

Static quark-antiquark interactions in zero and finite temperature QCD: I. Heavy quark free energies, running coupling, and quarkonium binding

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We analyze heavy quark free energies in 2-flavor QCD at finite temperature and the corresponding heavy quark potential at zero temperature. Static quark-antiquark sources in color singlet, octet and color averaged channels are used to probe thermal modifications of the medium. The temperature dependence of the running coupling, $\alpha_{qq}(r, T)$, is analyzed at short and large distances and is compared to zero temperature as well as quenched calculations. In part, we also compare our results to recent findings in 3-flavor QCD. We find that the characteristic length scale below which the running coupling shows almost no temperature dependence is almost twice as large as the Debye screening radius. Our analysis supports recent findings which suggest that χ_c and ψ' are suppressed already at the (pseudo) critical temperature and thus give a probe for quark gluon plasma production in heavy ion collision experiments, while J/ψ may survive the transition and will dissolve at higher temperatures.

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I. INTRODUCTION

The study of the fundamental forces between quarks and gluons is an essential key to the understanding of QCD and the occurrence of different phases which are expected to show up when going from low to high temperatures (T) and/or baryon number densities. For instance, at small or vanishing temperatures quarks and gluons get confined by the strong force while at high temperatures asymptotic freedom suggests a quite different QCD medium consisting of rather weakly coupled quarks and gluons, the so-called quark gluon plasma (QGP) [1]. On quite general grounds it is therefore expected that the interactions get modified by temperature. For the analysis of these modifications of the strong forces the change in free energy due to the presence of a static quark-antiquark pair separated by a distance r in a QCD-like thermal heat bath has often been used since early work [2,3]. In fact, the static quark-antiquark free energy which is obtained from Polyakov loop correlation functions calculated at finite temperature plays a similar important role in the discussion of properties of the strong force as the static quark potential does at zero temperature.

The properties of this observable (at $T = 0$: potential, at $T \neq 0$: free energy) at short and intermediate distances ($rT \lesssim 1$) is important for the understanding of in-medium modifications of heavy quark bound states. A quantitative analysis of heavy quark free energies becomes of considerable importance for the discussion of possible signals for the quark gluon plasma formation in heavy ion collision experiments [4,5]. For instance, recent studies of heavy quarkonium systems within potential models use the

quark-antiquark free energy to define an appropriate finite temperature potential which is considered in the nonrelativistic Schrödinger equation [6–9]. Such calculations, however, do not quite match the results of direct lattice calculations of the quarkonium dissociation temperatures which have been obtained so far only for the pure gauge theory [10,11]. It was pointed out [12] that the free energy (F) of a static quark-antiquark pair can be separated into two contributions, the internal energy (U) and the entropy (S). The separation of the entropy contribution from the free energy, i.e. the variable $U = F + TS$, could define an appropriate effective potential at finite temperature¹ [12,14], $V_{\text{eff}}(r, T) \equiv U$, to be used as input in model calculations and might explain, in part, the quantitative differences found when comparing solutions of the Schrödinger equation with direct calculations of spectral functions [10,11]. First calculations which use the internal energy obtained in our calculations [15–18] support this expectation. Most of these studies so far consider quenched QCD. Using potentials from the quenched theory, however, will describe the interaction of a heavy quark-antiquark pair in a thermal medium made up of gluons only. It is then important to understand how these results might change for the case of a thermal heat bath which also contains dynamical quarks.

On the other hand, it is the large distance property of the heavy quark interaction which is important for our understanding of the bulk properties of the QCD plasma phase,

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¹While a definition of the quark-antiquark potential can be given properly at zero temperature using large Wilson loops, at finite temperature a definition of the thermal modification of an appropriate potential energy between the quark-antiquark pair is complicated [13].

e.g. the screening property of the quark gluon plasma [19,20], the equation of state [21,22], and the order parameter (Polyakov loop) [12,23–25]. In all of these studies deviations from perturbative calculations and the ideal gas behavior are expected and were indeed found at temperatures which are only moderately larger than the deconfinement temperature. This calls for quantitative nonperturbative calculations. Also in this case most of today’s discussions of the bulk thermodynamic properties of the QGP and its apparent deviations from the ideal gas behavior rely on results obtained in lattice studies of the pure gauge theory, although several qualitative differences are to be expected when taking into account the influence of dynamical fermions; for instance, the phase transition in full QCD will appear as a crossover rather than a “true” phase transition with related singularities in thermodynamic observables. Moreover, in contrast to a steadily increasing confinement interaction in the quenched QCD theory, in full QCD the strong interaction below deconfinement will show a qualitatively different behavior at large quark-antiquark separations. Because of the possibility of pair creation, the stringlike interaction between the two test quarks can break, leading to a constant potential and/or free energy already at temperatures below deconfinement [26].

Thus it is quite important to extend our recently developed concepts for the analysis of the quark-antiquark free energies and internal energies in pure gauge theory [12,20,27,28] to the more complex case of QCD with dynamical quarks, and to quantify the qualitative differences which will show up between pure gauge theories and QCD.

For our study of the strong interaction in terms of the quark-antiquark free energies in full QCD, lattice configurations were generated for 2-flavor QCD ($N_f = 2$) on $16^3 \times 4$ lattices with bare quark mass $ma = 0.1$, i.e. $m/T = 0.4$, corresponding to a ratio of pion to rho masses (m_π/m_ρ) at the (pseudo) critical temperature of about 0.7 (a denotes the lattice spacing) [29]. We have used Symanzik-improved gauge and p4-improved staggered fermion actions. This combination of lattice actions is known

TABLE I. Sample sizes at each β value and the temperature in units of the (pseudo) critical temperature T_c .

β	T/T_c	# conf.	β	T/T_c	# conf.
3.52	0.76	2000	3.72	1.16	2000
3.55	0.81	3000	3.75	1.23	1000
3.58	0.87	3500	3.80	1.36	1000
3.60	0.90	2000	3.85	1.50	1000
3.63	0.96	3000	3.90	1.65	1000
3.65	1.00	4000	3.95	1.81	1000
3.66	1.02	4000	4.00	1.98	4000
3.68	1.07	3600	4.43	4.01	1600
3.70	1.11	2000			

to reduce the lattice cutoff effects in Polyakov loop correlation functions at small quark-antiquark separations seen as an improved restoration of the broken rotational symmetry. For any further details of the simulations with these actions, see [30,31]. In Table I we summarize our simulation parameters, i.e. the lattice coupling β , the temperature T/T_c in units of the pseudocritical temperature, and the number of configurations used at each β value. The pseudocritical coupling for this action is $\beta_c = 3.649(2)$ [30]. To set the physical scale we use the string tension, σa^2 ,

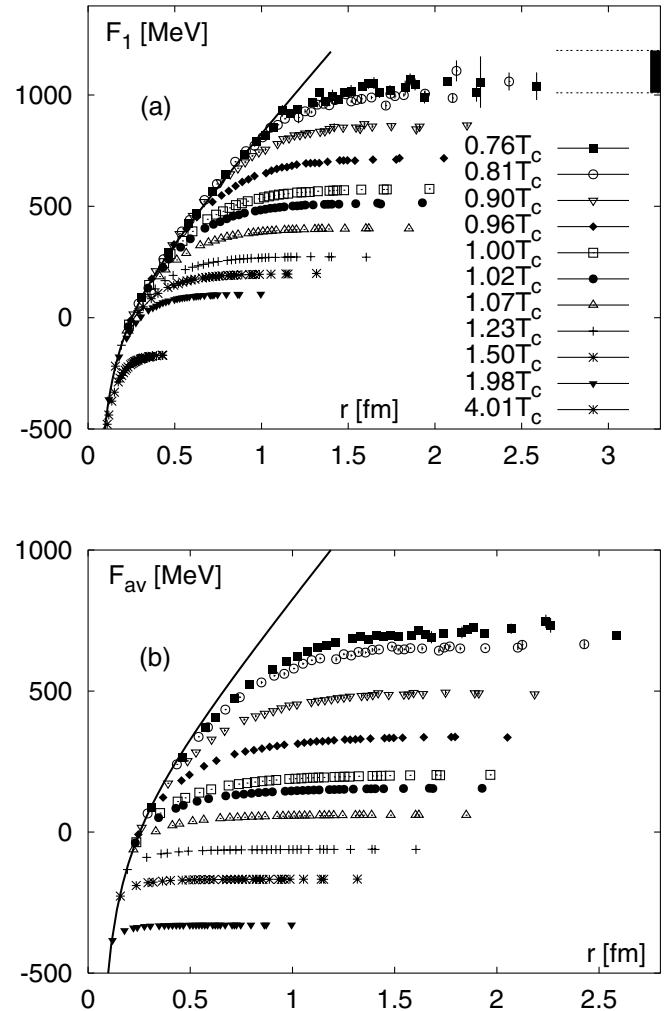


FIG. 1. (a) The color singlet quark-antiquark free energies, $F_1(r, T)$, at several temperatures close to the phase transition as function of distance in physical units. Shown are results from lattice studies of 2-flavor QCD. The solid line represents in each figure the $T = 0$ heavy quark potential, $V(r)$. The dashed error band corresponds to the string-breaking energy at zero temperature, $V(r_{\text{breaking}}) \approx 1000-1200$ MeV, based on the estimate of the string-breaking distance, $r_{\text{breaking}} \approx 1.2-1.4$ fm [37]. (b) The color averaged free energy, $F_{\bar{q}q}(r, T)$, normalized such that $F_{av}(r, T) \equiv F_{\bar{q}q}(r, T) - T \ln 9$ [12] approaches the heavy quark potential, $V(r)$ (line), at the smallest distance available on the lattice. The symbols are chosen as in (a).

measured in units of the lattice spacing, obtained from the large distance behavior of the heavy quark potential calculated from smeared Wilson loops at zero temperature [29]. This is also used to define the temperature scale and $a\sqrt{\sigma}$ is used for setting the scale for the free energies and the physical distances. For the conversion to physical units, $\sqrt{\sigma} = 420$ MeV is used. For instance, we get $T_c = 202(4)$ MeV calculated from $T_c/\sqrt{\sigma} = 0.48(1)$ [29]. In parts of our analysis of the quark-antiquark free energies we are also interested in the flavor and finite quark mass dependence. For this reason we also compare our 2-flavor QCD results to the available recent findings in quenched ($N_f = 0$) [12,20] and 3-flavor QCD ($m_\pi/m_\rho \approx 0.4$ [32]) [33]. Here we use $T_c = 270$ MeV for quenched and $T_c = 193$ MeV [33] for the 3-flavor case.

