One-loop renormalization group equations for two left-right supersymmetric models

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In this paper we present the renormalization group equations to one-loop order for all the parameters of two supersymmetric left-right theories that are softly broken. Both models are based upon the gauge group $SU(3)^c \times SU(2)_L \times SU(2)_R \times U(1)_{B-L}$ and both contain an arbitrary number of bidoublets as well as singlets; however, one model uses doublets to break $SU(2)_R$ and the other uses triplets.

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I. INTRODUCTION

The recent discovery of oscillating neutrinos (implying that neutrinos are massive) has created definitive experimental evidence of a flaw in the standard model. This flaw can be rectified by adding an $SU(2)_R$ group to the standard model group structure. This will allow for a Dirac mass term for the neutrinos and will also provide a Majorana mass term for the right-handed neutrino via the seesaw mechanism [1] when $SU(2)_R$ is broken. These extensions are called left-right [2–9] and they can also be imbedded in supersymmetry (SUSY). With SUSY, the models are dubbed SUSYLR [10–16] and contain the attractive features of the supersymmetric standard model (e.g. providing a solution for the hierarchy problem and allowing for gauge coupling unification [17]). SUSYLR models have the additional appealing characteristics of solving the strong *CP* problem [18–23], asymptotic parity invariance and automatic *R*-Parity Conservation [24–29].

The parameters in these models are written down at a high scale of new physics such as the grand unified theory (GUT) or Planck scale. In order to make predictions at lower energy levels, renormalization group equations (RGEs) must be calculated for these parameters, and their values extrapolated to the energy realm of current experiments. In this paper the RGEs for two instances of left-right models are presented (non-SUSY Triplet left-right model equations can be found in [30]). These equations were calculated to one-loop order using the general $N = 1$ supersymmetry RGEs given in [31,32] and agree (after accounting for the absence of $SU(2)_R$) with the subset previously published in [33]. The following equations represent a completion and extension of those RGEs, and provide a valuable tool for extrapolating down from higher scale physics to the scale of $SU(2)_R$ breaking—at which point the model contains the minimal supersymmetric standard model (MSSM) and all couplings of interest can be extrapolated using the RGEs of the MSSM found in [32].

The two models used in this paper differ in their $SU(2)_R$ breaking fields: one uses $SU(2)_R$ doublets and the other $SU(2)_R$ triplets. Theoretical consequences of these models can be found in various papers including [10–13,16,34,35], respectively. In Section II we will present the doublet model starting with the specifics and continuing with the RGEs. In Section III we will follow a similar format for the triplet model.

II. DOUBLET MODEL

A. Particle Content

In Table I is the particle content for the doublet implementation of SUSYLR and the particle representations under the non-Abelian gauge groups. The particle quantum numbers are stated for the $U(1)_{B-L}$ gauge group (The $B - L$ number used in the RGEs follows the GUT normalization scheme; the values in the table do not. To get the GUT-normalized value, multiply the number in the table by $\sqrt{3/8}$.)

The Q and L are the quark and lepton fields of the MSSM and Q^c and L^c are the equivalent $SU(2)_R$ fields. In order to keep this model general, we allow for an arbitrary amount of singlet fields, n_S and so in S^α, $\alpha = 1 \dots n_S$. Likewise there are n_Φ bidoublet fields and so in Φ_a, $a = 1 \dots n_\Phi$. Note that while including one Φ bidoublet does give mass to the fermions, it does not produce quark mixings at tree level (thus another method is required for $V_{CKM} \neq 1$—see, for instance [35]) and so most models set $n_\Phi = 2$. The SU(2) doublets and bidoublets are represented in the following manner (with color and generational indices suppressed):

TABLE I. This table shows the representations for the non-Abelian gauge groups and the $B - L$ number for $U(1)$.

	$SU(3)^c$	$\times$	$SU(2)_L$	$\times$	$SU(2)_R$	$\times$	$U(1)_{B-L}$
Q	3		2		1		$+\frac{1}{3}$
Q^c	3		1		2		$-\frac{1}{3}$
L	1		2		1		-1
L^c	1		1		2		$+1$
Φ_a	1		2		2		0
χ	1		2		1		$+1$
χ^c	1		1		2		-1
$\bar{\chi}$	1		2		1		-1
$\bar{\chi}^c$	1		1		2		$+1$
S^α	1		1		1		0

$$Q = \begin{pmatrix} u \\ d \end{pmatrix}, \qquad Q^c = \begin{pmatrix} d^c \\ -u^c \end{pmatrix}, \qquad \Phi_a = \begin{pmatrix} \phi^0_{da} & \phi^+_{ua} \\ \phi^-_{da} & \phi^0_{ua} \end{pmatrix}.$$

Here Q and Q^c are used as an example for any $SU(2)$ doublet pair. The other doublets can be written in a similar fashion where the charges of the fields must obey the equation $Q = I_{L3} + I_{R3} + \frac{B-L}{2}$. Where I_{L3} and I_{R3} are the third component of the $SU(2)_L$ and $SU(2)_R$ quantum numbers.

Under $SU(2)$, these fields transform as:

$$\begin{aligned} &Q \to U_L Q, && Q^c \to U_R Q^c, \\ &L \to U_L L, && L^c \to U_R L^c, \\ &\chi \to U_L \chi, && \chi^c \to U_R \chi^c, \\ &\bar{\chi} \to U_L \bar{\chi}, && \bar{\chi}^c \to U_R \bar{\chi}^c, \\ &\Phi_a \to U_L \Phi_a U_R^\dagger, && S^\alpha \to S^\alpha. \end{aligned}$$

And their parity transformations are:

$$\begin{aligned} &Q \to -i\tau_2 Q^{c*}, && Q^c \to i\tau_2 Q^*, \\ &L \to -i\tau_2 L^{c*}, && L^c \to i\tau_2 L^*, \\ &\chi \to -i\tau_2 \chi^{c*}, && \chi^c \to i\tau_2 \chi^*, \\ &\bar{\chi} \to -i\tau_2 \bar{\chi}^{c*}, && \bar{\chi}^c \to i\tau_2 \bar{\chi}^*, \\ &\Phi_a \to \Phi_a^\dagger, && S^\alpha \to S^{\alpha *}. \end{aligned}$$

B. Superpotential and Soft Breaking Lagrangian

The most general superpotential and soft supersymmetry breaking Lagrangian for this model are:

$$\begin{aligned} W = {} & i h_a Q^T \tau_2 \Phi_a Q^c + i h'_a L^T \tau_2 \Phi_a L^c + i\lambda_a \chi^T \tau_2 \Phi_a \chi^c + i\bar{\lambda}_a \bar{\chi}^T \tau_2 \Phi_a \bar{\chi}^c + i\mu^\alpha_\chi S^\alpha \chi^T \tau_2 \bar{\chi} + i\mu^\alpha_{\chi^c} S^\alpha \chi^{cT} \tau_2 \bar{\chi}^c \\ & + \frac{1}{6} Y^{\alpha\beta\gamma} S^\alpha S^\beta S^\gamma + \mu^\alpha_{ab} S^\alpha \mathrm{Tr}(\Phi_a^T \tau_2 \Phi_b \tau_2) + i M_\chi \chi^T \tau_2 \bar{\chi} + i M_{\chi^c} \chi^{cT} \tau_2 \bar{\chi}^c + M_{\Phi ab} \mathrm{Tr}(\Phi_a^T \tau_2 \Phi_b \tau_2) \\ & + \frac{1}{2} M_S^{\alpha\beta} S^\alpha S^\beta + L^\alpha S^\alpha + W_{NR}, \end{aligned} \tag{2.1}$$

$$\begin{aligned} \mathcal{L}_{S\mathcal{B}} = {} & -\frac{1}{2}(M_3 \tilde{g}\, \tilde{g} + M_L \tilde{W}_L\, \tilde{W}_L + M_R \tilde{W}_R\, \tilde{W}_R + M_I \tilde{B}\, \tilde{B} + \text{h.c.}) - \Big[i A_{Qa} \tilde{Q}^T \tau_2 \Phi_a \tilde{Q}^c + i A_{La} \tilde{L}^T \tau_2 \Phi_a \tilde{L}^c + i A_{\lambda a} \chi^T \tau_2 \Phi_a \chi^c \\ & + i A_{\bar{\lambda} a} \bar{\chi}^T \tau_2 \Phi_a \bar{\chi}^c + i A^\alpha_\chi S^\alpha \chi^T \tau_2 \bar{\chi} + i A^\alpha_{\chi^c} S^\alpha \chi^{cT} \tau_2 \bar{\chi}^c + \frac{1}{6} A_S^{\alpha\beta\gamma} S^\alpha S^\beta S^\gamma + A^\alpha_{\Phi ab} S^\alpha \mathrm{Tr}(\Phi_a^T \tau_2 \Phi_b \tau_2) + \text{h.c.} \Big] \\ & - \Big[i B_\chi \chi^T \tau_2 \bar{\chi} + i B_{\chi^c} \chi^{cT} \tau_2 \bar{\chi}^c + B_{ab} \mathrm{Tr}(\Phi_a^T \tau_2 \Phi_b \tau_2) + \frac{1}{2} B_S^{\alpha\beta} S^\alpha S^\beta \Big] \\ & - [m_Q^2 \tilde{Q}^T \tilde{Q}^* + m_{Q^c}^2 \tilde{Q}^{c\dagger} \tilde{Q}^c + m_L^2 \tilde{L}^T \tilde{L}^* + m_{L^c}^2 \tilde{L}^{c\dagger} \tilde{L}^c + m_\chi^2 \chi^\dagger \chi + m_{\chi^c}^2 \chi^{c\dagger} \chi^c + m_{\bar{\chi}}^2 \bar{\chi}^\dagger \bar{\chi} + m_{\bar{\chi}^c}^2 \bar{\chi}^{c\dagger} \bar{\chi}^c \\ & + m_{\Phi ab}^2 \mathrm{Tr}(\Phi_a^\dagger \Phi_b) + (m_{s^2}^{\alpha\beta}) S^{\alpha^*} S^\beta], \end{aligned} \tag{2.2}$$

where we have suppressed the generational and $SU(2)$ indices. If these were to be included, the term $m_Q^2 \tilde{Q}^T \tilde{Q}^*$ would be written as $(m_Q^2)_i^j \tilde{Q}_{j\alpha} \tilde{Q}^i_\alpha$, where the lower case English letters run over generations and the Greek letters run over $SU(2)_L$ index in this case. W_{NR} denotes nonrenormalizable terms arising from higher scale physics and would include $f(L^T \tau_2 \chi) \times (L^T \tau_2 \chi) + f^*(L^{cT} \tau_2 \chi^c)(L^{cT} \chi^c \tau_2 \chi^c)/M_{pl}$ the term that gives rise to the seasaw mechanism (M_{pl} refers to the higher scale physics, e.g. the planck scale). Since coefficients of this form are suppressed by the scale of higher physics, their contributions to the renormalization group equations may be ignored and will not be included below.

By demanding parity invariance from this theory, we also find the following relations among the parameters [18,19,21]: Yukawa couplings are Hermitian except for μ^α_χ and $\mu^\alpha_{\chi^c}$, trilinear couplings are Hermitian except for A^α_χ and $A^\alpha_{\chi^c}$, soft breaking mass terms for an $SU(2)_L$ doublet are equal to those of the corresponding $SU(2)_R$ doublet, and

$$\begin{aligned} &\mu^\alpha_\chi = \mu^{\alpha *}_{\chi^c}, \qquad A^\alpha_\chi = A^{\alpha *}_{\chi^c}, \qquad M_\chi = M^*_{\chi^c}, \\ &M_{\Phi ab} = M^*_{\Phi ab}, \qquad M_S^{\alpha\beta} = M_S^{\alpha\beta *}, \qquad L^\alpha = L^{\alpha *}, \\ &g_L = g_R, \qquad M_1 = M_1^*, \qquad M_L = M_R^*, \qquad M_3 = M_3^*, \\ &B_\chi = B^*_{\chi^c}, \qquad B_{\Phi ab} = B^*_{\Phi ab}, \qquad B_S^{\alpha\beta} = B_S^{\alpha\beta *}, \end{aligned}$$

where g_L and g_R are the $SU(2)_L$ and $SU(2)_R$ gauge coupling constants, respectively.

C. RGEs

In this section we present our results: one-loop renormalization group equations for this model. The equations are broken up into subsections corresponding to their coupling type.

