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Combining information from $B \to \pi\pi$ and $B \to \pi\rho, \pi\omega$ decays

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We consider the $B \to \pi\pi$ and $B \to \pi\rho, \pi\omega$ decays alongside each other, taking into account the contributions from all individual penguin amplitudes generated by the internal t, c, and u quarks. We argue that three ratios of penguin amplitudes, each for a different internal quark, formed by dividing the individual penguin amplitude in $B \to \pi\pi$ by the corresponding amplitude in $B \to \pi\rho, \pi\omega$, should be equal. We study the implications of the assumed existence of this connection between $B \to \pi\pi$ and $B \to \pi\rho, \pi\omega$. First, accepting that in the $B \to \pi\pi$ decays the ratio C/T of the color-suppressed factorization amplitude C to the tree factorization amplitude T is negligible, we determine the ratio of individual penguin amplitudes. Then, from the $B \to \pi\rho, \pi\omega$ data, we extract the effective (i.e. possibly containing some penguin terms) tree and the effective color-suppressed amplitudes relevant for these processes, and the corresponding solutions for the factorization amplitudes. Finally, we argue that the C/T ratio in $B \to \pi\pi$ should be identical to its counterpart in $B \to \pi\rho, \pi\omega$ (relevant for pion emission from the decaying b quark). This constraint permits the determination of C/T and of other amplitude ratios directly from the data. Although the $|C/T|$ ratio extracted from the available data still carries a substantial error, it is consistent with the expected value of 0.25–0.5.

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I. INTRODUCTION

Decays of B mesons to charmless final states provide us with a lot of information concerning the phases of the Cabibbo-Kobayashi-Maskawa (CKM) matrix. Consequently, these decays have been the subject of numerous studies. The problem is extracting relevant information from the data, since any such procedure involves serious uncertainties resulting from our poor knowledge of the effects of strong interactions. Many papers have been devoted to the analysis of $B \to PP$ decays (with P denoting a pseudoscalar meson), in the hope that the abundance of data will permit the determination of several weak and strong parameters involved.

In recent papers [1] it was shown that the data on $B \to \pi\pi$ decays require the presence of important nonfactorizable corrections and hadronic interference effects if the SM value of $\gamma \approx 65^\circ$ and the CP-averaged $B_d \to \pi^0\pi^0$ branching ratios recently measured by *BABAR* and Belle [2] are used. The authors of Ref. [1] show in a theoretically clean way that the effective tree and color-suppressed amplitudes $\tilde{T}$ and $\tilde{C}$ governing the $B \to \pi\pi$ decays are roughly equal in absolute magnitudes, thus contradicting the expectation that the latter amplitude should be substantially suppressed. In order to conform to this expectation, one has to admit that the corrections in the effective amplitudes $\tilde{T}$ and $\tilde{C}$ arising from the usually neglected penguin contributions are substantial (see also [3,4]). Thus, hadronic-level effects appear to invalidate the naive expectation of the factorization prescription. Indeed, final-state strong interaction effects should contribute to the redefinition of the original quark-diagram amplitudes, thus generating the usually neglected penguin contributions referred to above (see Refs. [5,6]).

In general, better extraction of the relevant strong and weak parameters requires considering a larger body of data. Recently, several analyses appeared in which extraction of the angle γ of the unitarity triangle (UT) was attempted from a fit to all currently available data on $B \to PP$ or $B \to PV$ decays (see e.g. [6–8]). Such analyses in the $B \to PP$ and $B \to PV$ sectors are usually performed separately from one another. On the other hand, arguments may be given that some of the parameters, introduced in these two sectors to take account of the effects of strong interactions, are actually related. By combining in a single analysis the information from both sectors, one could then hopefully get an additional handle on the previously undetermined parameters.

In this paper, we analyze the decays of B to $\pi\pi$, $\pi\rho$, and $\pi\omega$ in an approach modeled on Ref. [1]. The data on these decays should provide sufficient information on which to base the analysis and comparison of the size of all factorization amplitudes involved. In principle, there is no need here to use the data on strangeness-changing two-body decays of B mesons, a welcome feature since $B \to \pi K$ decays exhibit various puzzles (in addition to substantial contributions from electroweak penguins), as analyzed in [1]. In practice, however, the data on $B \to \pi\pi$ are still not good enough, and we find it necessary to determine the magnitude of penguin amplitude P from $B^+ \to \pi^+ K^0$. This transition and the related $B^+ \to \pi^+ K^{*0}$ decay constitute the only places where information from the strangeness-changing sector enters into our analysis. The knowledge of $|P|$ permits the extraction of several $B \to \pi\rho, \pi\omega$ parameters from the data and the determination of amplitude ratios, such as e.g. $\tilde{T}/P$ etc. (i.e. it gives the size of various amplitudes in units of $|P|$).

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In Sec. II, we set out our notation following Refs. [1,4] and, using the Summer 2004 data given by the HFAG [9] we repeat this part of the $B \to \pi\pi$ analysis of Ref. [4], which is relevant for our purposes. We then present a simple yet illuminating formula which expresses the ratio of penguin amplitudes involving loops with different quarks in terms of the ratio of factorization amplitudes C/T and the parameters extracted from the data. In Sec. III, using the data on $B^+ \to \pi^+ K^{(*)0}$ decays, we determine the absolute magnitude of penguin amplitudes and formulate our main assumption concerning their ratios. In Sec. IV, we proceed with an analysis of the $B \to \pi\rho$ and $B \to \pi\omega$ decays. We find that there are two acceptable sets of solutions for the effective color-suppressed and tree amplitudes. Assuming that the ratio of penguin amplitudes involving loops with different quarks is well approximated by the formula of Sec. II with $C/T = 0$, we determine the tree and color-suppressed factorization amplitudes in $B \to \pi\rho, \pi\omega$. In Sec. V, we give a $B \to PV$ analog of the formula given in Sec. II, expressing the ratio of penguin amplitudes in terms of the ratio of factorization amplitudes in the $B \to \pi\rho, \pi\omega$ sector. We combine this formula with its counterpart from Sec. II and directly from the data determine both C/T and the ratio of penguin amplitudes involving loops with different quarks. Our conclusions are contained in Sec. VI.

II. DECAYS $B \to \pi\pi$

The $B \to \pi\pi$ amplitudes may be expressed in terms of amplitudes P (penguin), $\tilde{T}$ (effective tree), and $\tilde{C}$ (effective color-suppressed):

$$\sqrt{2}A(B^+ \to \pi^+\pi^0) = -[\tilde{T} + \tilde{C}] \tag{1}$$

$$A(B_d^0 \to \pi^+\pi^-) = -[\tilde{T} + P] \tag{2}$$

$$\sqrt{2}A(B_d^0 \to \pi^0\pi^0) = -[\tilde{C} - P], \tag{3}$$

with

$$P \equiv P_c = A\lambda^3\mathcal{P}_{tc} \tag{4}$$

$$\tilde{T} \equiv e^{i\gamma}(T - R_bP_u) = A\lambda^3R_be^{i\gamma}(\mathcal{T} - \mathcal{P}_{tu}) \tag{5}$$

$$\tilde{C} \equiv e^{i\gamma}(C + R_bP_u) = A\lambda^3R_be^{i\gamma}(\mathcal{C} + \mathcal{P}_{tu}), \tag{6}$$

where the right-most forms involve the definitions of Refs. [1,4], from which we omitted the contributions due to the exchange amplitudes. In principle, the latter could be included by a mere redefinition of P_{tu}. However, the general idea of this paper would then require an analogous redefinition of penguin amplitudes in $B \to \pi\rho, \pi\omega$, with the relative size of penguin and exchange amplitudes unlikely to be the same as in $B \to \pi\pi$. Only if exchange amplitudes are neglected (as is usually done and is assumed hereafter) and P_{tu} represents just the difference between the top- and up- penguins, our approach does not need additional parameters.