Our results for the color singlet quark-antiquark free energies, F_1 , and color averaged free energies, F_{av} , are summarized in Fig. 1 as a function of distance at several temperatures close to the transition. At distances much smaller than the inverse temperature ($rT \ll 1$) the dominant scale is set by distance and the QCD running coupling will be controlled by the distance. In this limit the thermal modification of the strong interaction will become negligible and the finite temperature free energy will be given by the zero temperature heavy quark potential (solid line). With increasing quark-antiquark separation, however, thermal effects will dominate the behavior of the finite temperature free energies ($rT \gg 1$). Qualitative and quantitative differences between quark-antiquark free energy and internal energy will appear and clarify the important role of the entropy contribution still present in free energies. The quark-antiquark internal energy will provide a different look on the interquark interaction and thermal modifications of the finite temperature quark-antiquark potential. Further details of these modifications on the quark-antiquark free and internal energies will be discussed.

This paper is organized as follows: we start in Sec. II with a discussion of the zero temperature heavy quark potential and the coupling. Both will be calculated from 2-flavor lattice QCD simulations. We analyze in Sec. III the thermal modifications on the quark-antiquark free energies and discuss quarkonium binding. Section IV contains our summary and conclusions. A detailed discussion of the quark-antiquark internal energy and entropy will be given separately [34].

II. THE ZERO TEMPERATURE HEAVY QUARK POTENTIAL AND COUPLING

A. Heavy quark potential at $T = 0$

For the determination of the heavy quark potential at zero temperature, $V(r)$, we have used the measurements of large smeared Wilson loops given in [29] for the same simulation parameters ($N_f = 2$ and $ma = 0.1$) and action. To eliminate the divergent self-energy contributions we

matched these data for all β values (different β values correspond to different values of the lattice spacing a) at large distances to the bosonic string potential,

$$V(r) = -\frac{\pi}{12} \frac{1}{r} + \sigma r \equiv -\frac{4}{3} \frac{\alpha_{\text{str}}}{r} + \sigma r, \quad (1)$$

where we already have separated the Casimir factor so that $\alpha_{\text{str}} \equiv \pi/16$. In this normalization any divergent contributions to the lattice potential are eliminated uniquely. In Fig. 2 we show our results together with the heavy quark potential from the string picture (dashed line). One can see that the data are well described by Eq. (1) at large distances, i.e. $r\sqrt{\sigma} \gtrsim 0.8$, corresponding to $r \gtrsim 0.4$ fm. At these distances we see no major difference between the 2-

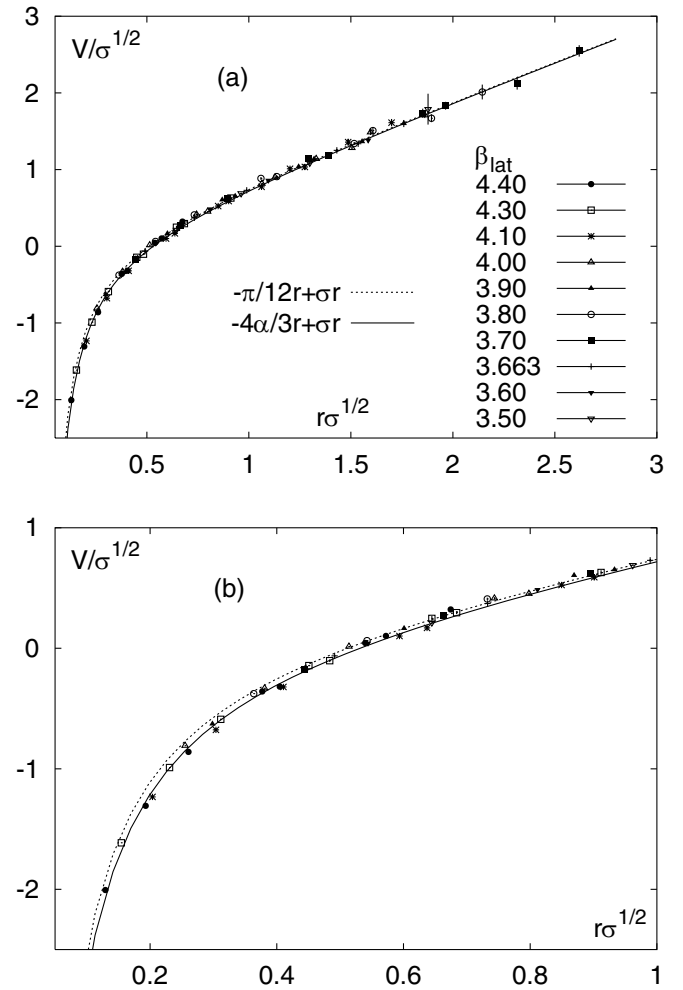


FIG. 2. (a) The heavy quark potential at $T = 0$ from [29] obtained from 2-flavor QCD lattice simulations with quark masses $ma = 0.1$ for different values of the lattice coupling β . (b) shows an enlargement of the short distance regime. The data are matched to the bosonic string potential (dashed line) at large distances. Also included is the fit to the Cornell form (solid line) given in Eq. (4). Note here that the heavy quark potential from quenched lattice QCD and the string model potential coincide already at $r\sqrt{\sigma} \gtrsim 0.8$ [35,36] ($r \gtrsim 0.4$ fm).

flavor QCD potential obtained from Wilson loops and the quenched QCD potential which can be well parametrized within the string model already for $r \gtrsim 0.4$ fm [35,36]. In fact, we also do not see any signal for string breaking in the zero temperature QCD heavy quark potential. This is expected due to the fact that the Wilson loop operator used here for the calculation of the $T = 0$ potential has only small overlap with states where string breaking occurs [37,38]. Moreover, the distances for which we analyze the data for the QCD potential are below $r \lesssim 1.2$ fm at which string breaking is expected to set in at zero temperature and similar quark masses [37].

B. The coupling at $T = 0$

Deviations from the string model and from the pure gauge potential, however, are clearly expected to become apparent in the 2-flavor QCD potential at small distances and may already be seen from the short distance part in Fig. 2. These deviations are expected to arise from an asymptotic weakening of the QCD coupling, i.e. $\alpha = \alpha(r)$, and to some extent also due to the effect of including dynamical quarks, i.e. from leading order perturbation theory one expects

$$\alpha(r) \simeq \frac{1}{8\pi} \frac{1}{\beta_0 \log(1/(r\Lambda_{\text{QCD}}))}, \quad (2)$$

with

$$\beta_0 = \frac{33 - 2N_f}{48\pi^2}, \quad (3)$$

where N_f is the number of flavors and Λ_{QCD} denotes the corresponding QCD- Λ scale. The data in Fig. 2(b) show a slightly steeper slope at distances below $r\sqrt{\sigma} \simeq 0.5$ compared to the pure gauge potential given in Ref. [35] indicating that the QCD coupling gets stronger in the entire distance range analyzed here when including dynamical quarks. This is in qualitative agreement with (2). To include the effect of a stronger Coulombic part in the QCD potential we test the Cornell parametrization,

$$\frac{V(r)}{\sqrt{\sigma}} = -\frac{4}{3} \frac{\alpha}{r\sqrt{\sigma}} + r\sqrt{\sigma}, \quad (4)$$

with a free parameter α . From a best-fit analysis of Eq. (4) to the data ranging from $0.2 \lesssim r\sqrt{\sigma} \lesssim 2.6$ we find

$$\alpha = 0.212(3). \quad (5)$$

This already may indicate that the logarithmic weakening of the coupling with decreasing distance will not too strongly influence the properties of the QCD potential at these distances, i.e. at $r \gtrsim 0.1$ fm. However, the value of α is moderately larger than $\alpha_{\text{str}} \simeq 0.196$ introduced above. To compare the relative size of α in full QCD to α in the quenched theory we again have performed a best-fit analysis of the quenched zero temperature potential given in [35] using the Ansatz given in Eq. (4) and a similar distance

range. Here we find $\alpha_{\text{quenched}} = 0.195(1)$ which is again smaller than the value for the QCD coupling but quite comparable to α_{str} . In earlier studies of the heavy quark potentials in pure gauge theories and full QCD even larger values for the couplings were reported [39–44]. To avoid here any confusions concerning the value of α we should stress that α should not be mixed with some value for the QCD coupling constant α_{QCD} ; it simply is a fit parameter indicating the “average strength” of the Coulomb part in the Cornell potential. The QCD coupling could be identified properly only in the entire perturbative distance regime and will be a running coupling, i.e. $\alpha_{\text{QCD}} = \alpha_{\text{QCD}}(r)$.

