Massive neutrinos and (heterotic) string theory

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String theories in principle address the origin and values of the quark and lepton masses. Perhaps the small values of neutrino masses could be explained generically in string theory even if it is more difficult to calculate individual values, or perhaps some string constructions could be favored by generating small neutrino masses. We examine this issue in the context of the well-known three-family standard-like Z_3 heterotic orbifolds, where the theory is well enough known to construct the corresponding operators allowed by string selection rules, and analyze the D- and F-flatness conditions. Surprisingly, we find that a simple seesaw mechanism does not arise. It is not clear whether this is a property of this construction, or of orbifolds more generally, or of string theory itself. Extended seesaw mechanisms may be allowed; more analysis will be needed to settle that issue. We briefly speculate on their form if allowed and on the possibility of alternatives, such as small Dirac masses and triplet seesaws. The smallness of neutrino masses may be a powerful probe of string constructions in general. We also find further evidence that there are only 20 inequivalent models in this class, which affects the counting of string vacua.

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I. INTRODUCTION

String theory proposes to provide a well-defined underlying theory for elementary particle physics. As such it is obligated to provide an understanding for the phenomena we see at accessible energy scales, including the origin of fermion masses and mixings. In particular, one should be able to identify the mechanism that explains the smallness of neutrino masses as a natural outcome in some class of explicit string constructions. In this paper we perform a study of a particular class of real string constructions in a top-down manner, and search for the couplings necessary to generate the "minimal seesaw" mechanism for neutrino masses (to be defined more precisely below). Though we are mindful that this is not the only possible method of achieving very light neutrinos, it does lead naturally to very small masses (though not necessarily to large mixing angles) and it is the basis of the vast majority of phenomenological studies of neutrinos in the literature.[1] The minimal seesaw requires a well-defined set of fields and couplings to be present in the low-energy theory. In particular, it requires the simultaneous presence of both Dirac mass terms and large Majorana mass terms for the right-handed neutrinos. There is no standard model symmetry to forbid such couplings. Large Majorana masses might, however, be forbidden by extensions of the low-energy theory, such as an additional $U(1)'$ gauge symmetry [6]. Their possible existence forms a useful probe of the much more restrictive string constructions.

Sadly, string theory has been largely silent on the issue of neutrino mass since the subject was first raised in the context of heterotic strings nearly 20 years ago [7,8]. The reason for this silence is not hard to understand: the issue of flavor is perhaps the most difficult phenomenological problem to study in explicit, top-down string constructions—and neutrino masses are just one aspect of this problem. To begin such a study requires that many things be worked out: one needs not just the spectrum of massless states, but also their charges under all Abelian symmetries (properly redefined so that only one linear combination of $U(1)$ factors is anomalous). To obtain the superpotential couplings to very high order the string selection rules for the particular construction must be worked out and put into a form amenable to automation. Obtaining these working ingredients takes time and effort, though the techniques are well known. Certain parts of this process have been completed and discussed in the literature for several string models. The most comprehensive study of weakly-coupled heterotic models with semirealistic gauge groups and particle content are the free-fermionic constructions (see for example [9–12] and references therein) and the bosonic standard-like Z_3 orbifold constructions (see for example [8,13] and references therein). In particular, a systematic study of the spectra in the phenomenologically promising BSL_A class of the Z_3 orbifold has been performed by one of

[1] For recent reviews of the neutrino oscillation data and models of their masses and mixings, see [1–5].

the authors of this paper. Thus we have at our disposal these results and here we will exploit them to perform a systematic study of the superpotential couplings and flat directions[2]—with particular emphasis on the issue of neutrino mass. We will define this BSL_A class more properly in Sec. II.

But merely working out the allowed superpotential couplings (itself a tedious task) is not sufficient for studying neutrino masses. The minimal seesaw calls for a very special type of coupling: a supersymmetric bilinear Majorana mass term. Such terms do not arise from string theory for the states in the massless spectrum. Thus it must be that this term arises *dynamically* through the vacuum expectation value (vev) of some field or fields. This means we must consider the issue of flat directions in the space of chiral matter fields. To be more precise, in semirealistic string constructions we are inevitably faced with an enormous vacuum degeneracy that we do not know how to resolve from first principles. So even once we have

(i) assumed a particular string construction,
(ii) assumed a particular compactification, and
(iii) assumed (or better yet, determined) the background values for string moduli, we are still
(iv) faced with a wealth of D- and F-flat directions in the space of chiral matter fields.

These flat directions are combinations of background field values for which the classical scalar potential vanishes and supersymmetry is maintained. There are typically many such directions, all degenerate and all consistent vacuum configurations of the string construction. By setting certain chiral superfields to background values that do *not* lie along such a flat direction, one is attempting to expand about an inappropriate configuration—in particular, a configuration in which supersymmetry is spontaneously broken at a high scale (not to mention a configuration which is not a valid point for a saddle-point expansion). A minimum of the classical potential, and thus a classical vacuum, will be the supersymmetric one.[3] Therefore, to the extent that superpotential couplings of the minimal supersymmetric standard model (MSSM) arise from terms involving one or more vevs, they must occur along such flat directions, and the issue of fermion masses and flavor is intricately tied to the issue of vacuum selection in string theory—hence the great difficulty in studying the issue of neutrino masses.

Only a handful of investigations into neutrino masses in *explicit* top-down string constructions have been performed, though there are many more examples of "string inspired" bottom-up studies, and it is worthwhile to review these instances before proceeding. Two noteworthy examples in intersecting brane constructions are that of Ibañez *et al.* [14] and that of Antoniadis *et al.* [15]. These are both nonsupersymmetric constructions with low string scales. In the first case Majorana neutrino masses are forbidden by a residual global symmetry broken only by chiral symmetry breaking effects so that masses can only be of the Dirac type. In the second case Majorana couplings are again forbidden, but a large internal dimension is used to justify the smallness of neutrino masses. Heterotic examples come closer to realizing the standard seesaw paradigm. The most complete top-down analyses involve free-fermionic constructions of Ellis *et al.* [16,17] and Faraggi *et al.* [18–20] in which a detailed treatment of flat directions was performed. In both cases some assumptions about strong dynamics in the hidden sector need to be made in order to populate the neutrino mass matrix. In addition, the latter set of models involves an extended set of fields that are not of the minimal seesaw variety. In both of these heterotic cases (as well as the recent heterotic construction of Kobayashi *et al.* [21]) several right-handed neutrino species are involved, where by "different species" we mean that right-handed neutrinos with different gauge charges with respect to some extension of the standard model gauge group are involved. The point to be made here is not to say that these are impossibilities or that these examples cannot explain the smallness of neutrino masses. It is rather to emphasize that in the (very few) extant string examples neutrino and lepton mass matrices arise that look very different from those expected from a typical grand unified theory (GUT) ansatz, and in some cases different from those stemming from a minimal seesaw ansatz.

Our focus will be very much different. In the heterotic-based papers mentioned above one or two flat directions were chosen for further study on the basis of certain phenomenological virtues that are not directly concerned with neutrino masses, such as the projecting of certain exotic matter states from the spectrum or the desire to accommodate realistic quark masses. Neutrino masses are an afterthought, and it is not clear whether the lack of a simple seesaw is indicative of the string construction or simply the flat direction chosen. Here we will take the uniqueness of the neutrino self-coupling of the minimal seesaw as a guide and study a very large class of flat directions, in a large set of models, in search of precisely this coupling. By putting the issue of neutrino masses as the primary consideration we are thus examining the question of whether it is possible to infer the existence of small neutrino masses as a reasonable outcome of a consistent, explicit string construction independent of other questions of low-energy phenomenology. (Of course, our preference and ultimate goal would be to find a theory in which the conspiracy of operators, couplings, charges, etc., was such that neutrinos almost "had" to be light, rather than it being an accident.) If the answer is positive then it also would be of interest to study whether the string constraints provide

[2]These flat directions are only approximate (i.e. through degree 9); see Appendix B.

[3]Here we assume that there is a supersymmetric minimum. Also, in the presence of low-scale supersymmetry-breaking effects, the minimum of the effective potential may be shifted slightly away from a supersymmetric minimum.

any insight into such issues as the existence of two large mixing angles, the nature of the mass hierarchy (ordinary or inverted or approximately degenerate) and the relation (if any) to the quark and charged lepton masses and mixings.

We now summarize the content and results of this article. In Sec. II we define and motivate the class of heterotic orbifold models that we analyze and explain the remarkable fact that from a starting point of thousands of possibilities one is left to consider only 20 inequivalent cases. A definition of the minimal seesaw and a description of the algorithm we use for finding it is described in Sec. III, where we also provide various details regarding flat directions and couplings in the models. Two cases of the 20 allowed for the possibility of a Majorana coupling along a flat direction, though both ultimately fail to provide the minimal seesaw or realistic neutrino masses for a variety of reasons. Nevertheless we investigate both in some detail and describe their successes and ultimate failures in Sec. IV. We comment on the possible implications and alternatives in the concluding Sec. V. Supporting material is provided in three appendices. In Appendix A we list the relevant fields in the spectra for representative models of the two promising cases of Sec. IV. In Appendix B we discuss the extent to which the flat directions we identify should be considered approximately flat. Finally, in Appendix C we provide a brief and accessible review of the string selection rules as they apply to superpotential couplings in the effective supergravity. We show that it is possible to reduce these (conveniently) to gauge invariance and a set of triality (i.e., Z_3) invariances.

II. BSL$_A$ MODELS OF THE Z_3 ORBIFOLD

As mentioned in the introduction, to systematically study the issue of flavor in general—and neutrino masses in particular—requires a thorough knowledge of many aspects of the low-energy (4D) theory. To date one of the few classes of string constructions where this process has been systematically performed are standard-like models obtained from Z_3 orbifolds [22,23] of the $E_8 \times E_8$ heterotic string [24]. The construction is *bosonic* because of the way in which fields in the underlying 2D conformal field theory are realized, and it is *symmetric* because left-moving and right-moving 2D conformal field theory degrees of freedom associated with the compact 6D space are treated symmetrically. There is an Abelian embedding of the orbifold action (space group) into the gauge degrees of freedom through a shift embedding V with two discrete Wilson lines a_1 and a_3 [25,26]. Note that there are only three independent Wilson lines in the Z_3 orbifold. Because the third Wilson line is set to zero, one automatically obtains a three-generation model—part of the reason for the phenomenological interest in the Z_3 construction.

The twist vector which represents the orbifold action and the Wilson lines are embedded in the $E_8 \times E_8$ root torus, and the result is a breaking of this group to a product of subgroups. The surviving groups emerge according to $E_8^{(1)} \rightarrow G_{\text{obs}}$ and $E_8^{(2)} \rightarrow G_{\text{hid}}$. There are a vast number of such consistent embeddings, but it has been shown in the Z_3 case that many of them are actually equivalent [27]. The bosonic standard-like models of type A (BSL$_A$ models—a designation coined in [13]) are those models of this class which have $G_{\text{obs}} = SU(3) \times SU(2) \times U(1)^5$ and (three generations of) (3,2) representations in order to accommodate the quark doublets. As it turns out, with the choice of G_{obs} that has been imposed, the (3,2) representations always occur in the untwisted sector [28].