In Eqs. (4) and (5), $A = 0.83 \pm 0.02$, $\lambda = 0.224 \pm 0.0036$, $R_b = 0.37 \pm 0.04$, and γ parametrize the CKM matrix. $T \equiv A\lambda^3R_b\mathcal{T}$ and $C \equiv A\lambda^3R_b\mathcal{C}$ involve the strong amplitudes of color-allowed and color-suppressed tree diagrams. Finally, $P_q \equiv -\lambda_c^{(d)}\mathcal{P}_{tq} = A\lambda^3\mathcal{P}_{tq} \equiv A\lambda^3(\mathcal{P}_t - \mathcal{P}_q)$ with $\mathcal{P}_k$ describing penguin strong amplitudes corresponding to internal k-quark exchanges ($k \in \{t, c, u\}$), i.e. with the full penguin amplitude given by $\lambda_u^{(d)}\mathcal{P}_u + \lambda_c^{(d)}\mathcal{P}_c + \lambda_t^{(d)}\mathcal{P}_t$, where $\lambda_q^{(k)} = V_{qk}V_{qb}^*$, and V is the CKM matrix. For flavor-symmetric final-state interactions (FSI), the above formulas encompass all elastic and inelastic FSI with the exception of those represented by crossed diagrams (see [5,6]). When the latter are taken into account, the amplitudes T and C become mixtures of the color-allowed and color-suppressed factorization amplitudes [5,6].

A. Extraction of hadronic parameters

Introducing the hadronic parameters of Refs. [1,4]:

$$de^{i\theta} \equiv -e^{i\gamma}P/\tilde{T} = -|P/\tilde{T}|e^{-i\delta_{\tilde{T}}} = -P_c/(T - R_bP_u) \tag{7}$$

$$xe^{i\Delta} \equiv \tilde{C}/\tilde{T} = |\tilde{C}/\tilde{T}|e^{i(\delta_{\tilde{C}} - \delta_{\tilde{T}})} = (C + R_bP_u)/(T - R_bP_u) \tag{8}$$

where $\delta_{\tilde{T}}$, $\delta_{\tilde{C}}$ denote strong phases (in the convention in which the strong phase of the penguin amplitude P_c is assumed zero), one can derive the following formulas relating the parameters just introduced to the branching ratios $\mathcal{B}$ and asymmetries in $B \to \pi\pi$ decays [1]:

$$R_{+-}^{\pi\pi} = \frac{1 + 2x\cos\Delta + x^2}{1 - 2d\cos\theta\cos\gamma + d^2} \tag{9}$$

$$R_{00}^{\pi\pi} = \frac{d^2 + 2dx\cos(\Delta - \theta)\cos\gamma + x^2}{1 - 2d\cos\theta\cos\gamma + d^2} \tag{10}$$

$$A_{\pi^+\pi^-}^{\mathrm{dir}} = -\frac{2d\sin\theta\sin\gamma}{1 - 2d\cos\theta\cos\gamma + d^2} \tag{11}$$

$$A_{\pi^+\pi^-}^{\mathrm{mix}} = \frac{\sin(2\beta + 2\gamma) - 2d\cos\theta\sin(2\beta + \gamma) + d^2\sin(2\beta)}{1 - 2d\cos\theta\cos\gamma + d^2}, \tag{12}$$

with [9]

$$R_{+-}^{\pi\pi} \equiv 2\frac{\mathcal{B}(B^\pm \to \pi^\pm\pi^0)}{\mathcal{B}(B_d \to \pi^+\pi^-)}\frac{\tau_{B_d^0}}{\tau_{B^+}} = 2.20 \pm 0.31 \tag{13}$$

$$R_{00}^{\pi\pi} \equiv 2\frac{\mathcal{B}(B_d \to \pi^0\pi^0)}{\mathcal{B}(B_d \to \pi^+\pi^-)} = 0.66 \pm 0.13 \tag{14}$$

$$A^{\rm dir}_{\pi^+\pi^-} = -0.37 \pm 0.11 \tag{15}$$

$$A^{\rm mix}_{\pi^+\pi^-} = +0.61 \pm 0.14, \tag{16}$$

where, as in original papers [1,4], the asymmetries are estimated as weighted averages, in spite of the *BABAR* and Belle results still not being fully consistent. Since these averages have not changed much in comparison to information available before Summer 2004, we believe that performing the analysis of this paper for the *BABAR* and Belle asymmetries separately is unwarranted.

Assuming $\beta = 24^\circ$ and $\gamma = 65^\circ$, one can determine d and θ using Eqs. (11) and (12) and the experimental values of the asymmetries from Eqs. (15) and (16) [1]. Two solutions for (d, θ) are obtained, of which Refs. [1,4] accept only the one with small d, the other one (with $d \approx 4.6$) being excluded as it leads to complex solutions.

With the updated values of asymmetry averages, we obtain

$$d = 0.52^{+0.22}_{-0.14} \tag{17}$$

$$\theta = +(141^{+11}_{-12})^\circ. \tag{18}$$

In [1] ([4]) the corresponding values were: $d = 0.49^{+0.33}_{-0.21}$ ($0.51^{+0.26}_{-0.20}$), and $\theta = +(137^{+19}_{-23})^\circ$ ($+ (140^{+14}_{-18})^\circ$). We determine the errors as in [1], i.e. the errors associated with the specific input parameter (here: asymmetry or a $R^{\pi\pi}$ ratio) are estimated by varying its value within 1σ while keeping the other input parameters at their central values, with the individual errors thus obtained subsequently added in quadrature.

The solution of Eqs. (9), (10), (13), and (14) yields

$$x = 1.13^{+0.16}_{-0.15} \tag{19}$$

$$\Delta = -(55^{+17}_{-26})^\circ, \tag{20}$$

to be compared with $x = 1.22^{+0.25}_{-0.21}$ ($1.13^{+0.17}_{-0.16}$), and $\Delta = -(71^{+19}_{-25})^\circ$ ($- (57^{+20}_{-30})^\circ$) in Refs. [1,4] respectively. As in [1], we have discarded the second solution for x and Δ (with $x \approx 0.96$, and $\Delta \approx +33^\circ$), since the $A_{CP}(B^+ \to K^0\pi^+)$ asymmetry it yields is of the order of 0.2 (using Eq. (33) below), too large when compared with the experimental data of [9]: $A^{\rm exp}_{CP}(B^+ \to K^0\pi^+) = -0.02 \pm 0.034$. A similar argument was originally used in [1] in connection with $A_{CP}(B^+ \to K^+\pi^0)$. The experimental value of the $A_{CP}(B^+ \to K^0\pi^+)$ asymmetry puts a much stronger bound on the error in $\Delta - \theta$ (see below).

B. Ratio of penguin amplitudes

From Eqs. (7) and (8) one derives:

$$\frac{P_c}{P_u} = \frac{\mathcal{P}_{tc}}{\mathcal{P}_{tu}} = -\left(1 + \frac{C}{T}\right)\frac{R_b d e^{i\theta}}{x e^{i\Delta} - \frac{C}{T}} \tag{21}$$

Since the value of $|C/T|$ is expected to be of the order of 0.25 only, a good estimate of the ratio of penguin terms should be obtained by setting $C/T = 0$ above:

$$\frac{P_c}{P_u} \approx -\frac{R_b d e^{i\theta}}{x e^{i\Delta}} \approx (0.17^{+0.08}_{-0.05}) e^{i(16^{+21}_{-28})^\circ} \tag{22}$$

with

$$\Delta - \theta = (+164^{+28}_{-21})^\circ \tag{23}$$

where from now on we treat the errors of d, θ, x, and Δ as independent, adding in quadrature the corresponding errors to the moduli and phases of quantities depending on these parameters. Since the errors on d, θ, x, and Δ are actually interrelated, another possible treatment of errors would be to vary the original four parameters ($R^{\pi\pi}_{+-}$, $R^{\pi\pi}_{00}$, $A^{\rm dir}_{\pi^+\pi^-}$, $A^{\rm mix}_{\pi^+\pi^-}$) within their error bars. We have not chosen this more involved route since possible errors stemming from the lack of consistency among the *BABAR* and Belle asymmetry results for $A^{\rm dir}_{\pi^+\pi^-}$, $A^{\rm mix}_{\pi^+\pi^-}$ might render its higher quality questionable. Error analysis of this paper, with $R^{\pi\pi}_{+-}$, $R^{\pi\pi}_{00}$, $A^{\rm dir}_{\pi^+\pi^-}$, $A^{\rm mix}_{\pi^+\pi^-}$ varied within their error bars, could be repeated when the input data are under better control.