When approaching the short distance perturbative regime the Cornell form will overestimate the value of the coupling due to the perturbative logarithmic weakening of the latter, $\alpha_{\text{QCD}} = \alpha_{\text{QCD}}(r)$. To analyze the short distance properties of the QCD potential and the coupling in more detail, i.e. for $r \lesssim 0.4$ fm, and to firmly establish here the onset of its perturbative weakening with decreasing distance, it is customary to do so using nonperturbative definitions of running couplings. Following the discussions on the running of the QCD coupling [35,45–48], it appears most convenient to study the QCD force, i.e. $dV(r)/dr$, rather than the QCD potential. In this case one defines the QCD coupling in the so-called qq scheme,

$$\alpha_{qq}(r) \equiv \frac{3}{4} r^2 \frac{dV(r)}{dr}. \quad (6)$$

In this scheme any undetermined constant contribution to the heavy quark potential cancels out. Moreover, the large distance, nonperturbative confinement contribution to $\alpha_{qq}(r)$ is positive and allows for a smooth matching of the perturbative short distance coupling to the nonpertur-

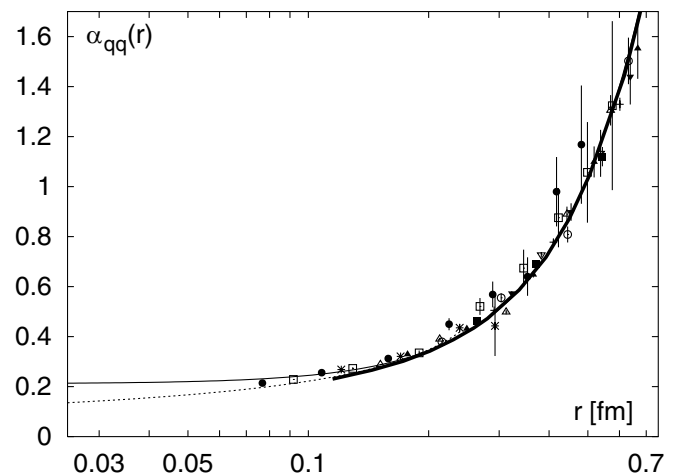


FIG. 3. The short distance part of the running coupling $\alpha_{qq}(r)$ in 2-flavor QCD at zero temperature defined in Eq. (6) as a function of the distance r (in physical units). The symbols for the different β values are chosen as indicated in Fig. 2(a). The lines are discussed in the text.

bative large distance confinement signal. In any case, however, in the nonperturbative regime the value of the coupling will depend on the observable used for its definition.

We have calculated the derivatives of the potential with respect to the distance, $dV(r)/dr$, by using finite difference approximations for neighboring distances on the lattice for each β value separately. Our results for $\alpha_{qq}(r)$ as a function of distance in physical units for 2-flavor QCD are summarized in Fig. 3. The symbols for the β values are chosen as in Fig. 2(a). We again show in that figure the corresponding line for the Cornell fit (solid line). At large distances, $r \gtrsim 0.4$ fm, the data clearly mimic the nonperturbative confinement part of the QCD force, $\alpha_{qq}(r) \simeq 3r^2\sigma/4$. We also compare our data to the recent high statistics calculation in pure gauge theory (thick solid line) [35]. These data are available for $r \gtrsim 0.1$ fm and within the statistics of the QCD data no significant differences could be identified between the QCD and pure gauge data for $r \gtrsim 0.4$ fm. At smaller distances ($r \lesssim 0.4$ fm), however, the data show some enhancement compared to the coupling in quenched QCD. The data below 0.1 fm, moreover, fall below the large distance Cornell fit. This may indicate the logarithmic weakening of the coupling. At smaller distances than 0.1 fm we therefore expect the QCD potential to be influenced by the weakening of the coupling and $\alpha_{qq}(r)$ will approach values clearly smaller than α deduced from the Cornell Ansatz. Unfortunately we can, at present, not go to smaller distances to clearly demonstrate this behavior with our data in 2-flavor QCD. Moreover, at small distances cutoff effects may also influence our analysis of the coupling and more detailed studies are required here. Despite these uncertainties, however, in earlier studies of the coupling in pure gauge theory [20,35,48] it is shown that the perturbative logarithmic weakening becomes already important at distances smaller than 0.2 fm and contact with perturbation theory could be established.

As most of our lattice data for the finite temperature quark-antiquark free energies do not reach distances smaller than 0.1 fm we use in the following the Cornell form deduced in (4) as reference to the zero temperature heavy quark potential.

III. QUARK-ANTIQUARK FREE ENERGY

We will analyze here the temperature dependence of the change in free energy due to the presence of a heavy (static) quark-antiquark pair in a 2-flavor QCD heat bath. The static quark sources are described by the Polyakov loop,

$$L(\vec{x}) = \frac{1}{3} \text{Tr}W(\vec{x}), \quad (7)$$

with

$$W(\vec{x}) = \prod_{\tau=1}^{N_\tau} U_0(\vec{x}, \tau), \quad (8)$$

where we already have used the lattice formulation with $U_0(\vec{x}, \tau) \in SU(3)$ being defined on the lattice link in time direction. The change in free energy due to the presence of the static color sources in color singlet (F_1) and color octet (F_8) states can be calculated in terms of Polyakov loop correlation functions [3,49–51],

$$e^{-F_1(r)/T+C} = \frac{1}{3} \text{Tr}\langle W(\vec{x})W^\dagger(\vec{y}) \rangle, \quad (9)$$

$$e^{-F_8(r)/T+C} = \frac{1}{8} \langle \text{Tr}W(\vec{x})\text{Tr}W^\dagger(\vec{y}) \rangle - \frac{1}{24} \text{Tr}\langle W(\vec{x})W^\dagger(\vec{y}) \rangle, \quad (10)$$

where $r = |\vec{x} - \vec{y}|$. As it stands, the correlation functions for the color singlet and octet free energies are gauge dependent quantities and thus gauge fixing is needed to define them properly. Here, we follow [49] and fix to Coulomb gauge. In part, we also consider the so-called color averaged free energy defined through the manifestly gauge independent correlation function of two Polyakov loops,

$$e^{-F_{\bar{q}q}(r)/T+C} = \frac{1}{9} \langle \text{Tr}W(\vec{x})\text{Tr}W^\dagger(0) \rangle = \langle L(\vec{x})L^\dagger(\vec{y}) \rangle. \quad (11)$$

The constant C appearing in (9)–(11) also includes divergent self-energy contributions which require renormalization. Following [12] the free energies have been normalized such that the color singlet free energy approaches the heavy quark potential (solid line) at the smallest distance available on the lattice, $F_1(r/a = 1, T) = V(r)$. In Sec. III B we will explain the connection

TABLE II. Renormalization constants, $Z(g, m)$, versus β and the renormalized Polyakov loop, L^{ren} , versus T/T_c for 2-flavor QCD with quark mass $m/T = 0.4$.

β	$Z(g, m)$	T/T_c	$L^{\text{ren}}(T)$
3.52	1.333(19)	0.76	0.033(2)
3.55	1.351(10)	0.81	0.049(2)
3.60	1.370(08)	0.90	0.093(2)
3.63	1.376(07)	0.96	0.160(3)
3.65	1.376(07)	1.00	0.241(5)
3.66	1.375(06)	1.02	0.290(5)
3.68	1.370(06)	1.07	0.398(7)
3.72	1.374(02)	1.16	0.514(3)
3.75	1.379(02)	1.23	0.575(2)
3.80	1.386(01)	1.36	0.656(2)
3.85	1.390(01)	1.50	0.722(2)
3.90	1.394(01)	1.65	0.779(1)
3.95	1.396(13)	1.81	0.828(3)
4.00	1.397(01)	1.98	0.874(1)
4.43	1.378(01)	4.01	1.108(2)

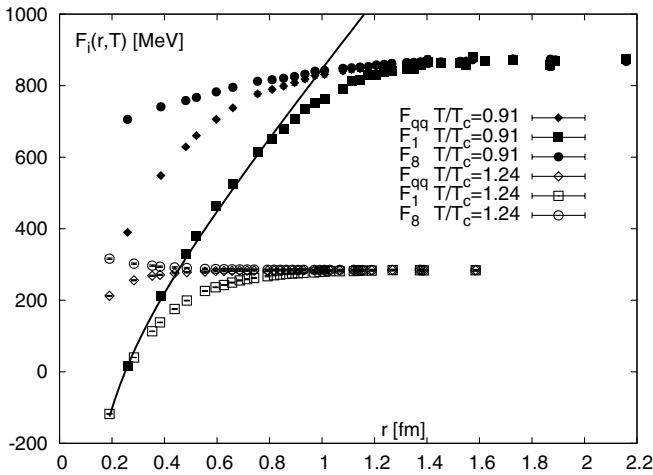


FIG. 4. Heavy quark free energies for 2 flavors of dynamical quarks at a quark mass of $m/T = 0.4$ calculated on $16^3 \times 4$ lattices. Shown are the free energies in different color channels, the singlet (F_1), octet (F_8), and color averaged ($F_{\bar{q}q}$) free energies normalized here as discussed in [12] to the zero temperature potential obtained in Sec. II (solid line).

of this procedure to the renormalized Polyakov loop and show the resulting renormalization constants in Table II.

Some results for the color singlet, octet, and averaged quark-antiquark free energies are shown in Fig. 4 for one temperature below and one temperature above deconfinement, respectively. The free energies calculated in different color channels coincide at large distances and clearly show the effects from string breaking below and color screening above deconfinement. The octet free energies above T_c are repulsive for all distances while below T_c the distances analyzed here are not small enough to show the (perturbatively) expected repulsive short distance part. Similar results are obtained at all temperatures analyzed here. In the remainder of this section we study in detail the thermal modifications of these free energies from short to large distances. We begin our analysis of the free energies at small distances in Sec. III A with a discussion of the running coupling which leads to the renormalization of the free energies in Sec. III B. The separation of small and large distances which characterizes sudden qualitative changes in the free energy will be discussed in Sec. III C. Large distance modifications of the quark-antiquark free energy will be studied in Sec. III D at temperatures above and in Sec. III E at temperatures below deconfinement.

Our analysis of thermal modifications of the strong interaction will mainly be performed for the color singlet free energy. In this case a rather simple Coulombic r dependence is suggested by perturbation theory at $T = 0$ and short distances as well as for large distances at high temperatures. In particular, a proper r dependence of $F_{\bar{q}q}$ is difficult to establish [12]. This may be attributed to contributions from higher excited states [52] or to the repulsive

contributions from states with static charges fixed in an octet configuration.