1. Gauge Couplings

$$16\pi^2 \frac{d}{dt} g_1 = 9g_1^3, \qquad 16\pi^2 \frac{d}{dt} g_L = (1+n_\Phi)g_L^3,$$
$$16\pi^2 \frac{d}{dt} g_R = (1+n_\Phi)g_R^3, \qquad 16\pi^2 \frac{d}{dt} g_3 = -3g_3^3 \tag{2.3}$$

2. Yukawa Couplings

$$16\pi^2 \frac{d}{dt} h_a = h_a\left[2h_b^\dagger h_b - \frac{1}{6}g_1^2 - 3g_L^2 - 3g_R^2 - \frac{16}{3}g_3^2\right] + h_b[\mathrm{Tr}(3h_b^\dagger h_a + h_b'^\dagger h_a') + 2h_b^\dagger h_a + \lambda_b^*\lambda_a + \bar{\lambda}_b^*\bar{\lambda}_a + 4(\mu_\Phi^{\alpha\dagger}\mu_\Phi^\alpha)_{ba}] \tag{2.4}$$

$$16\pi^2 \frac{d}{dt} h_a' = h_a'\left[2h_b'^\dagger h_b' - \frac{3}{2}g_1^2 - 3g_L^2 - 3g_R^2\right] + h_b'[\mathrm{Tr}(3h_b^\dagger h_a + h_b'^\dagger h_a') + 2h_b'^\dagger h_a' + \lambda_b^*\lambda_a + \bar{\lambda}_b^*\bar{\lambda}_a + 4(\mu_\Phi^{\alpha\dagger}\mu_\Phi^\alpha)_{ba}] \tag{2.5}$$

$$16\pi^2 \frac{d}{dt} \lambda_a = \lambda_a\left[\mu_\chi^{\alpha*}\mu_\chi^\alpha + \mu_{\chi^c}^{\alpha*}\mu_{\chi^c}^\alpha + 4\lambda_b^*\lambda_b - \frac{3}{2}g_1^2 - 3g_L^2 - 3g_R^2\right] + \lambda_b[\mathrm{Tr}(3h_b^\dagger h_a + h_b'^\dagger h_a') + \lambda_b^*\lambda_a + \bar{\lambda}_b^*\bar{\lambda}_a + 4(\mu_\Phi^{\alpha\dagger}\mu_\Phi^\alpha)_{ba}] \tag{2.6}$$

$$16\pi^2 \frac{d}{dt} \bar{\lambda}_a = \bar{\lambda}_a\left[\mu_\chi^{\alpha*}\mu_\chi^\alpha + \mu_{\chi^c}^{\alpha*}\mu_{\chi^c}^\alpha + 4\bar{\lambda}_b^*\bar{\lambda}_b - \frac{3}{2}g_1^2 - 3g_L^2 - 3g_R^2\right] + \bar{\lambda}_b[\mathrm{Tr}(3h_b^\dagger h_a + h_b'^\dagger h_a') + \lambda_b^*\lambda_a + \bar{\lambda}_b^*\bar{\lambda}_a + 4(\mu_\Phi^{\alpha\dagger}\mu_\Phi^\alpha)_{ba}] \tag{2.7}$$

$$16\pi^2 \frac{d}{dt} \mu_\chi^\alpha = \mu_\chi^\alpha\left[2\lambda_a^*\lambda_a + 2\bar{\lambda}_a^*\bar{\lambda}_a + 2\mu_\chi^{\beta*}\mu_\chi^\beta - \frac{3}{2}g_1^2 - 3g_L^2\right] + \mu_\chi^\beta\left[2\mu_\chi^{\beta*}\mu_\chi^\alpha + 2\mu_{\chi^c}^{\beta*}\mu_{\chi^c}^\alpha + \frac{1}{2}(Y^{\beta\gamma\delta})^* Y^{\alpha\gamma\delta} + 8\mathrm{Tr}(\mu_\Phi^{\beta\dagger}\mu_\Phi^\alpha)\right] \tag{2.8}$$

$$16\pi^2 \frac{d}{dt} \mu_{\chi^c}^\alpha = \mu_{\chi^c}^\alpha\left[2\lambda_a^*\lambda_a + 2\bar{\lambda}_a^*\bar{\lambda}_a + 2\mu_{\chi^c}^{\beta*}\mu_{\chi^c}^\beta - \frac{3}{2}g_1^2 - 3g_R^2\right] + \mu_{\chi^c}^\beta\left[2\mu_\chi^{\beta*}\mu_\chi^\alpha + 2\mu_{\chi^c}^{\beta*}\mu_{\chi^c}^\alpha + \frac{1}{2}(Y^{\beta\mu\nu})^* Y^{\alpha\mu\nu} + 8\mathrm{Tr}(\mu_\Phi^{\beta\dagger}\mu_\Phi^\alpha)\right] \tag{2.9}$$

$$\begin{aligned}16\pi^2 \frac{d}{dt} Y^{\alpha\beta\gamma} = {}& Y^{\alpha\beta\rho}\left[2\mu_\chi^{\rho*}\mu_\chi^\gamma + 2\mu_{\chi^c}^{\rho*}\mu_{\chi^c}^\gamma + \frac{1}{2}(Y^{\rho\mu\nu})^* Y^{\gamma\mu\nu} + 8\mathrm{Tr}(\mu_\Phi^{\rho\dagger}\mu_\Phi^\gamma)\right] \\ &+ Y^{\gamma\beta\rho}\left[2\mu_\chi^{\rho*}\mu_\chi^\alpha + 2\mu_{\chi^c}^{\rho*}\mu_{\chi^c}^\alpha + \frac{1}{2}(Y^{\rho\mu\nu})^* Y^{\alpha\mu\nu} + 8\mathrm{Tr}(\mu_\Phi^{\rho\dagger}\mu_\Phi^\alpha)\right] \\ &+ Y^{\gamma\alpha\rho}\left[2\mu_\chi^{\rho*}\mu_\chi^\beta + 2\mu_{\chi^c}^{\rho*}\mu_{\chi^c}^\beta + \frac{1}{2}(Y^{\rho\mu\nu})^* Y^{\beta\mu\nu} + 8\mathrm{Tr}(\mu_\Phi^{\rho\dagger}\mu_\Phi^\beta)\right]\end{aligned} \tag{2.10}$$

$$\begin{aligned}16\pi^2 \frac{d}{dt} \mu_{\Phi ab}^\alpha = {}& \mu_{\Phi ac}^\alpha[\mathrm{Tr}(3h_c^\dagger h_b + h_c'^\dagger h_b') + \lambda_c^*\lambda_b + \bar{\lambda}_c^*\bar{\lambda}_b + 4(\mu_\Phi^{\beta\dagger}\mu_\Phi^\beta)_{cb}] \\ &+ [\mathrm{Tr}(3h_a h_c^\dagger + h_a' h_c'^\dagger) + \lambda_a\lambda_c^* + \bar{\lambda}_a\bar{\lambda}_c^* + 4(\mu_\Phi^\beta\mu_\Phi^{\beta\dagger})_{ac}]\mu_{\Phi cb}^\alpha \\ &+ \mu_{\Phi ab}^\beta\left[2\mu_\chi^{\beta*}\mu_\chi^\alpha + 2\mu_{\chi^c}^{\beta*}\mu_{\chi^c}^\alpha + \frac{1}{2}(Y^{\beta\mu\nu})^* Y^{\alpha\mu\nu} + 8\mathrm{Tr}(\mu_\Phi^{\beta\dagger}\mu_\Phi^\alpha) - \delta^{\alpha\beta}(3g_L^2 + 3g_R^2)\right]\end{aligned} \tag{2.11}$$

3. Mass Couplings

$$16\pi^2 \frac{d}{dt} M_\chi = M_\chi\left[2\lambda_a^*\lambda_a + 2\bar{\lambda}_a^*\bar{\lambda}_a + 2\mu_\chi^{\alpha*}\mu_\chi^\alpha - \frac{3}{2}g_1^2 - 3g_L^2\right] \tag{2.12}$$

$$16\pi^2 \frac{d}{dt} M_{\chi^c} = M_{\chi^c}\left[2\lambda_a^*\lambda_a + 2\bar{\lambda}_a^*\bar{\lambda}_a + 2\mu_{\chi^c}^{\alpha*}\mu_{\chi^c}^\alpha - \frac{3}{2}g_1^2 - 3g_R^2\right] \tag{2.13}$$

$$16\pi^2 \frac{d}{dt} M_{\Phi ab} = M_{\Phi ac}[\mathrm{Tr}(3h_c^\dagger h_b + h_c'^\dagger h_b') + \lambda_c^* \lambda_b + \bar{\lambda}_c^* \bar{\lambda}_b + 4(\mu_\Phi^{\alpha\dagger} \mu_\Phi^\alpha)_{cb}] + [\mathrm{Tr}(3h_a h_c^\dagger + h_a' h_c'^\dagger) + \lambda_a \lambda_c^* + \bar{\lambda}_a \bar{\lambda}_c^* + 4(\mu_\Phi^\alpha \mu_\Phi^{\alpha\dagger})_{ac}] M_{\Phi cb} + M_{\Phi ab}(-3g_L^2 - 3g_R^2) \tag{2.14}$$

$$16\pi^2 \frac{d}{dt} M_s^{\alpha\beta} = M_s^{\alpha\rho}\left[2\mu_{\chi^c}^{\rho *}\mu_{\chi^c}^{\beta} + 2\mu_{\chi}^{\rho *}\mu_{\chi}^{\beta} + \frac{1}{2}(Y^{\rho\mu\nu})^* Y^{\beta\mu\nu} + 8\mathrm{Tr}(\mu_\Phi^{\rho\dagger}\mu_\Phi^{\beta})\right] + M_s^{\beta\rho}\left[2\mu_{\chi^c}^{\rho *}\mu_{\chi^c}^{\alpha} + 2\mu_{\chi}^{\rho *}\mu_{\chi}^{\alpha} + \frac{1}{2}(Y^{\rho\mu\nu})^* Y^{\alpha\mu\nu} + 8\mathrm{Tr}(\mu_\Phi^{\rho\dagger}\mu_\Phi^{\alpha})\right] \tag{2.15}$$

4. Linear Term

$$16\pi^2 \frac{d}{dt} L^\alpha = L^\beta\left[2\mu_{\chi^c}^{\beta *}\mu_{\chi^c}^{\alpha} + 2\mu_{\chi}^{\beta *}\mu_{\chi}^{\alpha} + \frac{1}{2}(Y^{\beta\mu\nu})^* Y^{\alpha\mu\nu} + 8\mathrm{Tr}(\mu_\Phi^{\beta\dagger}\mu_\Phi^{\alpha})\right] \tag{2.16}$$

5. Gaugino Masses

$$16\pi^2 \frac{d}{dt} M_1 = 18 M_1 g_1^2, \qquad 16\pi^2 \frac{d}{dt} g_L = 2(1+n_\Phi) M_L g_L^2, \qquad 16\pi^2 \frac{d}{dt} g_R = 2(1+n_\Phi) M_R g_R^2, \qquad 16\pi^2 \frac{d}{dt} g_3 = -6 M_3 g_3^2 \tag{2.17}$$

6. Soft Breaking Trilinear *A*'s

$$16\pi^2 \frac{d}{dt} A_{Qa} = A_{Qa}\left[2h_b^\dagger h_b - \frac{1}{6}g_1^2 - 3g_L^2 - 3g_R^2 - \frac{16}{3}g_3^2\right] + h_a\left[4h_b^\dagger A_{Qb} + \frac{1}{3}M_1 g_1^2 + 6M_L g_L^2 + 6M_R g_R^2 + \frac{32}{3}M_3 g_3^2\right] + A_{Qb}[4h_b^\dagger h_a + \mathrm{Tr}(3h_b^\dagger h_a + h_b'^\dagger h_a') + \lambda_b^*\lambda_a + \bar{\lambda}_b^*\bar{\lambda}_a + 4(\mu_\Phi^{\alpha\dagger}\mu_\Phi^\alpha)_{ba}] + h_b[2h_b^\dagger A_{Qa} + \mathrm{Tr}(6h_b^\dagger A_{Qa} + 2h_b'^\dagger A_{La}) + 2\lambda_b^* A_{\lambda a} + 2\bar{\lambda}_b^* A_{\bar{\lambda} a} + 8(\mu_\Phi^{\alpha\dagger} A_\Phi^\alpha)_{ba}] \tag{2.18}$$

$$16\pi^2 \frac{d}{dt} A_{La} = A_{La}\left[2h_b'^\dagger h_b' - \frac{3}{2}g_1^2 - 3g_L^2 - 3g_R^2\right] + h_a'[4h_b'^\dagger A_{Lb} + 3M_1 g_1^2 + 6M_L g_L^2 + 6M_R g_R^2] + A_{Lb}[4h_b'^\dagger h_a' + \mathrm{Tr}(3h_b^\dagger h_a + h_b'^\dagger h_a') + \lambda_b^*\lambda_a + \bar{\lambda}_b^*\bar{\lambda}_a + 4(\mu_\Phi^{\alpha\dagger}\mu_\Phi^\alpha)_{ba}] + h_b'[2h_b'^\dagger A_{La} + \mathrm{Tr}(6h_b^\dagger A_{Qa} + 2h_b'^\dagger A_{La}) + 2\lambda_b^* A_{\lambda a} + 2\bar{\lambda}_b^* A_{\bar{\lambda} a} + 8(\mu_\Phi^{\alpha\dagger} A_\Phi^\alpha)_{ba}] \tag{2.19}$$

