Heavy quark effective theory sum rules for higher excited charmed mesons: With a view on $D_{sJ}(2632)$

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Mass of the excited $(1^-, 2^-)$, $(2^-, 3^-)$ and $(2^+, 3^+)$ $c\bar{s}$ states is calculated to the $1/m_Q$ order within the framework of the heavy quark effective theory using QCD sum rule method. The obtained kinetic energy is comparable to the c-quark mass, thus results in large $1/m_Q$ correction for the c-quark case. With the sum rules for the chromomagnetic interaction term, the mass splitting for the three doublets is presented. Based on the predicted mass spectrum the exotic $D_{sJ}(2632)$ state, if really exists, is unlikely to be a conventional orbitally higher excited $c\bar{s}$ state, although the experimental center lies within the range for the mass of the 1^- meson.

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I. INTRODUCTION

The study of the exotic meson state has been a constant interest since the establishment of QCD. In the last few years several exotic meson states, especially the charm-strange $D_{sJ}(2632)$ state [1] if confirmed, with narrow width and bizarre decay mode, have been observed on many facilities. There exist various interpretations for this newly observed state $D_{sJ}(2632)$: four quark state [2–4], conventional $c\bar{s}$ excited state [3–7] and hybrid meson state [3,4]. In order to identify this exotic state and clarify the situation, the study of the mass spectroscopy of the higher excited meson states is decorous.

The system composed of heavy-light quarks exhibits a kind of spin-flavor symmetry in the limit of infinitely heavy quark mass. Thus it is convenient to work within the framework of the heavy quark effective theory (HQET) [8,9], which incorporates the heavy quark symmetry explicitly, for the problems concerning heavy meson. In HQET, the heavy quark spin decouples from the spin of the light degrees of freedom and the properties of the heavy-light system are determined by the spin of the light degrees. For the case of meson, the lowest lying states are the S-wave states with the spin of the light degrees $s_\ell = 1/2$: the $(0^-, 1^-)$ doublet which resulted from the heavy and light spin coupling. The P-wave excitation corresponds to two series of states, one is the $s_\ell = 1/2$ series, the $(0^+, 1^+)$ doublet; the other is the $s_\ell = 3/2$ series, the $(1^+, 2^+)$ doublet. For the D-wave states, those are $(1^-, 2^-)$ and $(2^-, 3^-)$ doublets, corresponding to the spin of the light degrees of freedom $s_\ell = 3/2$ and $s_\ell = 5/2$.

For the aim of making predictions on the spectrum of mesons, the asymptotic freedom characteristic special for QCD must be considered and the nonperturbative method must be employed. We will use QCD sum rule method [10] formulated within the framework of HQET [11] to determine the spectrum. Within this approach, the masses of the ground state heavy mesons are studied [12,13], the excited masses for the strange and nonstrange $(0^+, 1^+)$, $(1^+, 2^+)$ doublets are also obtained [14–17]. The full QCD analysis for those states has been done earlier [18]. There also exist other approaches to the spectrum of the excited heavy mesons [19]. In the current work we will take the $(1^-, 2^-)$, $(2^-, 3^-)$ and $(2^+, 3^+)$ doublets into account.

II. SUM RULES AT THE LEADING ORDER

In the leading order of HQET the only remaining parameter is the effective mass $\bar{\Lambda}$, which defined as the limit $\bar{\Lambda} = \lim_{m_Q\to\infty} m_M - m_Q$, where m_M and m_Q are mass of the meson and heavy quark in consideration, respectively. Within the QCD sum rule approach, the construction of an appropriate interpolating current for the state under consideration anteceding the determination of the corresponding mass is compulsory and necessary. Here we adopt the interpolating currents built in [14] based on the study of Bethe-Salpeter equation for heavy meson. For the above mentioned three doublets we are interested in, they bear the forms [14,20]:

$$
\begin{aligned}
J^{\alpha}_{1,-} &= \sqrt{\frac{3}{4}}\bar{h}_v\Big(D^{\alpha}_t - \frac{1}{3}\gamma^{\alpha}_t \not{D}_t\Big)s,\\
\tilde{J}^{\alpha\beta}_{2,-} &= \sqrt{\frac{1}{2}}T^{\alpha\beta,\mu\nu}\bar{h}_v\gamma_5\gamma_{t\mu}D_{t\nu}s,\\
J^{\alpha\beta}_{2,-} &= \sqrt{\frac{5}{6}}T^{\alpha\beta,\mu\nu}\bar{h}_v\gamma_5\Big(D_{t\mu}D_{t\nu} - \frac{2}{5}D_{t\mu}\gamma_{t\nu}\not{D}_t\Big)s,\\
\tilde{J}^{\alpha\beta\lambda}_{3,-} &= \sqrt{\frac{1}{2}}T^{\alpha\beta\lambda,\mu\nu\sigma}\bar{h}_v\gamma_{t\mu}D_{t\nu}D_{t\sigma}s,\\
J^{\alpha\beta}_{2,+} &= \sqrt{\frac{5}{6}}T^{\alpha\beta,\mu\nu}\bar{h}_v\Big(D_{t\mu}D_{t\nu} - \frac{2}{5}D_{t\mu}\gamma_{t\nu}\not{D}_t\Big)s,\\
\tilde{J}^{\alpha\beta\lambda}_{3,+} &= \sqrt{\frac{1}{2}}T^{\alpha\beta\lambda,\mu\nu\sigma}\bar{h}_v\gamma_5\gamma_{t\mu}D_{t\nu}D_{t\sigma}s,
\end{aligned}
\tag{1}
$$

where s denotes the s-quark field and v is the heavy quark velocity, h_v is the generic velocity-dependent heavy quark effective field in HQET and $D_{t\mu} = D_\mu - v \cdot D v_\mu$, with D_μ the covariant derivative. The tensors $T^{\alpha\beta,\mu\nu}$ and $T^{\alpha\beta\lambda,\mu\nu\sigma}$ are used to symmetrize indices and are given by

$$T^{\alpha\beta,\mu\nu} = \frac{1}{2}(g_t^{\alpha\beta} g_t^{\mu\nu} + g_t^{\alpha\nu} g_t^{\beta\mu}) - \frac{1}{3} g_t^{\alpha\beta} g_t^{\mu\nu},$$
$$T^{\alpha\beta\lambda,\mu\nu\sigma} = \frac{1}{3}(g_t^{\alpha\mu} g_t^{\beta\nu} g_t^{\lambda\sigma} + g_t^{\alpha\nu} g_t^{\beta\mu} g_t^{\lambda\sigma} + g_t^{\alpha\sigma} g_t^{\beta\nu} g_t^{\lambda\mu}) - \frac{1}{3}(g_t^{\alpha\beta} g_t^{\mu\nu} g_t^{\lambda\sigma} + g_t^{\alpha\lambda} g_t^{\mu\nu} g_t^{\beta\sigma} + g_t^{\beta\lambda} g_t^{\mu\nu} g_t^{\alpha\sigma}), \tag{2}$$

with the transverse metric $g_t^{\mu\nu} = g^{\mu\nu} - v^\mu v^\nu$. It is well known that the current interpolating a state is not unique [21], the currents used here are those with minimal derivatives. Those currents with the derivative acting on the heavy quark field is not adopted here, for there is no numerically significant difference [22].

With these interpolating currents the coupling constant of the corresponding state to the vacuum can be defined. Because of the spin symmetry between the two states within one doublet, only one coupling should be considered. Below we list the definition,

$$\langle D_1^*(v,\epsilon)|J_{1,-}^{\alpha}|0\rangle = \sqrt{2M} f_{3/2,-} \epsilon^{*\alpha},$$
$$\langle D_2(v,\epsilon)|J_{2,-}^{\alpha\beta}|0\rangle = \sqrt{2M} f_{5/2,-} \epsilon^{*\alpha\beta}, \tag{3}$$
$$\langle D_2^*(v,\epsilon)|J_{2,+}^{\alpha\beta}|0\rangle = \sqrt{2M} f_{5/2,+} \epsilon^{*\alpha\beta},$$

where M is the meson mass and ϵ is the corresponding polarization tensor.