A. The running coupling at $T \neq 0$

We extend here our studies of the coupling at zero temperature to finite temperatures below and above deconfinement following the conceptual approach given in [20]. In this case the appropriate observable is the color singlet quark-antiquark free energy and its derivative. We use the perturbative short and large distance relation from one gluon exchange [3,50,51], i.e. in the limit $r\Lambda_{\text{QCD}} \ll 1$ zero temperature perturbation theory suggests

$$F_1(r, T) \equiv V(r) \simeq -\frac{4}{3} \frac{\alpha(r)}{r}, \quad (12)$$

while high temperature perturbation theory, i.e. $rT \gg 1$ and T well above T_c , yields

$$F_1(r, T) \simeq -\frac{4}{3} \frac{\alpha(T)}{r} e^{-m_D(T)r}. \quad (13)$$

In both relations we have neglected any constant contributions to the free energies which, in particular, at high temperatures will dominate the large distance behavior of the free energies. Moreover, we already anticipated here the running of the couplings with the expected dominant scales r and T in both limits. At finite temperature we define the running coupling in analogy to $T = 0$ as (see [12,20])

$$\alpha_{qq}(r, T) \equiv \frac{3}{4} r^2 \frac{dF_1(r, T)}{dr}. \quad (14)$$

With this definition any undetermined constant contributions to the free energies are eliminated and the coupling defined here at finite temperature will recover the coupling at zero temperature defined in (6) in the limit of small distances. Therefore $\alpha_{qq}(r, T)$ will show the (zero temperature) weakening in the short distance perturbative regime. In the large distance limit, however, the coupling will be dominated by Eq. (13) and will be suppressed by color screening, $\alpha_{qq}(r, T) \simeq \alpha(T) \exp(-m_D(T)r)$, $rT \gg 1$. It thus will exhibit a maximum at some intermediate distance. Although in the large distance regime $\alpha_{qq}(r, T)$ will be suppressed by color screening and thus nonperturbative effects will strongly control the value of $\alpha_{qq}(r, T)$, in this limit the temperature dependence of the coupling, $\alpha(T)$, can be extracted by directly comparing the singlet free energy with the high temperature perturbative relation above deconfinement. Results from such an analysis will be given in Sec. III D.

We calculated the derivative, dF_1/dr , of the color singlet free energies with respect to distance by using cubic spline approximations of the r dependence of the free energies for each temperature. We then performed the derivatives on the basis of these splines. Our results for $\alpha_{qq}(r, T)$ calculated in this way are shown in Fig. 5 and are

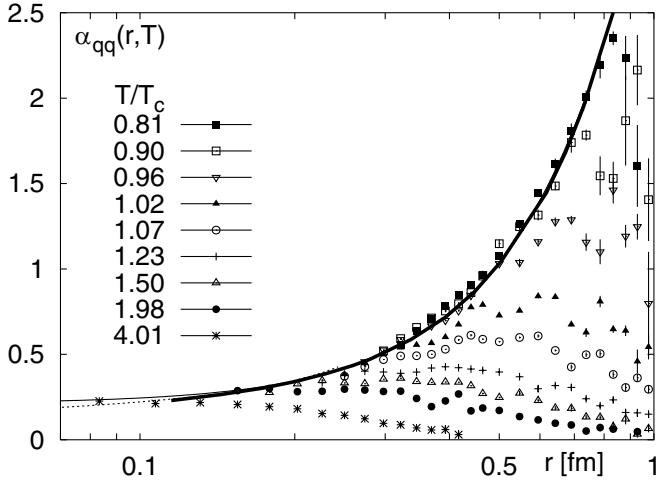


FIG. 5. The running coupling in the qq scheme defined in Eq. (14) calculated from derivatives of the color singlet free energies with respect to r at several temperatures as a function of distance below and above deconfinement. We also show the corresponding coupling at zero temperature (solid line) from Eq. (4) and compare the results again to the results in pure gauge theory (thick solid and dashed lines) [35,48].

compared to the coupling at zero temperature discussed already in Sec. II B. Here the thin solid line corresponds to the coupling in the Cornell Ansatz deduced in Eq. (4). We again show in this figure the results from $SU(3)$ -lattice (thick line) and perturbative (dashed line) calculations at zero temperature from [35,48]. The strong r dependence of the running coupling near T_c observed already in pure gauge theory [20] is also visible in 2-flavor QCD. Although our data for 2-flavor QCD do not allow for a detailed quantitative analysis of the running coupling at smaller distances, the qualitative behavior is in quite good agreement with the recent quenched results. At large distances the running coupling shows a strong temperature dependence which sets in at shorter separations with increasing temperature. At temperatures close to but above T_c , $\alpha_{qq}(r, T)$ coincides with $\alpha_{qq}(r)$ already at separations $r \approx 0.4$ fm and clearly mimics here the confinement part of $\alpha_{qq}(r)$. This is also apparent in quenched QCD [20]. Remnants of the confinement part of the QCD force may survive the deconfinement transition and could play an important role for the discussion of nonperturbative aspects of quark-antiquark interactions at temperatures moderately above T_c [15,17]. A clear separation of the different effects usually described by the concept of color screening ($T \gtrsim T_c$) and effects usually described by the concept of string breaking ($T \lesssim T_c$) is difficult to establish at temperatures in the close vicinity of the confinement-deconfinement crossover.

We also analyzed the size of the maximum that the running coupling $\alpha_{qq}(r, T)$ at fixed temperature exhibits at a certain distance, r_{\max} , i.e. we identify a temperature dependent coupling, $\tilde{\alpha}_{qq}(T)$, defined as

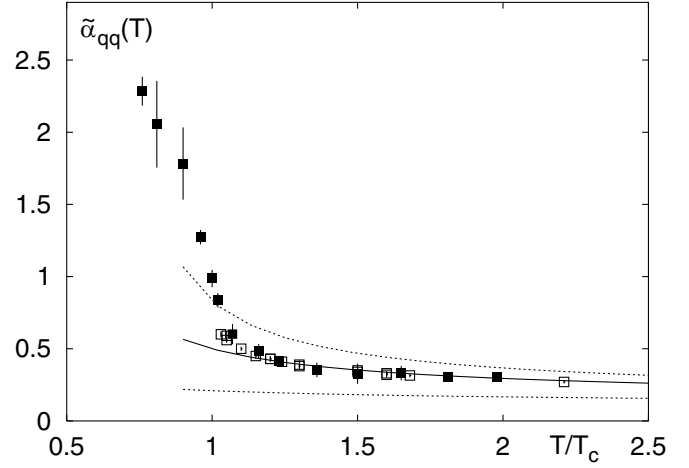


FIG. 6. The size of the maximum, $\tilde{\alpha}_{qq}(T)$, defined in Eq. (15), as a function of temperature in 2-flavor QCD (filled symbols) and pure gauge theory (open symbols) from [20]. The lines are explained in the text.

$$\tilde{\alpha}_{qq}(T) \equiv \alpha_{qq}(r_{\max}, T). \quad (15)$$

The values for r_{\max} will be discussed in Sec. III C [see Fig. 8]. Values for $\tilde{\alpha}_{qq}(T)$ are also available in pure gauge theory [20] at temperatures above deconfinement.² Our results for $\tilde{\alpha}_{qq}(T)$ in 2-flavor QCD and pure gauge theory are shown in Fig. 6 as a function of temperature, T/T_c . At temperatures above deconfinement we cannot identify significant differences between the data from pure gauge and 2-flavor QCD.³ Only at temperatures quite close to but above the phase transition small differences between full and quenched QCD become visible in $\tilde{\alpha}_{qq}(T)$. Nonetheless, the value of $\tilde{\alpha}_{qq}(T)$ drops from about 0.5 at temperatures only moderately larger than the transition temperature, $T \gtrsim 1.2T_c$, to a value of about 0.3 at $2T_c$. This change in $\tilde{\alpha}_{qq}(T)$ with temperature calculated in 2-flavor QCD does not appear to be too dramatic and might indeed be described by the 2-loop perturbative coupling,

$$g_{2\text{-loop}}^{-2}(T) = 2\beta_0 \ln\left(\frac{\mu T}{\Lambda_{\overline{MS}}}\right) + \frac{\beta_1}{\beta_0} \ln\left(2 \ln\left(\frac{\mu T}{\Lambda_{\overline{MS}}}\right)\right), \quad (16)$$

with

$$\beta_0 = \frac{1}{16\pi^2} \left(11 - \frac{2N_f}{3}\right),$$

$$\beta_1 = \frac{1}{(16\pi^2)^2} \left(102 - \frac{38N_f}{3}\right),$$

assuming vanishing quark masses. In view of the ambiguity in setting the scale in perturbation theory, μT , we per-

²In pure gauge theory r_{\max} and $\tilde{\alpha}_{qq}(T)$ would be infinite below T_c .
³Note here, however, the change in temperature scale from $T_c = 202$ MeV in full and $T_c = 270$ MeV in quenched QCD.

formed a best-fit analysis to fix the scale for the entire temperature range, $1.2 \leq T/T_c \leq 2$. We find here $\mu = 1.14(2)\pi$ with $T_c/\Lambda_{\overline{MS}} = 0.77(21)$ using $T_c \simeq 202(4)$ MeV [22] and $\Lambda_{\overline{MS}} \simeq 261(17)$ MeV [53], which is still in agreement with the lower limit of the range of scales one commonly uses to fix perturbative couplings, $\mu = \pi, \dots, 4\pi$. This is shown by the solid line (fit) in Fig. 6 including the error band estimated through $\mu = \pi$ to $\mu = 4\pi$ and the error on $T_c/\Lambda_{\overline{MS}}$ (dotted lines). We will turn back to a discussion of the temperature dependence of the coupling above deconfinement in Sec. III D.

At temperatures in the vicinity and below the phase transition temperature, $T \lesssim 1.2T_c$, the behavior of $\tilde{\alpha}_{qq}(T)$ is, however, quite different from the perturbative logarithmic change with temperature. The values for $\tilde{\alpha}_{qq}(T)$ rapidly grow here with decreasing temperature and approach nonperturbatively large values. This again shows that $\tilde{\alpha}_{qq}(r, T)$ mimics the confinement part of the zero temperature force still at relatively large distances and that this behavior persists up to temperatures close to but above deconfinement. This again demonstrates the persistence of confinement forces at $T \gtrsim T_c$ and intermediate distances and demonstrates the difficulty to separate clearly the different effects usually described by color screening and string breaking in the vicinity of the phase transition. We note here, however, that similar to the coupling in quenched QCD [20] the coupling which describes the short distance Coulombic part in the free energies is almost temperature independent in this temperature regime, i.e. even at relatively large distances the free energies shown in Fig. 1 show no or only little temperature dependence below deconfinement.

B. Renormalization of the quark-antiquark free energies and Polyakov loop

On the lattice the expectation value of the Polyakov loop and its correlation functions suffer from linear divergences. This leads to vanishing expectation values in the continuum limit, $a \rightarrow 0$, at all temperatures. To become a meaningful physical observable a proper renormalization is required [12,24,54]. We follow here the conceptual approach suggested in [12,28] and extend our earlier studies in pure gauge theory to the present case of 2-flavor QCD. First experiences with this renormalization method in full QCD were already reported in [23,33].