$$16\pi^2 \frac{d}{dt} A_{\lambda a} = A_{\lambda a}\left[4\lambda_b^*\lambda_b + \mu_\chi^{\alpha *}\mu_\chi^\alpha + \mu_{\chi^c}^{\alpha *}\mu_{\chi^c}^\alpha - \frac{3}{2}g_1^2 - 3g_L^2 - 3g_R^2\right] + \lambda_a[8\lambda_b^* A_{\lambda b} + 2\mu_\chi^{\alpha *}A_\chi^\alpha + 2\mu_{\chi^c}^{\alpha *}A_{\chi^c}^\alpha + 3M_1 g_1^2 + 6M_L g_L^2 + 6M_R g_R^2] + A_{\lambda b}[\mathrm{Tr}(3h_b^\dagger h_a + h_b'^\dagger h_a') + \lambda_b^*\lambda_a + \bar{\lambda}_b^*\bar{\lambda}_a + 4(\mu_\Phi^{\alpha\dagger}\mu_\Phi^\alpha)_{ba}] + \lambda_b[\mathrm{Tr}(6h_b^\dagger A_{Qa} + 2h_b'^\dagger A_{La}) + 2\lambda_b^* A_{\lambda a} + 2\bar{\lambda}_b^* A_{\bar{\lambda} a} + 8(\mu_\Phi^{\alpha\dagger} A_\Phi^\alpha)_{ba}] \tag{2.20}$$

$$16\pi^2 \frac{d}{dt} A_{\bar{\lambda} a} = A_{\bar{\lambda} a}\left[4\bar{\lambda}_b^*\bar{\lambda}_b + \mu_\chi^{\alpha *}\mu_\chi^\alpha + \mu_{\chi^c}^{\alpha *}\mu_{\chi^c}^\alpha - \frac{3}{2}g_1^2 - 3g_L^2 - 3g_R^2\right] + \bar{\lambda}_a[8\bar{\lambda}_b^* A_{\bar{\lambda} b} + 2\mu_\chi^{\alpha *}A_\chi^\alpha + 2\mu_{\chi^c}^{\alpha *}A_{\chi^c}^\alpha + 3M_1 g_1^2 + 6M_L g_L^2 + 6M_R g_R^2] + A_{\bar{\lambda} b}[\mathrm{Tr}(3h_b^\dagger h_a + h_b'^\dagger h_a') + \lambda_b^*\lambda_a + \bar{\lambda}_b^*\bar{\lambda}_a + 4(\mu_\Phi^{\alpha\dagger}\mu_\Phi^\alpha)_{ba}] + \bar{\lambda}_b[\mathrm{Tr}(6h_b^\dagger A_{Qa} + 2h_b'^\dagger A_{La}) + 2\lambda_b^* A_{\lambda a} + 2\bar{\lambda}_b^* A_{\bar{\lambda} a} + 8(\mu_\Phi^{\alpha\dagger} A_\Phi^\alpha)_{ba}] \tag{2.21}$$

$$16\pi^2\frac{d}{dt}A^{\alpha}_{\chi} = A^{\alpha}_{\chi}\Big[2\lambda^*_a\lambda_a + 2\bar{\lambda}^*_a\bar{\lambda}_a + 2\mu^{\beta*}_{\chi}\mu^{\beta}_{\chi} - \frac{3}{2}g_1^2 - 3g_L^2\Big] + \mu^{\alpha}_{\chi}[4\lambda^*_aA_{\lambda a} + 4\bar{\lambda}^*_aA_{\bar{\lambda}a} + 4\mu^{\beta*}_{\chi}A^{\beta}_{\chi} + 3M_1g_1^2 + 6M_Lg_L^2] + A^{\beta}_{\chi^c}\Big[2\mu^{\beta*}_{\chi}\mu^{\alpha}_{\chi} + 2\mu^{\beta*}_{\chi^c}\mu^{\alpha}_{\chi^c} + \frac{1}{2}(Y^{\beta\mu\nu})^*Y^{\alpha\mu\nu} + 8\mathrm{Tr}(\mu^{\beta\dagger}_{\Phi}\mu^{\alpha}_{\Phi})\Big] + \mu^{\beta}_{\chi}[4\mu^{\beta*}_{\chi}A^{\alpha}_{\chi} + 4\mu^{\beta*}_{\chi^c}A^{\alpha}_{\chi^c} + (Y^{\beta\mu\nu})^*A^{\alpha\mu\nu}_S + 16\mathrm{Tr}(\mu^{\beta\dagger}_{\Phi}A^{\alpha}_{\Phi})] \tag{2.22}$$

$$16\pi^2\frac{d}{dt}A^{\alpha}_{\chi^c} = A^{\alpha}_{\chi^c}\Big[2\lambda^*_a\lambda_a + 2\bar{\lambda}^*_a\bar{\lambda}_a + 2\mu^{\beta*}_{\chi^c}\mu^{\beta}_{\chi^c} - \frac{3}{2}g_1^2 - 3g_R^2\Big] + \mu^{\alpha}_{\chi^c}[4\lambda^*_aA_{\lambda a} + 4\bar{\lambda}^*_aA_{\bar{\lambda}a} + 4\mu^{\beta*}_{\chi^c}A^{\beta}_{\chi^c} + 3M_1g_1^2 + 6M_Rg_R^2] + A^{\beta}_{\chi^c}\Big[2\mu^{\beta*}_{\chi}\mu^{\alpha}_{\chi} + 2\mu^{\beta*}_{\chi^c}\mu^{\alpha}_{\chi^c} + \frac{1}{2}(Y^{\beta\mu\nu})^*Y^{\alpha\mu\nu} + 8\mathrm{Tr}(\mu^{\beta\dagger}_{\Phi}\mu^{\alpha}_{\Phi})\Big] + \mu^{\beta}_{\chi^c}[4\mu^{\beta*}_{\chi}A^{\alpha}_{\chi} + 4\mu^{\beta*}_{\chi^c}A^{\alpha}_{\chi^c} + (Y^{\beta\mu\nu})^*A^{\alpha\mu\nu}_S + 16\mathrm{Tr}(\mu^{\beta\dagger}_{\Phi}A^{\alpha}_{\Phi})] \tag{2.23}$$

$$16\pi^2\frac{d}{dt}A^{\alpha\beta\gamma}_S = A^{\alpha\beta\rho}_S\Big[2\mu^{\rho*}_{\chi}\mu^{\gamma}_{\chi} + 2\mu^{\rho*}_{\chi^c}\mu^{\gamma}_{\chi^c} + \frac{1}{2}(Y^{\rho\mu\nu})^*Y^{\gamma\mu\nu} + 8\mathrm{Tr}(\mu^{\rho\dagger}_{\Phi}\mu^{\gamma}_{\Phi})\Big] + Y^{\alpha\beta\rho}[4\mu^{\rho*}_{\chi}A^{\gamma}_{\chi} + 4\mu^{\rho*}_{\chi^c}A^{\gamma}_{\chi^c} + (Y^{\rho\mu\nu})^*A^{\gamma\mu\nu}_S + 16\mathrm{Tr}(\mu^{\rho\dagger}_{\Phi}A^{\gamma}_{\Phi})] + A^{\gamma\beta\rho}_S\Big[2\mu^{\rho*}_{\chi}\mu^{\alpha}_{\chi} + 2\mu^{\rho*}_{\chi^c}\mu^{\alpha}_{\chi^c} + \frac{1}{2}(Y^{\rho\mu\nu})^*Y^{\alpha\mu\nu} + 8\mathrm{Tr}(\mu^{\rho\dagger}_{\Phi}\mu^{\alpha}_{\Phi})\Big] + Y^{\gamma\beta\rho}[4\mu^{\rho*}_{\chi}A^{\alpha}_{\chi} + 4\mu^{\rho*}_{\chi^c}A^{\alpha}_{\chi^c} + (Y^{\rho\mu\nu})^*A^{\alpha\mu\nu}_S + 16\mathrm{Tr}(\mu^{\rho\dagger}_{\Phi}A^{\alpha}_{\Phi})] + A^{\gamma\alpha\rho}_S\Big[2\mu^{\rho*}_{\chi}\mu^{\beta}_{\chi} + 2\mu^{\rho*}_{\chi^c}\mu^{\beta}_{\chi^c} + \frac{1}{2}(Y^{\rho\mu\nu})^*Y^{\beta\mu\nu} + 8\mathrm{Tr}(\mu^{\rho\dagger}_{\Phi}\mu^{\beta}_{\Phi})\Big] + Y^{\gamma\alpha\rho}[4\mu^{\rho*}_{\chi}A^{\beta}_{\chi} + 4\mu^{\rho*}_{\chi^c}A^{\beta}_{\chi^c} + (Y^{\rho\mu\nu})^*A^{\beta\mu\nu}_S + 16\mathrm{Tr}(\mu^{\rho\dagger}_{\Phi}A^{\beta}_{\Phi})] \tag{2.24}$$

$$16\pi^2\frac{d}{dt}A^{\alpha}_{\Phi ab} = A^{\alpha}_{\Phi ac}[\mathrm{Tr}(3h^\dagger_ch_b + h'^\dagger_ch'_b) + \lambda^*_c\lambda_b + \bar{\lambda}^*_c\bar{\lambda}_b + 4(\mu^{\beta\dagger}_{\Phi}\mu^{\beta}_{\Phi})_{cb}] + \mu^{\alpha}_{\Phi ac}[\mathrm{Tr}(6h^\dagger_cA_{Qb} + 2h'^\dagger_cA_{Lb}) + 2\lambda^*_cA_{\lambda b} + 2\bar{\lambda}^*_cA_{\bar{\lambda}b} + 8(\mu^{\beta\dagger}_{\Phi}A^{\beta}_{\Phi})_{cb}] + [\mathrm{Tr}(3h_ah^\dagger_c + h'_ah'^\dagger_c) + \lambda_a\lambda^*_c + \bar{\lambda}_a\bar{\lambda}^*_c + 4(\mu^{\beta}_{\Phi}\mu^{\beta\dagger}_{\Phi})_{ac}]A^{\alpha}_{\Phi cb} + [\mathrm{Tr}(6A_{Qa}h^\dagger_c + 2A_{La}h'^\dagger_c) + 2A_{\lambda a}\lambda^*_c + 2A_{\bar{\lambda}a}\bar{\lambda}^*_c + 8(A^{\beta}_{\Phi}\mu^{\beta\dagger}_{\Phi})_{ac}]\mu^{\alpha}_{\Phi cb} + A^{\beta}_{\Phi ab}\Big[2\mu^{\beta*}_{\chi}\mu^{\alpha}_{\chi} + 2\mu^{\beta*}_{\chi^c}\mu^{\alpha}_{\chi^c} + \frac{1}{2}(Y^{\beta\mu\nu})^*Y^{\alpha\mu\nu} + 8\mathrm{Tr}(\mu^{\beta\dagger}_{\Phi}\mu^{\alpha}_{\Phi}) - \delta^{\alpha\beta}(3g_L^2 + 3g_R^2)\Big] + \mu^{\beta}_{\Phi ab}[4\mu^{\beta*}_{\chi}A^{\alpha}_{\chi} + 4\mu^{\beta*}_{\chi^c}A^{\alpha}_{\chi^c} + (Y^{\beta\mu\nu})^*A^{\alpha\mu\nu}_S + 16\mathrm{Tr}(\mu^{\beta\dagger}_{\Phi}A^{\alpha}_{\Phi}) + \delta^{\alpha\beta}(6M_Lg_L^2 + 6M_Rg_R^2)] \tag{2.25}$$