From Eq. (7) one further obtains :

$$\frac{T}{P_c} = R_b \frac{P_u}{P_c} - \frac{1}{d} e^{-i\theta} \approx -\frac{1 + x e^{i\Delta}}{d e^{i\theta}} \approx (3.6^{+1.4}_{-1.1}) e^{i(10^{+18}_{-15})^\circ} \tag{24}$$

and

$$\frac{T}{R_b P_u} \approx 1 + \frac{1}{x} e^{-i\Delta} \approx (1.68^{+0.19}_{-0.18}) e^{i(26^{+8}_{-12})^\circ} \tag{25}$$

which demonstrates that the size of the penguin correction term ($R_b P_u$) with respect to the tree amplitude T is large, as discussed in [1].

When small nonzero values of C/T are admitted, the ratio P_c/P_u receives a correction term

$$\delta(P_c/P_u) \approx \frac{R_b d e^{i\theta}}{x e^{i\Delta}}\left(\frac{1}{x e^{i\Delta}} - 1\right)\frac{C}{T} \approx 0.15 e^{-i40^\circ} \frac{C}{T} \tag{26}$$

whose inclusion, as shown by the right-hand side above (obtained by inserting the central values of x, d, etc.), increases the errors in our estimate of $|P_c/P_u|$ given in Eq. (22) by some 20% (for $|C/T| \approx 0.2$).

III. SIZE OF PENGUIN AMPLITUDES

Given the current errors in the determination of the $|\tilde{T}/P|$ ratio [c.f. Eq. (17)], further information on the size of amplitudes considered in Sec. II is best obtained from the decays of $B^+ \to \pi^+ K^0$ ($B^- \to \pi^- \bar{K}^0$) and $B^+ \to \pi^+ K^{*0}$ ($B^- \to \pi^- \bar{K}^{*0}$) as these are expressed in terms of penguin amplitudes alone:

$$A(B^+ \to \pi^+ K^0) = \tilde{P}' \tag{27}$$

$$A(B^+ \to \pi^+ K^{*0}) = \tilde{P}'_P, \quad (28)$$

with

$$\tilde{P}' = -\lambda_u^{(s)} \mathcal{P}_{tu} - \lambda_c^{(s)} \mathcal{P}_{tc} \quad (29)$$

$$\tilde{P}'_P = -\lambda_u^{(s)} \mathcal{P}_{P,tu} - \lambda_c^{(s)} \mathcal{P}_{P,tc}, \quad (30)$$

where the subscript P (V) for $B \to PV$ amplitudes denotes amplitudes in which the spectator quark ends in the final P (V) meson. As already remarked in the Introduction, this constitutes an implicit use of SU(3), at present necessary, given the size of errors in the $\pi\pi$ sector. In fact, knowing the absolute size of penguin amplitudes well is important in the formulas of Sec. IV [see e.g. Eqs. (69)–(72)], with the route via $K\pi$ etc. decays yielding much smaller errors. With the size of penguin amplitude depending upon the SU(3) assumption in question, the latter affects also the extracted values of tree and color-suppressed amplitudes.

A. $B^+ \to \pi^+ K^0$

Let us consider the $B^+ \to \pi^+ K^0$ and $B^- \to \pi^- \bar{K}^0$ decays. We introduce P'_c (the analog of P_c):

$$P'_c = -\lambda_c^{(s)} \mathcal{P}_{tc} = -A\lambda^2(1 - \lambda^2/2)\mathcal{P}_{tc}. \quad (31)$$

Using Eq. (22) for the ratio of $\mathcal{P}_{tu}/\mathcal{P}_{tc}$ one obtains from Eqs. (29) and (31):

$$\tilde{P}' = P'_c\left[1 + \epsilon R_b \frac{\mathcal{P}_{tu}}{\mathcal{P}_{tc}}\right] \approx P'_c\left[1 - \epsilon \frac{x}{d} \frac{e^{i\Delta}}{e^{i\theta}} e^{i\gamma}\right], \quad (32)$$

where $\epsilon = \lambda^2/(1 - \lambda^2) = 0.05$.

The CP-asymmetry $A_{CP}(B^+ \to \pi^+ K^0)$ is approximately

$$A_{CP}(B^+ \to \pi^+ K^0) \approx \frac{-2\epsilon \frac{x}{d} \sin(\Delta - \theta) \sin\gamma}{1 - 2\epsilon \frac{x}{d} \cos(\Delta - \theta) \cos\gamma} \quad (33)$$

which, together with its experimental value of -0.02 ± 0.034 and $x/d = 2.17^{+0.86}_{-0.70}$, points towards $\Delta - \theta \approx (+5^{+10}_{-9})^\circ$ or $(+174 \pm 11)^\circ$, significantly improving upon the value of Eq. (23). Thus, the experimental value of the $A_{CP}(B^+ \to \pi^+ K^0)$ asymmetry forces $\Delta - \theta$ to be close to 0° or 180°, rejecting the solution with $\Delta \approx 33^\circ$ (for which $\Delta - \theta \approx -108^\circ$). Consequently, from now on, whenever only $\Delta - \theta$ appears instead of Δ and θ, we shall use the average of the two determinations:

$$\Delta - \theta = +(173 \pm 10)^\circ. \quad (34)$$

Thus, the right-hand side of Eq. (22) is replaced with

$$\frac{P_c}{P_u} \approx (0.17^{+0.08}_{-0.05}) e^{i(7\pm 10)^\circ}. \quad (35)$$

The CP-averaged branching ratio of $B^+ \to \pi^+ K^0$ is approximately:

$$\mathcal{B}(B^\pm \to \pi^\pm K^0(\bar{K}^0)) \approx |P'_c|^2[1 - 2\epsilon \frac{x}{d} \cos(\Delta - \theta) \cos\gamma]. \quad (36)$$

Using the experimental value of this branching ratio: $\mathcal{B}_{\rm exp}(B^\pm \to \pi^\pm K^0(\bar{K}^0)) = 24.1 \pm 1.3$ (in units of 10^{-6}), $x/d = 2.17^{+0.86}_{-0.70}$, and $\Delta - \theta = +(173 \pm 10)^\circ$, one finds:

$$|P'_c|^2 \approx 22.1^{+1.3}_{-1.4} \quad (37)$$

i.e.

$$P'_c = -4.70^{+0.14}_{-0.15}, \quad (38)$$

where the strong phase of P'_c (originating from $\mathcal{P}_{tc}$) is assumed zero by SU(3) symmetry with P_c. One then finds

$$P_c = -\frac{\lambda}{1 - \lambda^2/2} P'_c = -(0.230 \pm 0.004) P'_c = 1.08 \pm 0.04 \quad (39)$$

B. $B^+ \to \pi^+ K^{*0}$

For the description of $B^+ \to \pi^+ K^{*0}$ and $B^- \to \pi^- \bar{K}^{*0}$ decays, in analogy to P'_c and P_c, we introduce:

$$P'_{P,c} = -\lambda_c^{(s)} \mathcal{P}_{P,tc} = -A\lambda^2(1 - \lambda^2/2)\mathcal{P}_{P,tc}. \quad (40)$$

To proceed we assume that

$$\frac{\mathcal{P}_{P,tu}}{\mathcal{P}_{P,tc}} = \frac{\mathcal{P}_{tu}}{\mathcal{P}_{tc}}. \quad (41)$$

The above equality follows if one accepts that the formation of the final PP or PV pair is independent of penguin transition occurring before that formation takes place. This should be so if the intermediate $s\bar{u}$ state does not remember how it was produced. The relevant penguin-induced decay amplitude becomes then a product of a penguin term and the amplitude describing the formation of the final state, with the latter amplitude canceling out in the ratios $\mathcal{P}_{P,tu}/\mathcal{P}_{P,tc}$ and $\mathcal{P}_{tu}/\mathcal{P}_{tc}$. The above assumption is a crucial assumption upon which the rest of this paper is based.