In the limit of short distances, $r \ll 1/T$, thermal modifications of the quark-antiquark free energy become negligible and the running coupling is controlled by distance only. Thus we can fix the free energies at small distances to the heavy quark potential, $F_1(r \ll 1/T, T) \simeq V(r)$, and the renormalization group equation (RGE) will lead to

$$\lim_{r \rightarrow 0} T \frac{dF_1(r, T)}{dT} = 0, \quad (17)$$

where we already have assumed that the continuum limit,

$a \rightarrow 0$, has been taken. On the basis of the analysis of the coupling in Sec. III A and experiences with the quark-antiquark free energy in pure gauge theory [12,20] we assume here that the color singlet free energies in 2-flavor QCD calculated on finite lattices with temporal extent $N_\tau = 4$ already have approached appropriate small distances, $r \ll 1/T$, allowing for renormalization.

The (renormalized) color singlet quark-antiquark free energies, $F_1(r, T)$, and the heavy quark potential, $V(r)$ (line), were already shown in Fig. 1(a) as a function of distance at several temperatures close to the phase transition. From that figure it can be seen that the quark-antiquark free energy fixed at small distances approaches finite, temperature dependent plateau values at large distances signaling color screening ($T \gtrsim T_c$) and string breaking ($T < T_c$). These plateau values, $F_\infty(T) \equiv F_1(r \rightarrow \infty, T)$, are decreasing with increasing temperature in the temperature range analyzed here. In general, it is expected that $F_\infty(T)$ will continue to increase at smaller temperature and will smoothly match $V(r \equiv \infty)$ [6] at zero temperature while it will become negative at high temperature and asymptotically is expected to become proportional to $g^3 T$ [12,55]. The plateau value of the quark-antiquark free energy at large distances can be used to define nonperturbatively the renormalized Polyakov loop [12], i.e.

$$L^{\text{ren}}(T) = \exp\left(-\frac{F_\infty(T)}{2T}\right). \quad (18)$$

As the unrenormalized free energies approach $|\langle L \rangle|^2$ at large distances, this may be reinterpreted in terms of a renormalization constant that has been determined by demanding (17) to hold at short distances [25,56]

$$L^{\text{ren}} \equiv |\langle (Z(g, m))^{N_\tau} L \rangle|. \quad (19)$$

The values for $Z(g, m)$ for our simulation parameters are summarized in Table II. The normalization constants for the free energies appearing in (9)–(11) are then given by

$$C = -2N_\tau Z(g, m). \quad (20)$$

An analysis of the renormalized Polyakov loop expectation value in high temperature perturbation theory [55] suggests at (resummed) leading order,⁴ the behavior

$$L^{\text{ren}}(T) \simeq 1 + \frac{2}{3} \frac{m_D(T)}{T} \alpha(T) \quad (21)$$

⁴In Ref. [55] the Polyakov loop expectation value is calculated in pure gauge theory and the Debye mass, $m_D(T)/T = \sqrt{N_c/3}g(T)$, enters here through the resummation of the gluon polarization tensor. When changing from pure gauge to full QCD quark loops will contribute to the polarization tensor. In this case resummation will lead to the Debye mass given in (24). Thus the flavor dependence in Eq. (21) at this level is given only by the Debye mass.

in the fundamental representation. Thus high temperature perturbation theory suggests that the limiting value at infinite temperature, $L^{\text{ren}}(T \rightarrow \infty) = 1$, is approached from above. An expansion of (18) then suggests $F_\infty(T) \simeq -\frac{4}{3}m_D(T)\alpha(T) \simeq -\mathcal{O}(g^3T)$. We thus expect $F_\infty(T) \rightarrow -\infty$ in the high temperature limit.

To avoid here any fit to the complicated r and T dependence of the quark-antiquark free energy we estimate the value of $F_\infty(T)$ from the quark-antiquark free energies at the largest separation available on a finite lattice, $r = N_\sigma/2$. As the free energies in this renormalization scheme coincide at large distances in the different color channels, we determine $F_\infty(T)$ from the color averaged free energies, i.e. $F_\infty(T) \equiv F_{\bar{q}q}(r = N_\sigma/2, T)$. This is a manifestly gauge invariant quantity. In Fig. 7 we show the results for L^{ren} in 2-flavor QCD (filled symbols) compared to the quenched results (open symbols) obtained in [12]. In quenched QCD L^{ren} is zero below T_c as the quark-antiquark free energy signals permanent confinement, i.e. $F_\infty(T \lesssim T_c) = \infty$ in the infinite volume limit, while it jumps to a finite value just above T_c . The singularity in the temperature dependence of $L^{\text{ren}}(T)$ located at T_c clearly signals the first order phase transition in $SU(3)$ gauge theory. The renormalized Polyakov loop in 2-flavor QCD, however, is no longer zero below T_c . Because of string breaking the quark-antiquark free energies approach constant values leading to nonzero values of L^{ren} . Although the renormalized Polyakov loop calculated in full QCD is no longer an order parameter for the confinement-deconfinement phase transition, it still shows a quite different behavior in the two phases and a clear signal for a qualitative change in the vicinity of the transition. Above deconfinement $L^{\text{ren}}(T)$ yields finite values also in quenched QCD.

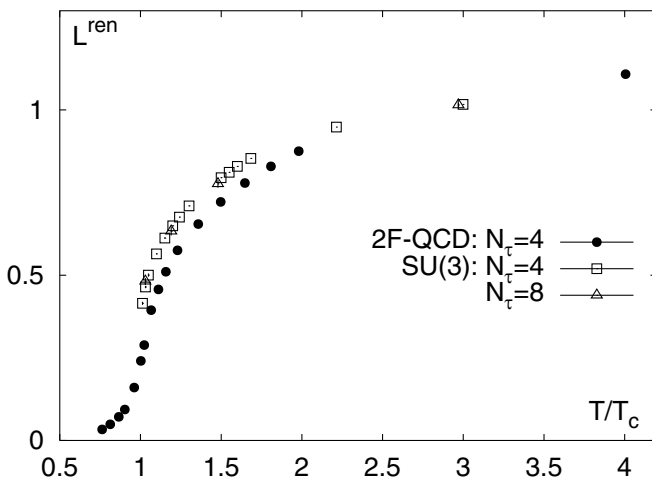


FIG. 7. The renormalized Polyakov loop in 2-flavor QCD (filled circles) compared to the quenched results (open symbols) from [12] as a function of temperature in units of the (different) transition temperatures.

In the temperature range $1 \lesssim T/T_c \lesssim 2$ we find that in 2-flavor QCD L^{ren} lies below the results in quenched QCD. This, however, may change at higher temperatures. The value for L^{ren} at $4T_c$ is larger than unity and we find indication for $L^{\text{ren}}_{2\text{-flavor}}(4T_c) \gtrsim L^{\text{ren}}_{\text{quenched}}(4T_c)$. The properties of L^{ren} , however, clearly depend on the relative normalization of the quark-antiquark free energies in quenched and full QCD.

C. Short vs large distances

Having discussed the quark-antiquark free energies at quite small distances where no or only little temperature effects influence the behavior of the free energies and at quite large distances where aside from T no other scale controls the free energy, we now turn to a discussion of medium effects at intermediate distances. The aim is to gain insight into distance scales that can be used to quantify at which distances temperature effects in the quark-antiquark free energies set in and may influence the in-medium properties of heavy quark bound states in the quark gluon plasma.

It can be seen from Fig. 1(a) that the color singlet free energy changes rapidly from the Coulomb-like short distance behavior to an almost constant value at large distances. This change reflects the in-medium properties of the heavy quark-antiquark pair, i.e. the string-breaking property and color screening. To characterize this rapid onset of in-medium modifications in the free energies we introduced in Ref. [12] a scale, r_{med} , defined as the distance at which the value of the $T = 0$ potential reaches the value $F_\infty(T)$, i.e.

$$V(r_{\text{med}}) \equiv F_\infty(T). \quad (22)$$

As $F_\infty(T)$ is a gauge invariant observable this relation provides a nonperturbative, gauge invariant definition of the scale r_{med} . While in pure gauge theory the color singlet free energies signal permanent confinement at temperatures below T_c leading to a proper definition of this scale only above deconfinement, in full QCD it can be deduced in the whole temperature range. On the other hand, the change in the coupling $\alpha_{qq}(r, T)$ as a function of distance at fixed temperature mimics the qualitative change in the interaction when going from small to large distances and the coupling exhibits a maximum at some intermediate distance. The location of this maximum, r_{max} , can also be used to identify a scale that characterizes separation between the short distance vacuumlike and the large distance medium modified interaction between the static quarks [20]. Because of the rapid crossover from short to large distance behavior [see Fig. 1(a)] it should be obvious that r_{med} and r_{max} define similar scales, however, by construction $r_{\text{max}} \lesssim r_{\text{med}}$.

To gain important information about the flavor and quark mass dependence of our analysis of the scales in

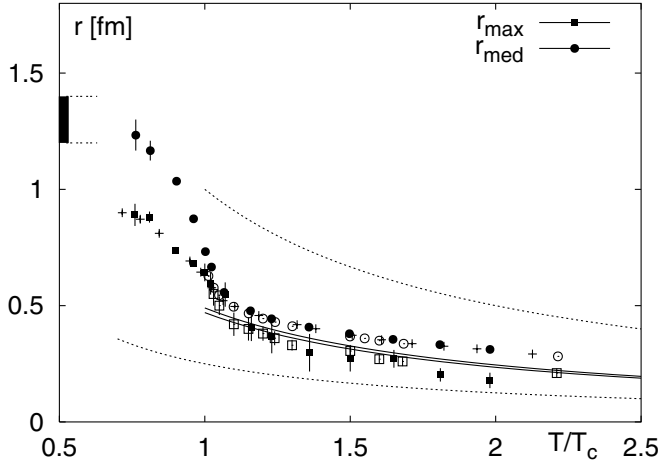


FIG. 8. The location of the maximum in $\alpha_{qq}(r, T)$ at fixed temperature, r_{\max} ($N_f = 0$: open squares; $N_f = 2$: filled squares), and our results for r_{med} ($N_f = 0$: open circles; $N_f = 2$: filled circles; $N_f = 3$: crosses) defined in Eq. (22) versus T/T_c . The band at the left frame indicates the distance range at which string breaking is expected to occur in 2-flavor QCD at $T = 0$ and quark mass $m_\pi/m_\rho \approx 0.7$ [37]. The various lines are explained in the text.