Three-generation models of this type have appeared extensively in the literature on semirealistic heterotic orbifolds [8,29–32]. A complete enumeration of consistent embeddings into the first E_8 was given in [27]. However, it is in general not possible to place all the fields of the MSSM matter content exclusively in the untwisted sector. Thus it is necessary to work out the spectrum of twisted sector states as well, and to therefore provide all possible completions of the embeddings in [27] to the hidden sector E_8 factor as well.[4] A complete enumeration of consistent completions of these embeddings into the second E_8 was given in [33]. There it was found that only five possibilities for G_{hid} exist. For one of the five possibilities, the non-Abelian part of G_{hid} only contains $SU(2)$ factors. We do not regard this as a viable hidden sector for dynamical supersymmetry breaking by gaugino condensation, since the condensation scale will be far too low to provide a reasonable scale of supersymmetry breaking [34]. Therefore only the four remaining possibilities for G_{hid} are of interest to us. With this restriction, it was found that there are just 175 models.

A systematic study of several properties of these models was given in [13]. In particular the complete massless spectrum of chiral matter was determined for each of the 175 models, where it was found that only 20 distinct sets of representations occur for the 175 models. Furthermore, the 4D theories generated by the different embeddings in each of the 20 classes had several identical physical properties. This was interpreted as an indication that models in the same pattern are actually equivalent. We will refer to these 20 cases as *patterns* of the BSL$_A$ master class.

We give a summary of these 20 patterns in Table I, organized by the hidden sector gauge group G_{hid}, with

[4]When a nontrivial embedding is chosen, the low-energy gauge theory results from a twisted affine Lie algebra in the underlying 2D conformal field theory. In this case the weights of the states under the original $E_8 \times E_8$ Cartan elements are shifted by fractional amounts 0 mod 1/3. The twisted sector states are then typically charged under both E_8 factors, so that it is necessary to know the "hidden" sector embedding to obtain the states that are also charged under the "observable" sector Cartan elements; i.e., one loses the clear distinction between hidden and observable sector states.

TABLE I. Summary of BSL_A patterns. For each pattern we give the number of models in the pattern, the hidden sector gauge group, the scale of the anomalous $U(1)$ FI-term, and the number of distinct species of chiral matter superfields.

Pattern	No.	G_{hid}	r_{FI}	Species
1.1	7	$SO(10) \times U(1)^3$	No $U(1)_X$	51
1.2	7	$SO(10) \times U(1)^3$	0.15	76
2.1	10	$SU(5) \times SU(2) \times U(1)^3$	0.09	64
2.2	10	$SU(5) \times SU(2) \times U(1)^3$	0.10	66
2.3	7	$SU(5) \times SU(2) \times U(1)^3$	0.10	65
2.4	7	$SU(5) \times SU(2) \times U(1)^3$	0.13	60
2.5	6	$SU(5) \times SU(2) \times U(1)^3$	0.14	61
2.6	6	$SU(5) \times SU(2) \times U(1)^3$	0.12	51
3.1	12	$SU(4) \times SU(2)^2 \times U(1)^3$	0.07	58
3.2	5	$SU(4) \times SU(2)^2 \times U(1)^3$	0.12	57
3.3	10	$SU(4) \times SU(2)^2 \times U(1)^3$	0.12	57
3.4	5	$SU(4) \times SU(2)^2 \times U(1)^3$	0.13	53
4.1	7	$SU(3) \times SU(2)^2 \times U(1)^4$	0.10	61
4.2	12	$SU(3) \times SU(2)^2 \times U(1)^4$	0.09	62
4.3	7	$SU(3) \times SU(2)^2 \times U(1)^4$	0.07	63
4.4	15	$SU(3) \times SU(2)^2 \times U(1)^4$	0.12	59
4.5	17	$SU(3) \times SU(2)^2 \times U(1)^4$	0.11	61
4.6	13	$SU(3) \times SU(2)^2 \times U(1)^4$	0.12	60
4.7	6	$SU(3) \times SU(2)^2 \times U(1)^4$	0.11	62
4.8	6	$SU(3) \times SU(2)^2 \times U(1)^4$	0.12	53

the number of different embeddings in each pattern given in the first column. In each of the 175 individual embeddings the $U(1)$ charges of the spectrum can be obtained and the anomaly isolated to one Abelian factor $U(1)_X$. This anomaly is cancelled [35–37] by the Green-Schwarz mechanism [38], which involves a Fayet-Iliopoulos (FI) term in the 4D Lagrangian. This term can be calculated from the known spectrum and is given by

$$\xi_{\mathrm{FI}} = \frac{g_{\mathrm{STR}}^2 \operatorname{Tr} Q_X}{192\pi^2} M_{\mathrm{PL}}^2, \tag{2.1}$$

where g_{STR} is the (unified) string coupling just below the compactification scale and M_{PL} is the reduced Planck mass. We will discuss the relevance of this particular mass scale in Sec. IV. The value of ξ_{FI} depends on the vacuum expectation value of the dilaton, which provides the determination of g_{STR}. If we take the unification of couplings in the MSSM as a rough guide to what this vev might be, we can make the approximation $g_{\mathrm{STR}}^2 \simeq 0.5$. Then the value of the ratio $r_{\mathrm{FI}} = \sqrt{|\xi_{\mathrm{FI}}|}/M_{\mathrm{PL}}$ can be calculated; we provide the numerical value of this factor in the third column of Table I. In the final column we give the total number of different species of chiral superfield in each of the individual models for the pattern. Note that the total number of fields would then involve a three-fold replication of these species (except that for twisted oscillator states there is a 9-fold multiplicity).

When the gauge anomaly is isolated to a single Abelian factor, the only nominal difference between different members of each pattern is the apparent charges under the various $U(1)$ factors in $G_{\mathrm{obs}} \times G_{\mathrm{hid}}$. Yet the value of $\operatorname{Tr} Q_X$ is identical for each member of a given pattern, suggesting that a basis exists for which these charges—and perhaps those of all the Abelian factors—would, in fact, be identical. In this case the members of each pattern would truly be redundant models; in this work we will find further evidence for this conjecture.

Clearly, then, the bosonic standard-like models of the Z_3 orbifold are an ideal starting point for a dedicated study of neutrino masses: they already contain the standard model particle content and gauge group (though with much more besides), the vast number of possibilities has been reduced to a tractable number through much past research, and many of the key ingredients needed for our analysis are already known. Indeed, these properties have made this class a laboratory for other recent work in string phenomenology [39–43].

III. THE SEARCH FOR NEUTRINO MASS COUPLINGS

We now know with certainty that some neutrinos have mass and that the different flavors of neutrinos mix with one another with large mixing angles. We further know that the differences in the squared masses of the physical eigenstates are extraordinarily small: on the order of 10^{-3} eV^2 for the mass difference that explains the atmospheric oscillation data and 10^{-5} eV^2 for the mass difference that explains the solar neutrino oscillation data. One possible explanation is that the masses themselves are of this order, and indeed cosmological observations of large scale structure constrain the sum of the physical masses to be on the order of a few $\times 0.1$ eV [44]. The fantastic smallness of these numbers, in comparison to the masses of the quarks and charged leptons, seems to call for an explanation dramatically different from those of other standard model fields.

A. The minimal seesaw

While it is logically possible that neutrinos get their masses solely through electroweak symmetry breaking, with extremely small Yukawa couplings to Higgs states and right-handed (Dirac) neutrinos, the preferred explanation has long been the seesaw mechanism [45–48]. In this scenario one assumes the existence of heavy (Majorana) neutrinos which are singlets under the standard model gauge group G_{SM} which play the role of right-handed neutrinos. If these heavy states have $\mathcal{O}(1)$ Yukawa couplings to the lepton and Higgs doublets, then integrating them out of the effective theory produces sufficiently small effective neutrino masses for the light states. In a supersymmetric context we can cast this as an effective neutrino mass superpotential which takes the form

$$W_{\rm eff} = (\nu_i, N_i)\begin{pmatrix} 0 & (m_D)_{ij} \\ (m_D)_{ji} & (m_M)_{ij} \end{pmatrix}\begin{pmatrix} \nu_j \\ N_j \end{pmatrix}, \tag{3.1}$$

where one assumes $m_D \ll m_M$ in order to produce the desired light eigenvalues. Here, the ν_i are the neutrino superfields associated with the minimal supersymmetric standard model, and the N_i are (charge-conjugated) right-handed neutrino superfields. One typically assumes that ν_i runs over three generations of fields and that there are three (or possibly more) N_j. We will assume three generations of each type of field, given the three-generation construction that we appeal to. We will refer to the neutrino system defined by these assumptions and the matrix (3.1) as the "minimal seesaw." The overwhelming majority of the vast literature on neutrino phenomenology is based on this minimal paradigm. We wish to study whether it is possible to embed this scenario in a string-derived model—say, a BSL_A model.

Once we introduce string theory, we are confronted with a number of chiral superfields beyond the states of the MSSM. There are many potential candidates for the right-handed neutrino fields N_i. In fact, typically half the species in the models of Table I are singlets of the standard model gauge group—though none of them are singlets under *all* of the Abelian gauge factors simultaneously. In this they are distinct from the various moduli of the string theory. These states are also represented by chiral superfields and are singlets under all gauge symmetries. Could these be candidates for right-handed neutrinos?

While not a logical impossibility, we argue that a viable model of neutrino mass is unlikely to involve these fields. Moduli fields have no superpotential couplings at the perturbative level, so the types of Yukawa interactions that can give rise to the matrix (3.1) are absent at this level. Furthermore, the string moduli are likely to receive a mass only after supersymmetry is broken, and thus we might expect typical values in the matrix $(m_M)_{ij}$ to be $\mathcal{O}$ (TeV). The entries in the matrix $(m_D)_{ij}$ would then need to be extremely small to explain the observed neutrino mass differences. Thus we will search for the needed couplings among the fields that have been summarized in Table I.

As mentioned above, bare mass terms $W = m_M \Phi\Phi$ with $m_M \ll M_{\rm PL}$ do not arise in a natural way from the underlying string theory. Thus our first task is to identify a degree $n \geq 3$ coupling that would yield an effective Majorana mass term

$$\langle S_1 \cdots S_{n-2}\rangle NN, \tag{3.2}$$

where we have suppressed the generation labels associated with the 3-fold degeneracy of the spectrum. The principal questions that we address in this section are

(1) Is it possible to get the simplest sort of Majorana mass couplings (3.2) in the BSL_A models?

(2) Since vevs of $G_{\rm SM}$ singlets $\langle S_i\rangle$ are necessary, we must simultaneously ask: Are these vevs consistent with D- and F-flatness?

B. Flat direction scan and analysis

To obtain answers to questions (1) and (2), we have studied in detail all allowed superpotential couplings and an elementary class of flat directions up to a certain order (described below) for a representative sample (3 models from each of the 20 representation patterns) of the 175 models in the BSL_A class. Though straightforward, the computation is very tedious and impossible without automation; it took weeks for the C/C + + routines to run on a Pentium 4 processor. Some idea of the scale of the project will be evident in the discussion below, since a fringe benefit of the analysis is a count of couplings and flat directions for each model in which interesting aspects of the BSL_A models emerge. It also should be stated that none of the analysis made here requires a detailed knowledge of the strength of couplings. For a study of the class of flat directions that we consider, it is sufficient to know the selection rules.