7. Soft Breaking Bilinear B's

$$16\pi^2\frac{d}{dt}B_{\chi} = B_{\chi}\Big[2\lambda^*_a\lambda_a + 2\bar{\lambda}^*_a\bar{\lambda}_a + 2\mu^{\alpha*}_{\chi}\mu^{\alpha}_{\chi} - \frac{3}{2}g_1^2 - 3g_L^2\Big] + M_{\chi}[4\lambda^*_aA_{\lambda a} + 4\bar{\lambda}^*_aA_{\bar{\lambda}a} + 4\mu^{\alpha*}_{\chi}A^{\alpha}_{\chi} + 3M_1g_1^2 + 6M_Lg_L^2] + \mu^{\alpha}_{\chi}[4\mu^{\alpha*}_{\chi}B_{\chi} + 4\mu^{\alpha*}_{\chi^c}B_{\chi^c} + (Y^{\alpha\mu\nu})^*B^{\mu\nu}_S + 16\mathrm{Tr}(\mu^{\alpha\dagger}_{\Phi}B_{\Phi})] \tag{2.26}$$

$$16\pi^2\frac{d}{dt}B_{\chi^c} = B_{\chi^c}\Big[2\lambda^*_a\lambda_a + 2\bar{\lambda}^*_a\bar{\lambda}_a + 2\mu^{\alpha*}_{\chi^c}\mu^{\alpha}_{\chi^c} - \frac{3}{2}g_1^2 - 3g_R^2\Big] + M_{\chi^c}[4\lambda^*_aA_{\lambda a} + 4\bar{\lambda}^*_aA_{\bar{\lambda}a} + 4\mu^{\alpha*}_{\chi^c}A^{\alpha}_{\chi^c} + 3M_1g_1^2 + 6M_Rg_R^2] + \mu^{\alpha}_{\chi^c}[4\mu^{\alpha*}_{\chi}B_{\chi} + 4\mu^{\alpha*}_{\chi^c}B_{\chi^c} + (Y^{\alpha\mu\nu})^*B^{\mu\nu}_S + 16\mathrm{Tr}(\mu^{\alpha\dagger}_{\Phi}B_{\Phi})] \tag{2.27}$$

$$16\pi^2 \frac{d}{dt} B_{\Phi ab} = B_{\Phi ac}[\mathrm{Tr}(3h_c^\dagger h_b + h_c'^\dagger h_b') + \lambda_c^* \lambda_b + \bar{\lambda}_c^* \bar{\lambda}_b + 4(\mu_\Phi^{\alpha\dagger} \mu_\Phi^\alpha)_{cb}]$$
$$+ M_{\Phi ac}[\mathrm{Tr}(6h_c^\dagger A_{Qb} + 2h_c'^\dagger A_{Lb}) + 2\lambda_c^* A_{\lambda b} + 2\bar{\lambda}_c^* A_{\bar{\lambda} b} + 8(\mu_\Phi^{\beta\dagger} A_\Phi^\beta)_{cb}]$$
$$+ \mu_{\Phi ab}^\alpha [4\mu_\chi^{\alpha *} B_\chi + 4\mu_{\chi^c}^{\alpha *} B_{\chi^c} + (Y^{\alpha\mu\nu})^* B_S^{\mu\nu} + 16\mathrm{Tr}(\mu_\Phi^{\alpha\dagger} B_\Phi)]$$
$$+ [\mathrm{Tr}(3h_a h_c^\dagger + h_a' h_c'^\dagger) + \lambda_a \lambda_c^* + \bar{\lambda}_a \bar{\lambda}_c^* + 4(\mu_\Phi^\alpha \mu_\Phi^{\alpha\dagger})_{ac}] B_{\Phi cb}$$
$$+ [\mathrm{Tr}(6A_{Qa} h_c^\dagger + 2A_{La} h_c'^\dagger) + 2A_{\lambda a} \lambda_c^* + 2A_{\bar{\lambda} a} \bar{\lambda}_c^* + 8(A_\Phi^\alpha \mu_\Phi^{\alpha\dagger})_{ac}] M_{\Phi cb} - B_{\Phi ab}(3g_L^2 + 3g_R^2)$$
$$+ M_{\Phi ab}(6M_L g_L^2 + 6M_R g_R^2) \tag{2.28}$$

$$16\pi^2 \frac{d}{dt} B_S^{\alpha\beta} = B_S^{\alpha\rho}\left[2\mu_\chi^{\rho *} \mu_\chi^\beta + 2\mu_{\chi^c}^{\rho *} \mu_{\chi^c}^\beta + \frac{1}{2}(Y^{\rho\mu\nu})^* Y^{\beta\mu\nu} + 8\mathrm{Tr}(\mu_\Phi^{\rho\dagger} \mu_\Phi^\beta)\right]$$
$$+ M_S^{\alpha\rho}[4\mu_\chi^{\rho *} A_\chi^\beta + 4\mu_{\chi^c}^{\rho *} A_{\chi^c}^\beta + (Y^{\rho\mu\nu})^* A_S^{\beta\mu\nu} + 16\mathrm{Tr}(\mu_\Phi^{\rho\dagger} A_\Phi^\beta)]$$
$$+ Y^{\alpha\beta\rho}[4\mu_\chi^{\rho *} B_\chi + 4\mu_{\chi^c}^{\rho *} B_{\chi^c} + (Y^{\rho\mu\nu})^* B_S^{\mu\nu} + 16\mathrm{Tr}(\mu_\Phi^{\rho\dagger} B_\Phi)]$$
$$+ \left[2\mu_\chi^\alpha \mu_\chi^{\rho *} + 2\mu_{\chi^c}^\alpha \mu_{\chi^c}^{\rho *} + \frac{1}{2} Y^{\alpha\mu\nu}(Y^{\rho\mu\nu})^* + 8\mathrm{Tr}(\mu_\Phi^\alpha \mu_\Phi^{\rho\dagger})\right] B_S^{\rho\beta}$$
$$+ [4A_\chi^\alpha \mu_\chi^{\rho *} + 4A_{\chi^c}^\alpha \mu_{\chi^c}^{\rho *} + A_S^{\alpha\mu\nu}(Y^{\rho\mu\nu})^* + 16\mathrm{Tr}(A_\Phi^\alpha \mu_\Phi^{\rho\dagger})] M_S^{\rho\beta} \tag{2.29}$$

8. Soft Breaking Masses

For convenience, we define the quantity:

$$S_2 \equiv 4[\mathrm{Tr}(m_Q^2 - m_{Q^c}^2 - m_L^2 + m_{L^c}^2) + m_\chi^2 - m_{\chi^c}^2 + m_{\bar{\chi}^c}^2 - m_{\bar{\chi}}^2] \tag{2.30}$$

which is used in the soft breaking mass equations below.

$$16\pi^2 \frac{d}{dt} m_Q^2 = 2m_Q^2 h_a h_a^\dagger + h_a(2h_a^\dagger m_Q^2 + 4m_{Q^c}^2 h_a^\dagger + 4m_{\Phi ab}^2 h_b^\dagger) + 4A_{Qa} A_{Qa}^\dagger - \frac{1}{3} M_1 M_1^\dagger g_1^2 - 6M_L M_L^\dagger g_L^2$$
$$- \frac{32}{3} M_3 M_3^\dagger g_3^2 + \frac{1}{8} g_1^2 S_2 \tag{2.31}$$

$$16\pi^2 \frac{d}{dt} m_{Q^c}^2 = 2m_{Q^c}^2 h_a^\dagger h_a + h_a^\dagger(2h_a m_{Q^c}^2 + 4m_Q^2 h_a + 4h_b m_{\Phi ba}^2) + 4A_{Qa}^\dagger A_{Qa} - \frac{1}{3} M_1 M_1^\dagger g_1^2 - 6M_R M_R^\dagger g_R^2$$
$$- \frac{32}{3} M_3 M_3^\dagger g_3^2 - \frac{1}{8} g_1^2 S_2 \tag{2.32}$$

$$16\pi^2 \frac{d}{dt} m_L^2 = 2m_L^2 h_a' h_a'^\dagger + h_a'(2h_a'^\dagger m_L^2 + 4m_{L^c}^2 h_a'^\dagger + 4m_{\Phi ab}^2 h_b'^\dagger) + 4A_{La} A_{La}^\dagger - 3M_1 M_1^\dagger g_1^2 - 6M_L M_L^\dagger g_L^2 - \frac{3}{8} g_1^2 S_2 \tag{2.33}$$

$$16\pi^2 \frac{d}{dt} m_{L^c}^2 = 2m_{L^c}^2 h_a'^\dagger h_a' + h_a'^\dagger(2h_a' m_{L^c}^2 + 4m_L^2 h_a' + 4h_b' m_{\Phi ba}^2) + 4A_{La}^\dagger A_{La} - 3M_1 M_1^\dagger g_1^2 - 6M_R M_R^\dagger g_R^2 + \frac{3}{8} g_1^2 S_2 \tag{2.34}$$

$$16\pi^2 \frac{d}{dt} m_\chi^2 = \lambda_a[4m_\chi^2 \lambda_a^* + 4m_{\chi^c}^2 \lambda_a^* + 4m_{\Phi ab}^2 \lambda_b^*] + \mu_\chi^\alpha[2m_\chi^2 \mu_\chi^{\alpha *} + 2m_{\bar{\chi}}^2 \mu_\chi^{\alpha *} + 2(m_S^2)^{\alpha\beta} \mu_\chi^{\beta *}] + 4A_{\lambda a}^* A_{\lambda a}$$
$$+ 2A_\chi^{\alpha *} A_\chi^\alpha - 3M_1 M_1^\dagger g_1^2 - 6M_L M_L^\dagger g_L^2 + \frac{3}{8} g_1^2 S_2 \tag{2.35}$$

$$16\pi^2 \frac{d}{dt} m^2_{\bar{\chi}} = \bar{\lambda}_a[4m^2_{\bar{\chi}}\bar{\lambda}^*_a + 4m^2_{\bar{\chi}^c}\bar{\lambda}^*_a + 4m^2_{\Phi ab}\bar{\lambda}^*_b] + \mu^\alpha_{\bar{\chi}}[2m^2_{\bar{\chi}}\mu^{\alpha*}_{\chi} + 2m^2_{\chi}\mu^{\alpha*}_{\chi} + 2(m^2_S)^{\alpha\beta}\mu^{\beta*}_{\chi}] + 4A^*_{\bar{\lambda}a}A_{\bar{\lambda}a} + 2A^{\alpha*}_{\chi}A^\alpha_{\chi} - 3M_1M_1^\dagger g_1^2 - 6M_LM_L^\dagger g_L^2 - \frac{3}{8}g_1^2S_2 \tag{2.36}$$

$$16\pi^2 \frac{d}{dt} m^2_{\chi^c} = \lambda^*_a[4m^2_{\chi^c}\lambda_a + 4m^2_{\chi}\lambda_a + 4m^2_{\Phi ba}\lambda_b] + \mu^{\alpha*}_{\chi^c}[2m^2_{\chi^c}\mu^\alpha_{\chi^c} + 2m^2_{\bar{\chi}^c}\mu^\alpha_{\chi^c} + 2\mu^\beta_{\chi^c}(m^2_S)^{\beta\alpha}] + 4A^*_{\lambda a}A_{\lambda a} + 2A^{\alpha*}_{\chi^c}A^\alpha_{\chi^c} - 3M_1M_1^\dagger g_1^2 - 6M_RM_R^\dagger g_R^2 - \frac{3}{8}g_1^2S_2 \tag{2.37}$$

$$16\pi^2 \frac{d}{dt} m^2_{\bar{\chi}^c} = \bar{\lambda}^*_a[4m^2_{\bar{\chi}^c}\bar{\lambda}_a + 4m^2_{\bar{\chi}}\bar{\lambda}_a + 4m^2_{\Phi ba}\bar{\lambda}_b] + \mu^{\alpha*}_{\chi^c}[2m^2_{\bar{\chi}^c}\mu^\alpha_{\chi^c} + 2m^2_{\chi^c}\mu^\alpha_{\chi^c} + 2\mu^\beta_{\chi^c}(m^2_S)^{\beta\alpha}] + 4A^*_{\bar{\lambda}a}A_{\bar{\lambda}a} + 2A^{\alpha*}_{\chi^c}A^\alpha_{\chi^c} - 3M_1M_1^\dagger g_1^2 - 6M_RM_R^\dagger g_R^2 + \frac{3}{8}g_1^2S_2 \tag{2.38}$$

$$16\pi^2 \frac{d}{dt}(m^2_S)^{\alpha\beta} = (m^2_S)^{\alpha\rho}\left[2\mu^{\rho*}_{\chi^c}\mu^\beta_{\chi^c} + 2\mu^{\rho*}_{\chi}\mu^\beta_{\chi} + \frac{1}{2}(Y^{\rho\mu\nu})^*Y^{\beta\mu\nu} + 8\mathrm{Tr}(\mu^\rho_\Phi\mu^\beta_\Phi)\right] + \left[2\mu^{\alpha*}_{\chi^c}\mu^\rho_{\chi^c} + 2\mu^{\alpha*}_{\chi}\mu^\rho_{\chi} + \frac{1}{2}(Y^{\alpha\mu\nu})^*Y^{\rho\mu\nu} + 8\mathrm{Tr}(\mu^{\alpha\dagger}_\Phi\mu^\rho_\Phi)\right](m^2_S)^{\rho\beta} + 4\mu^{\alpha*}_{\chi}\mu^\beta_{\chi}(m^2_{\bar{\chi}} + m^2_{\chi}) + 4\mu^{\alpha*}_{\chi^c}\mu^\beta_{\chi^c}(m^2_{\bar{\chi}^c} + m^2_{\chi^c}) + 2(Y^{\alpha\rho\mu})^*Y^{\beta\rho\nu}(m^2_S)^{\nu\mu} + 32\mathrm{Tr}(\mu^\beta_\Phi m^2_\Phi\mu^{\alpha\dagger}_\Phi) + 4A^{\alpha*}_{\chi}A^\beta_{\chi} + 4A^{\alpha*}_{\chi^c}A^\beta_{\chi^c} + (A^{\alpha\mu\nu}_S)^*A^{\beta\mu\nu}_S + 16\mathrm{Tr}(A^{\alpha\dagger}_\Phi A^\beta_\Phi) \tag{2.39}$$