Assuming Eq. (41), an analog of Eq. (32) follows:

$$\tilde{P}'_P = P'_{P,c}\left[1 + \epsilon R_b \frac{\mathcal{P}_{tu}}{\mathcal{P}_{tc}}\right] \approx P'_{P,c}\left[1 - \epsilon \frac{x}{d} \frac{e^{i\Delta}}{e^{i\theta}} e^{i\gamma}\right]. \quad (42)$$

Consequently, from the experimental branching ratio of $\mathcal{B}_{\rm exp}(B^\pm \to \pi^\pm K^{*0}(\bar{K}^{*0})) = 9.76^{+1.16}_{-1.22}$ and using the analog of Eq. (36):

$$\mathcal{B}(B^\pm \to \pi^\pm K^{*0}(\bar{K}^{*0})) \approx |P'_{P,c}|^2\left[1 - 2\epsilon \frac{x}{d} \cos(\Delta - \theta) \cos\gamma\right] \quad (43)$$

one determines that

$$\left|\frac{P'_{P,c}}{P'_c}\right|^2 = \frac{\mathcal{B}_{\exp}(B^\pm \to \pi^\pm K^{*0}(\bar{K}^{*0}))}{\mathcal{B}_{\exp}(B^\pm \to \pi^\pm K^0(\bar{K}^0))} \quad (44)$$

Introducing the ratio $\xi \equiv P_{P,c}/P_c$ so that the $B \to PV$ amplitudes may be later expressed in units of the $B \to PP$ penguin amplitude P_c [to be compared with Eqs. (24) and (25)], one then finds that

$$\xi = P_{P,c}/P_c = \mathcal{P}_{P,tc}/\mathcal{P}_{tc} = \mathcal{P}_{P,tu}/\mathcal{P}_{tu} = P'_{P,c}/P'_c = 0.64 \pm 0.04 \quad (45)$$

with

$$P'_{P,c} = \xi P'_c = -2.99 \pm 0.21, \quad (46)$$

where we adopted the convention of a vanishing strong phase for the $P'_{P,c}$ penguin amplitude ($\mathcal{P}_{P,tc}$). One then finds

$$P_{P,c} = -(0.230 \pm 0.004) P'_{P,c} = 0.69^{+0.05}_{-0.05}. \quad (47)$$

IV. DECAYS $B \to \pi\rho, \pi\omega$

Besides the amplitudes $P_{P(V)}$ [$P_{P(V),c}$] already considered in the previous section, strangeness-conserving decays of B mesons into a pseudoscalar-vector meson pair introduce several further amplitudes: $T_{P(V)}$ (tree), $C_{P(V)}$ (color-suppressed), $S_{P(V)}$ (singlet penguin), etc., of which $T_{P(V)}, P_{P(V)}, C_{P(V)}$ are considered to be the dominant ones. The Zweig rule suggests that S_P should be negligible. On the other hand, S_V does not need to be, in analogy to the situation in the $B \to PP$ sector where decays $B \to K\eta(\eta')$ seem to indicate the non-negligible size of S' [10,11]. Since we want to restrict our analysis to a group of decays akin to $B \to \pi\pi$, i.e. not involving singlet penguin amplitudes, we are left with the following B_d^0, B^+ decay channels to be considered: $\pi^+\rho^-$, $\pi^-\rho^+$, $\pi^0\rho^+$, $\pi^+\rho^0$, $\pi^0\rho^0$, $\pi^+\omega$, and $\pi^0\omega$ (together with CP conjugate processes). By restricting our analysis to these decays, we do not need to introduce the additional parameters related to the singlet amplitudes. Our omission of the strangeness-changing $B \to PV$ decays is deliberate, since such amplitudes seem to exhibit some anomalous behavior already in the $B \to PP$ sector [12].

The respective $B \to \pi\rho, \pi\omega$ amplitudes are given by (in sign convention used e.g. in Ref. [8])

$$A(B_d^0 \to \pi^+\rho^-) = -[\tilde{T}_V + P_V] \quad (48)$$

$$A(B_d^0 \to \pi^-\rho^+) = -[\tilde{T}_P + P_P] \quad (49)$$

$$A(B^+ \to \pi^0\rho^+) = -\frac{1}{\sqrt{2}}[\tilde{T}_P + \tilde{C}_V + P_P - P_V] \quad (50)$$

$$A(B^+ \to \pi^+\rho^0) = -\frac{1}{\sqrt{2}}[\tilde{T}_V + \tilde{C}_P - P_P + P_V] \quad (51)$$

$$A(B_d^0 \to \pi^0\rho^0) = -\frac{1}{2}[\tilde{C}_P + \tilde{C}_V - P_P - P_V] \quad (52)$$

$$A(B^+ \to \pi^+\omega) = \frac{1}{\sqrt{2}}[\tilde{T}_V + \tilde{C}_P + P_P + P_V] \quad (53)$$

$$A(B_d^0 \to \pi^0\omega) = \frac{1}{2}[\tilde{C}_P - \tilde{C}_V + P_P + P_V] \quad (54)$$

where

$$P_{P(V)} \equiv P_{P(V),c} = A\lambda^3 \mathcal{P}_{P(V),tc} \quad (55)$$

and $\tilde{T}_{P(V)}$, $\tilde{C}_{P(V)}$ involve expressions similar to Eqs. (5) and (6).

Following the arguments given in [10] and used in previous discussions [11], we assume that $P_V = -P_P$, or, more precisely, that:

$$\mathcal{P}_{V,tc(u)} = -\mathcal{P}_{P,tc(u)}. \quad (56)$$

Then, defining $P_{P,u}$ and $P_{V,u}$ in analogy to Eq. (55)

$$P_{P(V),u} = A\lambda^3 \mathcal{P}_{P(V),tu} \quad (57)$$

and inserting $\mathcal{P}_{P,tc(u)} = \xi \mathcal{P}_{tc(u)}$ from Eq. (45), the amplitudes $\tilde{T}_{P(V)}$ and $\tilde{C}_{P(V)}$ may be written as

$$\tilde{T}_V = e^{i\gamma}(T_V - R_b P_{V,u}) = A\lambda^3 R_b e^{i\gamma}(\mathcal{T}_V + \xi \mathcal{P}_{tu}) \quad (58)$$

$$\tilde{T}_P = e^{i\gamma}(T_P - R_b P_{P,u}) = A\lambda^3 R_b e^{i\gamma}(\mathcal{T}_P - \xi \mathcal{P}_{tu}) \quad (59)$$

$$\tilde{C}_V = e^{i\gamma}(C_V + R_b P_{V,u}) = A\lambda^3 R_b e^{i\gamma}(\mathcal{C}_V - \xi \mathcal{P}_{tu}) \quad (60)$$

$$\tilde{C}_P = e^{i\gamma}(C_P + R_b P_{P,u}) = A\lambda^3 R_b e^{i\gamma}(\mathcal{C}_P + \xi \mathcal{P}_{tu}) \quad (61)$$

with $P_{P,u} = \xi P_u = -P_{V,u}$. In the above equations, the right-most entries are given in the form completely analogous to that used in Ref. [1], with $T_{V(P)} = A\lambda^3 R_b \mathcal{T}_{V(P)}$ and $C_{V(P)} = A\lambda^3 R_b \mathcal{C}_{V(P)}$ involving the strong amplitudes $\mathcal{T}_{V(P)}$ and $\mathcal{C}_{V(P)}$ of color-allowed and color-suppressed tree diagrams.

Since the amplitude $P_P = P_{P,c} = -P_V$ is known [Eq. (47)], from Eq. (48) and the knowledge of experimental asymmetries and branching ratios for the $B_d^0 \to \pi^+\rho^-$ decay one can determine the magnitude and (relative) phase of amplitude $\tilde{T}_V$. Then, using Eq. (58) with $P_{V,u} = -P_{P,u}$ and the estimate $P_{P,u} = P_{P,c}P_u/P_c \approx -P_{P,c} x e^{i(\Delta-\theta)}/(R_b d)$ one can extract the tree amplitude T_V. A similar procedure applied to Eqs. (49) and (59) yields tree amplitude T_P. A subsequent use of Eqs. (50) and (51), should permit the determination of C_V and C_P. The remaining three equations (52)–(54) provide additional constraints/check on the extracted values of color-suppressed amplitudes. We now turn to the extraction of the relevant amplitudes from the data.