As is well known, D-flat directions are easily and completely classified by analytic invariants [49–52]. To each holomorphic gauge-invariant $I(\Phi)$ of the chiral superfields $\Phi_1, \ldots, \Phi_n$ in the theory corresponds a D-flat direction, given by

$$\langle K_i\rangle = c\langle I_i\rangle, \tag{3.3}$$

where c is a universal constant, $K_i = \partial K/\partial\Phi_i$ with K the Kähler potential, and $I_i = \partial I/\partial\Phi_i$. Of course, c can be absorbed into the definition of I. It is an undetermined parameter, whose magnitude corresponds to the scale of the breaking. Energetically, all scales are equally favored.

In the case where there is an anomalous $U(1)_X$, a slight modification is required. We choose a basis of $U(1)$ charges where only one, Q_X, is anomalous and $\mathrm{Tr}\, Q_X > 0$. We express the invariant I as a sum of monomials $I^{(A)}$ in the fields

$$I = \sum_A c_A I^{(A)}. \tag{3.4}$$

D-flatness is satisfied if and only if: (i) each $I^{(A)}$ is gauge invariant with respect to all nonanomalous factors of the gauge group, and (ii) at least one of the $I^{(A)}$ has strictly negative Q_X charge. The vanishing of the Q_X D-term imposes one real constraint on the c_A; an overall phase among the c_A can be removed by going to unitarity gauge with respect to the $U(1)_X$. The remaining degeneracy of solutions corresponds to flat directions, termed elsewhere as *D-moduli* [53,54].

In our analysis of effective Majorana neutrino mass couplings, we restrict our attention to the case where I is a single monomial satisfying this invariant condition, with $Q_X(I) < 0$ if an anomalous $U(1)$ exists. We refer to this product of fields as an *I-monomial*. Thus we examine only special points in the D-moduli space. Polynomials that are linear combinations of the I-monomials correspond to a more general class of D-flat directions [55,56]. Since these generalizations allow for more fields to get vevs, they might provide new paths to obtain (3.2). However, this more complicated scenario involves significantly more analysis. While it is a sensible follow-up to the present study, for practical reasons we leave it to future work. To simplify our analysis, we impose *stringent F-flatness* [12]. That is to say, we do not permit the vev of any monomial in $\partial W/\partial \phi_i$ to contribute a nonzero term (to the order we study). This sufficient but not necessary condition is a further restriction to special points in D-moduli space, which is nevertheless a class with a large number of elements.

In our automated search, we adopted the following procedure:

Step 1.—We generated a complete list of I-monomials that (i) contained only fields neutral under $SU(3) \times SU(2) \in G_{\rm obs}$ and, (ii) had degree less than or equal to ten.

Step 2.—All superpotential couplings allowed by selection rules (see Appendix C) were generated, up to and including degree 9.

Step 3.—We eliminated from the list of I-monomials all those that would violate stringent F-flatness with respect to the superpotential couplings generated in Step 2. The remaining I-monomials specify our list of D- and F-flat directions.

Step 4.—For each flat direction that survived Step 3, we searched for couplings from the list generated in Step 2 that would provide an effective Majorana mass coupling of the form (3.2), where the vevs $\langle S_i \rangle$ were each contained in the I-monomial of the given flat direction. The repeated field in the coupling then becomes a candidate right-handed neutrino N.

Step 5.—If the candidate N fields were not singlets of the non-Abelian factors of $G_{\rm hid}$, we also checked that the flat direction broke those factors of the gauge group so that N could be an effective gauge singlet along the flat direction.

The result of this procedure was a success for only 2 of the 20 patterns in these models: pattern 1.1 and 2.6. This is already a remarkable result. But this is not sufficient to claim that the minimal seesaw has been discovered. In cases where we find in the affirmative on the two questions just posed at the end of Sec. III A, other questions remain:

(3) The vevs required for (3.2), and any others that are required for flatness, generally break some of the $U(1)$ factors in the model. A question connected with this is: Does a $U(1)$ survive that will serve as electroweak hypercharge $U(1)_Y$, and, in particular, is N a singlet under this group?

(4) Does the candidate Majorana neutrino N also have the requisite $H_u L N$ Dirac couplings to $SU(2)$ doublets so as to produce the m_D entries in (3.1)?

(5) Do the remainder of the standard model particles have the proper charge assignments under the candidate $U(1)_Y$?

If questions (3)–(5) can be answered affirmatively, then we have the minimal seesaw. Note that we are not demanding anything about the remaining Yukawa couplings of the MSSM. Of course a truly realistic model must not only possess such superpotential terms, it must also possess them in such a way as to give rise to the observed hierarchies between quark, charged lepton and neutral lepton masses, and the observed large (small) leptonic (quark) mixings. As mentioned in the introduction, we are here making neutrinos our principal focus.

The two patterns that were successful in answering questions (1) and (2) will be discussed in more detail in Sec. IV. Here we wish to remark on a few aspects of the flat direction analysis that deserve some comment. Our procedure clearly produced a wealth of data on couplings and flat directions for all of the models in the BSL_A class. A sense of the size of the project can be seen in the number of superpotential couplings, allowed by all selection rules, that needed to be studied. These are the results of Step 2 above, and are listed in Table II. Many of the higher order

TABLE II. Number of allowed superpotential couplings by degree. For each pattern of Table I we give the number of superpotential coupling at leading order (degree 3) through degree 9 allowed by the string selection rules (note that there were no degree 5 couplings allowed for any pattern).

Pattern	3	4	6	7	8	9
1.1	113	24	21 329	23 768	1697	3 380 308
1.2	97	12	13 968	4418	498	1 552 812
2.1	67	10	5188	3515	162	342 186
2.2	80	11	7573	3066	272	582 326
2.3	75	10	6508	2874	250	467 020
2.4	53	0	2795	360	0	119 454
2.5	58	6	3363	688	26	150 838
2.6	31	0	642	0	0	10 976
3.1	54	4	2749	768	21	119 973
3.2	43	2	1758	291	9	59 182
3.3	48	4	2187	393	20	81 497
3.4	31	8	750	375	42	15 074
4.1	50	3	2090	693	14	81 222
4.2	62	6	3206	793	38	143 257
4.3	55	5	2516	613	15	100 793
4.4	38	2	1137	147	3	28 788
4.5	48	0	1872	0	0	62 597
4.6	47	0	1738	50	0	51 970
4.7	53	0	2219	0	0	76 244
4.8	21	0	301	0	0	4120

couplings are just products of lower order invariants. However, even taking this into account, the number of invariants is impressive. That Patterns 2.6, 4.5, 4.7, and 4.8 only have superpotential couplings whose degree is a multiple of three follows directly from the string selection rules, as discussed in Appendix C. It is unclear to us why Patterns 2.4 and 4.6 lack superpotential couplings whose degree is a multiple of four. It also is interesting that degree 5 couplings were never allowed. There is a unique form allowed by string selection rules, but in the BSL_A class of models this is never a gauge-invariant coupling. It would be of interest to understand this result more fundamentally; we leave it for future considerations.

The second significant result is the extent to which stringent F-flatness restricts the number of I-monomials. That is to say, F-flatness is a powerful restriction on flat directions—perhaps not a great surprise, but we are able to quantify this in Table III. The first column in that table, in which no condition of F-flatness is imposed, is the result of Step 1 above, while the final column is the result of Step 3. It is interesting that in some models either there is a unique stringently F-flat direction or no such directions at all, to the order considered here. A further analysis of these cases is warranted to understand what is the true nature of the vacuum in these models;[5] however, this is beyond the scope of the present study. It also can be observed that stringent F-flatness with respect to the degree 3 couplings is already very limiting. In every model the higher order couplings only reduce the number of flat directions by a factor of $\mathcal{O}(1)$.

TABLE III. Restriction of D-flat directions due to stringent F-flatness. The column ‘‘w/o’’ indicates the number of I-monomials that were found without imposing stringent F-flatness. The column ‘‘w/3’’ contains the number that remained after imposing stringent F-flatness solely with respect to the degree 3 superpotential couplings. The column ‘‘w/3-9’’ provides the final number of I-monomials that survive our analysis, having imposed stringent F-flatness up to degree 9.

Pattern	w/o	w/3	w/3-9
1.1	1 486 616	16 283	489
1.2	11 656	188	28
2.1	155 555	1239	245
2.2	96 932	737	249
2.3	43 884	670	115
2.4	5195	114	12
2.5	12	0	0
2.6	825	9	9
3.1	16 927	80	27
3.2	2443	18	10
3.3	9871	74	22
3.4	1303	59	41
4.1	17 413	106	26
4.2	78 819	513	199
4.3	14 715	310	163
4.4	26	0	0
4.5	5126	32	25
4.6	128	8	5
4.7	5285	15	15
4.8	49	1	1

But by far the most significant and intriguing result was the following: We analyzed the first 3 models from each of the 20 BSL_A representation patterns.[6] For each model that we studied of a given pattern, our results were identical in terms of the number of couplings of each degree, the number of initial I-monomials obtained in Step 1, and the number of I-monomials that survived Step 3. This provides further support to what was already indicated in the results of [13]: the models of a given pattern are in fact equivalent and that the BSL_A class only contains 20 inequivalent models. This is a drastic reduction from the tens of thousands that would be expected from naïve considerations of all the different embeddings one can construct that would yield the same G_{obs}. Furthermore, the restrictiveness of F-flatness is responsible for isolated vacua in $N = 1$ models and is often invoked in the counting of string vacua. We wish to emphasize the relevance of our analysis to ‘‘landscape’’ analyses: merely counting free parameters in some moduli space is not really a counting of physically distinct vacua.

IV. TWO PROMISING CASES

Two of the 20 patterns were capable of producing a candidate Majorana neutrino mass as an effective operator of the form (3.2) along one or more flat directions. Neither pattern was ultimately able to generate realistic neutrino masses, however. In this section we will consider each pattern by choosing a representative model from the set (with the implicit assumption that all models in a pattern are actually equivalent).

A. Pattern 2.6

We will first consider pattern 2.6 by choosing one of the six models in the pattern for explicit examination: model 2.8.[7] This model is defined by the following embedding vectors

[5]The true minimum of the scalar potential may involve nontrivial cancellations between terms contributing to the F-terms, so they would correspond to the larger class of flat directions that are not stringently F-flat.

[6]Compare with Table 13 of [13]. We were not able to check all models for all patterns due to the rather lengthy run-time for the automated analysis.

[7]The numbering system for the 175 individual models derives from [13] but is otherwise irrelevant for our discussion here.

$$V = \frac{1}{3}(-1, -1, 0, 0, 0, 2, 0, 0; 2, 1, 1, 0, 0, 0, 0, 0),$$
$$a_1 = \frac{1}{3}(1, 1, -1, -1, -1, -1, 0, 0; -1, 0, 0, 1, 1, 1, 1, -1),$$
$$a_3 = \frac{1}{3}(0, 0, 0, 0, 0, 0, 2, 0; -1, 1, 1, 1, 1, 1, 1, -1), \qquad (4.1)$$

and the resulting gauge group is $G = SU(3) \times SU(2) \times SU(5) \times SU(2) \times U(1)^8$. Our choice for the eight $U(1)$ generators, in terms of the canonical momenta of the $E_8 \times E_8$ root torus, is given in Table IV. Note that these generators have been redefined so that only the last generator Q_8 is anomalous, with $\mathrm{Tr}\, Q_8 = 3024$. (This charge is not canonically normalized; see Table IV.) The spectrum of chiral superfields and their charges under these eight $U(1)$ factors is given in Table VI of Appendix A.