$$16\pi^2 \frac{d}{dt} m^2_{\Phi ab} = m^2_{\Phi ac}[\mathrm{Tr}(3h_c^\dagger h_b + h_c'^\dagger h_b') + \lambda^*_c\lambda_b + \bar{\lambda}^*_c\bar{\lambda}_b + 4(\mu^{\alpha\dagger}_\Phi\mu^\alpha_\Phi)_{cb}] + [\mathrm{Tr}(3h_a^\dagger h_c + h_a'^\dagger h_c') + \lambda^*_a\lambda_c + \bar{\lambda}^*_a\bar{\lambda}_c + 4(\mu^{\alpha\dagger}_\Phi\mu^\alpha_\Phi)_{ac}]m^2_{\Phi cb} + \mathrm{Tr}[6h_a^\dagger h_b m^2_{Q^c} + 6h_a^\dagger m^2_Q h_b + 2h_a'^\dagger h_b' m^2_{L^c} + 2h_a'^\dagger m^2_L h_b' + 6A^\dagger_{Qa}A_{Qb} + 2A^\dagger_{La}A_{Lb}] + 2\lambda^*_a\lambda_b(m^2_{\chi^c} + m^2_{\chi}) + 2\bar{\lambda}^*_a\bar{\lambda}_b(m^2_{\bar{\chi}^c} + m^2_{\bar{\chi}}) + 2A^*_{\lambda a}A_{\lambda b} + 2A^*_{\bar{\lambda}a}A_{\bar{\lambda}b} + [8\mu^{\alpha\dagger}_\Phi(m^2_\Phi)^*\mu^\alpha_\Phi + 8\mu^{\alpha\dagger}_\Phi\mu^\beta_\Phi(m^2_S)^{\beta\alpha} + 8A^{\alpha\dagger}_\Phi A^\alpha_\Phi - 6M_LM_L^\dagger g_L^2 - 6M_RM_R^\dagger g_R^2]_{ab} \tag{2.40}$$

III. CONCERNING TRIPLETS

A. Particle Content and Quantum Numbers

Table II shows the various particles of the triplet version of the SUSYLR model and their representations—except for the $U(1)_{B-L}$ group where the $B-L$ number is given (The $B-L$ number used in the RGEs follows the GUT normalization scheme; the values in the table do not. To get the GUT-normalized value, multiply the number in the table by $\sqrt{3/8}$.)

The Q and the L are the standard quarks and leptons of the MSSM while the Q^c and L^c contain the corresponding right-handed conjugate fields. In order to keep this model general, we allow for an arbitrary number of singlet fields and bidoublet fields. These values are n_S and n_Φ, respectively. Thus, for S^α, we have $\alpha = 1, 2, \ldots, n_S$; for Φ_a, we have $a = 1, 2, \ldots, n_\Phi$ (for further comments on n_Φ see section II A).

For the following work the particles have been chosen to have the form shown below, where the Q and the Q^c fields serve as templates to construct the other $SU(2)$ doublets (note that the color and generations have been suppressed here). The charge is determined by the equation $Q = I_{3L} + I_{3R} + \frac{B-L}{2}$ and the standard I_3 ordering is used (row one has the highest I_3 value, row two the next highest, etc).

TABLE II. This table shows the representations for the non-Abelian gauge groups and the $B-L$ number for $U(1)$. The $B-L$ number as presented needs to be normalized; when using the GUT normalization (as this paper does), this means multiplying it by $\sqrt{3/8}$.

	$SU(3)^c$	×	$SU(2)_L$	×	$SU(2)_R$	×	$U(1)_{B-L}$
Q	3		2		1		$+\frac{1}{3}$
Q^c	3		1		2		$-\frac{1}{3}$
L	1		2		1		-1
L^c	1		1		2		$+1$
Φ_a	1		2		2		0
Δ	1		3		1		$+2$
Δ^c	1		1		3		-2
$\bar{\Delta}$	1		3		1		-2
$\bar{\Delta}^c$	1		1		3		$+2$
S^α	1		1		1		0

$$Q = \begin{pmatrix} u \\ d \end{pmatrix}, \qquad Q^c = \begin{pmatrix} d^c \\ -u^c \end{pmatrix}, \qquad \Delta = \begin{pmatrix} \frac{\delta^+}{\sqrt{2}} & \delta^{++} \\ \delta^0 & -\frac{\delta^+}{\sqrt{2}} \end{pmatrix}, \qquad \Delta^c = \begin{pmatrix} -\frac{\delta^{c-}}{\sqrt{2}} & -\delta^{c0} \\ -\delta^{c--} & \frac{\delta^{c-}}{\sqrt{2}} \end{pmatrix}$$

$$\bar{\Delta} = \begin{pmatrix} \frac{\bar{\delta}^-}{\sqrt{2}} & \bar{\delta}^0 \\ \bar{\delta}^{--} & -\frac{\bar{\delta}^-}{\sqrt{2}} \end{pmatrix}, \qquad \bar{\Delta}^c = \begin{pmatrix} -\frac{\bar{\delta}^{c+}}{\sqrt{2}} & -\bar{\delta}^{c++} \\ -\bar{\delta}^{c0} & \frac{\bar{\delta}^{c+}}{\sqrt{2}} \end{pmatrix}, \qquad \Phi_a = \begin{pmatrix} \phi^0_{da} & \phi^+_{ua} \\ \phi^-_{da} & \phi^0_{ua} \end{pmatrix}$$

These fields transform under $SU(2)$ as

$$Q \to U_L Q, \qquad Q^c \to U_R Q^c, \qquad L \to U_L L, \qquad L^c \to U_R L^c, \qquad \Delta \to U_L \Delta U_L^\dagger, \qquad \Delta^c \to U_R \Delta^c U_R^\dagger,$$
$$\bar{\Delta} \to U_L \bar{\Delta} U_L^\dagger, \qquad \bar{\Delta}^c \to U_R \bar{\Delta}^c U_R^\dagger, \qquad \Phi_a \to U_L \Phi_a U_R^\dagger, \qquad S^\alpha \to S^\alpha$$

and under Parity as

$$Q \to -i\tau_2 Q^{c*}, \qquad Q^c \to i\tau_2 Q^*, \qquad L \to -i\tau_2 L^{c*}, \qquad L^c \to i\tau_2 L^*, \qquad \Delta \to \tau_2 \Delta^{c*}\tau_2, \qquad \Delta^c \to \tau_2 \Delta^* \tau_2,$$
$$\bar{\Delta} \to \tau_2 \bar{\Delta}^{c*}\tau_2, \qquad \bar{\Delta}^c \to \tau_2 \bar{\Delta}^* \tau_2, \qquad \Phi_a \to \Phi_a^\dagger, \qquad S^\alpha \to S^{\alpha *}$$

B. Superpotential and Soft Breaking Lagrangian

With the transformations and representations given above, the most general superpotential and soft breaking terms are

$$\begin{aligned} W = {} & i h_a Q^T \tau_2 \Phi_a Q^c + i h'_a L^T \tau_2 \Phi_a L^c + i f L^T \tau_2 \Delta L + i f_c L^{cT} \tau_2 \Delta^c L^c + M_\Delta \mathrm{Tr}(\Delta \bar{\Delta}) + M_{\Delta^c} \mathrm{Tr}(\Delta^c \bar{\Delta}^c) \\ & + M_{\Phi ab} \mathrm{Tr}(\Phi_a^T \tau_2 \Phi_b \tau_2) + \mu^\alpha_\Delta S^\alpha \mathrm{Tr}(\Delta\bar{\Delta}) + \mu^\alpha_{\Delta^c} S^\alpha \mathrm{Tr}(\Delta^c \bar{\Delta}^c) + \mu^\alpha_{\Phi ab} S^\alpha \mathrm{Tr}(\Phi_a^T \tau_2 \Phi_b \tau_2) + \frac{1}{6} Y^{\alpha\beta\gamma} S^\alpha S^\beta S^\gamma \\ & + \frac{1}{2} M_S^{\alpha\beta} S^\alpha S^\beta + L^\alpha S^\alpha \end{aligned} \tag{3.1}$$

$$\begin{aligned} -\mathcal{L}_{SB} = {} & \frac{1}{2}(M_3 \tilde{g}\,\tilde{g} + M_L \tilde{W}_L\,\tilde{W}_L + M_R \tilde{W}_R\,\tilde{W}_R + M_1 \tilde{B}\,\tilde{B} + \mathrm{h.c.}) \\ & + \Big[i A_{Qa} \tilde{Q}^T \tau_2 \Phi_a \tilde{Q}^c + i A_{La} \tilde{L}^T \tau_2 \Phi_a \tilde{L}^c + i A_f \tilde{L}^T \tau_2 \Delta \tilde{L} + i A_{f^c} \tilde{L}^{cT} \tau_2 \Delta^c \tilde{L}^c + A^\alpha_\Delta S^\alpha \mathrm{Tr}(\Delta\bar{\Delta}) + A^\alpha_{\Delta^c} S^\alpha \mathrm{Tr}(\Delta^c \bar{\Delta}^c) \\ & + A^\alpha_{\Phi ab} S^\alpha \mathrm{Tr}(\Phi_a^T \tau_2 \Phi_b \tau_2) + \frac{1}{6} A_S^{\alpha\beta\gamma} S^\alpha S^\beta S^\gamma + \mathrm{h.c.} \Big] \\ & + \Big[B_\Delta \mathrm{Tr}(\Delta\bar{\Delta}) + B_{\Delta^c} \mathrm{Tr}(\Delta^c \bar{\Delta}^c) + B_{\Phi ab} \mathrm{Tr}(\Phi_a^T \tau_2 \Phi_b \tau_2) + \frac{1}{2} B_S^{\alpha\beta} S^\alpha S^\beta + \mathrm{h.c.} \Big] \\ & + [m_Q^2 \tilde{Q}^T \tilde{Q}^* + m_{Q^c}^2 \tilde{Q}^{c\dagger} \tilde{Q}^c + m_L^2 \tilde{L}^T \tilde{L}^* + m_{L^c}^2 \tilde{L}^{c\dagger} \tilde{L}^c + m_\Delta^2 \mathrm{Tr}(\Delta^\dagger \Delta) + m_{\bar{\Delta}}^2 \mathrm{Tr}(\bar{\Delta}^\dagger \bar{\Delta}) + m_{\Delta^c}^2 \mathrm{Tr}(\Delta^{c\dagger}\Delta^c) \\ & + m_{\bar{\Delta}^c}^2 \mathrm{Tr}(\bar{\Delta}^{c\dagger} \bar{\Delta}^c) + m_{\Phi ab}^2 \mathrm{Tr}(\Phi_a^\dagger \Phi_b) + (m_S^2)^{\alpha\beta} (S^\alpha)^* S^\beta] \end{aligned} \tag{3.2}$$

where the generational and color indices have been suppressed and the transposes and τ_2's belong to $SU(2)$. Thus the first term in W is actually

$$i(h_a)^j_k (\, u_{jA} \quad d_{jA}\,) \tau_2 \Phi_a \begin{pmatrix} d^{ck}_A \\ -u^{ck}_A \end{pmatrix}$$

with the lowercase Latin indices specifying the generation and the uppercase Latin indices specifying color.

By demanding parity invariance from this theory, we also find the following relations among the parameters [18,19,21]: soft breaking mass terms for an $SU(2)_L$ doublet are equal to those of the corresponding $SU(2)_R$ doublet and

$$h_a = h_a^\dagger, \qquad h'_a = h'^\dagger_a, \qquad f = f_c^*, \qquad \mu^\alpha_\Delta = \mu^{\alpha *}_{\Delta^c},$$
$$\mu_{\Phi ab} = \mu^*_{\Phi ab}, \qquad M_\Delta = M^*_{\Delta^c}, \qquad M_{\Phi ab} = M^*_{\Phi ab},$$
$$M_S^{\alpha\beta} = M_S^{\alpha\beta *}, \qquad L^\alpha = L^{\alpha *}, \qquad g_L = g_R,$$
$$M_1 = M_1^*, \qquad M_L = M_R^*, \qquad M_3 = M_3^*,$$
$$B_\Delta = B^*_{\Delta^c}, \qquad B_{\Phi ab} = B^*_{\Phi ab}, \qquad B_S^{\alpha\beta} = B_S^{\alpha\beta *}$$

where g_L and g_R are the $SU(2)_L$ and $SU(2)_R$ coupling constants, respectively.