A. Extraction of tree amplitudes T_V and T_P

In analogy with Eq. (7), we first introduce the following parameters in the $B \to PV$ sector:

$$d_P e^{i\theta_P} = -e^{i\gamma}\frac{P_P}{\tilde{T}_P} = -\frac{|P_P|}{|\tilde{T}_P|}e^{-i\delta_{\tilde{T}_P}} = -\frac{P_{P,c}}{T_P - R_b P_{P,u}} \quad (62)$$

$$d_V e^{i\theta_V} = -e^{i\gamma}\frac{P_V}{\tilde{T}_V} = +\frac{|P_P|}{|\tilde{T}_V|}e^{-i\delta_{\tilde{T}_V}} = +\frac{P_{P,c}}{T_V + R_b P_{P,u}}. \quad (63)$$

For the CP-averaged branching ratio

$$\overline{\mathcal{B}}(B_d^0 \to \pi^+\rho^-) = \frac{1}{2}[\mathcal{B}(B_d^0 \to \pi^+\rho^-) + \mathcal{B}(\bar{B}_d^0 \to \pi^-\rho^+)] \quad (64)$$

and the CP-asymmetry

$$A(B_d^0 \to \pi^+\rho^-) = \frac{\mathcal{B}(\bar{B}_d^0 \to \pi^-\rho^+) - \mathcal{B}(B_d^0 \to \pi^+\rho^-)}{\mathcal{B}(B_d^0 \to \pi^+\rho^-) + \mathcal{B}(\bar{B}_d^0 \to \pi^-\rho^+)} \quad (65)$$

one derives:

$$\overline{\mathcal{B}}(B_d^0 \to \pi^+\rho^-) = \left[1 + \frac{1}{d_V^2} - \frac{2}{d_V}\cos\theta_V\cos\gamma\right]|P_P|^2 \quad (66)$$

$$A(B_d^0 \to \pi^+\rho^-) = \frac{\frac{4}{d_V}\sin\theta_V\sin\gamma}{1 + \frac{1}{d_V^2} - \frac{2}{d_V}\cos\theta_V\cos\gamma} \quad (67)$$

with [Eq. (47)]

$$P_P = 0.69 \pm 0.05. \quad (68)$$

Solving Eqs. (66) and (67) one gets

$$\tan\theta_V = \frac{k_V}{\sin\gamma(\cos\gamma \pm \sqrt{\cos^2\gamma + l_V - 1 - k_V^2/\sin^2\gamma})} \quad (69)$$

$$d_V = \frac{\sin\theta_V\sin\gamma}{k_V} \quad (70)$$

where we defined

$$k_V \equiv \frac{A(B_d^0 \to \pi^+\rho^-)\overline{\mathcal{B}}(B_d^0 \to \pi^+\rho^-)}{4|P_P|^2} \quad (71)$$

$$l_V \equiv \overline{\mathcal{B}}(B_d^0 \to \pi^+\rho^-)/|P_P|^2. \quad (72)$$

With the experimental values:

$$\overline{\mathcal{B}}(B_d^0 \to \pi^+\rho^-) = 10.1^{+2.1}_{-1.9} \quad (73)$$

$$A(B_d^0 \to \pi^+\rho^-) = -0.47^{+0.13}_{-0.14} \quad (74)$$

taken from [9], after neglecting the correlations with $B_d^0 \to \pi^-\rho^+$, one obtains the following two solutions:

$$d_V = d_{V,1} \equiv 0.206^{+0.025}_{-0.022} \quad (75)$$

$$\theta_V = \theta_{V,1} \equiv (-34^{+12}_{-15})^\circ \quad (76)$$

or

$$d_V = d_{V,2} \equiv 0.239^{+0.037}_{-0.032} \quad (77)$$

$$\theta_V = \theta_{V,2} \equiv (-139^{+17}_{-13})^\circ. \quad (78)$$

From the CP-averaged branching ratio $\overline{\mathcal{B}}(B_d^0 \to \pi^-\rho^+)$ and asymmetry $A(B_d^0 \to \pi^-\rho^+)$ defined in analogy to Eqs. (64) and (65), using the experimental values

$$\overline{\mathcal{B}}(B_d^0 \to \pi^-\rho^+) = 13.9^{+2.2}_{-2.1} \quad (79)$$

$$A(B_d^0 \to \pi^-\rho^+) = -0.15 \pm 0.09 \quad (80)$$

from [9], one similarly obtains:

$$d_P = d_{P,1} \equiv 0.174^{+0.019}_{-0.017} \quad (81)$$

$$\theta_P = \theta_{P,1} \equiv (-12^{+7}_{-8})^\circ \quad (82)$$

or

$$d_P = d_{P,2} \equiv 0.203^{+0.026}_{-0.023} \quad (83)$$

$$\theta_P = \theta_{P,2} \equiv (-166^{+9}_{-9})^\circ. \quad (84)$$

Further experimental constraints are given by the parameters $S_{\rho\pi}$ and $\Delta S_{\rho\pi}$ extracted from the time-dependent studies of $(B_d^0, \bar{B}_d^0) \to \rho^\pm\pi^\mp$, and providing information on the relative phases of effective tree amplitudes $\tilde{T}_V$ and $\tilde{T}_P$. In terms of our amplitudes of Eqs. (48) and (49), and their CP-counterparts, these parameters are expressed as follows:

$$S_{\rho\pi} = (S_{+-} + S_{-+})/2 \quad (85)$$

$$\Delta S_{\rho\pi} = (S_{+-} - S_{-+})/2 \quad (86)$$

with

$$S_{+-} \equiv \frac{2\mathrm{Im}\lambda^{+-}}{1 + |\lambda^{+-}|^2} \quad (87)$$

$$S_{-+} \equiv \frac{2\mathrm{Im}\lambda^{-+}}{1 + |\lambda^{-+}|^2} \quad (88)$$

$$\lambda^{+-} \equiv e^{-2i\beta} \frac{A(\bar{B}_d^0 \to \rho^+\pi^-)}{A(B_d^0 \to \rho^+\pi^-)} = -e^{i(\theta_P-\theta_V-2\beta)} \frac{d_P}{d_V} \frac{e^{-i\gamma} - d_V e^{i\theta_V}}{e^{i\gamma} - d_P e^{i\theta_P}} \quad (89)$$

$$\lambda^{-+} \equiv e^{-2i\beta} \frac{A(\bar{B}_d^0 \to \rho^-\pi^+)}{A(B_d^0 \to \rho^-\pi^+)} = -e^{i(\theta_V-\theta_P-2\beta)} \frac{d_V}{d_P} \frac{e^{-i\gamma} - d_P e^{i\theta_P}}{e^{i\gamma} - d_V e^{i\theta_V}} \quad (90)$$

The four pairs of solutions, i.e. (P_1, V_1), (P_1, V_2), (P_2, V_1), and (P_2, V_2), give the four predictions for $S_{\rho\pi}$ and $\Delta S_{\rho\pi}$ gathered in Table I.

Since, according to the data [9]:

$$S_{\rho\pi} = -0.15 \pm 0.13 \quad (91)$$

$$\Delta S_{\rho\pi} = +0.25 \pm 0.13 \quad (92)$$

cases (c) and (d) may be rejected. Although case (a) is clearly the best, case (b) cannot be ruled out: the difference of the two determinations of $\Delta S_{\rho\pi}$ [case (b), Eq. (92)] is consistent with zero at (slightly above) 2σ.

In both cases (a) and (b), we have

$$e^{-i\gamma}\tilde{T}_P/P_c = (3.68^{+0.46}_{-0.43}) e^{i(-168^{+8}_{-7})^\circ}. \quad (93)$$

For $e^{-i\gamma}\tilde{T}_V/P_c$, the relative phase $\delta \equiv \mathrm{Arg}(\tilde{T}_V/\tilde{T}_P) = \theta_P - \theta_V - 180^\circ$, and the relative size $|\tilde{T}_P/\tilde{T}_V| = d_V/d_P$ of tree amplitudes, one finds the results gathered in Table II. These solutions should be compared with $\delta \approx -22^\circ$ and $|\tilde{T}_P/\tilde{T}_V| \approx 1.46$ obtained in the favored fit of Ref. [8], and corresponding to our case (b).

