\infty}^{\infty} n(\omega_0 - n\Omega)\Gamma \frac{\hbar(\omega_0 - n\Omega)}{E_{\rm det}} \times [J_n^2[(\omega_0 - n\Omega)\tau_g] + J_n[(\omega_0 - n\Omega)\tau_g] \times J_{n+N}[(\omega_0 - n\Omega)\tau_g]\cos N\Omega t]. \quad (66)$$

Then for the special choice $N = 2$ assuming that the argument of the Bessel function is typically large one can put $J_{n+2}(x) \approx -J_n(x)$ for $x \gg 1$. Then the photo-detection probability can be written as

$$P(t) \approx 4\alpha \frac{S}{r^2}(1 - \cos 2\Omega t) \sum_{n=-\infty}^{\infty} n(\omega_0 - n\Omega) \times \Gamma \frac{\hbar(\omega_0 - n\Omega)}{E_{\rm det}} J_n^2[(\omega_0 - n\Omega)\tau_g]. \quad (67)$$

One can see that the term which is alternating in time in $P(t)$ arises due to interference of the different components of the photon wave packet (26).

D. Analysis of intensity correlations

In the previous section, we showed that by appropriate filtering of the light signal, it is possible to extract oscillations associated with the past interaction with the GW. While this previous proposal is promising and attractive, one can take an alternative route for the detection of the GW signal: the measurement of intensity-intensity correlations of the photo-detection signal. Consider the correlator $\langle\langle I(t_1)I(t_2)\rangle\rangle$: because of the time modulation due to the presence of the GW, it no longer depends on the variable $t_1 - t_2$ only. Using the definition the intensity operator, one can see qualitatively that the Fourier components of Eq. (43) will lead to a two particle interference effect.

The quantity to be measured on the Earth is thus the intensity correlator of the photo-detection signal which is the time ordered product of electrical field operators:

$$\langle\langle:\hat{I}(t_1)\hat{I}(t_2):\rangle\rangle dt_1 dt_2 = \alpha^2 \frac{S^2 c^2 dt_1 dt_2}{E_{\rm det}^2} \times \langle\langle \hat{\mathbf{E}}^{(-)}(t_1)\hat{\mathbf{E}}^{(-)}(t_2) \times \hat{\mathbf{E}}^{(+)}(t_2)\hat{\mathbf{E}}^{(+)}(t_1)\rangle\rangle. \quad (68)$$

Substituting here the expression for the electrical field operators (36) and performing the quantum average one gets

$$\langle\langle:\hat{I}(t_1)\hat{I}(t_2):\rangle\rangle dt_1 dt_2 = 2\alpha^2 \frac{S^2 dt_1 dt_2}{r^4} \int d\omega_1 d\omega_2 \times \frac{\hbar^2 \omega_1 \omega_2}{E_{\rm det}^2} n(\omega_1)n(\omega_2) \times e^{i\omega_1[t_1 - t_2 + \tau_g(\sin\Omega t_1 - \sin\Omega t_2)]} \times \times e^{-i\omega_2[t_1 - t_2 + \tau_g(\sin\Omega t_1 - \sin\Omega t_2)]} \quad (69)$$

This correlator can be rewritten as

$$\langle\langle:\hat{I}(t_1)\hat{I}(t_2):\rangle\rangle dt_1 dt_2 = 2\alpha^2 \frac{S^2 dt_1 dt_2}{r^4} |f[t_1 - t_2 + \tau_g(\sin\Omega t_1 - \sin\Omega t_2)]|^2, \quad (70)$$

where

$$f(t) = \int \frac{\hbar\omega}{E_{\rm det}} n(\omega) e^{-i\omega t} d\omega. \quad (71)$$

The function $f(t)$ typically decays exponentially on a time scale given by the inverse of the line width $\Gamma_0 \gg \Omega$. Thus one can safely linearize the dependence of the intensity-intensity correlator with respect to the relative time $\tau = t_1 - t_2$:

$$\langle\langle:\hat{I}(t + \tau)\hat{I}(t):\rangle\rangle = 2\alpha^2 \frac{S^2}{r^4} |f[\tau(1 + \Omega\tau_g \cos\Omega t)]|^2, \quad (72)$$

where $\tau = t_1 - t_2$ is the relative time and t is the total time. Let us now calculate the spectral power, $S(t)$ of the photons' noise defined as

$$S(t) = \int \langle\langle:\hat{I}(t + \tau)\hat{I}(t):\rangle\rangle d\tau. \quad (73)$$

Substituting here the expression for photon intensity correlator one can show that

$$S(t) = \frac{4\pi\alpha^2}{1 + \Omega\tau_g \cos\Omega t} \frac{S^2}{r^4} \int \frac{(\hbar\omega)^2}{E_{\rm det}^2} n^2(\omega) d\omega. \quad (74)$$

One can see from this equation that the intensity correlation contains periodic oscillations—although with a small amplitude—in the total time. Note that contrary to the Fabry-Perot diagnosis, here a rather accurate measurement is implied, as the contribution to other noise sources (due to scattering, etc.) needs to be minimized compared to this periodic signal. In particular, it requires a large time acquisition window to filter out spurious fluctuations.