C. RGEs

The renormalization group equations to one-loop order for all the parameters of the above theory are presented below and are categorized by the type of coupling

1. Gauge Couplings

$$16\pi^2 \frac{d}{dt} g_1 = 24 g_1^3, \qquad 16\pi^2 \frac{d}{dt} g_L = (4 + n_\Phi) g_L^3,$$
$$16\pi^2 \frac{d}{dt} g_R = (4 + n_\Phi) g_R^3, \qquad 16\pi^2 \frac{d}{dt} g_3 = -3 g_3^3 \tag{3.3}$$

2. Yukawa Couplings

$$16\pi^2 \frac{d}{dt} h_a = h_a\left[2h_b^\dagger h_b - \frac{1}{6}g_1^2 - 3g_R^2 - 3g_L^3 - \frac{16}{3}g_3^2\right] + h_b[\mathrm{Tr}(3h_b^\dagger h_a + h_b'^\dagger h_a') + 2h_b^\dagger h_a + 4(\mu_\Phi^{\alpha\dagger}\mu_\Phi^\alpha)_{ba}] \tag{3.4}$$

$$16\pi^2 \frac{d}{dt} h_a' = h_a'\left[6f_c^\dagger f_c + 2h_b'^\dagger h_b' - \frac{3}{2}g_1^2 - 3g_R^2 - 3g_L^2\right] + 6ff^\dagger h_a' + h_b'[2h_b'^\dagger h_a' + \mathrm{Tr}(3h_b^\dagger h_a + h_b'^\dagger h_a') + 4(\mu_\Phi^{\alpha\dagger}\mu_\Phi^\alpha)_{ba}] \tag{3.5}$$

$$16\pi^2 \frac{d}{dt} f = f\left[6f^\dagger f + 2h_a'^* h_a'^T + 2\mathrm{Tr}(f^\dagger f) + \mu_\Delta^{\alpha *}\mu_\Delta^\alpha - \frac{9}{2}g_1^2 - 7g_L^2\right] + [6ff^\dagger + 2h_a' h_a'^\dagger]f \tag{3.6}$$

$$16\pi^2 \frac{d}{dt} f_c = f_c\left[6f_c^\dagger f_c + 2h_a'^\dagger h_a' + 2\mathrm{Tr}(f_c^\dagger f_c) + \mu_{\Delta^c}^{\alpha *}\mu_{\Delta^c}^\alpha - \frac{9}{2}g_1^2 - 7g_R^2\right] + [6f_c f_c^\dagger + 2h_a'^T h_a'^*]f_c \tag{3.7}$$

$$16\pi^2 \frac{d}{dt} \mu_\Delta^\alpha = \mu_\Delta^\alpha[2\mathrm{Tr}(f^\dagger f) + 2\mu_\Delta^{\beta *}\mu_\Delta^\beta - 6g_1^2 - 8g_L^2] + \mu_\Delta^\beta\left[3\mu_\Delta^{\beta *}\mu_\Delta^\alpha + 3\mu_{\Delta^c}^{\beta *}\mu_{\Delta^c}^\alpha + 8\mathrm{Tr}(\mu_\Phi^{\beta\dagger}\mu_\Phi^\alpha) + \frac{1}{2}(Y^{\beta\mu\nu})^* Y^{\alpha\mu\nu}\right] \tag{3.8}$$

$$16\pi^2 \frac{d}{dt} \mu_{\Delta^c}^\alpha = \mu_{\Delta^c}^\alpha[2\mathrm{Tr}(f_c^\dagger f_c) + 2\mu_{\Delta^c}^{\beta *}\mu_{\Delta^c}^\beta - 6g_1^2 - 8g_R^2] + \mu_{\Delta^c}^\beta\left[3\mu_\Delta^{\beta *}\mu_\Delta^\alpha + 3\mu_{\Delta^c}^{\beta *}\mu_{\Delta^c}^\alpha + 8\mathrm{Tr}(\mu_\Phi^{\beta\dagger}\mu_\Phi^\alpha) + \frac{1}{2}(Y^{\beta\mu\nu})^* Y^{\alpha\mu\nu}\right] \tag{3.9}$$

$$16\pi^2 \frac{d}{dt} \mu_{\Phi ab}^\alpha = \mu_{\Phi ac}^\alpha[\mathrm{Tr}(3h_c^\dagger h_b + h_c'^\dagger h_b') + 4(\mu_\Phi^{\beta\dagger}\mu_\Phi^\beta)_{cb}] + [\mathrm{Tr}(3h_a h_c^\dagger + h_a' h_c'^\dagger) + 4(\mu_\Phi^\beta \mu_\Phi^{\beta\dagger})_{ac}]\mu_{\Phi cb}^\alpha + \mu_{\Phi ab}^\beta\left[3\mu_\Delta^{\beta *}\mu_\Delta^\alpha + 3\mu_{\Delta^c}^{\beta *}\mu_{\Delta^c}^\alpha + 8\mathrm{Tr}(\mu_\Phi^{\beta\dagger}\mu_\Phi^\alpha) + \frac{1}{2}(Y^{\beta\mu\nu})^* Y^{\alpha\mu\nu} - \delta^{\beta\alpha}(3g_L^2 + 3g_R^2)\right] \tag{3.10}$$

$$16\pi^2 \frac{d}{dt} Y^{\alpha\beta\gamma} = Y^{\alpha\beta\rho}\left[3\mu_\Delta^{\rho *}\mu_\Delta^\gamma + 3\mu_{\Delta^c}^{\rho *}\mu_{\Delta^c}^\gamma + 8\mathrm{Tr}(\mu_\Phi^{\rho\dagger}\mu_\Phi^\gamma) + \frac{1}{2}(Y^{\rho\mu\nu})^* Y^{\gamma\mu\nu}\right] + Y^{\gamma\beta\rho}\left[3\mu_\Delta^{\rho *}\mu_\Delta^\alpha + 3\mu_{\Delta^c}^{\rho *}\mu_{\Delta^c}^\alpha + 8\mathrm{Tr}(\mu_\Phi^{\rho\dagger}\mu_\Phi^\alpha) + \frac{1}{2}(Y^{\rho\mu\nu})^* Y^{\alpha\mu\nu}\right] + Y^{\alpha\gamma\rho}\left[3\mu_\Delta^{\rho *}\mu_\Delta^\beta + 3\mu_{\Delta^c}^{\rho *}\mu_{\Delta^c}^\beta + 8\mathrm{Tr}(\mu_\Phi^{\rho\dagger}\mu_\Phi^\beta) + \frac{1}{2}(Y^{\rho\mu\nu})^* Y^{\beta\mu\nu}\right] \tag{3.11}$$

3. Mass Terms

$$16\pi^2 \frac{d}{dt} M_\Delta = M_\Delta[2\mathrm{Tr}(f^\dagger f) + 2\mu_\Delta^{\alpha *}\mu_\Delta^\alpha - 6g_1^2 - 8g_L^2] \tag{3.12}$$

$$16\pi^2 \frac{d}{dt} M_{\Delta^c} = M_{\Delta^c}[2\mathrm{Tr}(f_c^\dagger f_c) + 2\mu_{\Delta^c}^{\alpha *}\mu_{\Delta^c}^\alpha - 6g_1^2 - 8g_R^2] \tag{3.13}$$

$$16\pi^2 \frac{d}{dt} M_{\Phi ab} = M_{\Phi ac}[\mathrm{Tr}(3h_c^\dagger h_b + h_c'^\dagger h_b') + 4(\mu_\Phi^{\beta\dagger}\mu_\Phi^\beta)_{cb}] + M_{\Phi ab}(-6g_L^2 - 6g_R^2) + [\mathrm{Tr}(3h_a h_c^\dagger + h_a' h_c'^\dagger) + 4(\mu_\Phi^\beta \mu_\Phi^{\beta\dagger})_{ac}]M_{\Phi cb} \tag{3.14}$$

$$16\pi^2 \frac{d}{dt} M_S^{\alpha\beta} = M_S^{\alpha\rho}\left[3\mu_\Delta^{\rho *}\mu_\Delta^\beta + 3\mu_{\Delta^c}^{\rho *}\mu_{\Delta^c}^\beta + 8\mathrm{Tr}(\mu_\Phi^{\rho\dagger}\mu_\Phi^\beta) + \frac{1}{2}(Y^{\rho\mu\nu})^* Y^{\beta\mu\nu}\right] + M_S^{\beta\rho}\left[3\mu_\Delta^{\rho *}\mu_\Delta^\alpha + 3\mu_{\Delta^c}^{\rho *}\mu_{\Delta^c}^\alpha + 8\mathrm{Tr}(\mu_\Phi^{\rho\dagger}\mu_\Phi^\alpha) + \frac{1}{2}(Y^{\rho\mu\nu})^* Y^{\alpha\mu\nu}\right] \tag{3.15}$$

4. Linear Term

$$16\pi^2 \frac{d}{dt} L^\alpha = L^\rho\left[3\mu_\Delta^{\rho*}\mu_\Delta^\alpha + 3\mu_{\Delta^c}^{\rho*}\mu_{\Delta^c}^\alpha + 8\mathrm{Tr}(\mu_\Phi^{\rho\dagger}\mu_\Phi^\alpha) + \frac{1}{2}(Y^{\rho\mu\nu})^* Y^{\alpha\mu\nu}\right] \tag{3.16}$$

5. Gaugino Masses

$$16\pi^2 \frac{d}{dt} M_1 = 48 M_1 g_1^2, \qquad 16\pi^2 \frac{d}{dt} g_L = (8 + 2n_\Phi) M_L g_L^2, \qquad 16\pi^2 \frac{d}{dt} g_R = (8 + 2n_\Phi) M_R g_R^2,$$
$$16\pi^2 \frac{d}{dt} g_3 = -6 M_3 g_3^2 \tag{3.17}$$

6. Soft Breaking Trilinear A's

$$\begin{aligned}16\pi^2 \frac{d}{dt} A_{Qa} = {}& A_{Qa}\left[2h_b^\dagger h_b - \frac{1}{6} g_1^2 - 3g_L^2 - 3g_R^2 - \frac{16}{3} g_3^2 \right] + 2h_b h_b^\dagger A_{Qa} \\ &+ h_a\left[4h_b^\dagger A_{Qb} + \frac{1}{3} g_1^2 M_1 + 6g_L^2 M_L + 6g_R^2 M_R + \frac{32}{3} g_3^2 M_3\right] + 4A_{Qb} h_b^\dagger h_a \\ &+ [\mathrm{Tr}(3h_a h_b^\dagger + h'_a h_b'^\dagger) + 4(\mu_\Phi^\alpha \mu_\Phi^{\alpha\dagger})_{ab}] A_{Qb} + [\mathrm{Tr}(6A_{Qa} h_b^\dagger + 2A_{La} h_b'^\dagger) + 8(A_\Phi^\alpha \mu_\Phi^{\alpha\dagger})_{ab}] h_b\end{aligned} \tag{3.18}$$

$$\begin{aligned}16\pi^2 \frac{d}{dt} A_{La} = {}& A_{La}\left[6f_c^\dagger f_c + 2h_b'^\dagger h'_b - \frac{3}{2} g_1^2 - 3g_L^2 - 3g_R^2 \right] + h'_a[12 f_c^\dagger A_{f_c} + 4h_b'^\dagger A_{Lb} + 3g_1^2 M_1 + 6g_L^2 M_L + 6g_R^2 M_R] \\ &+ [6ff^\dagger + 2h'_b h_b'^\dagger] A_{La} + [12 A_f f^\dagger + 4A_{Lb} h_b'^\dagger] h'_a + A_{Lb}[\mathrm{Tr}(3h_b^\dagger h_a + h_b'^\dagger h'_a) + 4(\mu_\Phi^{\alpha\dagger}\mu_\Phi^\alpha)_{ba}] \\ &+ h'_b[\mathrm{Tr}(6h_b^\dagger A_{Qa} + 2h_b'^\dagger A_{La}) + 8(\mu_\Phi^{\alpha\dagger} A_\Phi^\alpha)_{ba}]\end{aligned} \tag{3.19}$$