In this model, and for other models in this pattern, we find just 9 I-monomials that survive the requirement of F-flatness to degree 9 in the superpotential (cf. Table III). There are 14 effective Majorana masses for candidate right-handed neutrinos along six of the nine flat directions. These effective mass terms can be divided into two subsets. In the first we have an effective Majorana mass at the trilinear order. An example is

$$\begin{aligned} &\mathrm{I-monomial:}\ (4, 4, 6, 7, 18, 35, 43, 43),\\ &\mathrm{Eff.\ Maj.\ mass:}\ (\underline{4}, 5, 5), \end{aligned} \qquad (4.2)$$

where we underline the field(s) that get vev(s) to yield an effective mass coupling; repeated entries indicate so many powers of the repeated field. Recall that each field also carries a suppressed family index. There are six such examples of the coupling $(\underline{4}, 5, 5)$ along six different flat directions.

Using the values for the charges in Table VI it is easy to show that the combination of fields in the I-monomial of (4.2) is indeed gauge invariant. Our candidate right-handed neutrino is thus field #5, which we will label N_5. But from Table VI we see that the field N_5 is not a complete gauge singlet, but is actually a (10,2) representation of the hidden sector $SU(5) \times SU(2)$ gauge group. The putative "Majorana mass" term is seen to be the coupling **5 10 10** of the $SU(5)$ part of this group. What is more, fields charged under both of these groups are required to obtain vacuum expectation values along this particular flat direction. This is true of all six flat directions that allow such a candidate Majorana term. Thus the right-handed neutrino would need to be identified with a singlet of the surviving gauge group.

However, the minimal seesaw of (3.1) also requires the coupling of this N_5 field to some doublets of the observable sector $SU(2)$ group. But the presence of N_5 in the untwisted sector of this model prevents any such coupling at the leading order (as does the requirement of gauge invariance under all the non-Abelian factors). There are no couplings at all in the superpotential at degree 4 and 5 (see Table I), so the earliest opportunity for this important Dirac coupling is degree 6. Of the 642 allowed couplings at degree 6 only three involve the coupling of the field N_5 to doublets of the observable sector $SU(2)$. These three nonrenormalizable terms take the form

$$\mathrm{Coupling\ (1)}{:}\underline{S}_4 N_5 N_5 L_{12} B_{30} L'_{44}, \qquad (4.3)$$

$$\mathrm{Coupling\ (2)}{:}\underline{S}_4 N_5 N_5 L_{40} B_{30} L'_{22}, \qquad (4.4)$$

$$\mathrm{Coupling\ (3)}{:}\underline{S}_4 N_5 N_5 S_{29} B_{30} B_{30}. \qquad (4.5)$$

The fields are labeled according to their type and entry number in Table VI: S for singlets of all non-Abelian groups, N for the candidate right-handed neutrino, L for doublets of $SU(2)_{\mathrm{obs}}$, L' for doublets of $SU(2)_{\mathrm{hid}}$, and B for fields bifundamental under both $SU(2)$ factors. Clearly these are not standard Dirac mass terms for the field N_5.

Even if effective Dirac mass terms that could give rise to the matrix (m_D) in (3.1) were present at degree 6 in the superpotential, it is still unlikely that we would obtain an adequate set of neutrino masses. We can estimate the typical scale of the three light eigenvalues in the following manner. It is natural to assume that fields such as S_4 above obtain a vev near the scale given by ξ_{FI} in (2.1). Then the typical entry in the matrix m_M of (3.1) is $r_{\mathrm{FI}} M_{\mathrm{PL}} \sim 0.1 M_{\mathrm{PL}}$, where we have used the information from Table I. An effective Dirac mass term m_D at degree 6 would presum-

TABLE IV. $U(1)$ charge basis for model 2.8. The eight Abelian factors are defined in terms of the canonical momenta of the $E_8 \times E_8$ root lattice, with normalization given in the last column. Canonically normalized generators $\hat{Q}_a$ are obtained from $\hat{Q} = Q_a/\sqrt{k_a}$.

a	Q_a	$k_Q/36$
1	$6(3, 3, -4, -4, -4, 0, 0, 0; 0, 0, 0, 0, 0, 0, 0, 0)$	132
2	$6(2, 2, 1, 1, 1, -11, 0, 0; 0, 0, 0, 0, 0, 0, 0, 0)$	264
3	$6(0,0,0,0,0,0,1,0;0,0,0,0,0,0,0,0)$	2
4	$6(0,0,0,0,0,0,0,1;0,0,0,0,0,0,0,0)$	2
5	$6(-10, -10, -5, -5, -5, -5, 0, 0; 0, 0, 0, -12, -12, -12, -12, 12)$	2040
6	$6(0,0,0,0,0,0,0,0;0,1,1,0,0,0,0,0)$	4
7	$6(-10, -10, -5, -5, -5, -5, 0, 0; -17, 0, 0, 5, 5, 5, 5, -5)$	1428
8	$6(-2, -2, -1, -1, -1, -1, 0, 0; 5, 0, 0, 1, 1, 1, 1, -1)$	84

ably involve three such vevs, suggesting a set of light eigenvalues for the matrix (3.1) of the form

$$m_\nu \sim \frac{(r_{\rm FI}^3 v_u)^2}{r_{\rm FI} M_{\rm PL}} \sim r_{\rm FI}^5 \times 10^{-5}\ {\rm eV}, \tag{4.6}$$

where we have used $v_u \sim 100$ GeV. This suggests neutrino masses in the nano eV range—clearly far too small to fit the measured squared mass differences.

The second subset of candidate Majorana mass terms involves much higher-degree superpotential couplings, so one might expect a better fit to the data. For example, one of the eight remaining flat direction/Majorana coupling pairs is

$$\begin{aligned} &\text{I}-\text{monomial: } (4, 4, 7, 18, 19, 27, 43, 43),\\ &\text{Eff. Maj. mass: } (\underline{7, 7, 19, 27, 43, 43, 43}, 34, 34). \end{aligned} \tag{4.7}$$

Here the candidate right-handed neutrino is state N_{34} from one of the twisted sectors of the theory. Yet it is still charged under the hidden sector gauge group. In this case it is a $\bar{\mathbf{5}}$ of the hidden $SU(5)$ group. This will again make it impossible to generate a gauge-invariant Dirac mass term at the leading trilinear order. In this case the field N_{34} does not appear *in any allowed couplings whatsoever* at degree 6 in the superpotential, let alone couplings to $SU(2)$ doublets. The next allowed order for such a coupling is then degree 9, but a dimension-counting argument again gives rise to the same effective scale as in (4.6) for such a Dirac term with a degree 9 effective Majorana mass term. We thus conclude that (i) the required couplings in the minimal seesaw of Sec. III A do not arise in this model and (ii) the peculiarities associated with the fact that all candidate right-handed neutrinos in this model are charged under the hidden sector $SU(5)$ prevent viable neutrino masses even if they did.

B. Pattern 1.1

To exhibit the properties of the candidate neutrino sectors of pattern 1.1 we will choose model 1.2. This model is defined by the following set of embedding vectors

$$\begin{aligned} V &= \frac{1}{3}(-1, -1, 0, 0, 0, 2, 0, 0; 2, 1, 1, 0, 0, 0, 0, 0),\\ a_1 &= \frac{1}{3}(1, 1, -1, -1, 2, 0, 0, 0; 0, 2, 0, 0, 0, 0, 0, 0),\\ a_3 &= \frac{1}{3}(0, 0, 0, 0, 0, 0, 2, 0; -1, 0, -1, 0, 0, 0, 0, 0), \end{aligned} \tag{4.8}$$

and the resulting gauge group is $SU(3) \times SU(2) \times SO(10) \times U(1)^8$. In this model none of the $U(1)$ factors is anomalous, so we choose a simple basis, in terms of the canonical momenta of the $E_8 \times E_8$ root torus, for the $U(1)$ generators as given in Table V. The spectrum of chiral superfields and their charges under these eight $U(1)$ factors is given in Table VII of Appendix A.

TABLE V. $U(1)$ charge basis for model 1.2. The eight Abelian factors are defined in terms of the canonical momenta of the $E_8 \times E_8$ root lattice, with normalization given in the last column.

a	Q_a	$k_Q/36$
1	6(−3, −3, 2, 2, 2, 0, 0, 0; 0, 0, 0, 0, 0, 0, 0, 0)	60
2	6(1,1,1,1,1,0,0,0;0,0,0,0,0,0,0,0)	10
3	6(0,0,0,0,0,1,0,0;0,0,0,0,0,0,0,0)	2
4	6(0,0,0,0,0,0,1,0;0,0,0,0,0,0,0,0)	2
5	6(0,0,0,0,0,0,0,1;0,0,0,0,0,0,0,0)	2
6	6(0,0,0,0,0,0,0,0;0,0,1,0,0,0,0,0)	2
7	6(0,0,0,0,0,0,0,0;0,1,0,0,0,0,0,0)	2
8	6(0,0,0,0,0,0,0,0;1,0,0,0,0,0,0,0)	2

The lack of an anomalous $U(1)_X$ suggests that we can no longer assume vevs for fields in a particular flat direction are at the scale $\xi_{\rm FI}$. Nevertheless, the existence of flat directions (or nearly flat directions) allows us to consistently choose scalar fields to have large vevs [56,57]. The determination of the exact size of these vevs requires minimization of the effective scalar potential for these D-moduli.

As shown in Table III, a total of 489 I-monomials that satisfy stringent F-flatness to degree 9 were found. From these, and the 3 427 239 couplings that appear in Table II for this model, 42 instances of effective Majorana mass couplings were found along 18 of the 489 flat directions. For brevity, we do not enumerate all of these flat directions and effective Majorana mass couplings, but confine ourselves to a discussion of some representative examples. Remarkably, though there are nominally 42 different pairs of flat directions/effective Majorana operators, these pairs form patterns that repeat themselves—the labels on the fields may change, but the representations and structure do not. Thus a very small number of actual possibilities exist. All of the Majorana couplings/candidate neutrinos fall into one or the other of the two cases given below.

For our first example, let us consider the flat direction characterized by the following pair of invariant and Majorana operator:

$$\begin{aligned} &\text{I}-\text{monomial: } (2, 2, 3, 3, 8, 8, 34, 46, 61, 77),\\ &\text{Eff. Maj. mass: } (\underline{2, 3, 8, 8, 34, 46}, 74, 74). \end{aligned} \tag{4.9}$$

It can be seen from the spectrum of Table VII that all the fields getting vevs in the flat direction are non-Abelian singlets. The candidate right-handed neutrino is field #74 which we denote N_{74}, using the notation described above. This flat direction leaves two $U(1)$ factors unbroken: that is, there are two linear combinations of $U(1)$ generators such that the charges of all the fields in the first line of (4.9) can be made simultaneously zero. Therefore, if the standard model hypercharge generator can be identified with one of these linear combinations, the gauge invariance requirement of the superpotential terms will automatically enforce $Q_Y^{N_{74}} = 0$.