E. Conclusion

Let us now summarize all results obtained through the article and give some estimations and limitations for the proposed gravitational waves detection method. The primary effect we have dealt with is the modulation of the photon phase due to interaction with gravitational field have been found in the geometrical optics approximation and described by Eq. (26) for the photon wave function:

$$\phi_k(\xi) = C(\xi) e^{-i\omega_k \xi - i\omega_k \tau_g \sin\Omega\xi}, \quad (75)$$

where $\xi = t - r/c$ and $C(\xi) = [1 + \Omega\tau_g \cos\Omega\xi]^{-1}$ is the preexponential factor. This effect of phase modulation leads immediately to the well-known Shapiro effect of the time delay of the signal propagating near the source of gravitational field. For the case of the source of gravitational waves this time delay is an alternating function of time $\tau_{ar} = r/c - \tau_g \sin\Omega\tau$, where τ_{ar} is an arrival time of the signal, r/c the traveling time of the signal in flat Minkovski space, and

$$\tau_g = -\frac{h_{xx}(\Lambda)}{\pi}\left(\frac{\Lambda}{D}\right)^2 \Omega^{-1}. \qquad (76)$$

Here $h_{xx}(\Lambda)$ is the dimensionless strength of gravitational wave near the GW source, D impact parameter, $\Omega = 2\pi c/\Lambda$ is the frequency of gravitational waves. The Shapiro effect in fact seems to be a very promising candidate for gravity wave detection experiment since the time delay (or the additional alternating in time phase) mainly accumulated near the source of gravitational waves where the strength of GW are many orders of magnitude greater when they reach the Earth. However in the proposed experiment [16] it requires a pulsar on the line of sight which passes near the source of gravitational radiation. In this paper we propose a gravitational waves detection method based on the Fabry-Perot interference analysis (or equivalently time correlation measurement) of the light signal from an arbitrarily light source (not necessarily a pulsar) passing near the strong source of gravitational radiation. Then the photodetector signal contains an alternating in time component (with GW frequency) proportional to the strength of gravitational radiation near the GW source (see Eqs. (56), (61)):

$$P(t) = P_0(1 - \Omega\tau_g \cos\Omega t), \qquad (77)$$

with P_0 a photodetector signal corresponding to a light coming to a FP filter from a distant star. To estimate an effectiveness of the proposed method let us first estimate a brightness of the light source needed to resolve an alternating component in the photo-detection signal. Collecting the signal over a long time τ_{obs} allows to better resolve the alternating component. Assuming that during the observation time N_{obs} photons coming from a distant star hit the photodetector, in the limit of Poissonian statistics, $\langle \delta N_{obs}^2 \rangle = N_{obs}$ one has the following limiting requirement:

$$\Omega\tau_g(D)\sqrt{N_{obs}} \gg 1. \qquad (78)$$

In the extreme case where the light passes in immediate proximity ($D \approx \Lambda$) to the GW source one has the condition:

$$h(\Lambda)\sqrt{N_{obs}} \gg 1. \qquad (79)$$

Then assuming that the distant star emits in the frequency band $\Delta\omega$ and that the diameter of the telescope equals to d one has:

$$N_{obs} \sim L\frac{d^2}{R^2}\frac{\Gamma}{\Delta\omega}\frac{\tau_{obs}}{\hbar\omega_0}, \qquad (80)$$

where L is the brightness of the distant star, R is the distance to the Earth.

The most promising candidates of gravitational waves sources appropriate for proposed detection method are the periodic sources of gravitational radiation such as close binaries at the final stage of evolution or asymmetric rotating neutron stars, allowing to collect the signal for long enough time to resolve the tiny gravitational wave modulation on the stochastic Poisson noise. Let us give some estimations, assuming the total mass m of binary system of the order of the mass of the Sun, and rotating with a frequency $\nu \sim 10^{-2}$ Hz. Then the dimensionless strength of gravitational waves at the distance r from the emitter, according to quadrupole approximation, is given by

$$h \approx 4\frac{G^{5/3}}{c^4}\frac{1}{r}m^{5/3}\nu^{2/3}, \qquad (81)$$

assuming that the role of the distant star plays one of the components of the binary system $r = (Gm/\nu^2)^{1/3}$ one has $h(r) \approx 4(Gm\nu)^{4/3}/c^4 \sim 10^{-9}$. In order to resolve this modulation one needs about $N_{obs} \sim 10^{18}$ photons coming to a photodetector during τ_{obs}. Assuming the typical distance from the binary system to the Earth of the order 1 kpa with $d \approx 10m$, $\omega_0 \sim 10^{14}$ Hz, and $\Gamma/\Delta\omega \sim 1$, $\tau_{obs} \sim 10^3\nu^{-1} \sim 10^5 s$ one has the required brightness of the binary component, according to Eq. (80) of the order $L \sim 10^5 L_S$ where L_S is the brightness of the Sun. Of course the requirement to have such a bright component of the binary system makes the observation of the proposed effect rather problematic in practice.