$$\begin{aligned}16\pi^2 \frac{d}{dt} A_f = {}& A_f\left[6f^\dagger f + 2h_a'^* h_a'^T + 2\mathrm{Tr}(f^\dagger f) + \mu_\Delta^{\alpha*}\mu_\Delta^\alpha - \frac{9}{2} g_1^2 - 7g_L^2\right] + f[12 f^* A_f + 4h_a'^* A_{La}^T + 4\mathrm{Tr}(f^\dagger A_f) \\ &+ 2\mu_\Delta^{\alpha*} A_\Delta^\alpha + 9g_1^2 M_1 + 14 g_L^2 M_L] + [6ff^\dagger + 2h'_a h_a'^\dagger] A_f + [12 A_f f^\dagger + 4A_{La} h_a'^\dagger] f\end{aligned} \tag{3.20}$$

$$\begin{aligned}16\pi^2 \frac{d}{dt} A_{f^c} = {}& A_{f^c}\left[6f_c^\dagger f_c + 2h_a'^\dagger h'_a + 2\mathrm{Tr}(f_c^\dagger f_c) + \mu_{\Delta^c}^{\alpha*}\mu_{\Delta^c}^\alpha - \frac{9}{2} g_1^2 - 7g_R^2\right] + f_c[12 f_c^\dagger A_{f^c} + 4h_a'^\dagger A_{La} \\ &+ 4\mathrm{Tr}(f_c^\dagger A_{f^c}) + 2\mu_{\Delta^c}^{\alpha*} A_{\Delta^c}^\alpha + 9g_1^2 M_1 + 14 g_R^2 M_R] + [6 f_c f_c^\dagger + 2h_a'^T h_a'^*] A_{f^c} + [12 A_{f^c} f_c^\dagger + 4A_{La}^T h_a'^*] f_c\end{aligned} \tag{3.21}$$

$$\begin{aligned}16\pi^2 \frac{d}{dt} A_\Delta^\alpha = {}& A_\Delta^\alpha[2\mathrm{Tr}(f^\dagger f) + 2\mu_\Delta^{\beta*}\mu_\Delta^\beta - 6g_1^2 - 8g_L^2] + \mu_\Delta^\alpha[4\mathrm{Tr}(f^\dagger A_f) + 4\mu_\Delta^{\beta*} A_\Delta^\beta + 12 g_1^2 M_1 + 16 g_L^2 M_L] \\ &+ A_\Delta^\beta\left[3\mu_\Delta^{\beta*}\mu_\Delta^\alpha + 3\mu_{\Delta^c}^{\beta*}\mu_{\Delta^c}^\alpha + 8\mathrm{Tr}(\mu_\Phi^{\beta\dagger}\mu_\Phi^\alpha) + \frac{1}{2}(Y^{\beta\mu\nu})^* Y^{\alpha\mu\nu}\right] \\ &+ \mu_\Delta^\beta[6\mu_\Delta^{\beta*} A_\Delta^\alpha + 6\mu_{\Delta^c}^{\beta*} A_{\Delta^c}^\alpha + 16\mathrm{Tr}(\mu_\Phi^{\beta\dagger} A_\Phi^\alpha) + (Y^{\beta\mu\nu})^* A_S^{\alpha\mu\nu}]\end{aligned} \tag{3.22}$$

$$\begin{aligned}16\pi^2 \frac{d}{dt} A_{\Delta^c}^\alpha = {}& A_{\Delta^c}^\alpha[2\mathrm{Tr}(f_c^\dagger f_c) + 2\mu_{\Delta^c}^{\beta*}\mu_{\Delta^c}^\beta - 6g_1^2 - 8g_R^2] + \mu_{\Delta^c}^\alpha[4\mathrm{Tr}(f_c^\dagger A_{f^c}) + 4\mu_{\Delta^c}^{\beta*} A_{\Delta^c}^\beta + 12 g_1^2 M_1 + 16 g_R^2 M_R] \\ &+ A_{\Delta^c}^\beta\left[3\mu_\Delta^{\beta*}\mu_\Delta^\alpha + 3\mu_{\Delta^c}^{\beta*}\mu_{\Delta^c}^\alpha + 8\mathrm{Tr}(\mu_\Phi^{\beta\dagger}\mu_\Phi^\alpha) + \frac{1}{2}(Y^{\beta\mu\nu})^* Y^{\alpha\mu\nu}\right] + \mu_{\Delta^c}^\beta[6\mu_\Delta^{\beta*} A_\Delta^\alpha + 6\mu_{\Delta^c}^{\beta*} A_{\Delta^c}^\alpha \\ &+ 16\mathrm{Tr}(\mu_\Phi^{\beta\dagger} A_\Phi^\alpha) + (Y^{\beta\mu\nu})^* A_S^{\alpha\mu\nu}]\end{aligned} \tag{3.23}$$

$$16\pi^2 \frac{d}{dt} A^{\alpha}_{\Phi ab} = A^{\alpha}_{\Phi ac}[\mathrm{Tr}(3h_c^{\dagger} h_b + h_c'^{\dagger} h_b') + 4(\mu_{\Phi}^{\beta\dagger} \mu_{\Phi}^{\beta})_{cb}] + \mu^{\alpha}_{\Phi ac}[\mathrm{Tr}(6h_c^{\dagger} A_{Qb} + 2h_c'^{\dagger} A_{Lb}) + 8(\mu_{\Phi}^{\beta\dagger} A_{\Phi}^{\beta})_{cb}]$$
$$+ [\mathrm{Tr}(3h_a h_c^{\dagger} + h_a' h_c'^{\dagger}) + 4(\mu_{\Phi}^{\beta} \mu_{\Phi}^{\beta\dagger})_{ac}]A^{\alpha}_{\Phi cb} + [\mathrm{Tr}(6A_{Qa} h_c^{\dagger} + 2A_{La} h_c'^{\dagger}) + 8(A_{\Phi}^{\beta} \mu_{\Phi}^{\beta\dagger})_{ac}]\mu^{\alpha}_{\Phi cb}$$
$$+ A^{\alpha}_{\Phi ab}[-3g_L^2 - 3g_R^2] + \mu^{\alpha}_{\Phi ab}[6g_L^2 M_L + 6g_R^2 M_R]$$
$$+ A^{\beta}_{\Phi ab}\left[3\mu_{\Delta}^{\beta *} \mu_{\Delta}^{\alpha} + 3\mu_{\Delta^c}^{\beta *} \mu_{\Delta^c}^{\alpha} + 8\mathrm{Tr}(\mu_{\Phi}^{\beta\dagger} \mu_{\Phi}^{\alpha}) + \frac{1}{2}(Y^{\beta\mu\nu})^* Y^{\alpha\mu\nu}\right]$$
$$+ \mu^{\beta}_{\Phi ab}[6\mu_{\Delta}^{\beta *} A_{\Delta}^{\alpha} + 6\mu_{\Delta^c}^{\beta *} A_{\Delta^c}^{\alpha} + 16\mathrm{Tr}(\mu_{\Phi}^{\beta\dagger} A_{\Phi}^{\alpha}) + (Y^{\beta\mu\nu})^* A_S^{\alpha\mu\nu}] \quad (3.24)$$

$$16\pi^2 \frac{d}{dt} A_S^{\alpha\beta\gamma} = A_S^{\alpha\beta\rho}\left[3\mu_{\Delta}^{\rho *} \mu_{\Delta}^{\gamma} + 3\mu_{\Delta^c}^{\rho *} \mu_{\Delta^c}^{\gamma} + 8\mathrm{Tr}(\mu_{\Phi}^{\rho\dagger} \mu_{\Phi}^{\gamma}) + \frac{1}{2}(Y^{\rho\mu\nu})^* Y^{\gamma\mu\nu}\right]$$
$$+ Y^{\alpha\beta\rho}[6\mu_{\Delta}^{\rho *} A_{\Delta}^{\gamma} + 6\mu_{\Delta^c}^{\rho *} A_{\Delta^c}^{\gamma} + 16\mathrm{Tr}(\mu_{\Phi}^{\rho\dagger} A_{\Phi}^{\gamma}) + (Y^{\rho\mu\nu})^* A_S^{\gamma\mu\nu}]$$
$$+ A_S^{\gamma\beta\rho}\left[3\mu_{\Delta}^{\rho *} \mu_{\Delta}^{\alpha} + 3\mu_{\Delta^c}^{\rho *} \mu_{\Delta^c}^{\alpha} + 8\mathrm{Tr}(\mu_{\Phi}^{\rho\dagger} \mu_{\Phi}^{\alpha}) + \frac{1}{2}(Y^{\rho\mu\nu})^* Y^{\alpha\mu\nu}\right]$$
$$+ Y^{\gamma\beta\rho}[6\mu_{\Delta}^{\rho *} A_{\Delta}^{\alpha} + 6\mu_{\Delta^c}^{\rho *} A_{\Delta^c}^{\alpha} + 16\mathrm{Tr}(\mu_{\Phi}^{\rho\dagger} A_{\Phi}^{\alpha}) + (Y^{\rho\mu\nu})^* A_S^{\alpha\mu\nu}]$$
$$+ A_S^{\alpha\gamma\rho}\left[3\mu_{\Delta}^{\rho *} \mu_{\Delta}^{\beta} + 3\mu_{\Delta^c}^{\rho *} \mu_{\Delta^c}^{\beta} + 8\mathrm{Tr}(\mu_{\Phi}^{\rho\dagger} \mu_{\Phi}^{\beta}) + \frac{1}{2}(Y^{\rho\mu\nu})^* Y^{\beta\mu\nu}\right]$$
$$+ Y^{\alpha\gamma\rho}[6\mu_{\Delta}^{\rho *} A_{\Delta}^{\beta} + 6\mu_{\Delta^c}^{\rho *} A_{\Delta^c}^{\beta} + 16\mathrm{Tr}(\mu_{\Phi}^{\rho\dagger} A_{\Phi}^{\beta}) + (Y^{\rho\mu\nu})^* A_S^{\beta\mu\nu}] \quad (3.25)$$

7. Soft Breaking Bilinear *B*'s

$$16\pi^2 \frac{d}{dt} B_{\Delta} = B_{\Delta}[2\mathrm{Tr}(f^{\dagger} f) + 2\mu_{\Delta}^{\alpha *} \mu_{\Delta}^{\alpha} - 6g_1^2 - 8g_L^2] + M_{\Delta}[4\mathrm{Tr}(f^{\dagger} A_f) + 4\mu_{\Delta}^{\alpha *} A_{\Delta}^{\alpha} + 12g_1^2 M_1 + 16g_L^2 M_L]$$
$$+ \mu_{\Delta}^{\alpha}[6\mu_{\Delta}^{\alpha *} B_{\Delta} + 6\mu_{\Delta^c}^{\alpha *} B_{\Delta^c} + 16\mathrm{Tr}(\mu_{\Phi}^{\alpha\dagger} B_{\Phi}) + (Y^{\alpha\mu\nu})^* B_S^{\mu\nu}] \quad (3.26)$$

$$16\pi^2 \frac{d}{dt} B_{\Delta^c} = B_{\Delta^c}[2\mathrm{Tr}(f_c^{\dagger} f_c) + 2\mu_{\Delta^c}^{\alpha *} \mu_{\Delta^c}^{\alpha} - 6g_1^2 - 8g_R^2] + M_{\Delta^c}[4\mathrm{Tr}(f_c^{\dagger} A_{f^c}) + 4\mu_{\Delta^c}^{\alpha *} A_{\Delta^c}^{\alpha} + 12g_1^2 M_1 + 16g_R^2 M_R]$$
$$+ \mu_{\Delta^c}^{\alpha}[6\mu_{\Delta}^{\alpha *} B_{\Delta} + 6\mu_{\Delta^c}^{\alpha *} B_{\Delta^c} + 16\mathrm{Tr}(\mu_{\Phi}^{\alpha\dagger} B_{\Phi}) + (Y^{\alpha\mu\nu})^* B_S^{\mu\nu}] \quad (3.27)$$

$$16\pi^2 \frac{d}{dt} B_{\Phi ab} = B_{\Phi ac}[\mathrm{Tr}(3h_c^{\dagger} h_b + h_c'^{\dagger} h_b') + 4(\mu_{\Phi}^{\alpha\dagger} \mu_{\Phi}^{\alpha})_{cb}] + M_{\Phi ac}[\mathrm{Tr}(6h_c^{\dagger} A_{Qb} + 2h_c'^{\dagger} A_{Lb}) + 8(\mu_{\Phi}^{\alpha\dagger} A_{\Phi}^{\alpha})_{cb}]$$
$$+ [\mathrm{Tr}(3h_a h_c^{\dagger} + h_a' h_c'^{\dagger}) + 4(\mu_{\Phi}^{\alpha} \mu_{\Phi}^{\alpha\dagger})_{ac}]B_{\Phi cb} + [\mathrm{Tr}(6A_{Qa} h_c^{\dagger} + 2A_{La} h_c'^{\dagger}) + 8(A_{\Phi}^{\alpha} \mu_{\Phi}^{\alpha\dagger})_{ac}]M_{\Phi cb}$$
$$+ \mu^{\rho}_{\Phi ab}[6\mu_{\Delta}^{\rho *} B_{\Delta} + 6\mu_{\Delta^c}^{\rho *} B_{\Delta^c} + 16\mathrm{Tr}(\mu_{\Phi}^{\rho\dagger} B_{\Phi}) + (Y^{\rho\mu\nu})^* B_S^{\mu\nu}] + B_{\Phi ab}[-3g_L^2 - 3g_R^2]$$
$$+ M_{\Phi ab}[6g_L^2 M_L + 6g_R^2 M_R] \quad (3.28)$$