One of the essential differences with Ref. [8] is the fact that in our approach the effective tree amplitudes $\tilde{T}_P$ and $\tilde{T}_V$ do not correspond to the tree amplitudes T_P and T_V of the factorization picture. Instead, the effective amplitudes $\tilde{T}_P$ and $\tilde{T}_V$ involve substantial corrections to the factorization terms, due to the presence of the $P_{P,u}$ ($P_{V,u}$) part of the penguin amplitude [Eqs. (58) and (59)]. Using Eqs. (62) and (63) one can determine the tree amplitudes:

$$\frac{T_P}{P_c} = +\xi\left(R_b \frac{P_u}{P_c} - \frac{e^{-i\theta_P}}{d_P}\right) \quad (94)$$

$$\frac{T_V}{P_c} = -\xi\left(R_b \frac{P_u}{P_c} - \frac{e^{-i\theta_V}}{d_V}\right) \quad (95)$$

TABLE I. Four predictions for $S_{\rho\pi}$ and $\Delta S_{\rho\pi}$

	Case (a) (P_1, V_1)	Case (b) (P_1, V_2)	Case (c) (P_2, V_1)	Case (d) (P_2, V_2)
$S_{\rho\pi}$	$-0.27^{+0.07}_{-0.04}$	$-0.04^{+0.04}_{-0.04}$	$0.00^{+0.04}_{-0.04}$	$+0.32^{+0.04}_{-0.07}$
$\Delta S_{\rho\pi}$	$+0.38^{+0.25}_{-0.24}$	$+0.73^{+0.16}_{-0.17}$	$-0.73^{+0.19}_{-0.16}$	$-0.38^{+0.22}_{-0.24}$

TABLE II. Relative sizes and phases of effective tree amplitudes

	Case (a)	Case (b)
$e^{-i\gamma}\tilde{T}_V/P_c$	$(3.11^{+0.42}_{-0.39})e^{i(+34^{+15}_{-12})^\circ}$	$(2.68^{+0.45}_{-0.40})e^{i(+139^{+13}_{-17})^\circ}$
δ	$(-158^{+14}_{-17})^\circ$	$(-53^{+19}_{-15})^\circ$
$\lvert\tilde{T}_P/\tilde{T}_V\rvert$	$1.18^{+0.20}_{-0.16}$	$1.37^{+0.26}_{-0.22}$

From Eq. (94), using the estimate [see Eq. (22)]

$$R_b \frac{P_u}{P_c} \approx -\frac{x}{d} e^{i(\Delta-\theta)} \quad (96)$$

and Eq. (34), for both cases (a) and (b) one obtains:

$$\frac{T_P}{P_c} = (2.40^{+0.61}_{-0.61}) e^{i(-157^{+15}_{-13})^\circ}. \quad (97)$$

Similarly, from Eq. (95) one gets:

$$\text{case (a)} \quad \frac{T_V}{P_c} = (2.25^{+0.60}_{-0.50}) e^{i(+58^{+23}_{-20})^\circ} \quad (98)$$

$$\text{case (b)} \quad \frac{T_V}{P_c} = (3.91^{+0.71}_{-0.64}) e^{i(+150^{+10}_{-12})^\circ} \quad (99)$$

For case (a) one finds $|T_V/T_P| = 0.94^{+0.41}_{-0.29}$, while for case (b): $|T_V/T_P| = 1.63^{+0.63}_{-0.38}$. If the $B \to \rho$ and $B \to \pi$ form factors are similar, one expects (see [8]) that the ratio of $|T_V/T_P|$ should be approximately equal to the ratio of $f_\pi/f_\rho \approx 0.63$, as T_V (T_P) involves a weak current producing $\pi^\pm$ ($\rho^\pm$). Thus, case (a) seems favored again.

For both cases (a) and (b), however, one also estimates from Eqs. (24) and (97) that $|T_P/T| \approx 0.67^{+0.34}_{-0.25}$, which disagrees with the simple expectation (c.f. [8]) of $|T_P/T| \approx f_\rho/f_\pi = 1.59$ (while $|\tilde{T}_P/\tilde{T}| = \xi d/d_P = 1.91^{+0.84}_{-0.56}$). Still, one has to keep in mind that the above estimates are based on Eq. (96) which neglects terms of order C/T.

B. Extraction of color-suppressed amplitudes C_P and C_V

In analogy with Eqs. (62) and (63), we introduce the following parameters involving color-suppressed amplitudes $\tilde{C}_P$ and $\tilde{C}_V$:

$$y_P e^{i\Gamma_P} = -\frac{P_P}{\tilde{T}_V + \tilde{C}_P} e^{i\gamma} \quad (100)$$

$$y_V e^{i\Gamma_V} = -\frac{P_V}{\tilde{T}_P + \tilde{C}_V} e^{i\gamma}. \quad (101)$$

The CP-averaged branching ratio for the $B^+ \to \pi^+\rho^0$ decay and the corresponding asymmetry are given by

$$\bar{\mathcal{B}}(B^+ \to \pi^+\rho^0) = \left(4 + \frac{1}{y_P^2} + 4\cos\gamma \frac{\cos\Gamma_P}{y_P}\right)\frac{P_P^2}{2} \quad (102)$$

$$A(B^+ \to \pi^+ \rho^0) = -4 \sin\gamma \frac{\sin\Gamma_P}{y_P} \frac{1}{4 + \frac{1}{y_P^2} + 4\cos\gamma \frac{\cos\Gamma_P}{y_P}}. \quad (103)$$

Using the experimental numbers of

$$\bar{\mathcal{B}}(B^+ \to \pi^+ \rho^0) = 9.1 \pm 1.3 \quad (104)$$

$$A(B^+ \to \pi^+ \rho^0) = -0.19 \pm 0.11 \quad (105)$$

one finds two solutions:

Sol. (P1)
$$y_P = y_{P,1} \equiv 0.195^{+0.028}_{-0.024} \quad (106)$$

$$\Gamma = \Gamma_{P,1} \equiv (+23 \pm 14)^\circ \quad (107)$$

and

Sol. (P2)
$$y_P = y_{P,2} \equiv 0.149^{+0.015}_{-0.014} \quad (108)$$

$$\Gamma_P = \Gamma_{P,2} \equiv (+163^{+10}_{-11})^\circ. \quad (109)$$

From Eq. (100) one has:

$$\frac{\tilde{C}_P}{P_c} = -\xi\left(\frac{1}{y_P} e^{-i\Gamma_P} + \frac{1}{d_V} e^{-i\theta_V}\right) e^{i\gamma}. \quad (110)$$

Putting the estimates of y_P, d_V etc. into Eq. (110), one obtains the values of $\frac{\tilde{C}_P}{P_c} e^{-i\gamma}$ given in Table III.

For the $B^+ \to \pi^0 \rho^+$ decays, one obtains formulas completely analogous to (102) and (103), with $y_P \to y_V$ and $\Gamma_P \to \Gamma_V$. Using the experimental branching ratio and asymmetry:

$$\bar{\mathcal{B}}(B^+ \to \pi^0 \rho^+) = 12.0 \pm 2.0 \quad (111)$$

$$A(B^+ \to \pi^0 \rho^+) = +0.16 \pm 0.13 \quad (112)$$

one finds two solutions:

Sol. (V1)
$$y_V = y_{V,1} \equiv 0.165^{+0.025}_{-0.021} \quad (113)$$

$$\Gamma_V = \Gamma_{V,1} \equiv (-21^{+18}_{-19})^\circ \quad (114)$$

and

Sol. (V2)
$$y_V = y_{V,2} \equiv 0.130^{+0.015}_{-0.013} \quad (115)$$

$$\Gamma_V = \Gamma_{V,2} \equiv (-163^{+15}_{-14})^\circ. \quad (116)$$

From Eq. (101) one has:

$$\frac{\tilde{C}_V}{P_c} = \xi\left(\frac{1}{y_V} e^{-i\Gamma_V} + \frac{1}{d_P} e^{-i\theta_P}\right) e^{i\gamma}. \quad (117)$$

Putting the estimates of y_V, d_P, etc. into Eq. (117), one obtains the values of $\frac{\tilde{C}_V}{P_c} e^{-i\gamma}$ given in Table IV.

By comparing the experimental branching ratio for $B_d^0 \to \pi^0 \rho^0$ and the bound on $B_d^0 \to \pi^0 \omega$ (from [9]) with the predictions of all combinations of entries in Tables III and IV, one finds that only cases (a,P2,V2) and (b,P1,V2) may be admitted. We shall refer to them as Solutions I and II, respectively. The corresponding predictions for the branching ratios of $B_d^0 \to \pi^0 \rho^0$ and $B_d^0 \to \pi^0 \omega$ are compared with the data in Table V.

Discrepancies with experiment observed in Table V suggest that the assumptions [in particular the SU(3) assumption of Sec. III and/or possibly Eq. (41)], which lead to the value of P_P given in Eq. (68) thereby affecting the extracted size of color-suppressed $B \to PV$ effective amplitudes, might not be wholly adequate. One needs here a way of estimating the size of P_P in the strangeness-preserving sector, which would be both sufficiently precise and less assumption-dependent.