For the determination of the effective mass, we consider the two-point correlator,

$$\Pi(\omega) = i \int d^4x e^{ikx} \langle 0|TJ^+(x)J(0)|0\rangle, \tag{4}$$

in which the interpolating current J can generically be any of the above defined currents and $\omega = 2v \cdot k$ is the residual momentum. Then following the standard procedure of QCD sum rule method, after straightforward calculation and Borel transformation we arrive at the following sum rules, up to dimension 6 in the OPE:

$$2f_{3/2,-}^2 e^{-2\bar{\Lambda}/T} = \frac{3T^2}{16\pi^2}[2T^3\delta_4(\omega_c/T) - m_s T^2 \delta_3(\omega_c/T) - m_s^2 T \delta_2(\omega_c/T) + m_s^3 \delta_1(\omega_c/T)] - \frac{T}{32}\delta_0(\omega_c/T)\left\langle \frac{\alpha_s}{\pi} G^2 \right\rangle,$$
$$2f_{5/2,-}^2 e^{-2\bar{\Lambda}/T} = \frac{3T^4}{16\pi^2}[6T^3\delta_6(\omega_c/T) + 2m_s T^2 \delta_5(\omega_c/T) - 2m_s^2 T \delta_4(\omega_c/T) - m_s^3 \delta_3(\omega_c/T)] - \frac{T^2}{48}[4T\delta_2(\omega_c/T) - m_s\delta_1(\omega_c/T)]\left\langle \frac{\alpha_s}{\pi} G^2 \right\rangle, \tag{5}$$
$$2f_{5/2,+}^2 e^{-2\bar{\Lambda}/T} = \frac{3T^4}{16\pi^2}[6T^3\delta_6(\omega_c/T) - 2m_s T^2 \delta_5(\omega_c/T) - 2m_s^2 T \delta_4(\omega_c/T) + m_s^3 \delta_3(\omega_c/T)] - \frac{T^2}{48}[4T\delta_2(\omega_c/T) + m_s\delta_1(\omega_c/T)]\left\langle \frac{\alpha_s}{\pi} G^2 \right\rangle,$$

where the functions δ_n arise from the continuum subtraction and are defined as

$$\delta_n(\omega_c/T) = \frac{1}{n!}\int_{2m_s/T}^{\omega_c/T} ds s^n e^{-s}. \tag{6}$$

III. SUM RULES AT THE $1/m_Q$ ORDER

For the determination of the mass spectrum, it is not sufficient to get the leading order parameter, the effective mass, $\bar{\Lambda}$. The next order corrections come into play via the Lagrangian. To the next-to-leading order the HQET Lagrangian is

$$\mathcal{L}_{\rm eff} = \bar{h}_v iv \cdot D h_v + \frac{\mathcal{K}}{2m_Q} + \frac{S}{2m_Q} + \mathcal{O}(1/m_Q^2), \tag{7}$$

where $\mathcal{K}$ is the nonrelativistic kinetic energy operator, defined as

$$\mathcal{K} = \bar{h}_v (iD_\perp)^2 h_v \tag{8}$$

with $D_\perp^2 = D_\mu D^\mu - (v \cdot D)^2$, and S is the chromomagnetic interaction term

$$S = C_{\rm mag}(m_Q/\mu)\bar{h}_v \frac{g_s}{2}\sigma_{\mu\nu}G^{\mu\nu}h_v, \tag{9}$$

where $C_{\rm mag}(m_Q/\mu) = [\alpha_s(m_Q)/\alpha_s(\mu)]^{3/\beta_0}$ and $\beta_0 = 11 - 2n_f/3$ is the first coefficient of the β function.