QCD, we also took data for $F_\infty(T)$ from Ref. [33] at smaller quark mass, $m_\pi/m_\rho \approx 0.4$ [32], and calculated r_{med} in 3-flavor QCD with respect to the parametrization of $V(r)$ given in [33]. It is interesting to note here that a study of the flavor and quark mass dependence of r_{med} and r_{\max} is independent of any undetermined and maybe flavor and/or quark mass dependent overall normalization of the corresponding $V(r)$ at zero temperature. Our results for r_{\max} ($N_f = 0, 2$) and r_{med} ($N_f = 0, 2, 3$) are summarized in Fig. 8 as a function of T/T_c . It can be seen that the value $r_{\max} \approx 0.6$ fm is approached almost in common in quenched and 2-flavor QCD at the phase transition and it commonly drops to about 0.25 fm at temperatures around $2T_c$. No or only little differences between r_{\max} calculated from pure gauge and 2-flavor QCD could be identified at temperatures above deconfinement. The temperature dependence of r_{med} is similar to that of r_{\max} and again we see no major differences between pure gauge ($N_f = 0$) and QCD ($N_f = 2, 3$) results. In the vicinity of the transition temperature and above, both scales almost coincide. In fact, above deconfinement the flavor and finite quark mass dependence of r_{med} appears quite negligible. At high temperature we expect $r_{\text{med}} \approx 1/gT$ [12] while in terms of r_{\max} we found agreement with $r_{\max} = 0.48(1)$ fm T_c/T (solid lines) at temperatures up to $12T_c$ [20]. Note that both scales clearly lie well above the smallest distance attainable by us on the lattice, $rT \equiv 1/N_\tau = 1/4$. This distance is shown by the lower dashed line in Fig. 8.

At temperatures below deconfinement r_{\max} and r_{med} rapidly increase and fall apart when going to smaller

temperatures. In fact, at temperatures below deconfinement we clearly see a difference between r_{med} calculated in 2- and 3-flavor QCD. To some extent this is expected due to the smaller quark mass used in the 3-flavor QCD study as the string-breaking energy gets reduced. It is, however, difficult to clearly separate here a finite quark mass effect from flavor dependence. In both cases r_{med} approaches, already at $T \approx 0.8T_c$, quite similar values to those reported for the distance where string breaking at $T = 0$ is expected at similar quark masses. In 2-flavor QCD at $T = 0$ and quark mass $m_\pi/m_\rho \approx 0.7$ the string is expected to break at about 1.2–1.4 fm [37] while at smaller quark mass, $m_\pi/m_\rho \approx 0.4$, it might break earlier [38].

In contrast to the complicated r and T dependence of the free energy at intermediate distances, high temperature perturbation theory suggests a color screened Coulomb behavior for the singlet free energy at large distances. To analyze this in more detail we show in Fig. 9 the subtracted free energies, $r(F_1(\infty, T) - F_1(r, T))$. It can be seen that this quantity indeed decays exponentially at large distances, $rT \gtrsim 1$. This allows us to study the temperature dependence of the parameters $\alpha(T)$ and $m_D(T)$ given in Eq. (13). At intermediate and small distances, however, deviations from this behavior are expected and can clearly be seen and are to some extent due to the onset of the r dependence of the coupling at small distances. These deviations from the simple exponential decay become important already below some characteristic scale, r_d , which we can roughly identify here as $r_d T \approx 0.8$ –1. This scale, which defines a lower limit for the applicability of high temperature perturbation theory, is shown by the upper dashed line in Fig. 8 ($r_d T = 1$). It lies well above the scales r_{med} and r_{\max} which characterize the onset of medium modifications on the quark-antiquark free energy.

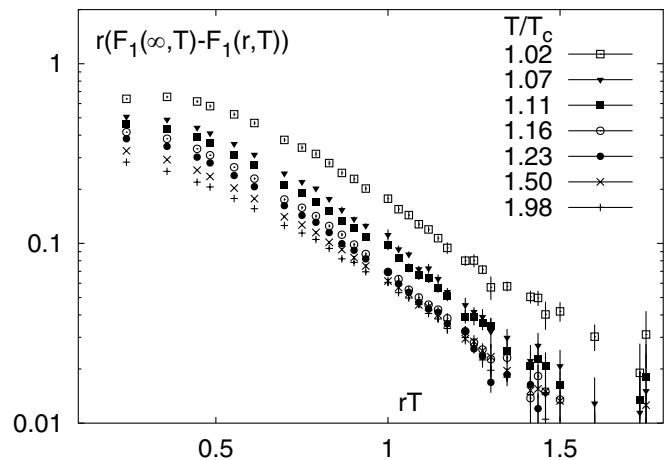


FIG. 9. The color singlet free energy versus rT obtained from lattice calculations in 2-flavor QCD at several temperatures above deconfinement.

D. Screening properties above deconfinement and the coupling

1. Screening properties and quarkonium binding

We follow here the approach commonly used [20,57,58] and define the nonperturbative screening mass, $m_D(T)$, and the temperature dependent coupling, $\alpha(T)$, from the exponential fall-off of the color singlet free energies at large distances, $rT \geq 0.8-1$. A consistent definition of screening masses, however, is accompanied by a proper definition of the temperature dependent coupling and only at sufficiently high temperatures is contact with perturbation theory expected [20,59]. A similar discussion of the color averaged quark-antiquark free energy is given in Refs. [19,60,61].

We used the Ansatz (13) to perform a best-fit analysis of the large distance part of the color singlet free energies, i.e. we used fit functions with the Ansatz

$$F_1(r, T) - F_1(r = \infty, T) = -\frac{4a(T)}{3r} e^{-m(T)r}, \quad (23)$$

where the two parameters $a(T)$ and $m(T)$ are used to estimate the coupling $\alpha(T)$ and the Debye mass $m_D(T)$, respectively. The fit range was chosen with respect to our discussion in Sec. III C, i.e. $rT \geq 0.8-1$, where we varied the lower fit limit within this range and averaged over the resulting values. The temperature dependent coupling $\alpha(T)$ defined here will be discussed later. Our results for the screening mass, $m_D(T)/T$, are summarized in Fig. 10 as a function of T/T_c and are compared to the results obtained in pure gauge theory [20]. The data obtained from our 2-flavor QCD calculations are somewhat larger than in quenched QCD. Although we are not expecting perturbation theory to hold at these small temperatures, this

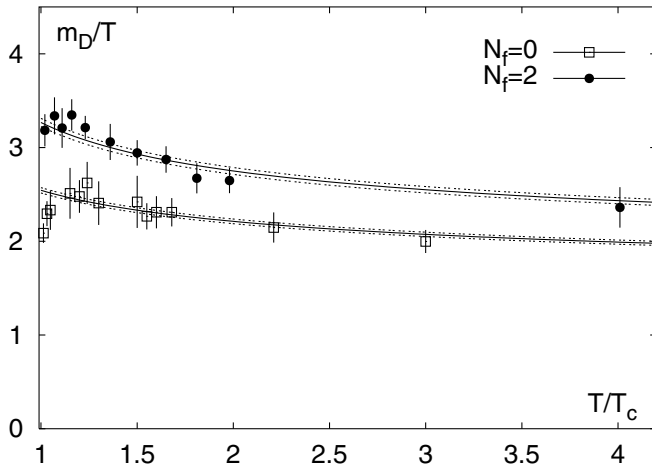


FIG. 10. The nonperturbatively defined screening masses from the large distance behavior of $F_1(r, T)$ calculated using $16^3 \times 4$ lattices as a function of temperature in 2-flavor QCD (filled circles) and in pure gauge theory (open squares [20]) using $32^3 \times 4$ lattices. The solid lines show the fit given in Eq. (23) with the corresponding error band (dotted lines).

enhancement is in qualitative agreement with leading order perturbation theory, i.e.

$$\frac{m_D(T)}{T} = \left(1 + \frac{N_f}{6}\right)^{1/2} g(T). \quad (24)$$

However, using the 2-loop formula (16) to estimate the temperature dependence of the coupling leads to significantly smaller values for m_D/T even when setting the scale by $\mu = \pi$ which commonly is used as an upper bound for the perturbative coupling. We therefore follow [19,20] and introduce a multiplicative constant, A , i.e. we allow for a nonperturbative correction defined as

$$\frac{m_D(T)}{T} \equiv A \left(1 + \frac{N_f}{6}\right)^{1/2} g_{2\text{-loop}}(T), \quad (25)$$

and fix this constant by best agreement with the nonperturbative data for $m_D(T)/T$ at temperatures $T \geq 1.2$. Here the scale in the perturbative coupling is fixed by $\mu = 2\pi$. This analysis leads to $A = 1.417(19)$ and is shown as a solid line with error band (dotted lines). Similar results were already reported in [19,20] for screening masses in pure gauge theory. Using the same fit range, i.e. $T = 1.2T_c - 4.1T_c$, for the quenched results, we obtain $A = 1.515(17)$. To avoid here any confusion concerning A we note that its value will crucially depend on the temperature range used to determine it. When approaching the perturbative high temperature limit, $A \rightarrow 1$ is expected.