This candidate right-handed neutrino does not appear in any allowed trilinear superpotential coupling. A careful analysis of the degree 4 superpotential couplings shows that the flat direction in (4.9) does not produce an effective Dirac coupling. This conclusion is also true with respect to the 21 329 couplings at degree 6. Since the $U(1)$'s are nonanomalous in this case, we cannot estimate the scale of the vevs in the flat direction, other than that it is below $M_{\rm PL}$. If their typical scale is $\langle S\rangle/M_{\rm PL} \equiv c < 1$, then the light neutrino eigenvalues will be of the order

$$m_\nu \sim \frac{(c^{d-3}v_u)^2}{c^6 M_{\rm PL}} \sim c^{2d-12}10^{-5}\ \text{eV}, \tag{4.10}$$

where d is the degree of the effective Dirac mass term. Thus an acceptable mass would require d smaller than 6.

A much more promising case is the one characterized by the following invariant:

$$\text{I} - \text{monomial: } (3, 3, 8, 21, 22, 29, 46, 72). \tag{4.11}$$

Once again, it can be seen from the spectrum of Table VII that all the fields getting vevs in the flat direction are non-Abelian singlets. Along this direction there are two effective Majorana mass operators, one at degree 6 and the other at degree 8

$$\begin{aligned} &\text{Eff. Maj. mass (1): } (\underline{8, 22, 46, 72}, 9, 9), \\ &\text{Eff. Maj. mass(2): } (\underline{3, 3, 8, 22, 46, 72}, 13, 13). \end{aligned} \tag{4.12}$$

The two Majorana operators differ only by the insertion of two untwisted sector fields S_3. The candidate right-handed neutrinos are thus N_9 and/or N_{13}. Along this flat direction three $U(1)$ factors remain unbroken to low energies, and all linear combinations of these three $U(1)$'s allow $Q_Y^{N_{9,13}} = 0$.

Once again, identifying either field as a bona fide neutrino requires looking for the requisite Dirac couplings to $SU(2)$ doublets. Here the outlook is much brighter: there are several couplings involving $SU(2)$ doublets and both N_9 and N_{13} at degree 3 and degree 4 in the superpotential. In fact, each admits two such couplings

$$(A)\begin{cases} N_g L_{36} L_{64} \\ \underline{S}_3 N_{13} L_{36} L_{64} \end{cases} \qquad (B)\begin{cases} N_g L_{52} L_{71} \\ \underline{S}_3 N_{13} L_{52} L_{71}, \end{cases} \tag{4.13}$$

where we use L to denote doublets of $SU(2)$. At this stage we are not yet in a position to distinguish lepton doublets from up-type Higgs doublets as we have not yet identified other Yukawa interactions or designated a unique hypercharge assignment. Thus we use a common notation for both doublets.

We thus appear to have the two essential ingredients for forming the matrix of couplings in (3.1) and, in fact, we have the potential for embedding the entire leptonic sector of the MSSM superpotential. For instance, if we make the identification $L_{36} = L$ and $L_{64} = H_u$, then we find trilinear couplings of the form

$$W \ni \lambda_1 L_{36} L_5 S_{60} + \lambda_2 L_{64} L_5 S_{38}. \tag{4.14}$$

If we further identify $L_5 = H_d$ we see that S_{60} could play the role of the right-handed charged lepton field E^c while S_{38} could generate an effective μ term through some additional low-energy dynamics. If we were instead to make the identification $L_{36} = H_u$ and $L_{64} = L$ we would merely need to exchange the interpretation of the fields S_{38} and S_{60}. There are several possible systems such as (4.14) for either choice of couplings (A) or (B) of (4.13).

Of course this discussion assumes that the correct hypercharge can simultaneously be assigned to each of these fields along a particular surviving $U(1)$ combination. In many cases this is indeed possible. For example, in the system given by (4.14) and the identification $\{N_9, L_{36}, L_{64}, L_5, S_{60}, S_{38}\} \leftrightarrow \{N, L, H_u, H_d, E^c, S\}$ the particular linear combination of $U(1)$ factors that gives rise to the correct hypercharge assignments $\{0, -1/2, 1/2, -1/2, 1, 0\}$ is given by

$$\begin{aligned} U(1)_Y = &-\frac{7}{180}U(1)_1 - \frac{1}{30}U(1)_2 + \frac{1}{6}U(1)_4 + \frac{1}{4}U(1)_6 \\ &- \frac{1}{4}U(1)_7 + \frac{1}{12}U(1)_8. \end{aligned} \tag{4.15}$$

This also accommodates the quarks of the MSSM. However, the hypercharge normalization is $k_Y = 91/6$ rather than the $SU(5)$-based GUT value of $k_Y = 5/3$. This is not consistent (perturbatively) with the observed couplings, even allowing for the effects of additional matter states in the running of the gauge couplings. However, our purpose in this study is to focus on the neutrino sector and examine how many, if any, of flat directions allow a minimal seesaw, irrespective of whether they are fully realistic in other ways.

So far, so good. We cannot say anything very definite about the typical scale of the vevs $\langle S_i\rangle$ that give rise to the effective Majorana couplings in (4.12) since there is no anomalous $U(1)$ factor in the model. But given that Dirac mass terms can arise at degree 3 or 4, a scale somewhere between the GUT and string scales for these vevs would be welcome. This is not impossible to imagine, since the standard model singlet fields involved in the flat direction (4.11) do couple to several doublets of $SU(2)$ and triplets/antitriplets of $SU(3)$ at the trilinear order. These are just the sorts of ingredients that can give rise to a high, radiatively generated intermediate scale [56]. It would be tempting, then, to declare victory and begin to calculate the possible mass textures for both the Majorana and Dirac matrices—perhaps by assuming only third generation Higgs fields and singlets in (4.11) receive vevs so that selection rules would then enforce texture zeros in the effective mass matrices.

But this would be premature. To begin with, we should note that there are no quark masses in this model at the leading order; by placing the up-type Higgs doublet in the twisted sector (as all of the examples in this class require to generate the neutrino Dirac mass) it becomes impossible to couple it to the (untwisted sector) quark doublet at the trilinear order.[8] The desired quark masses do not appear at degree 4 either, and at degree 6 we find only one new coupling involving quark doublets

$$W^6 \ni \underline{S_3 S_8 S_{72}} Q_1 Q_1 D_{56}, \tag{4.16}$$

where three of the fields are participants in the flat direction, the quark doublet is repeated, and the coupling is to field #56 which is a **3** of $SU(3)$. This is certainly not the quark sector of the standard model, and clearly there are no GUT relations between the neutrino Yukawa interactions and those of the up-type quarks. But our analysis was based on answering the sole question of whether the minimal seesaw can be found in an explicit string construction, so we will not consider the quarks further.

Of greater concern is the redundancy evidenced by the multiple neutrino candidates and multiple Higgs candidates in this example. Can these extra states be projected out of the light spectrum along the flat direction, perhaps leaving only one of the sets of couplings in (4.13)? Do the remaining light states, and, in particular, the candidate right-handed neutrinos, mix with one another? To fully understand the nature of neutrino masses in this set of examples a thorough analysis that considers all the relevant fields of the system must be performed. When we do so we will see that our earlier enthusiasm for this set of flat directions and couplings was misplaced.

A careful consideration of all degree 3 and 4 couplings in the superpotential indicates that many of the extra $SU(2)$ doublets [and, incidentally, all of the exotic **3**'s of $SU(3)$] are projected out of the spectrum—a welcome development. For example we find the couplings

$$W = \lambda_1 \underline{S}_{21} L_{49} L_{70} + \lambda_2 \underline{S}_{22} L_{12} L_{24} + \lambda_3 \underline{S}_{29} L_{51} L_{80} + \lambda_4 \underline{S}_{46} L_{47} L_{48}, \tag{4.17}$$

which eliminates all the possible combinations of $W = \lambda N L H_u$ associations but the two listed in (4.13). As there is no reason to choose N_9 versus N_{13} as our right-handed neutrino, we must therefore conclude that the neutrino sector of this theory involves at least two *species* of neutrino, each with three generations. So too we must accept two species of lepton doublets, and without loss of generality we may choose them to be L_{36} and L_{52}, with fields #64 and #71 being two species of up-type Higgs doublets.

So this model does not give rise to a minimal seesaw after all. In fact, there are terms that mix our fields with Dirac couplings (N_9 and N_{13}) with other standard model singlets that do not. We will refer to these additional states with the notation $\tilde{N}$. In particular we have the couplings

$$\begin{aligned} W_{\rm mix} = {} & \lambda \underline{S}_8 N_9 \tilde{N}_{14} + \lambda \underline{S}_{22} N_9 \tilde{N}_{27} + \lambda \underline{S}_{72} N_9 \tilde{N}_{50} \\ & + \lambda \underline{S}_{46} N_9 \tilde{N}_{81} + \lambda \underline{S_3 S_8} N_{13} \tilde{N}_{14} + \lambda \underline{S_3 S_{22}} N_{13} \tilde{N}_{27} \\ & + \lambda \underline{S_3 S_{72}} N_{13} \tilde{N}_{50} + \lambda \underline{S_3 S_{46}} N_{13} \tilde{N}_{81}, \end{aligned} \tag{4.18}$$

which generate an extended seesaw structure. As mentioned in the introduction, this is not an uncommon feature of explicit string constructions, in part because of the large numbers of standard model singlets that are generally present. Nor need it imply that small neutrino masses are impossible to obtain.

In this particular example the effective neutrino system mass matrix is given in block matrix form by

$$(\nu_L \tilde{N} N) \begin{pmatrix} 0 & 0 & A \\ 0 & 0 & B \\ A & B & C \end{pmatrix} \begin{pmatrix} \nu_L \\ \tilde{N} \\ N \end{pmatrix}, \tag{4.19}$$

defined with the basis sets

$$\nu_L = \{(\nu_L)_{36}, (\nu_L)_{52}\}, \qquad \tilde{N} = \{\tilde{N}_{14}, \tilde{N}_{27}, \tilde{N}_{50}, \tilde{N}_{81}\}, \qquad N = \{N_9, N_{13}\}. \tag{4.20}$$

The individual submatrices in (4.19) are

$$\begin{aligned} A &= \begin{pmatrix} \langle (H_u)_{64} \rangle & \langle S_3 (H_u)_{64} \rangle \\ \langle (H_u)_{71} \rangle & \langle S_3 (H_u)_{71} \rangle \end{pmatrix}, \\ B &= \begin{pmatrix} \langle S_8 \rangle & \langle S_3 S_8 \rangle \\ \langle S_{22} \rangle & \langle S_3 S_{22} \rangle \\ \langle S_{72} \rangle & \langle S_3 S_{72} \rangle \\ \langle S_{46} \rangle & \langle S_3 S_{46} \rangle \end{pmatrix}, \\ C &= \begin{pmatrix} \langle S_8 S_{22} S_{46} S_{72} \rangle & 0 \\ 0 & \langle S_3^2 S_8 S_{22} S_{46} S_{72} \rangle \end{pmatrix}, \end{aligned} \tag{4.21}$$

with the general expectation that the pattern of vevs would be such that $A \ll C \ll B$. But the matrix (4.19) has vanishing determinant and gives rise to three precisely massless eigenvalues; the ''seesaw'' serves only to split the masses of the very heavy eigenstates. This mechanism, in which the addition of off-diagonal terms in an extended right-handed sector destroys what appeared to be a successful construction, could easily occur in other constructions and therefore should be checked for in such.