Taking into account the $1/m_Q$ corrections in the Lagrangian, the meson mass formula in HQET is expressed as

$$M = m_Q + \bar{\Lambda} - \frac{1}{2m_Q}(\lambda_1 + d_M \lambda_2), \tag{10}$$

where the two additional parameters λ_1 and λ_2 at the $1/m_Q$ order are defined by two matrix elements

$$2M\lambda_1 = \langle M|\mathcal{K}|M\rangle, \qquad 2d_M C_{\rm mag} M\lambda_2 = \langle M|S|M\rangle, \tag{11}$$

the constant d_M is spin-related $d_M = d_{j,j_l}$ and $d_{j_l-1/2,j_l} = 2j_l + 2$, $d_{j_l+1/2,j_l} = -2j_l$.

The evaluation of λ_1 and λ_2 needs the consideration of the three-point correlator

$$\Sigma(\omega, \omega') = i^2 \int d^4x d^4y e^{ikx-ik'y} \langle 0|TJ^+(x)O(0)J(y)|0\rangle, \tag{12}$$

where the operator O can be $\mathcal{K}$ or S and J is still the generic interpolating current. What method we use here is following from the work of Ball and Braun [13] and is similar to our work on excited heavy baryons [22], so we will not dwell on the technical details and just give the resulted sum rules directly

$$\begin{aligned}
-4\lambda_1 f_{3/2,-}^2 e^{-2\bar{\Lambda}/T} &= \frac{15T^6}{8\pi^2}[3T\delta_6(\omega_c/T) - m_s\delta_5(\omega_c/T)] + \frac{T^2}{96}[T\delta_2(\omega_c/T) - 6m_s\delta_1(\omega_c/T)]\left\langle\frac{\alpha_s}{\pi}G^2\right\rangle,\\
-4\lambda_1 f_{5/2,-}^2 e^{-2\bar{\Lambda}/T} &= \frac{63T^8}{8\pi^2}[4T\delta_8(\omega_c/T) + m_s\delta_7(\omega_c/T)] + \frac{T^4}{16}[11T\delta_4(\omega_c/T) - 5m_s\delta_3(\omega_c/T)]\left\langle\frac{\alpha_s}{\pi}G^2\right\rangle,\\
-4\lambda_1 f_{5/2,+}^2 e^{-2\bar{\Lambda}/T} &= \frac{63T^8}{8\pi^2}[4T\delta_8(\omega_c/T) - m_s\delta_7(\omega_c/T)] + \frac{T^4}{16}[11T\delta_4(\omega_c/T) + 5m_s\delta_3(\omega_c/T)]\left\langle\frac{\alpha_s}{\pi}G^2\right\rangle,\\
4\lambda_2 f_{3/2,-}^2 e^{-2\bar{\Lambda}/T} &= \frac{4\alpha_s T^6}{3\pi^3}\left[T\delta_6(\omega_c/T) + \frac{127-40\ln 2}{75}m_s\delta_5(\omega_c/T)\right] + \frac{T^2}{72}[T\delta_2(\omega_c/T) - m_s\delta_1(\omega_c/T)]\left\langle\frac{\alpha_s}{\pi}G^2\right\rangle\\
&\quad + \frac{m_0^2}{108\pi^2}[2T\delta_1(\omega_c/T) - m_s\delta_0(\omega_c/T)]\langle\bar{s}s\rangle,\\
4\lambda_2 f_{5/2,-}^2 e^{-2\bar{\Lambda}/T} &= \frac{12T^8\alpha_s}{5\pi^3}[T\delta_8(\omega_c/T) + \frac{32\ln 2 - 1}{70}m_s\delta_7(\omega_c/T)] + \frac{T^4}{80}[2T\delta_4(\omega_c/T) - m_s\delta_3(\omega_c/T)]\left\langle\frac{\alpha_s}{\pi}G^2\right\rangle,\\
4\lambda_2 f_{5/2,+}^2 e^{-2\bar{\Lambda}/T} &= \frac{12T^8\alpha_s}{5\pi^3}\left[T\delta_8(\omega_c/T) - \frac{32\ln 2 - 1}{70}m_s\delta_7(\omega_c/T)\right] + \frac{T^4}{80}[2T\delta_4(\omega_c/T) + m_s\delta_3(\omega_c/T)]\left\langle\frac{\alpha_s}{\pi}G^2\right\rangle.
\end{aligned} \tag{13}$$