It is interesting to note here that the difference in m_D/T apparent in Fig. 10 between 2-flavor QCD and pure gauge

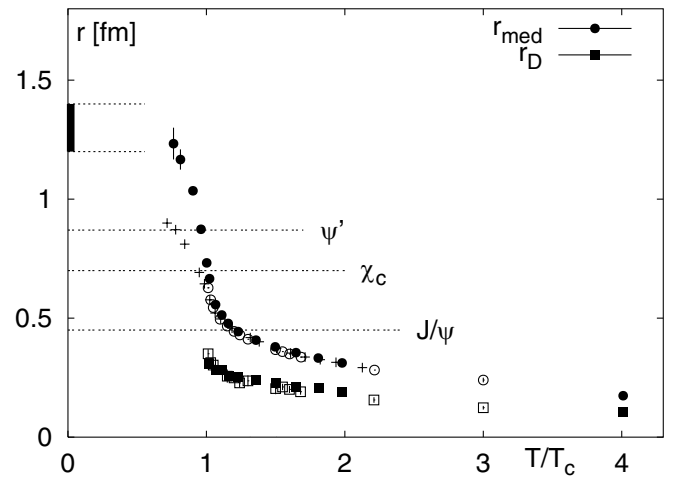


FIG. 11. The screening radius estimated from the inverse Debye mass, $r_D \equiv 1/m_D$ ($N_f = 0$: open squares; $N_f = 2$ filled squares), and the scale r_{med} ($N_f = 0$: open circles; $N_f = 2$: filled circles; $N_f = 3$: crosses) defined in (22) as a function of T/T_c . The dotted line indicates the smallest separation available on lattices with temporal extent $N_\tau = 4$. The horizontal lines give the mean squared charge radii of some charmonium states, J/ψ , χ_c , and ψ' (see also [13]) and the band at the left frame shows the distance at which string breaking is expected in 2-flavor QCD at $T = 0$ and quark mass $m_\pi/m_\rho \approx 0.7$ [37].

theory disappears when converting $m_D(T)$ to physical units. This is obvious from Fig. 11 which shows the Debye screening radius, $r_D \equiv 1/m_D$. In general, r_D is used to characterize the distance at which medium modifications of the quark-antiquark interaction become dominant. It often is used to describe the screening effects in phenomenological interquark potentials at high temperatures. From perturbation theory one expects that the screening radius will drop like $1/gT$. A definition of a screening radius, however, will again depend on the ambiguities present in the nonperturbative definition of a screening mass, $m_D(T)$. A different quantity that characterizes the onset of medium effects, r_{med} , has already been introduced in Sec. III C; this quantity is also expected to drop like $1/(gT)$ at high temperatures and could be considered to give an upper limit for the screening radius [20]. In Fig. 11 we compare both length scales as a function of temperature, T/T_c , and compare them to the findings in quenched QCD [12,20]. It can be seen that in the temperature range analyzed here, $r_D(T) < r_{\text{med}}(T)$ and no or only little differences between the results from quenched ($N_f = 0$) and full ($N_f = 2, 3$) QCD could be identified. Again we stress that in the perturbative high temperature limit differences are expected to arise as expressed by Eq. (24).

It is important to realize that at distances well below r_{med} , medium effects become suppressed and the color singlet free energy almost coincides with the zero temperature heavy quark potential [see Fig. 1(a)]. In particular, the screening radius estimated from the inverse Debye mass corresponds to distances which are only moderately larger than the smallest distance available in our calculations (compare with the lower dotted line in Fig. 8). In view of the almost temperature independent behavior of the color singlet free energies at small distances [Fig. 1(a)] it could be misleading to quantify the dominant screening length of the medium in terms of $r_D \equiv 1/m_D$. On the other hand, the color averaged free energies show already strong temperature dependence at distances similar to r_D [see Fig. 1(b)].

Following [13] we also included in Fig. 11 the mean charge radii of the most prominent charmonium states, J/ψ , χ_c , and ψ' , as horizontal lines. These lines characterize the averaged separation r which enters the effective potential in potential model calculations. It thus is reasonable to expect that the temperature at which these radii equal r_{med} could give a rough estimate for the onset of thermal effects in the charmonium states. It appears quite reasonable from this view that J/ψ indeed may survive the phase transition [10,11], while χ_c and ψ' are supposed to show significant thermal modifications at temperatures close to the transition. Recent potential model calculations support this analysis [16]. The wave functions for these states, however, will also reach out to larger distances [62] and this estimate can only be taken as a first indication for the relevant temperatures. Further details on this issue, including also bottomonium states, have been given in

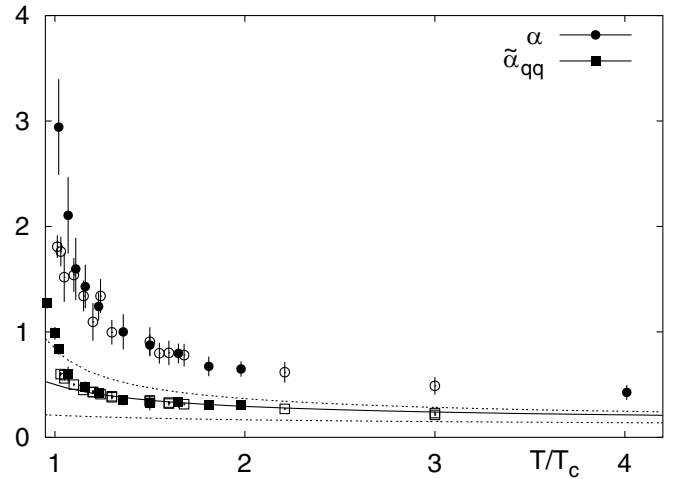


FIG. 12. The temperature dependence of the coupling $\alpha(T)$ as a function of temperature (filled circles) from fits of the large distance behavior of free energies using Eq. (23). We also show again the values of $\tilde{\alpha}_{qq}(T)$ defined as the maximum value of the running coupling $\alpha_{qq}(r, T)$ (filled squares) discussed in Sec. III A and the perturbative 2-loop coupling, Eq. (16), with scales $\mu = \pi, \dots, 4\pi$ (dashed lines). The open symbols indicate results from corresponding quenched QCD calculations [20].

Ref. [13]. We will turn again to a discussion of thermal modifications of quarkonium states in Ref. [34] using finite temperature quark-antiquark energies.

2. Temperature dependence of α_s

We finally discuss here the temperature dependence of the QCD coupling, $\alpha(T)$, extracted from the fits used to also determine m_D , i.e. from Eq. (23). From fits of the free energies above deconfinement we find the values shown in Fig. 12 as a function of T/T_c given by the filled circles. We again show in this figure the temperature dependent coupling $\tilde{\alpha}_{qq}(T)$ introduced in Sec. III A. It can clearly be seen that the values for both couplings are quite different, $\tilde{\alpha}_{qq}(T) \geq \alpha(T)$, at temperatures close to but above deconfinement while this difference rapidly decreases with increasing temperature. This again demonstrates the ambiguity in defining the coupling in the nonperturbative temperature range due to the different nonperturbative contributions to the observable used for its definition [20]. In fact, at temperatures close to the phase transition temperature we find quite large values for $\alpha(T)$, i.e. $\alpha(T) \simeq 2-3$ in the vicinity of T_c , while it drops rapidly to values smaller than unity, i.e. $\alpha(T) \lesssim 1$ already at temperatures $T/T_c \gtrsim 1.5$. A similar behavior was also found in [20] for the coupling in pure gauge theory (open symbols). In fact, no or only a marginal enhancement of the values calculated in full QCD compared to the values in quenched QCD could be identified here at temperatures $T \lesssim 1.5T_c$. We stress again that the large values for $\alpha(T)$ found here should not be confused with the coupling that characterizes the short distance Coulomb part of $F_1(r, T)$. The latter is

almost temperature independent at small distances and can to some extent be described by the zero temperature coupling.

E. String breaking below deconfinement

We finally discuss the large distance properties of the free energies below T_c . In contrast to the quark-antiquark free energy in quenched QCD where the string between the quark-antiquark pair cannot break and the free energies are linearly rising at large separations, in full QCD the string between two static color charges can break due to the possibility of spontaneously generating $q\bar{q}$ pairs from the vacuum. Therefore the quark-antiquark free energy reaches a constant value also below T_c . In Fig. 1 this behavior is clearly seen.

The distances at which the quark-antiquark free energies approach an almost constant value move to smaller separations at higher temperatures. This can also be seen from the temperature dependence of r_{med} in Fig. 8 at temperature below T_c . By construction r_{med} describes a distance which can be used to estimate a lower limit for the distance where the string breaking will set in. An estimate of the string-breaking radius at $T = 0$ can be calculated from the lightest heavy-light meson, $r_{\text{breaking}} \approx 1.2\text{--}1.4$ fm [37] and is shown on the left side in Fig. 8 within the dotted band. It can be seen that r_{med} in 2-flavor QCD does indeed approach such values at temperatures $T \lesssim 0.8T_c$. This suggests that the dependence on temperature in 2-flavor QCD

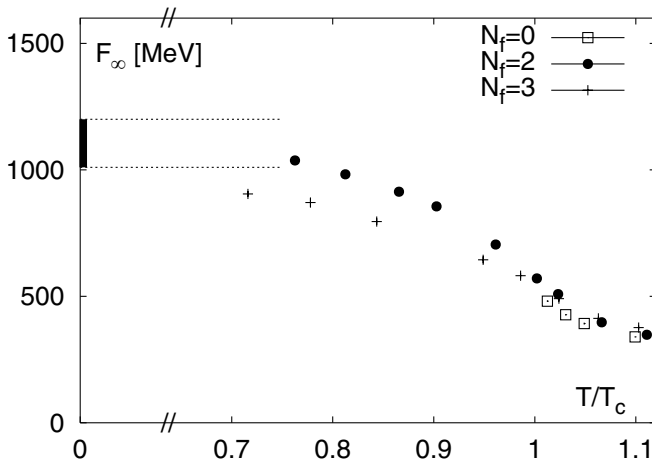


FIG. 13. The plateau value of the quark-antiquark free energy, $F_\infty(T)$, calculated in 2-flavor QCD as a function of T/T_c at temperatures in the vicinity and below the phase transition. The dashed lines show the expected value at zero temperature, $V(r_{\text{breaking}})$, with $r_{\text{breaking}} \approx 1.2 \sim 1.4$ fm using quark mass $m_\pi/m_\rho \approx 0.7$ [37]. The open symbols ($T \gtrsim T_c$) correspond to $F_\infty(T)$ in quenched QCD (from Ref. [12]) and the crosses to 3-flavor QCD studies (from [33]). The relative normalization of the corresponding $V(r)$ used for renormalization of $F_1(r \rightarrow \infty, T)$ in quenched and full QCD is such that the Cornell parametrization of $V(r)$ does not contain any constant at large distances.