$$16\pi^2 \frac{d}{dt} B_S^{\alpha\beta} = B_S^{\alpha\rho}\left[3\mu_{\Delta}^{\rho *} \mu_{\Delta}^{\beta} + 3\mu_{\Delta^c}^{\rho *} \mu_{\Delta^c}^{\beta} + 8\mathrm{Tr}(\mu_{\Phi}^{\rho\dagger} \mu_{\Phi}^{\beta}) + \frac{1}{2}(Y^{\rho\mu\nu})^* Y^{\beta\mu\nu}\right] + M_S^{\alpha\rho}[6\mu_{\Delta}^{\rho *} A_{\Delta}^{\beta} + 6\mu_{\Delta^c}^{\rho *} A_{\Delta^c}^{\beta}$$
$$+ 16\mathrm{Tr}(\mu_{\Phi}^{\rho\dagger} A_{\Phi}^{\beta}) + (Y^{\rho\mu\nu})^* A_S^{\beta\mu\nu}] + B_S^{\beta\rho}\left[3\mu_{\Delta}^{\rho *} \mu_{\Delta}^{\alpha} + 3\mu_{\Delta^c}^{\rho *} \mu_{\Delta^c}^{\alpha} + 8\mathrm{Tr}(\mu_{\Phi}^{\rho\dagger} \mu_{\Phi}^{\alpha}) + \frac{1}{2}(Y^{\rho\mu\nu})^* Y^{\alpha\mu\nu}\right]$$
$$+ M_S^{\beta\rho}[6\mu_{\Delta}^{\rho *} A_{\Delta}^{\alpha} + 6\mu_{\Delta^c}^{\rho *} A_{\Delta^c}^{\alpha} + 16\mathrm{Tr}(\mu_{\Phi}^{\rho\dagger} A_{\Phi}^{\alpha}) + (Y^{\rho\mu\nu})^* A_S^{\alpha\mu\nu}]$$
$$+ Y^{\alpha\beta\rho}[6\mu_{\Delta}^{\rho *} B_{\Delta} + 6\mu_{\Delta^c}^{\rho *} B_{\Delta^c} + 16\mathrm{Tr}(\mu_{\Phi}^{\rho\dagger} B_{\Phi}) + (Y^{\rho\mu\nu})^* B_S^{\mu\nu}] \quad (3.29)$$

8. Soft Breaking Masses

Since each of the RGEs for the soft breaking masses have the following term in common, it is convenient to define

$$S_3 \equiv 4\mathrm{Tr}(m_Q^2 - m_{Q^c}^2 - m_L^2 + m_{L^c}^2) + 12(m_\Delta^2 - m_{\bar\Delta}^2 - m_{\Delta^c}^2 + m_{\bar\Delta^c}^2)$$

$$16\pi^2 \frac{d}{dt} m_Q^2 = 2m_Q^2 h_a h_a^\dagger + h_a[2h_a^\dagger m_Q^2 + 4h_b^\dagger m_{\Phi ab}^2 + 4m_{Q^c}^2 h_a^\dagger] + 4A_{Qa}A_{Qa}^\dagger - \frac{1}{3}M_1M_1^\dagger g_1^2 - 6M_LM_L^\dagger g_L^2 - \frac{32}{3}M_3M_3^\dagger g_3^2 + \frac{1}{8}g_1^2 S_3 \tag{3.30}$$

$$16\pi^2 \frac{d}{dt} m_{Q^c}^2 = 2m_{Q^c}^2 h_a^\dagger h_a + h_a^\dagger[2h_a m_{Q^c}^2 + 4h_b m_{\Phi ba}^2 + 4m_Q^2 h_a] + 4A_{Qa}^\dagger A_{Qa} - \frac{1}{3}M_1M_1^\dagger g_1^2 - 6M_RM_R^\dagger g_R^2 - \frac{32}{3}M_3M_3^\dagger g_3^2 - \frac{1}{8}g_1^2 S_3 \tag{3.31}$$

$$16\pi^2 \frac{d}{dt} m_L^2 = 6m_L^2 ff^\dagger + f[6f^\dagger m_L^2 + 12(m_L^2)^T f^\dagger + 12f^\dagger m_\Delta^2] + 2m_L^2 h'_a h'^\dagger_a + h'_a[2h'^\dagger_a m_L^2 + 4m_{L^c}^2 h'^\dagger_a + 4h'^\dagger_b m_{\Phi ab}^2] + 12A_fA_f^\dagger + 4A_{La}A_{La}^\dagger - 3M_1M_1^\dagger g_1^2 - 6M_LM_L^\dagger g_L^2 - \frac{3}{8}g_1^2 S_3 \tag{3.32}$$

$$16\pi^2 \frac{d}{dt} m_{L^c}^2 = 6m_{L^c}^2 f_c^\dagger f_c + f_c^\dagger[6f_c m_{L^c}^2 + 12(m_{L^c}^2)^T f_c + 12f_c m_{\Delta^c}^2] + 2m_{L^c}^2 h'^\dagger_a h'_a + h'^\dagger_a[2h'_a m_{L^c}^2 + 4m_L^2 h'_a + 4h'_b m_{\Phi ba}^2] + 12A_{f^c}^\dagger A_{f^c} + 4A_{La}^\dagger A_{La} - 3M_1M_1^\dagger g_1^2 - 6M_RM_R^\dagger g_R^2 + \frac{3}{8}g_1^2 S_3 \tag{3.33}$$

$$16\pi^2 \frac{d}{dt} m_\Delta^2 = \mathrm{Tr}[4f^\dagger f m_\Delta^2 + 8f^\dagger m_L^2 f] + \mu_\Delta^{\alpha *}[2\mu_\Delta^\alpha m_\Delta^2 + 2\mu_\Delta^\alpha m_{\bar\Delta}^2 + 2\mu_\Delta^\beta (m_S^2)^{\beta\alpha}] + 4\mathrm{Tr}(A_f^\dagger A_f) + 2A_\Delta^{\alpha *}A_\Delta^\alpha - 12M_1M_1^\dagger g_1^2 - 16M_LM_L^\dagger g_L^2 + \frac{3}{4}g_1^2 S_3 \tag{3.34}$$

$$16\pi^2 \frac{d}{dt} m_{\bar\Delta}^2 = \mu_\Delta^{\alpha *}[2\mu_\Delta^\alpha m_{\bar\Delta}^2 + 2\mu_\Delta^\alpha m_\Delta^2 + 2\mu_\Delta^\beta (m_S^2)^{\beta\alpha}] + 2A_\Delta^{\alpha *}A_\Delta^\alpha - 12M_1M_1^\dagger g_1^2 - 16M_LM_L^\dagger g_L^2 - \frac{3}{4}g_1^2 S_3 \tag{3.35}$$

$$16\pi^2 \frac{d}{dt} m_{\Delta^c}^2 = \mathrm{Tr}[4f_cf_c^\dagger m_{\Delta^c}^2 + 8f_c m_{L^c}^2 f_c^\dagger] + \mu_{\Delta^c}^{\alpha *}[2\mu_{\Delta^c}^\alpha m_{\Delta^c}^2 + 2\mu_{\Delta^c}^\alpha m_{\bar\Delta^c}^2 + 2\mu_{\Delta^c}^\beta (m_S^2)^{\beta\alpha}] + 4\mathrm{Tr}(A_{f^c}A_{f^c}^\dagger) + 2A_{\Delta^c}^{\alpha *}A_{\Delta^c}^\alpha - 12M_1M_1^\dagger g_1^2 - 16M_RM_R^\dagger g_R^2 - \frac{3}{4}g_1^2 S_3 \tag{3.36}$$

$$16\pi^2 \frac{d}{dt} m_{\bar\Delta^c}^2 = \mu_{\Delta^c}^{\alpha *}[2\mu_{\Delta^c}^\alpha m_{\bar\Delta^c}^2 + 2\mu_{\Delta^c}^\beta (m_S^2)^{\beta\alpha} + 2\mu_{\Delta^c}^\alpha m_{\Delta^c}^2] + 2A_{\Delta^c}^{\alpha *}A_{\Delta^c}^\alpha - 12M_1M_1^\dagger g_1^2 - 16M_RM_R^\dagger g_R^2 + \frac{3}{4}g_1^2 S_3 \tag{3.37}$$

$$16\pi^2 \frac{d}{dt} m_{\Phi ab}^2 = m_{\Phi ac}^2[\mathrm{Tr}(3h_c^\dagger h_b + h'^\dagger_c h'_b) + 4(\mu_\Phi^{\beta\dagger}\mu_\Phi^\beta)_{cb}] + [\mathrm{Tr}(3h_a^\dagger h_c + h'^\dagger_a h'_c) + 4(\mu_\Phi^{\beta\dagger}\mu_\Phi^\beta)_{ac}]m_{\Phi cb}^2 + \mathrm{Tr}[6h_a^\dagger h_b m_{Q^c}^2 + 6h_a^\dagger m_Q^2 h_b + 2h'^\dagger_a h'_b m_{L^c}^2 + 2h'^\dagger_a m_L^2 h'_b + 6A_{Qa}^\dagger A_{Qb} + 2A_{La}^\dagger A_{Lb}] + 8[\mu_\Phi^\alpha m_\Phi^2 \mu_\Phi^{\alpha\dagger}]_{ba} + [8\mu_\Phi^{\alpha\dagger}\mu_\Phi^\beta (m_S^2)^{\beta\alpha} + 8A_\Phi^{\beta\dagger}A_\Phi^\beta - 6g_L^2M_LM_L^\dagger - 6g_R^2M_RM_R^\dagger]_{ab} \tag{3.38}$$

$$
\begin{aligned}
16\pi^2 \frac{d}{dt}(m_S^2)^{\alpha\beta} = {}& (m_S^2)^{\alpha\rho}\left[3\mu_\Delta^{\rho *}\mu_\Delta^\beta + 3\mu_{\Delta^c}^{\rho *}\mu_{\Delta^c}^\beta + 8\mathrm{Tr}(\mu_\Phi^{\rho\dagger}\mu_\Phi^\beta) + \frac{1}{2}(Y^{\rho\mu\nu})^* Y^{\beta\mu\nu}\right] \\
&+ \left[3\mu_\Delta^{\alpha *}\mu_\Delta^\rho + 3\mu_{\Delta^c}^{\alpha *}\mu_{\Delta^c}^\rho + 8\mathrm{Tr}(\mu_\Phi^{\alpha\dagger}\mu_\Phi^\rho) + \frac{1}{2}(Y^{\alpha\mu\nu})^* Y^{\rho\mu\nu}\right](m_S^2)^{\rho\beta} + 6\mu_\Delta^{\alpha *}\mu_\Delta^\beta m_{\bar\Delta}^2 \\
&+ 6\mu_\Delta^{\alpha *}\mu_\Delta^\beta m_\Delta^2 + 6\mu_{\Delta^c}^{\alpha *}\mu_{\Delta^c}^\beta m_{\bar\Delta^c}^2 + 6\mu_{\Delta^c}^{\alpha *}\mu_{\Delta^c}^\beta m_{\Delta^c}^2 + 32\mathrm{Tr}(\mu_\Phi^{\alpha\dagger}\mu_\Phi^\beta m_\Phi^2) + 2(Y^{\alpha\rho\mu})^* Y^{\beta\rho\nu}(m_S^2)^{\nu\mu} \\
&+ 6A_\Delta^{\alpha *}A_\Delta^\beta + 6A_{\Delta^c}^{\alpha *}A_{\Delta^c}^\beta + 16\mathrm{Tr}(A_\Phi^{\alpha\dagger}A_\Phi^\beta) + (A_S^{\alpha\mu\nu})^* A_S^{\beta\mu\nu} \qquad (3.39)
\end{aligned}
$$

IV. CONCLUSION

In this paper we have calculated the RGEs to one-loop order for two different types of SUSYLR models–one which breaks $SU(2)_R$ via doublets and the other using triplets. These equations should prove to be useful tools for relating the details of SUSYLR models to observable phenomena, thereby constraining the parameter space and perhaps verifying if SUSYLR models are viable extensions of the standard model.

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