What is worth noting is that on arriving at the above listed sum rules we have made the substitution $\omega_+ = (\omega + \omega')/2$ and $\omega_- = \omega - \omega'$ in the obtained spectral densities, and the quark-hadron duality is employed after the integration over ω_-, which is similar to those previous sum rule applications [23]. In the sum rules for λ_1 and λ_2 we only include the second term in the expansion of the propagator due to the finite mass of the s quark, which is proportional to the mass m_s. On doing this we assume that the higher order contribution in the m_s expansion is negligible, which is just the case as in [16]: the numerical result changes little if the m_s term is omitted.

IV. NUMERICAL ANALYSIS

In the numerical analysis we use the standard values for the vacuum condensates

$$\langle\bar{s}s\rangle = 0.8 \times (-0.23\ \text{GeV})^3,$$
$$\left\langle\frac{\alpha_s}{\pi}G^2\right\rangle = 0.012\ \text{GeV}^4, \qquad m_0^2 = 0.8\ \text{GeV}^2. \tag{14}$$

The s-quark mass we used is $m_s = 0.15$ GeV. For three active flavors we adopt $\Lambda_{\text{QCD}} = 375$ MeV and $\Lambda_{\text{QCD}} = 220$ MeV for four active flavors.

In the numerical analysis we find for the effective mass $\bar{\Lambda}$ sum rules there exists a stability window for the Borel parameter in the intermediate region $T \sim 1$ GeV, where the continuum threshold ω_c is $\sim$3 GeV. Within this region the condensate contribution is well under control, typically less than 10% of the perturbative one. But the other criterion in sum rule applications, i.e. the ground state dominance, cannot be satisfied simultaneously. When it goes to the sum rules for λ_1 and λ_2, due to the high power of the spectral density, this phenomenon is well known from experience. So we are content with the existence of the plateau of stability.

For the numerical results, we have

$$\begin{aligned}
\bar{\Lambda}_{3/2,-} &= 1.11 \pm 0.07\ \text{GeV},\\
\bar{\Lambda}_{5/2,-} &= 1.29 \pm 0.07\ \text{GeV},\\
\bar{\Lambda}_{5/2,+} &= 1.40 \pm 0.07\ \text{GeV}
\end{aligned} \tag{15}$$

for the effective mass sum rules,

$$\lambda_1^{3/2,-} = 1.3 \pm 0.2\ \text{GeV}^2, \qquad \lambda_1^{5/2,-} = 1.5 \pm 0.2\ \text{GeV}^2,$$
$$\lambda_1^{5/2,+} = 1.5 \pm 0.2\ \text{GeV}^2 \tag{16}$$

for the λ_1 sum rules, and

$$\begin{aligned}
\lambda_2^{3/2,-} &= (10.1 \pm 2.7) \times 10^{-2}\ \text{GeV}^2,\\
\lambda_2^{5/2,-} &= (3.4 \pm 0.8) \times 10^{-2}\ \text{GeV}^2,\\
\lambda_2^{5/2,+} &= (3.1 \pm 0.4) \times 10^{-2}\ \text{GeV}^2
\end{aligned} \tag{17}$$

for the λ_2 sum rules. The dependence on the Borel parameter T and the continuum threshold ω_c is shown in Figs. 1–3.

From those numerical results it is obvious that the kinetic energy λ_1 is considerably large, which in number is almost equal to the magnitude of the c-quark mass. This may be interpreted as the signal of the break down of the heavy quark expansion for the c-quark case at the $1/m_Q$