We might hope to populate some of the zero blocks in (4.19) to salvage this example (though we are already far from a minimal seesaw), but there are no Dirac couplings of doublets L_{36} or L_{64} to the fields $\tilde{N}$ at degree 3, 4, or 6. There are also no couplings at degree $d < 6$ that couple the fields in the $\tilde{N}$ system to themselves—that is, the entire matrix of values represented by the (2,2) entry in (4.19) is vanishing to this degree. We could imagine expanding the system yet again, and looking for couplings of an expanded $\tilde{N}$ system where effective mass terms can arise, say when one of the remaining $U(1)'$ factors is spontaneously bro-

[8]We do not consider the possibility of different families of up-type Higgs doublets involved in generating the neutrino and quark masses, respectively.

ken. But here again by considering all allowed operators of this form at degree $d < 6$ in the superpotential, we find the determinant of this submatrix always vanishes, indicating vanishing eigenvalues for the full matrix (4.19). By arguments similar to those that gave rise to (4.6) and (4.10) it is easy to see that realistic masses for the light neutrinos would require either a Dirac-type coupling of the $\tilde{N}$ to the ν_L fields or Majorana masses for the $\tilde{N}$ fields at no higher degree than the trilinear order. Thus we have succeeded in finding the right operators for the (ν_L, N) system in isolation, but by considering the full lepton system we find that we have failed to account for the finite and small neutrino masses observed in nature.

C. Why so many zeros?

It is of interest to understand why the zeros have appeared in the mass matrices (4.19) and (4.21). Are they exact? Are they the consequence of a symmetry? If they are not exact, at what order do nonzero contributions first appear, and what would be the effects? We have studied these questions, and here we will summarize the answers.

Since the fields of type N and $\tilde{N}$ are $U(1)_Y$ neutral, the zeros do not follow from this symmetry. However, one might ask: Are the zeros explained by the two extra $U(1)$'s that survive along the flat direction (4.11)? A useful basis for this $U(1)^2$ subgroup consists of the following (canonically normalized) generators:

$$A = -\frac{4}{15}Q_1 + \frac{67}{60}Q_2 - \frac{15}{4}Q_3 + \frac{23}{12}Q_4 - \frac{15}{4}Q_5 - \frac{11}{6}Q_6 + \frac{11}{6}Q_7 + \frac{17}{3}Q_8, \tag{4.22}$$

$$B = -\frac{119}{30}Q_1 - \frac{139}{60}Q_2 + \frac{91}{12}Q_3 - \frac{43}{12}Q_4 + \frac{91}{12}Q_5 - \frac{55}{6}Q_6 + \frac{55}{6}Q_7 + 2Q_8. \tag{4.23}$$

The N and $\tilde{N}$ fields are all neutral with respect to these generators. It follows that none of the zeros in the mass matrix are a consequence of gauge invariance.

Further investigation finds that there are discrete symmetries that survive along the flat direction that we study here. Along the flat direction (4.11) there is a breaking $U(1)^8 \to U(1)^3$. However, a discrete subgroup of the broken $U(1)^5$ survives, when it is combined with the trialities of the original theory. This subgroup is found by demanding that the fields in (4.11) are left invariant. However, we find that these surviving discrete symmetries do not explain the zeros either.

In fact, we find that the zeros are not exact but are violated by high order terms. For instance, mass terms of the form $m_{AB}\tilde{N}_A\tilde{N}_B$ are allowed by all symmetries and appear with m_{AB} being a degree 10 polynomial of the fields (4.11). This translates into Majorana mass terms of order 10^8 GeV or less. By arguments similar to those made above, such entries would only generate neutrino masses of order $m_\nu \sim c^8 \times 10^{-5}$ eV, where $c = S/M_P < 1$. Thus $m_\nu < 10^{-13}$ eV for $c < 0.1$, a negligible effect.

In summary, the zeros that we find are not exact, but they may as well be, since the allowed violations of them are of such high order. This is a consequence of the fact that we have studied all terms of the superpotential to a very high order.

V. CONCLUSIONS

Our systematic search of the BSL_A class of otherwise phenomenologically promising, top-down constructions of the heterotic string failed to reveal a minimal seesaw mechanism. This despite our placing the existence of an effective Majorana mass operator at the fore of our search—a search through literally millions of superpotential couplings along thousands of flat directions in a class with dozens of potentially realistic models. What are we to conclude from this null result?

It may be that our search, as computationally intensive as it was, was not sufficiently broad to find the elusive couplings. One might imagine generalizations in which several flat directions, or I-monomials, are simultaneously "turned on." This is, of course, a very real physical possibility that might allow for more effective mass terms (since more vevs are nonvanishing). But it also will tend to be even more severely constrained by F-flatness conditions. We might, in addition, explore directions that are not stringently F-flat; that is, directions where various nonvanishing terms cancel to give $\langle \partial W/\partial \phi_i \rangle = \langle W \rangle = 0$, or in which the flatness breaking terms are small enough to be harmless (as would occur when the vevs are at an intermediate scale). This would allow for more of the D-flat directions to survive in the generalization of Step 3 of Sec. III B above, yielding more effective mass couplings. This would be an interesting starting point for a future study, but it should be clear from the extensive discussion in Secs. III and IV that the computational demands would grow significantly if one were to depart from the simple rules we followed.

But one might have thought that if the minimal seesaw—or indeed a seesaw mechanism at all—is the answer to the problem of small neutrino masses then it should arise with great frequency, even in a simple search such as ours. That it did not might be merely a reflection of the peculiarities of the Z_3 orbifold itself, or perhaps of orbifold constructions more generally. Alas, we are unable to address such a question since the starting point to this work does not even exist for other orbifolds, much less more general heterotic constructions. Yet the Z_3 has been well studied in the past precisely because it has so many other desirable features. That it does not seem to possess a minimal seesaw mechanism should perhaps give us pause.

Thus we might consider again the most successful case we found in our study, that of pattern 1.1 as described in Sec. IV B. Here we find a structure that is not minimal, but of the extended seesaw variety. While this particular example was unable to give mass to the neutrino eigenstates, it may well be that string theory prefers, or at least can accommodate, neutrino mass mechanisms that are not minimal. We dedicated our analysis to the search for couplings of the form (3.2), but we might instead have considered the more general extended form

$$\langle S_1 \cdots S_{n-2} \rangle N N' \tag{5.1}$$

where N and N' are different *species* in the sense defined in [13]. That is, N and N' differ by something more than the third fixed point label that corresponds to the 3-fold degeneracy of this class of models. Needless to say, a systematic search would immediately have to confront the enormous increase in combinatorics involved in such couplings.

Generically, (5.1) corresponds to an effective theory with six right-handed neutrinos. Given our difficulty in finding couplings of the form (3.2) we might expect such an extended mass matrix to have vanishing diagonal entries. Then achieving appropriate neutrino masses would require a mass matrix of the form

$$W_{\rm eff} = (\nu_i, N_i, N_i') \begin{pmatrix} 0 & (m_D)_{ij} & (m_D')_{ij} \\ (m_D)_{ji} & 0 & (m_M)_{ij} \\ (m_D')_{ji} & (m_M)_{ji} & 0 \end{pmatrix} \begin{pmatrix} \nu_j \\ N_j \\ N_j' \end{pmatrix}. \tag{5.2}$$

It is also worth noting that the m_D, m_D' can have a certain amount of texture zeros and still give all neutrinos mass. However, nonzero elements for m_D and m_D' would both have to be present. The seesaw still gives 3 light flavors of mass $\mathcal{O}(m_D^2/m_M)$ and now 6 heavy flavors of mass $\mathcal{O}(m_M)$. An extended seesaw model, though of a somewhat different form from (5.2), has previously appeared in free-fermionic constructions [18], as noted in the introduction.

It may also be that the standard seesaw ideas are not the answer to small neutrino masses. Twelve of the 20 cases listed in Table I contain fields which are bifundamental under two different $SU(2)$ factors. If these groups were to be broken to the diagonal subgroup, such states would be effective triplets under the surviving $SU(2)$. If they were sufficiently massive, they may form the basis for a Type II seesaw mechanism [47,58]. An investigation of this possibility in the Z_3 orbifold is underway [59]. Dirac-type couplings of the form NLH_u are much more common than effective Majorana mass-type couplings. It may therefore simply be that neutrinos are Dirac particles, with neutral lepton Yukawa couplings only arising at higher order in the effective superpotential. These terms would have to be of extremely high order if the relevant vevs are close to $M_{\rm PL}$, or could be as low as degree 4 if the vevs are at some intermediate scale [56,60]. This is contrary to most theoretical prejudice, but it is certainly a logical possibility. One of the lessons of this study is that couplings, such as those leading to Majorana masses, are determined to a greater degree by the underlying theory in explicit string constructions than by esoteric reasoning (naturalness, elegance, simplicity, etc.). While we would have preferred to have found models with flat directions where the simple seesaw works, we therefore regard our null result as significant.

Our principal result also demonstrates the power of the underlying string theory. It is remarkable that something as simple as (3.2) is not possible along a flat direction. Certainly one would not expect this from a "bottom-up" perspective. This result is a consequence of the wealth of symmetry constraints that arise from the underlying theory. These are features that one is unlikely to have ever guessed. We feel that this demonstrates the importance of attempting to connect effective field theory model building with an underlying theory—or in modern parlance, an "ultraviolet completion."

We have found further evidence that there are only 20 inequivalent models in the BSL_A class. This drastic reduction from a naive estimate—based on the number of seemingly different embedding vectors—can be given the following interpretation. It shows that surveys of classes of string constructions can be done, and, that they can produce meaningful results much the way that a scan over some significant section of parameter space in an effective field theory (such as the MSSM) can have meaning. We find this encouraging, because it hints that qualitative impressions gained in such a survey are a good guide to effective field theory model building.

We last note the importance of having a useful query in mind when surveying explicit string constructions. The unique nature of the coupling (3.2) made it an extremely powerful tool in directing our attention to only a handful of promising cases from a vacuum space that (prior to the pioneering work of a number of theorists) looked to include hundreds of thousands of possible vacuum configurations, with thousands of flat directions to study in each one.

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APPENDIX A: SPECTRA FOR PROMISING CASES

In this appendix we provide a partial spectrum for Models 2.8 and 1.2. These are the two examples from pattern 2.6 and 1.1, respectively, that were chosen for detailed study of the neutrino sector in the text. Each species of chiral superfield carries a sequential numerical label. For each species there is a three-fold replication of generations. Fields are identified by their irreducible representation under the non-Abelian parts of $SU(3) \times SU(2) \times G_{\rm hid}$ and by their charges under the Abelian gauge factors. We group states by sector of the string Hilbert space, beginning

TABLE VI. Partial spectrum of chiral matter for model 2.8. Chiral superfields are grouped by sector of the string Hilbert space. Irreducible representations under the non-Abelian gauge group $SU(3) \times SU(2) \times SU(5)_H \times SU(2)_H$ is given, along with charges under the eight $U(1)$ factors. Note that Q_8 is the anomalous $U(1)_X$.