In reality the situation with binary systems is even worse due to the presence of the Doppler effect for the light emitted from one of the rotating components. In fact the Doppler effect results in the same frequency modulation of the emitted light as the gravitational waves modulation and the amplitude of the frequency modulation due to the Doppler effect is of the order of

$$\left(\frac{\Delta\omega}{\omega}\right)_{\text{Doppler}} \sim \left(\frac{v}{c}\right)^2 \approx \frac{(Gm\nu)^{1/3}}{c}\frac{\nu}{c}. \qquad (82)$$

For parameters described above it results in an effect of the order 10^5 that is 5 orders of magnitude greater than the frequency modulation due to gravitational waves.

The situation however could be much better for very close neutron star binaries at the last stage of evolution. In this case the rotational frequency can be of the order of 10^2 Hz and results in a dimensionless strength of emitted gravitational waves of the order $h(\Lambda) \sim 10^{-6}$. Assuming that the distance to the binary neutron star system is of the order 1 kpa and that there is a bright distant star situated on the line of sight to the binary from the Earth with impact parameter $D \sim \Lambda$ and collecting the signal during the time $\tau_{obs} \sim 10^3\nu^{-1} \sim 10s$ one has the following lower limit for the brightness of distant star $L \sim 10^2 L_S$.

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[1] J. Weber, Phys. Rev. Lett. **22**, 1320 (1969); **24**, 276 (1970); **25**, 180 (1970).

[2] E. Amaldi *et al.*, Nuovo Cimento Soc. Ital. Fis. C **7**, 338 (1984).

[3] E. Amaldi *et al.*, Nuovo Cimento Soc. Ital. Fis. C **9**, 829 (1986).

[4] P. Aston *et al.*, Phys. Rev. D **47**, 362 (1993).

[5] R. E. Vogt, in*Sixth Marcel Grossmann Meeting on General Relativity, Kyoto, Japan, 1991*, edited by H. Sato and T. Nakamura (World Scientific, Singapore, 1992).

[6] C. Bradaschia *et al.*, Nucl. Instrum. Methods Phys. Res., Sect. A **289**, 518 (1990).

[7] R. L. Forward, Phys. Rev. D **17**, 379 (1978).

[8] K. S. Thorne, in *300 Years of Gravitation*, edited by S. W. Hawking and W. Israel (Cambridge University Press, Cambridge, England, 1987).

[9] J. Hough, B. J. Meers, G. P. Newton, N. A. Robertson, H. Ward, B. F. Schultz, I. F. Corbett, and R. W. P. Drever, Vistas Astron. **30**, 109 (1987).

[10] R. W. P. Drever, in *Gravitational Radiation*, edited by N. Deruelle and T. Piran (North-Holland, Amsterdam, 1983).

[11] B. J. Meers, Phys. Rev. D **38**, 2317 (1988).

[12] B. F. Schutz, Ann. N.Y. Acad. Sci. **571**, 27 (1980).

[13] L. D. Landau and E. M. Lifshits, *Course of Theoretical Physics*, Vol. 2 (Pergamon Press, London, 1958).

[14] I. I. Shapiro, Phys. Rev. Lett. **13**, 789 (1964).

[15] R. Fakir, Astrophys. J. **426**, 74 (1994).

[16] M. V. Sazhin, Sov. Astron. **22**, 36 (1978); S. Detweiler, Astrophys. J. **234**, 1100 (1979).

[17] R. Fakir, Phys. Rev. D **50**, 3795 (1994).

[18] T. Damour and G. Esposito-Farèse, Phys. Rev. D **58**, 042001 (1998).

[19] S. M. Kopeikin *et al.*, Phys. Rev. D **59**, 084023 (1999).

[20] J. T. Mendonca and L. O'C. Drury, Phys. Rev. D **65**, 024026 (2002).

[21] F. I. Cooperstock, Ann. Phys. (N.Y.) **47**, 173 (1968).

[22] F. B. Estabrook and H. D. Wahlquest, Gen. Relativ. Gravit. **6**, 439 (1975).

[23] V. B. Braginsky, N. S. Kardashev, I. D. Novikov, and A. G. Polnarev, ICTP Report No. IC/89/392, 1989 (to be published).

[24] J. A. Lobo, Classical Quantum Gravity **9**, 1385 (1992).

[25] M. V. Sazhin and S. N. Markova, Phys. Lett. A **233**, 43 (1997).

[26] L. Mandel and E. Wolf, *Optical Coherence and Quantum Optics*, (Cambridge University Press, Cambridge, England, 1995).