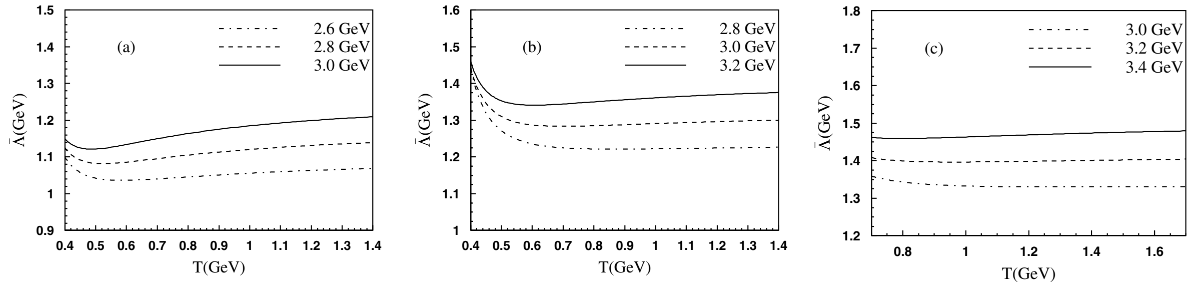

FIG. 1. QCD sum rules for the effective mass: (a) $(1^-, 2^-)$ doublet, the working region is $0.5 < T < 0.9$ GeV; (b) $(2^-, 3^-)$ doublet, the working region is $0.5 < T < 0.9$ GeV; (c) $(2^+, 3^+)$ doublet, the working region is $0.8 < T < 1.2$ GeV.

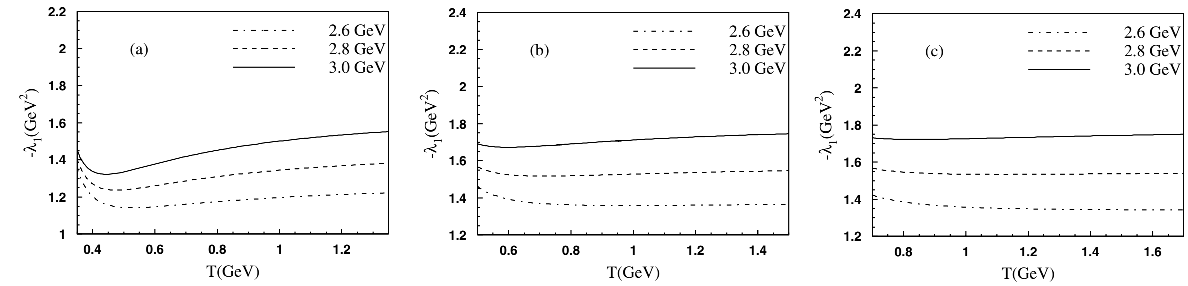

FIG. 2. QCD sum rules for the kinetic energy: (a) $(1^-, 2^-)$ doublet, the working region is $0.4 < T < 0.8$ GeV; (b) $(2^-, 3^-)$ doublet, the working region is $0.6 < T < 1.0$ GeV; (c) $(2^+, 3^+)$ doublet, the working region is $0.9 < T < 1.3$ GeV.

order, higher order corrections must be considered. For the b-quark case, this large kinetic energy is comparatively small and there is no problem at all. But it is also worthy of remark that the method we employed here tends to overestimate the kinetic energy λ_1, whose smallness is ensured by the analog of virial theorem in the field theory [24]. Neubert has proposed a procedure within the QCD sum rule approach which observes this restriction [25], but that is out of the scope of this paper. So as a compromise we can take half of the obtained value as the approximation of the real kinetic energy, this strategy is in accordance with the taking mean value solution [26,27] due to the smallness of the kinetic energy obtained in the covariant (Neubert's) approach. Thus modified kinetic energy becomes

$$\begin{aligned} \bar{\lambda}_1^{3/2,-} &= 0.65 \pm 0.08\ \mathrm{GeV}^2, \\ \bar{\lambda}_1^{5/2,-} &= 0.77 \pm 0.09\ \mathrm{GeV}^2, \\ \bar{\lambda}_1^{5/2,+} &= 0.77 \pm 0.10\ \mathrm{GeV}^2. \end{aligned} \tag{18}$$