	Irrep.	Q_1	Q_2	Q_3	Q_4	Q_5	Q_6	Q_7	Q_8
	sector: untwisted								
1	$(1,2,1,1)_0$	18	−54	0	0	−90	0	−90	−18
2	$(3,2,1,1)_0$	6	−18	0	0	90	0	90	18
3	$(\bar{3},1,1,1)_0$	−24	72	0	0	0	0	0	0
4	$(1,1,5,1)_0$	0	0	0	0	72	0	72	−36
5	$(1,1,10,2)_0$	0	0	0	0	−36	0	−36	18
	sector: $(-1,-1)$								
6	$(1,1,1,1)_0$	−12	14	2	0	−190	0	−54	6
7	$(1,1,1,2)_0$	−12	14	2	0	−10	0	−78	−24
	sector: $(-1,0)$								
12	$(1,2,1,1)_0$	−30	2	0	0	−70	−2	−36	18
	sector: $(-1,1)$								
18	$(1,1,1,2)_0$	−12	14	−2	0	−70	2	−36	−24
19	$(1,1,1,1)_0$	−12	14	−2	0	−70	−4	−138	6
22	$(1,1,1,2)_0$	6	−40	1	−3	20	2	54	−6
	sector: $(0,-1)$								
27	$(1,1,1,1)_0$	−12	14	2	0	−10	6	24	30
	sector: (0,0)								
29	$(1,1,1,1)_0$	−12	−52	0	0	20	4	−48	24
30	$(1,2,1,2)_0$	6	26	0	0	−10	−2	24	−12
31	$(1,2,1,1)_0$	6	26	0	0	−10	4	−78	18
	sector: (0,1)								
35	$(1,1,1,1)_0$	−12	14	−2	0	110	2	−60	30
	sector: (1,-1)								
38	$(\bar{3},1,1,1)_0$	0	−22	2	0	−10	0	−78	18
40	$(1,2,1,1)_0$	−12	14	−1	3	−10	0	−78	18
	sector: (1,0)								
43	$(1,1,\bar{5},1)_0$	24	−28	0	0	8	−2	42	0
44	$(1,1,1,2)_0$	24	−28	0	0	80	4	12	−6
	sector: (1,1)								
50	$(1,2,1,1)_0$	−12	14	1	−3	−70	2	−36	18

TABLE VII. Partial spectrum of chiral matter for model 1.2. Notation is identical to that of Table VI. This model has no anomalous $U(1)$ factor.

	Irrep.	Q_1	Q_2	Q_3	Q_4	Q_5	Q_6	Q_7	Q_8
	sector: untwisted								
1	$(3,2,1)_0$	6	−12	0	0	0	0	0	0
2	$(1,1,1)_0$	0	0	−6	0	−6	0	0	0
3	$(1,1,1)_0$	0	0	−6	0	6	0	0	0
4	$(1,1,1)_0$	0	0	0	0	0	−6	0	6
	sector: $(-1,-1)$								
5	$(1,2,1)_0$	18	4	−2	2	0	−2	−2	0
8	$(1,1,1)_0$	0	10	4	2	0	−2	−2	0
9	$(1,1,1)_0$	0	−5	−5	−1	3	−2	−2	0
12	$(1,2,1)_0$	−18	1	1	−1	3	−2	−2	0
13	$(1,1,1)_1$	0	−5	1	−1	−3	−2	−2	0
14	$(1,1,1)_0$	0	−5	1	−1	−3	4	4	0
	sector: $(-1,0)$								
21	$(1,1,1)_0$	0	−5	1	3	−3	−4	−2	−2
22	$(1,1,1)_0$	0	−5	1	3	−3	2	4	−2
	sector: $(-1,1)$								
24	$(1,2,1)_0$	18	4	−2	−2	0	0	−2	2
27	$(1,1,1)_0$	0	10	4	−2	0	0	−2	2
29	$(1,1,1)_0$	0	−5	1	−5	−3	0	−2	2
31	$(1,2,1)_0$	−18	1	1	1	−3	0	−2	2
	sector: $(0,-1)$								
34	$(1,1,1)_0$	12	−4	4	−4	0	−2	2	0
35	$(1,2,1)_0$	−6	2	−2	−4	0	−2	2	0
36	$(1,2,1)_0$	−6	2	4	2	0	−2	2	0
38	$(1,1,1)_0$	−24	−7	1	−1	3	−2	2	0
	sector: (0,0)								
46	$(1,1,1)_0$	12	−4	4	0	0	2	2	4
47	$(1,2,1)_0$	−6	2	−2	0	0	2	−4	−2
48	$(1,2,1)_0$	−6	2	−2	0	0	−4	2	−2
49	$(1,2,1)_0$	−6	2	−2	0	0	2	2	4
	sector: (0,1)								
50	$(1,1,1)_0$	12	−4	4	4	0	0	2	2
51	$(1,2,1)_0$	−6	2	−2	4	0	0	2	2
52	$(1,2,1)_0$	−6	2	4	−2	0	0	2	2
56	$(3,1,1)_0$	0	5	1	1	−3	0	2	2
	sector: $(1,-1)$								
60	$(1,1,1)_0$	−12	−6	−2	−4	0	4	0	0
61	$(1,1,1)_0$	−12	−6	4	2	0	4	0	0
64	$(1,2,1)_0$	6	3	1	−1	−3	4	0	0
	sector: (1,0)								
70	$(1,2,1)_0$	6	3	1	−3	3	2	0	−2
71	$(1,2,1)_0$	6	3	1	3	−3	2	0	−2
72	$(1,1,1)_0$	−12	9	1	−3	−3	2	0	−2
74	$(1,1,1)_1$	−12	−6	−2	0	0	2	0	−2
	sector: (1,1)								
77	$(1,1,1)_0$	−12	−6	4	−2	0	0	0	−4
80	$(1,2,1)_0$	6	3	1	1	3	0	0	−4
81	$(1,1,1)_0$	−12	9	1	1	−3	0	0	−4

with the untwisted sector and followed by each of the twisted sectors. These twisted states are labeled by two integers, indicating the fixed point location in each of the first two compact complex planes (the location in the third plane being the three-fold degeneracy that gives rise to the three generations). These integers may take the value 1, 0, or -1 in our convention. The subscript following the irreducible representation label denotes the string oscillator number, if any, for the state.

To keep these tables manageable we have included only those states which are doublets of the standard model $SU(2)$ factor or are otherwise mentioned in the text. Complete tables of spectra can be obtained from the authors by request.

APPENDIX B: VIOLATIONS OF F-FLATNESS

In $N = 1$ supersymmetric models such as the ones studied here, it is generally the case that only isolated minima of the scalar potential exist once all superpotential couplings, to all orders, are taken into account. For theories with a large number of fields, such as we study here, the number of isolated minima is typically quite vast. Most of these minima are highly nontrivial, involving cancellations between terms appearing in the $\mathcal{O}(150)$ vanishing F-term conditions. Thus the "flat directions" that we study here are most likely only approximate; i.e. they are violated by some very high order terms in the superpotential. However, provided that the vevs are small relative to the Planck scale (which we assume) the shift in the vacuum away from our approximately flat directions should be negligibly small.

For example, for the flat direction (4.11) in model 1.2 studied in Sec. IV B, it is easy to find superpotential terms will violate F-flatness for this model. Suppose a monomial m of each of the fields involved in (4.11) with powers p_3, p_8, etc.

$$m = S_3^{p_3} S_8^{p_8} S_{21}^{p_{21}} S_{22}^{p_{22}} S_{29}^{p_{29}} S_{46}^{p_{46}} S_{72}^{p_{72}}. \tag{B1}$$

Contributions to F-terms may occur if the $U(1)^8$ charge of m is zero or equal to the charge of any of the fields in the spectrum. The assumption of vanishing charges implies

$$(p_3, \ldots, p_{72}) = (a + b, b, a, b, b, a, a), \tag{B2}$$

where a, b are integers. Taking into account the string selection rules for m, we find they are only satisfied if and only if $a + b = 0 \bmod 3$, which implies a variety of degree 12 operators. Thus the violations of F-flatness from terms of the form (B1) first occur at degree 12. Taking $\langle S\rangle \sim 0.1$, we obtain F-term breaking of order $m_{\mathrm{soft}} = F/M_{\mathrm{PL}} \sim 10^{-11} M_{\mathrm{PL}} = 10^7$ GeV. (Here $F \sim W' \sim 10^{-11} M_{\mathrm{PL}}^2$ has been estimated for a degree 12 contribution to the superpotential, and the usual estimate $m_{\mathrm{soft}} = F/M_{\mathrm{PL}}$ for supergravity mediated supersymmetry breaking in the observable sector has been used.) This may seem large. However, it requires only a modest (relative to the Planck scale) shift in the vacuum if one is to cancel the F-term with other contributions from W'. For example, if a trilinear contribution is turned on, i.e. $\langle W'_{\mathrm{tri}}\rangle \sim v^2$, cancellation of the F-term implies that the vev v would be of order $v = 10^{-5.5} M_{\mathrm{PL}}$. This vev would not change the dominant features of the effective low-energy mass matrices and Yukawa couplings, unless it generates new couplings for light particles at low order. Whether or not this occurs must be studied on a case by case basis.

A similar (tedious) analysis could be carried out for all proposed flat directions, for all models, identifying the lowest order at which nonvanishing contributions to flat directions would occur. However, since we already have verified stringent F-flatness to degree 9, the lowest order we will ever find is degree 10. Generalizing the arguments made above, cancellation of F-terms would require shifts in the vacuum that are very small fractions of M_{PL}, which in most cases would have negligible effects in the low-energy theory. For this reason we believe that the high orders to which we have checked F-flatness should suffice for our purposes. In any event it provides a leading order approximation to F-flat vacua.

Admittedly then, the flat directions we study are only a tiny sample of the vast number of approximate minima. Nevertheless, since they are the easiest to classify and do not require detailed knowledge of the strengths of superpotential couplings, they are the most sensible class of approximate minima to study in a first detailed analysis of the low-energy couplings.

APPENDIX C: SELECTION RULES FOR SUPERPOTENTIAL COUPLINGS

In this appendix, we review constraints on superpotential couplings in Z_3 constructions such as the BSL_A models that we study. Orbifold selection rules are presented from the practical standpoint: we explain how they are implemented rather than why they are true. The origin of these rules in the underlying conformal field theory has been described in detail in [61] and reviewed in Appendix B of [8]. The presentation here rests on Refs. [62–65].

Gauge invariance.—This is very restrictive in the BSL_A models, where the gauge groups are rank 16. The $U(1)$ parts of the gauge group, $U(1)^8$ or $U(1)^9$ factors, greatly reduce the number of invariants beyond what would be allowed from non-Abelian factors alone. This is particularly true because of the large number of matter fields that are non-Abelian singlets. These matter fields are never singlets with respect to all of the $U(1)$ factors.

Point group selection rule.—This is a *twisted triality* invariance. Nonoscillator twisted fields T and oscillator twisted fields Y (*blowing up modes* of the (0,2) construction [61]) have twisted triality 1, whereas untwisted matter fields U and Kähler moduli $M^{k\bar{\ell}}$ have twisted triality 0. The point group selection rule for a Z_3 model states that only couplings of the form $M^{\ell} U^m T^{3n} Y^{3p}$, where $\ell, m, n, p \in \mathbf{Z}$, are allowed. These are just couplings of vanishing twisted triality; i.e., triality of 0 mod 3.