Solution II, with $y_P \approx 0.195$, is fully consistent with the information gained from the branching ratio $\mathcal{B}(B^+ \to \pi^+ \omega) = 5.9 \pm 0.8$, which yields $y_P = 0.20 \pm 0.02$. Solution I, with $y_P \approx 0.149$, agrees with y_P determined from $B^+ \to \pi^+ \omega$ at 2σ. Thus, the $B^+ \to \pi^+ \omega$ branching ratio favors Solution II over Solution I. We recall that it is just the opposite case with the values of $S_{\rho\pi}$ and $\Delta S_{\rho\pi}$ which favor case (a) (hence Solution I) over case (b) (Solution II) by 2σ. Since for the $B_d^0 \to \pi^0 \rho^0$ branching ratio, as Table V shows, the difference between experiment and theory is 1.7σ (2.1σ) for Solution I (II), one concludes that Solution I describes the data slightly better than Solution II.

The color-suppressed factorization amplitudes may be estimated from

$$\frac{C_P}{P_c} = -\xi\left[\frac{1}{y_P} e^{-i\Gamma_P} + \frac{1}{d_V} e^{-i\theta_V} + R_b \frac{P_u}{P_c}\right] \quad (118)$$

$$\approx -\xi\left[\frac{1}{y_P} e^{-i\Gamma_P} + \frac{1}{d_V} e^{-i\theta_V} - \frac{x}{d} e^{i(\Delta-\theta)}\right], \quad (119)$$

and

TABLE III. Effective color-suppressed amplitudes $\tilde{C}_P$ from $B^+ \to \pi^+ \rho^0$ decays

$e^{-i\gamma}\tilde{C}_P/P_c$	Case (a)	Case (b)
Sol. (P1)	$(5.62^{+0.77}_{-0.84})e^{i(-175^{+11}_{-10})^\circ}$	$(1.11^{+1.08}_{-0.69})e^{i(-155^{+37}_{-57})^\circ}$
Sol. (P2)	$(1.61^{+0.98}_{-0.65})e^{i(-17^{+39}_{-26})^\circ}$	$(6.15^{+0.77}_{-0.87})e^{i(-5^{+9}_{-9})^\circ}$

TABLE IV. Effective color-suppressed amplitudes $\tilde{C}_V$ from $B^+ \to \pi^0 \rho^+$ decays

	$\frac{\tilde{C}_V}{P_c} e^{-i\gamma}$
Sol. (V1)	$7.53^{+0.84}_{-0.81} e^{i(+17^{+11}_{-10})^\circ}$
Sol. (V2)	$2.47^{+1.17}_{-0.97} e^{i(+117^{+28}_{-22})^\circ}$

TABLE V. Branching ratios for $B_d^0 \to \pi^0\rho^0$ and $B_d^0 \to \pi^0\omega$ decays

	Exp	Sol. I	Sol. II
$B_d^0 \to \pi^0\rho^0$	5.0 ± 1.8	$1.68^{+0.83}_{-0.51}$	$0.86^{+0.90}_{-0.49}$
$B_d^0 \to \pi^0\omega$	<1.2	$2.28^{+0.81}_{-0.65}$	$2.31^{+0.87}_{-0.80}$

TABLE VI. Color-suppressed factorization amplitudes C_P and C_V obtained for $C/T = 0$

	Sol. I	Sol. II
C_P	$\mathrm{Re}(C_P/P_c) = 0.15^{+0.88}_{-0.86}$ $\mathrm{Im}(C_P/P_c) = -0.31^{+1.01}_{-1.01}$	$C_P/P_c = 2.40^{+1.09}_{-0.82} e^{i(-173^{+21}_{-24})^\circ}$
C_V	$C_V/P_c = 2.05^{+1.39}_{-1.30} e^{i(+82^{+22}_{-24})^\circ}$	

$$\frac{C_V}{P_c} = \xi\left[\frac{1}{y_V} e^{-i\Gamma_V} + \frac{1}{d_P} e^{-i\theta_P} + R_b \frac{P_u}{P_c}\right] \tag{120}$$

$$\approx \xi\left[\frac{1}{y_V} e^{-i\Gamma_V} + \frac{1}{d_P} e^{-i\theta_P} - \frac{x}{d} e^{i(\Delta-\theta)}\right]. \tag{121}$$

The values of C_P and C_V obtained for both Solutions I and II assuming $C/T = 0$ [Eqs. (119) and (121)] are gathered in Table VI.

Interestingly, with the central value of $|T_P/P_c|$ being 2.40, Solution I is consistent with a small value of C_P/T_P, while for Solution II the C_P/T_P ratio is of the order of 1. On the other hand, given the central T_V/P_c value of 2.25 (3.91) for Solution I(II), respectively, [Eqs. (98) and (99)], it is Solution II for which C_V/T_V seems to be smaller. One has to remember, however, that in our calculations we used the values of x and d determined from the averages of not fully consistent asymmetries in the $\pi\pi$ sector. Furthermore, our estimate of errors in the determination of P_c/P_u did not include the errors due to nonvanishing C/T. As remarked earlier, for $|C/T| \approx 0.2$ these corrections may increase the error of P_c/P_u by 20%, affecting the ensuing discussion correspondingly (see Sec. V).

V. EXTRACTION OF C/T AND P_c/P_u

In the analysis performed so far, the ratio C/T of the factorization amplitudes in $B \to \pi\pi$ decays has been assumed to be negligible. It turns out, however, that one can actually *determine* the value of C/T directly from the data, provided one is willing to make an additional very plausible assumption. Namely, we observe that the amplitudes C, T in $B \to \pi\pi$ decay and the amplitudes C_V, T_V in $B \to \pi\rho, \pi\omega$ transitions are due to the same process, namely, a decay of b quark into a pion and a light quark. The difference between the two color-suppressed amplitudes C and C_V (and between the two tree amplitudes T and T_V) should be due only to the fact that the amplitude for the recombination of the freshly produced light quark with the spectator quark depends on whether the two recombine into a pseudoscalar (in $B \to \pi\pi$) or a vector meson (in $B \to \pi\rho, \pi\omega$). However, this dependence on the recombination amplitude should cancel in the ratios, i.e. in C/T and C_V/T_V. Consequently, we may assume that

$$\frac{C}{T} = \frac{C_V}{T_V}. \tag{122}$$

We now recall Eq. (21), which correlates the ratio P_c/P_u with the size of C/T. We seek a similar connection for the $B \to \pi\rho, \pi\omega$ sector. To this end, we observe that using the expressions (59) and (60) for the effective amplitudes in Eq. (101) we can write:

$$\frac{T_P}{P_c} + \frac{C_V}{P_c} = \xi\left(2R_b \frac{P_u}{P_c} + \frac{1}{y_V} e^{-i\Gamma_V}\right). \tag{123}$$

Now, Eqs. (62) and (63) may be rewritten as

$$\frac{T_P}{P_c} = \xi\left(R_b \frac{P_u}{P_c} - \frac{1}{d_P} e^{-i\theta_P}\right) \tag{124}$$

$$\frac{T_V}{P_c} = \xi\left(-R_b \frac{P_u}{P_c} + \frac{1}{d_V} e^{-i\theta_V}\right). \tag{125}$$

From Eqs. (123) and (124) we determine

$$\frac{C_V}{P_c} = \xi\left(R_b \frac{P_u}{P_c} + \frac{1}{y_V} e^{-i\Gamma_V} + \frac{1}{d_P} e^{-i\theta_P}\right). \tag{126}$$

Dividing Eq. (126) by Eq. (125) we obtain

$$\frac{C_V}{T_V} = \frac{R_b \frac{P_u}{P_c} + \frac{1}{y_V} e^{-i\Gamma_V} + \frac{1}{d_P} e^{-i\theta_P}}{\frac{1}{d_V} e^{-i\theta_V} - R_b \frac{P_u}{P_c}}. \tag{127}$$

The above equation may be rewritten in the form completely analogous to Eq. (21), namely:

$$\frac{P_c}{P_u} = -\left(1 + \frac{C_V}{T_V}\right) \frac{R_b}{\frac{1}{d_P} e^{-i\theta_P} + \frac{1}{y_V} e^{-i\Gamma_V} - \frac{C_V}{T_V} \frac{1}{d_V} e^{-i\theta_V}}. \tag{128}$$

In the denominator above, the first two terms partially cancel. By assuming that C_V/T_V is so small that the third term may be neglected, we obtain a counterpart of the previous estimate of P_c/P_u given in Eq. (22):

$$\frac{P_c}{P_u} = -\frac{R_b}{\frac{1}{d_P} e^{-i\theta_P} + \frac{1}{y_V} e^{-i\Gamma_V}} \approx (0.10^{+0.05}_{-0.04}) e^{i(63^{+22}_{-28})^\circ}, \tag{129}$$

which, despite the approximation involved, is consistent with Eq. (22), and thus with a large value of P_u as compared with P_c (and a small value of C/T).