The d_M weighted average mass is dependent of the kinetic energy only and is independent of the chromomagnetic interaction term. Such averaged mass is

$$\frac{1}{8}(3m_{D_{s1}^*} + 5m_{D_{s2}}) = m_c + (1.11 \pm 0.07\ \mathrm{GeV}) + \frac{1}{m_c}[(0.65 \pm 0.08)\ \mathrm{GeV}^2] \tag{19}$$

for the $(1^-, 2^-)$ doublet,

$$\frac{1}{12}(5m_{D_{s2}} + 7m_{D_{s3}^*}) = m_c + (1.29 \pm 0.07\ \mathrm{GeV}) + \frac{1}{m_c}[(0.77 \pm 0.09)\ \mathrm{GeV}^2] \tag{20}$$

for the $(2^-, 3^-)$ doublet, and

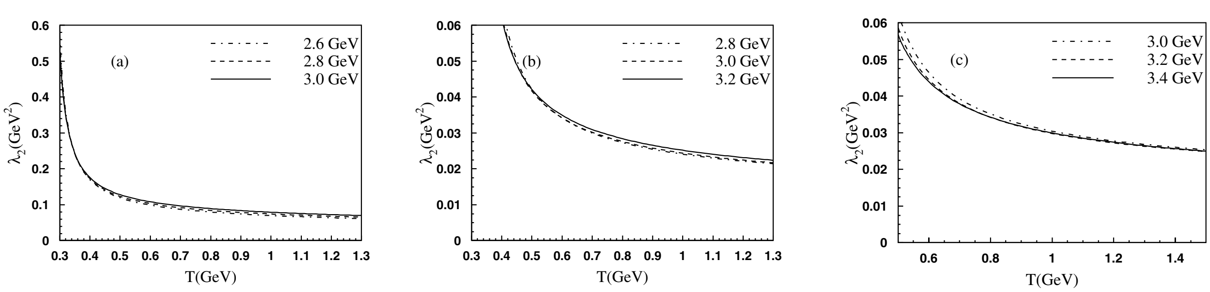

FIG. 3. QCD sum rules for the chromomagnetic interaction term: (a) $(1^-, 2^-)$ doublet, the working region is $0.5 < T < 0.9$ GeV; (b) $(2^-, 3^-)$ doublet, the working region is $0.5 < T < 0.9$ GeV; (c) $(2^+, 3^+)$ doublet, the working region is $0.8 < T < 1.2$ GeV.

$$\frac{1}{12}(5m_{D^*_{s2}} + 7m_{D_{s3}}) = m_c + (1.40 \pm 0.07\ \text{GeV}) + \frac{1}{m_c}[(0.77 \pm 0.10)\ \text{GeV}^2] \qquad (21)$$

for the $(2^+, 3^+)$ doublet. If the modified value of the kinetic energy is used, those expressions change to

$$\begin{aligned}\frac{1}{8}(3m_{D^*_{s1}} + 5m_{D_{s2}}) &= m_c + (1.11 \pm 0.07\ \text{GeV}) + \frac{1}{m_c}[(0.32 \pm 0.04)\ \text{GeV}^2],\\ \frac{1}{12}(5m_{D_{s2}} + 7m_{D^*_{s3}}) &= m_c + (1.29 \pm 0.07\ \text{GeV}) + \frac{1}{m_c}[(0.38 \pm 0.04)\ \text{GeV}^2],\\ \frac{1}{12}(5m_{D^*_{s2}} + 7m_{D_{s3}}) &= m_c + (1.40 \pm 0.07\ \text{GeV}) + \frac{1}{m_c}[(0.39 \pm 0.05)\ \text{GeV}^2].\end{aligned} \qquad (22)$$

To the order $1/m_Q$ the mass splitting is determined by chromomagnetic interaction. For the two states within one doublet, this splitting can be expressed as

$$m^2_{D_{s,j+1}} - m^2_{D_{s,j}} = 4(j+1)\lambda_2 + \mathcal{O}(1/m^2_Q). \qquad (23)$$

For the three states we are interested, they are

$$\begin{aligned} m^2_{D_{s2}} - m^2_{D^*_{s1}} &= 0.81 \pm 0.22\ \text{GeV}^2,\\ m^2_{D^*_{s3}} - m^2_{D_{s2}} &= 0.41 \pm 0.10\ \text{GeV}^2,\\ m^2_{D_{s3}} - m^2_{D^*_{s2}} &= 0.37 \pm 0.05\ \text{GeV}^2.\end{aligned} \qquad (24)$$

Changing from the c-quark case to the b-quark one, we only need to replace m_c by m_b and multiply λ_2 by a factor 0.8, since C_{mag} is approximately 0.8 when scaled up to the b-quark mass.