Lattice group selection rule.—This is a restriction on couplings between twisted sector fields and is a 3-fold triality. Each twisted matter field has fixed point labels (n_1, n_2, n_3), with each entry taking values $0, \pm 1$. An invariant coupling must have vanishing triality with respect to each of the entries. Thus, consider a coupling with m twisted fields

$$T_{n_1^{(1)},n_2^{(1)},n_3^{(1)}} \cdots T_{n_1^{(m)},n_2^{(m)},n_3^{(m)}}. \tag{C1}$$

The lattice group selection rule requires

$$n_i^{(1)} + \cdots + n_i^{(m)} = 0 \quad \text{mod } 3, \quad \forall\ i = 1, 2, 3. \tag{C2}$$

H-momentum conservation.—This takes its name from the bosonized description of Neveu-Schwarz/Ramond world-sheet fermions, $:\psi^{2m-1}\psi^{2m}:(z) \equiv \partial H^m(z)$. Here, z is the complex world-sheet coordinate of the underlying conformal field theory. The intricacies of this selection rule have been reviewed, for example, in [8]. Here we merely state the results as they pertain to superpotential couplings.[9] A general n-point amplitude associated with the superpotential vertex

$$\int d^2\theta W \ni \chi_1\chi_2\phi_3 \ldots \phi_n \tag{C3}$$

in the effective supergravity arises from a correlation function in the underlying conformal field theory of the form

$$\langle V_{-(1/2)}(z_1)V_{-(1/2)}(z_2)V_{-1}(z_3)V_0(z_4) \cdots V_0(z_n)\rangle. \tag{C4}$$

Here, the subscripts indicate ghost number q of vertex operators in the covariant formulation [62].

For the untwisted matter states, what is important is that they have a degeneracy of 3 corresponding to different internal $SO(2)^3$ weights. Each $SO(2)$ factor corresponds to one of the three complex planes of the compact space. The states carry a label of the degeneracy: U^i where $i = 1, 2, 3$. The different $SO(2)^3$ weights determine different combinations of $H(z)$ that appear in the vertex operators. It is this $H(z)$ dependence that is important to H-momentum conservation. Thus constraints arise on the labels i that can appear in an invariant coupling. It will later prove important that the operators with ghost number $q = 0$ contain a world-sheet derivative factor

$$V_0(U^i) \sim \partial X^i, \tag{C5}$$

whereas the others do not. Here, X^i are the (complex) world-sheet bosons corresponding to the 6D compact space. World-sheet derivatives such as ∂X^i are important in the final selection rule discussed below.

[9]While this selection rule is most often described in terms of the covariantly quantized string [62], it also is straightforward to derive in a lightcone description of the physical states.

For the twisted sector there is no such degeneracy of $SO(2)^3$ weights. Instead the degeneracy corresponds to the fixed point labels discussed above. These play no role in H-momentum conservation. What is important is that for nonoscillator twisted fields

$$V_0(T) \sim \sum_i f^i(H)\partial X^i, \tag{C6}$$

where $f^i(H)$ stands for details that we will not discuss here, but which are involved in H-momentum conservation. In the case of oscillator twisted fields Y^ℓ, all their vertex operators pick up an extra derivative factor, corresponding to left-moving oscillator number $N_L = 1/3$

$$V_q(Y^\ell) \sim V_q(T)\overline{\partial X}^\ell. \tag{C7}$$

The 9 Kähler moduli $M^{k\bar{\ell}}$ also play a role in the superpotential couplings. Since we are only interested in how they interact with matter, we can always include them through V_0 operators, which take the form

$$V_0(M^{k\bar{\ell}}) \sim \partial X^k \overline{\partial X}^{\bar{\ell}}. \tag{C8}$$

A trilinear coupling between untwisted matter fields, $U^iU^jU^k$, only conserves H-momentum if i, j, and k are different from each other. Thus

$$U^iU^jU^k \sim \epsilon^{ijk}. \tag{C9}$$

For a higher order coupling, the V_0 operators are not constrained by H-momentum

$$U^iU^jU^kU^{\ell_1} \cdots U^{\ell_n} \sim \epsilon^{ijk}\partial X^{\ell_1} \cdots \partial X^{\ell_n}. \tag{C10}$$

With respect to the degeneracy labels on the untwisted fields, which serve as generation labels in the effective supergravity, many different couplings are allowed, corresponding to different assignments of n fields to the $q = 0$ picture.

A trilinear coupling between twisted matter fields always conserves H-momentum. For a higher order twisted coupling, H-momentum conservation constraints on the $f^i(H)$ in (C6) picks out a certain combination of the derivatives:

$$T^{3m} \sim (\partial X^1\partial X^2\partial X^3)^{m-1}. \tag{C11}$$

If there are both twisted and untwisted fields, then it is convenient to take the untwisted fields in the $q = 0$ picture, so that

$$T^{3m}U^{i_1} \cdots U^{i_n} \sim (\partial X^1\partial X^2\partial X^3)^{m-1}\partial X^{i_1} \cdots \partial X^{i_n}. \tag{C12}$$

It is obvious how the above expressions are modified once oscillator twisted states or Kähler moduli are included. For example,

$$T^{3m-1}Y^\ell \sim (\partial X^1\partial X^2\partial X^3)^{m-1}\overline{\partial X}^\ell,$$
$$U^iU^jU^kM^{m\bar{\ell}} \sim \epsilon^{ijk}\partial X^m\overline{\partial X}^{\bar{\ell}}. \tag{C13}$$

Automorphism selection rule.—This last selection rule requires that couplings be invariant under automorphisms of the $SU(3)^3$ lattice. Here this amounts to examining the factors of ∂X^i and $\overline{\partial X^i}$ coming from the vertex operators. Suppose

$$\langle V_{-(1/2)}(z_1)V_{-(1/2)}(z_2)V_{-1}(z_3)V_0(z_4)\cdots V_0(z_n)\rangle \sim \prod_{i=1}^{3}(\partial X^i)^{m_i}(\overline{\partial X^i})^{p_i}. \tag{C14}$$

Then the automorphism selection rule states that the coupling will vanish unless

$$m_i - p_i = 0 \quad \mathrm{mod}\ 3 \quad \forall\ i. \tag{C15}$$

It is convenient to define powers that do not distinguish between the indices, just counting the number of ∂X's or $\overline{\partial X}$'s that appear:

$$m = \sum_i m_i, \qquad p = \sum_i p_i. \tag{C16}$$

A necessary but not sufficient condition is that

$$m - p = 0 \quad \mathrm{mod}\ 3. \tag{C17}$$

Twisted fields contribute to $m - p$, mod 3, only if they are oscillators, through (C7). This is because (C11) only gives multiples of 3. Each untwisted oscillator subtracts 1 from $m - p$, mod 3. Untwisted superfields contribute to $m - p$ through (C5). In couplings without twisted fields (C10), $m - p$ just counts the number of untwisted fields mod 3. In couplings with twisted fields (C12), we can always associate the untwisted fields with V_0 operators, so again $m - p$ counts the number of untwisted fields mod 3. Finally, the Kähler moduli do not contribute to $m - p$, mod 3. From these considerations it can be seen that a convenient way to encode the constraint (C17) is the following: we assign *untwisted triality* of 1 to fields U and -1 to fields Y. Fields T and $M^{k\bar{\ell}}$ have untwisted triality 0. The demand that couplings be invariant with respect to untwisted triality is equivalent to (C17).

It is easy to show that if a coupling has vanishing untwisted triality, so that it satisfies (C17), then it can be made to satisfy the stricter automorphism selection rule (C15) simply by supplementing with an appropriate combination of off-diagonal moduli. As an example, let us examine couplings of the form $TTTTTTY^1Y^1Y^2$, with none of the T fields an oscillator state. This involves nine twisted states, so remembering the derivative terms which come from Y^i and using (C11) we find

$$TTTTTTY^1Y^1Y^2 \sim (\partial X^1\partial X^2\partial X^3)^2\bar{\partial}\bar{X}^1\bar{\partial}\bar{X}^1\bar{\partial}\bar{X}^2. \tag{C18}$$

Now apply the rule (C15):

$$m_1 - p_1 = 0, \qquad m_2 - p_2 = 1, \qquad m_3 - p_3 = 2. \tag{C19}$$

Thus, the coupling is forbidden by the automorphism selection rule. However, notice that

$$M^{3\bar{2}} \sim \partial X^3\bar{\partial}\bar{X}^2 \tag{C20}$$

provides just the factors we need in order to satisfy the automorphism selection rule. Consequently, the coupling

$$M^{3\bar{2}}TTTTTTY^1Y^1Y^2 \tag{C21}$$

is allowed.

More generally, suppose

$$m_i - p_i = 3\ell_i + r_i, \qquad r_i \ni \{-1, 0, 1\}. \tag{C22}$$

Thus if the coupling has vanishing untwisted triality, $r_1 + r_2 + r_3 = 0$ mod 3. Then it is easy to check that for any choice of the r_i it is possible to find a combination of off-diagonal moduli that will cancel the r_i's. For example if $(r_1, r_2, r_3) = (-1, 0, 1)$, then $M^{1\bar{3}}$ will do the job. Or, if $(r_1, r_2, r_3) = (1, 1, 1)$ then $M^{3\bar{1}}M^{3\bar{2}}$ will suffice. The other nontrivial possibilities are just permutations of $(-1, 0, 1)$, or $(-1, -1, -1)$. It is easy to check that these can be compensated in a manner similar to the two examples just given.

The off-diagonal moduli parameterize angles between the three complex planes of the compact space. If we were to assume that the vacuum was maximally symmetric, then the vevs of the off-diagonal moduli would vanish. We will not make this assumption here, since it is a very special point in moduli space and hardly corresponds to a generic situation. Since we allow for nonvanishing off-diagonal moduli, any coupling that satisfies (C17) but not (C15) can be made to satisfy (C15) simply by adding some number of off-diagonal moduli. Thus in the presence of off-diagonal moduli, we need only check untwisted triality to ensure that both H-momentum conservation and the automorphism selection rules are satisfied.

A simple example is afforded by degree 5 superpotential couplings. A coupling of all untwisted fields will not work because the untwisted triality is $5 \simeq 2$. Given that twisted fields must be included, twisted triality requires exactly three. But then two untwisted remain, which give untwisted triality of 2. To cancel this, two of the twisted fields must be oscillators. Thus the unique type of trialities-allowed coupling is $UUYYT$. In our analysis of the BSL_A models, we find that this is never gauge invariant, and thus never allowed. A further simple consequence of untwisted triality is that models that do not contain twisted oscillators (all those that fall into patterns 2.6, 4.5, 4.7, and 4.8) can only have superpotential couplings whose degree is a multiple of 3. This is seen explicitly in Table II.

Summary.—Provided that we do not go to a special point in the moduli space where off-diagonal moduli vanish, the imposition of the selection rules just amounts to the simultaneous satisfaction of: (i) lattice triality, (ii) twisted triality, (iii) untwisted triality. This makes the selection rules very easy to automate and has greatly aided our analysis.

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