is small below the smallest temperature analyzed here, $0.76T_c$. This can also be seen from the behavior of $F_\infty(T)$ shown in Fig. 13 [see also Fig. 1(a)] compared to the value commonly expected at $T = 0$. We use $V(r_{\text{breaking}}) \approx 1000\text{--}1200$ MeV as a reference to the zero temperature string-breaking energy with quark mass $m_\pi/m_\rho \approx 0.7$. This estimate is shown on the left side in Fig. 13 as the dotted band. A similar behavior is expected for the free energies in 3-flavor QCD and smaller quark mass, $m_\pi/m_\rho \approx 0.4$. As seen also in Fig. 13 the values for $F_\infty(T)$ are smaller than in 2-flavor QCD and larger quark mass. This may indicate that string breaking sets in at smaller distances for smaller quark masses. However, in [29] no mass dependence (in the color averaged free energies) was observed below the quark mass analyzed by us ($m/T = 0.4$). At present it is, however, difficult to judge whether the differences seen for 2- and 3-flavor QCD for $T/T_c < 1$ are due to quark mass or flavor dependence of the string breaking. Although $F_\infty(T)$ still is close to $V(r_{\text{breaking}})$ at $T \sim 0.8T_c$, it rapidly drops to about half of this value in the vicinity of the phase transition, $F_\infty(T_c) \approx 575$ MeV. This value is almost the same in 2- and 3-flavor QCD; we find $F_\infty^{N_f=2}(T_c) \approx 575(15)$ MeV and $F_\infty^{N_f=3}(T_c) \approx 548(20)$ MeV. It is interesting to note that the values of $F_\infty(T)$ in quenched QCD ($N_f = 0$) also approach a similar value at temperatures just above T_c . We find $F_\infty(T_c^+) \approx 481(4)$ MeV where $T_c^+ \equiv 1.02T_c$ denotes the closest temperature above T_c analyzed in quenched QCD. Of course, the value for $F_\infty(T_c^+)$ will increase when going to temperatures even closer to T_c . The flavor and quark mass dependence of $F_\infty(T)$, including also higher temperatures, will be discussed in more detail in Ref. [34].

IV. SUMMARY AND CONCLUSIONS

Our analysis of the zero temperature heavy quark potential, $V(r)$, calculated in 2-flavor lattice QCD using large Wilson loops [29] shows no signal for string breaking at distances below 1.3 fm. This is quite consistent with earlier findings [37,38]. The r dependence of $V(r)$ becomes comparable to the potential from the bosonic string picture already at distances larger than 0.4 fm. Similar findings have also been reported in lattice studies of the potential in quenched QCD [35,36]. At those distances, $0.4 \text{ fm} \lesssim r \lesssim 1.5 \text{ fm}$, we find no or only little differences between lattice data for the potential in quenched ($N_f = 0$), given in Ref. [35], and full ($N_f = 2$) QCD. At smaller distances, however, deviations from the large distance Coulomb term predicted by the string picture, $\alpha_{\text{str}} \approx 0.196$, are found here when performing a best-fit analysis with a free Cornell Ansatz. We find $\alpha \approx 0.212(3)$ which could describe the data down to $r \gtrsim 0.1$ fm. By analyzing the coupling in the qq scheme defined through the force, $dV(r)/dr$, small enhancement compared to the coupling in quenched QCD is found for $r \lesssim 0.4$ fm. At distances substantially

smaller than 0.1 fm the logarithmic weakening of the coupling enters and will dominate the r dependence of $V(r)$. The observed running of the coupling may already signal the onset of the short distance perturbative regime. This is also evident from quenched QCD lattice studies of $V(r)$ [48].

The running coupling at finite temperature defined in the qq scheme using the derivative of the color singlet quark-antiquark free energy, $dF_1(r, T)/dr$, shows only little qualitative and quantitative differences when changing from pure gauge [20] to full QCD at temperatures well above deconfinement. Again, at small distances the running coupling is controlled by distance and becomes comparable to $\alpha_{qq}(r)$ at zero temperature. The properties of $\alpha_{qq}(r, T)$ at temperatures in the vicinity of the phase transition are to a large extent controlled by the confinement signal at zero temperature. A clear separation of the different effects usually described by the concepts of color screening ($T \gtrsim T_c$) and the concept of string breaking ($T \lesssim T_c$) is difficult in the crossover region. Remnants of the confinement part of the QCD forces may, in part, dominate the nonperturbative properties of the QCD plasma at temperatures only moderately larger than T_c . This supports similar findings in recent studies of the quark-antiquark free energies in quenched QCD [20].

The properties of the quark-antiquark free energy and the coupling at small distances thus again allow for non-perturbative renormalization of the free energy and Polyakov loop [12]. The crossover from confinement to deconfinement is clearly signaled by the Polyakov loop through a rapid increase at temperatures close to T_c . String breaking dominates the quark-antiquark free energies at temperatures well below deconfinement in all color channels leading to finite values of the Polyakov loop. The string-breaking energy, $F_\infty(T)$, and the distance where string breaking sets in are decreasing with increasing temperatures. The plateau value $F_\infty(T)$ approaches about 95% of the value one usually estimates at zero temperature, $V(r_{\text{breaking}}) \simeq 1.1$ GeV [37,38], already for $T \simeq 0.8T_c$. We thus expect that the change in quark-antiquark free energies is only small when going to smaller temperatures and the quark-antiquark free energy, $F_1(r, T)$, will show only small differences from the heavy quark potential at $T = 0$, $V(r)$. Significant thermal modifications on heavy quark bound states can thus be expected only for temperatures above $0.8T_c$. Our analysis of r_{med} suggests indeed a qualitative similar behavior for the free energies in 3-flavor QCD. This can also be seen from the behavior of r_{med} shown in Fig. 11.

At temperatures well above the (pseudo) critical temperature, i.e. $1.2 \lesssim T/T_c \lesssim 4$, no or only little qualitative differences in the thermal properties of the quark-antiquark free energies calculated in quenched ($N_f = 0$) and full ($N_f = 2, 3$) QCD could be established here when converting the observables to physical units. Color screening

clearly dominates the quark-antiquark free energy at large distances, and screening masses, which are nonperturbatively determined from the exponential fall-off of the color singlet free energies, could be extracted (for $N_f = 2$). In accordance with earlier findings in quenched QCD [19,20] we find substantially larger values for the screening masses than given by leading order perturbation theory. The values of the screening masses, $m_D(T)$, again show only marginal differences as a function of T/T_c compared to the values found in quenched QCD (see also Fig. 11). The large screening mass defines a rather small screening radius, $r_D \equiv 1/m_D$, which refers to a length scale where the singlet free energy shows almost no deviations from the heavy quark potential at zero temperature. It thus might be misleading to quantify the length scale of the QCD plasma where temperature effects dominate thermal modifications on heavy quark bound states with the observable $r_D \equiv 1/m_D$ in the nonperturbative temperature regime close to but above T_c . On the other hand, the color averaged free energies show indeed strong temperature dependence at distances which could be characterized by $1/m_D$. In view of color changing processes as a mechanism for direct quarkonium dissociation [63] the discussion of the color averaged free energy could become important.

We have also compared r_D and r_{med} in Fig. 11 to the expected mean squared charge radii of some charmonium states. It is reasonable that the temperatures at which these radii equal r_{med} give a first indication of the temperature at which thermal modifications become important in the charmonium states. It appears thus quite reasonable that J/ψ will survive the transition while χ_c and ψ' are expected to show strong thermal effects at temperatures in the vicinity of the transition and this may support recent findings [11,16,64]. Of course, the wave functions of these states will also reach out to larger distances and thus our analysis can only be taken as a first indication of the relevant temperatures. We will turn back to this issue in Ref. [34]. The analysis of bound states using, for instance, the Schrödinger equation will do better in this respect. It can, however, clearly be seen from Fig. 11 that although $r_{\text{med}}(T_c) \simeq 0.7$ fm is approached almost in common for $N_f = 0, 2, 3$, it falls apart for $N_f = 2, 3$ at smaller temperatures. It thus could be difficult to determine suppression patterns from free energies for quarkonium states which are substantially larger than 0.7 fm independently from N_f and/or finite quark mass.

The analysis presented here has been performed for a single quark mass value that corresponds to a pion mass of about 770 MeV ($m_\pi/m_\rho \simeq 0.7$). In Ref. [29], however, no major quark mass effects were visible in color averaged free energies below this quark mass value. The comparisons of r_{med} and $F_\infty(T)$ calculated in 2-flavor QCD ($m_\pi/m_\rho \simeq 0.7$) with results calculated in 3-flavor QCD ($m_\pi/m_\rho \simeq 0.4$ [32]) support this property. While at temperatures above deconfinement no or only little differences

in this observable can be identified, at temperatures below T_c differences can be seen. To what extent these are due to the smaller quark masses used in the 3-flavor case or whether these differences reflect a flavor dependence of the string-breaking distance requires further investigation. The present analysis was carried out on one lattice size ($16^3 \times 4$) and therefore performing an extrapolation to the continuum limit could not be done with the current data. However, the analysis of the quenched free energies [12,20], where no major differences between the $N_\tau = 4$ and $N_\tau = 8$ results were visible, and the use of improved actions suggests that cutoff effects might be small. Despite these uncertainties and the fact that parts of our comparisons to results from quenched QCD are on a qualitative level, we find quite important information for the study of heavy quark bound states in the QCD plasma phase. At temperatures well above T_c , i.e. $1.2 \lesssim T/T_c \lesssim 4$, no or only little differences appear between results calculated in quenched and QCD. This might suggest that using thermal parameters extracted from free or internal energy in quenched QCD as input for model calculations of heavy quark bound states [15,16] is a reasonable approximation. Furthermore this also supports the investigation of heavy quarkonia in quenched lattice QCD calculations using the analysis of spectral functions [10,11,65]. On the other

hand, however, most of our 2- and 3-flavor QCD results differ from quenched calculations at temperatures in the vicinity and below the phase transition. Because of these qualitative differences, results from quenched QCD could make a discussion of possible signals for the quark gluon plasma production in heavy ion collision experiments complicated when temperatures and/or densities close to the transition become important.

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