If we take the c-quark mass to be $m_c = 1.41 \pm 0.16$ GeV [27], then we have the following spectrum in Table I. Mass obtained in this table is based on the original numerical results for λ_1, if the mean value is used, then the corresponding mass will be lower by ~200 MeV. Then there arises an interesting thing: the experimental mass for $D_{sJ}(2632)$ almost coincides with the center value for the mass of the D^*_{s1} state in Table I. But caution must be made on concluding that $D_{sJ}(2632)$ is the predicted D^*_{s1} state, because the static properties are not decisive in the identification of excited states with rich spectrum structure and there is also considerable error in our numerical result. The obtained spectrum merely indicates that there is no contradiction if the $D_{sJ}(2632)$ state be identified as the predicted D^*_{s1} state. Further, if $D_{sJ}(2632)$ can be identified as an excited $c\bar{s}$ state, the anomalous decay pattern can only be explained by the nodal structure of the wave function, i.e. $D_{sJ}(2632)$ will be a radially excited state [3–5]. However, there also exists multichannel calculation supporting $D_{sJ}(2632)$ as being the conventional $c\bar{s}$ state [7], though within the same approach different opinion presents [28]. The situation is not clear now and subsequent study of the decay modes will clarify the current mess-up.

When the s-quark is substituted by a massless quark, the sum rules can be analyzed similarly. By extrapolating to the massless quark case, except for the substitution of m_s by 0, there emerges an additional term $m_0^2\langle\bar{q}q\rangle/12$ for the $f_{3/2,-}$ sum rule, which results from the Dirac δ function in the spectral density. The most pronounced feature for these sum rules is that in the massless quark case the $5/2^-$ and $5/2^+$ doublets degenerate up to dimension 6 in the OPE, which is the direct result of the fact that there is no quark condensate contribution to the sum rule for them. The numerical results are

$$\bar{\Lambda} = 1.42 \pm 0.06\ \text{GeV}, \qquad -\lambda_1 = 1.86 \pm 0.18\ \text{GeV}^2, \qquad \lambda_2 = (5.3 \pm 0.7) \times 10^{-2}\ \text{GeV}^2 \qquad (25)$$

for the $3/2^-$ doublet, and

$$\bar{\Lambda} = 1.40 \pm 0.07\ \text{GeV}, \qquad -\lambda_1 = 1.45 \pm 0.19\ \text{GeV}^2, \qquad \lambda_2 = (3.7 \pm 0.9) \times 10^{-2}\ \text{GeV}^2 \qquad (26)$$

for the two $5/2$ doublets. Those values are commonly larger than the strange-flavored case because the working region is not identical, for the massless quark case we work at somewhat higher continuum threshold value for the stability of the sum rule to appear.

To conclude, the mass of excited charmed-strange doublets $(1^-, 2^-)$, $(2^-, 3^-)$ and $(2^+, 3^+)$ is calculated to order $1/m_Q$ using QCD sum rule method formulated within the framework of HQET. From this case study we find the static properties are not decisive in identifying the observed exotic state $D_{sj}(2632)$ as conventional $c\bar{s}$ meson state, due to the large uncertainty in the numerical results, although the predicted center is very near the experimental value. Further theoretical study and experimental data on the decay modes are in urgent need.

ACKNOWLEDGMENTS

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TABLE I. The mass spectrum for the three orbitally excited doublets in units of GeV.

D^*_{s1}	D_{s2}	D_{s2}	D^*_{s3}	D^*_{s2}	D_{s3}
2.81 ± 0.27	3.11 ± 0.38	3.17 ± 0.32	3.32 ± 0.38	3.29 ± 0.35	3.43 ± 0.38

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