If C/T is assumed equal to C_V/T_V, Eqs. (21) and (128) may be solved for C/T with the final result:

$$\frac{C}{T} = \frac{\frac{1}{d_P}e^{-i\theta_P} + \frac{1}{y_V}e^{-i\Gamma_V} - \frac{x}{d}e^{i(\Delta-\theta)}}{\frac{1}{d_V}e^{-i\theta_V} - \frac{1}{d}e^{-i\theta}}. \tag{130}$$

Let us take the central values of the parameters and discuss the denominator first. Solutions I and II differ in their values for the parameters of the pair (d_V, θ_V). For Solution I (II), the first term in the denominator has an absolute value of around 4.9 (4.2). The second term has an absolute value of around 1.9. The sum of the terms in the denominator, with phases taken into account, has an absolute value of 6.8 (4.3) for Solution I (II), respectively. As for the numerator, our previous considerations uniquely determined the values of the three numerator terms. In particular, the absolute value of the first term is equal to $1/d_P = 1/d_{P,1} \approx 5.7$, that of the second term is $1/y_V = 1/y_{V,2} \approx 7.7$, while for the third term it is equal to $x/d \approx 2.2$. It is therefore nontrivial that with the central values of phases taken into account, the sum of these terms is not large and has the absolute value of around 3.2, leading to the central value of $|C/T|$ for Solution I being 0.47. The sum of the three terms in the numerator is most sensitive to the value of angle Γ_V. If Γ_V is set at its 1σ deviation value of -177°, the absolute value of the numerator becomes 1.3 only. For Solution I, the value of $|C/T|$ would then become equal to 0.2. When all of the errors are calculated, one obtains the values given in Table VII.

The determinations of $C_V/T_V = C/T$ given in Table VII may be compared with the central value of $C_V/T_V = 0.91e^{i24^\circ}$ $(0.52e^{-i68^\circ})$ for Solution I (II) obtained in Sec. IV for $C/T = 0$. Thus, the previously obtained central value of C_V/T_V gets significantly reduced (increased) for Solution I (II). Although in Solution II the central value of $|C/T|$ is now quite large, it is also compatible with $|C/T|$ of order 0.25. Better data are clearly required.

Other estimates of C/T also lead to values of order 0.5. For example, in Ref. [1] arguments in favor of $C/T = 0.5 \times e^{i290^\circ}$ are given. Similarly, in their recent SU(3)-symmetric fit to all $B \to PP$ decays, Chiang *et al.* [3] obtain the value $|C/T| = 0.46^{+0.43}_{-0.30}$.

With the central values of $|C/T|$ in Table VII significantly larger than the expected value of around 0.25, the original estimate of P_c/P_u, obtained in Eq. (22) upon assuming $C/T = 0$, could be substantially affected. Solving Eqs. (21) and (128) for P_c/P_u, one obtains

$$\frac{P_c}{P_u} = R_b d e^{i\theta} \frac{1 - \frac{\kappa}{d}e^{-i\theta}}{1 - \frac{\kappa}{d_V}e^{-i\theta_V}}, \tag{131}$$

TABLE VII. Extracted values of C/T

	Solution I	Solution II
C/T	$(0.47^{+0.33}_{-0.30})e^{i(+47^{+24}_{-27})^\circ}$	$(0.75^{+0.52}_{-0.49})e^{i(-30\pm24)^\circ}$

TABLE VIII. Extracted values of C_P/T_P

	Solution I	Solution II
$\mathrm{Re}(C_P/T_P)$	$-0.24^{+0.23}_{-0.21}$	$+0.40^{+0.38}_{-0.30}$
$\mathrm{Im}(C_P/T_P)$	$+0.03^{+0.36}_{-0.35}$	$+0.00^{+0.28}_{-0.26}$
C_P/T_P	$(0.25^{+0.31}_{-0.21})e^{i(173^{+67}_{-66})^\circ}$	$(0.40^{+0.40}_{-0.26})e^{i(0^{+36}_{-46})^\circ}$

where

$$\kappa = \frac{1 + xe^{i\Delta}}{\frac{1}{d_P}e^{-i\theta_P} + \frac{1}{y_V}e^{-i\Gamma_V} + \frac{1}{d_V}e^{-i\theta_V}}. \tag{132}$$

Numerically, for Solution I one finds:

$$\frac{P_c}{P_u} = (0.21^{+0.09}_{-0.06})e^{i(44^{+19}_{-23})^\circ}, \tag{133}$$

which still bears resemblance to $(0.17^{+0.08}_{-0.05})e^{i(16^{+21}_{-28})^\circ}$ of Eq. (22).

On the other hand, for Solution II one obtains:

$$\frac{P_c}{P_u} = (0.71^{+1.52}_{-0.66})e^{i(+20^{+87}_{-55})^\circ}, \tag{134}$$

with error estimates so large that they admit small values for both $|P_c/P_u|$ and $|P_u/P_c|$. Again, there is a strong dependence on Γ_V, with larger values of $|P_c/P_u|$ attained when Γ_V is set at its 1σ deviation value of -148°.

From Eqs. (94), (118), and (131), one can further determine the corresponding values of C_P/T_P. They are gathered in Table VIII.

From Tables VII and VIII we see that it is Solution I which prefers smaller central values of both $C_V/T_V = C/T$ and C_P/T_P. With present errors, however, both Solutions I and II are still compatible with $|C/T|$ and $|C_P/T_P|$ of around 0.25.

For completeness, we have also calculated the ratio $|T_V/T_P|$ obtaining $0.96^{+0.19}_{-0.18}$ $(0.88^{+0.37}_{-0.26})$ for Solution I (II), respectively, (to be compared with the value of $f_\pi/f_\rho \approx 0.63$ expected in [8]).

VI. SUMMARY

In this paper we performed a joint analysis of the $B \to \pi\pi$ and $B \to \pi\rho, \pi\omega$ decays with the aim of studying the effects of the presence of two independent superpositions of penguin amplitudes on the possible values of color-suppressed and tree factorization amplitudes. Our analysis assumes that the formation of the final PP or PV pair is independent of the penguin transition occurring before that formation takes place. This constitutes a crucial assumption of our approach. The analysis yields two sets of solutions for the effective color-suppressed $(\tilde{C}_V, \tilde{C}_P)$ and tree $(\tilde{T}_P, \tilde{T}_V)$ amplitudes in the $B \to \pi\rho, \pi\omega$ transitions, with one solution weakly favored over the other one.

Assuming the C/T ratio in $B \to \pi\pi$ to be negligible, we estimated the ratio of the two superpositions of penguin amplitudes, using it subsequently to determine the values of C_V, T_V, C_P, T_P from the data. This procedure yielded two sets of numerical estimates for C_P/T_P and C_V/T_V.

By imposing the condition of equality for the ratios of C/T and C_V/T_V we determined the value of C/T directly from the data. The two solutions obtained are compatible both with a value of $|C/T|$ of around 0.25 and with the estimates from literature yielding $|C/T| \approx 0.5$, with errors still of the order of 0.3–0.4. The corresponding solutions for P_c/P_u and C_P/T_P have been given as well. One of the solutions is preferred as it yields smaller central values of both $C/T = C_V/T_V$ and C_P/T_P. Discrimination between the solutions, and a more precise determination of C/T, require better data. When such data become available, a well-defined value may be extracted for C/T along the lines similar to those presented here and compared with expectations and other estimates, providing us with more information on the C/T ratio and the expected connection between penguin amplitudes in $B \to \pi\pi$ and $B \to \pi\rho, \pi\omega$ decays.

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