Implications of the nonuniversal Z boson in flavor changing neutral current mediated rare decays

R. Mohanta
School of Physics, University of Hyderabad, Hyderabad - 500 046, India
(Received 1 May 2005; published 28 June 2005)

We analyze the effect of the nonuniversal Z boson in the rare decays $B_s \to l^+ l^-$, $B_s \to l^+ l^- \gamma$, and $Z \to b\bar{s}$ decays. These decays involve the flavor changing neutral current (FCNC) mediated $b \to s$ transitions, and are found to be very small in the standard model. The smallness of these decays in the standard model makes them sensitive probe for new physics. We find an enhancement of at least an order in these branching ratios because of the nonuniversal Zbs coupling.

DOI: 10.1103/PhysRevD.71.114013 PACS numbers: 13.20.He, 12.60.−i, 13.20.−v, 13.38.−b

The rare decays induced by the flavor changing neutral current (FCNC) transitions are very important to probe the flavor sector of the standard model (SM). In the SM they arise from one-loop diagrams and hence are generally suppressed in comparison to the tree diagrams. This makes them as a sensitive probe for new physics. The rare B decays which are mediated by the FCNC transitions are of the kind $b \to s$ or $b \to d$. Prominent examples of rare B decays are $B \to K^* \gamma$, $B \to \rho\gamma$, $B \to K l^+ l^-$, $B_{s,d} \to l^+ l^-$. During the last few years, considerable theoretical attention has therefore been focused on these decays in view of the planned experiments at B factories, which are likely to measure branching fractions as low as 10^{-8} [1,2]. The results of the branching ratios of FCNC mediated B decays are very sensitive to new physics beyond the SM. Thus, a detailed investigation of the rare decays is a promising way to discover or severely constrain the new physics.

In this paper, we focus on a specific class of rare decay modes $B_s \to l^+ l^-$ and $B_s \to l^+ l^- \gamma$, which are mediated by the Z boson exchange and the rare Z decay mode $Z \to b\bar{s}$. We consider the effect of the nonuniversal Z boson which induces FCNC interaction at the tree level. It is well known that FCNC coupling of the Z boson can be generated at the tree level in various exotic scenarios. Two popular examples discussed in the literature are the models with an extra $U(1)$ symmetry [3] and those with the addition of nonsequential generation of quarks [4]. In the case of extra $U(1)$ symmetry the FCNC couplings of the Z boson are induced by $Z - Z'$ mixing, provided the SM quarks have family nonuniversal charges under the new $U(1)$ group. In the second case, adding a different number of up- and down-type quarks, the pseudo CKM matrix needed to diagonalize the charged currents is no longer unitary and this leads to tree-level FCNC couplings. It should be noted that, recently, there has been renewed interests shown in the literature concerning the nonuniversal Z induced new physics [5]. In light of this it necessitates also a detailed investigation of rare B decays, which are very promising to discover and/or to constrain new physics.

Here we will follow the second approach [4] to analyze some FCNC induced rare decays. It is a simple model beyond the standard model with an enlarged matter sector due to an additional vectorlike down quark D_4. The presence of an additional down quark implies a 4×4 matrix $V_{i\alpha}$ ($i = u, c, t, 4$, $\alpha = d, s, b, b'$), diagonalizing the down quark mass matrix. For our purpose the relevant information for the low energy physics is encoded in the extended mixing matrix. The charged currents are unchanged except that the V_{CKM} is now the 3×4 upper submatrix of V. However, the distinctive feature of this model is that the FCNC interaction enters neutral current Lagrangian of the left-handed down quarks as

$$\mathcal{L}_Z = \frac{g}{2\cos\theta_W}[\bar{u}_{Li}\gamma^\mu u_{Li} - \bar{d}_{L\alpha}U_{\alpha\beta}\gamma^\mu d_{L\beta} - 2\sin^2\theta_W J^\mu_{em}]Z_\mu, \tag{1}$$

with

$$U_{\alpha\beta} = \sum_{i=u,c,t} V^\dagger_{\alpha i} V_{i\beta} = \delta_{\alpha\beta} - V^*_{4\alpha}V_{4\beta}, \tag{2}$$

where U is the neutral current mixing matrix for the down sector, which is given above. As V is not unitary, $U \neq \mathbf{1}$. In particular the nondiagonal elements do not vanish

$$U_{\alpha\beta} = -V^*_{4\alpha}V_{4\beta} \neq 0 \quad \text{for} \quad \alpha \neq \beta. \tag{3}$$

Since the various $U_{\alpha\beta}$ are nonvanishing, they would signal new physics and the presence of FCNC at the tree level and this can substantially modify the predictions of SM for the FCNC processes.

Now let us consider the FCNC process $B_s \to l^+ l^-$. These decays, in particular, the process $B_s \to \mu^+\mu^-$ has attracted a lot of attention recently since it is very sensitive to the structure of SM and potential source of new physics beyond the SM. Furthermore, this process is very clean and the only nonperturbative quantity involved is the decay constant of B_s meson which can be reliably calculated by the well-known nonperturbative methods such as QCD sum rules, lattice gauge theory, etc. Therefore, it provides a good hunting ground to probe for new physics. The branching ratio for $B_s \to l^+ l^-$ has been calculated in the SM [6] and also in beyond the SM in a number of papers [7]. Let us start by recalling the result for $B_s \to l^+ l^-$ in QCD-improved standard model. The effective Hamiltonian describing this process is

$$\mathcal{H}_{eff} = \frac{G_F \alpha}{\sqrt{2}\pi} V_{tb} V_{ts}^* \Big[C_9^{\rm eff} (\bar{s}\gamma_\mu P_L b)(\bar{l}\gamma^\mu l) + C_{10}(\bar{s}\gamma_\mu P_L b)(\bar{l}\gamma^\mu \gamma_5 l) - \frac{2C_7 m_b}{q^2} \times (\bar{s} i\sigma_{\mu\nu} q^\nu P_R b)(\bar{l}\gamma^\mu l)\Big], \quad (4)$$

where $P_{L,R} = \frac{1}{2}(1 \mp \gamma_5)$ and q is the momentum transfer. C_i's are the Wilson coefficients evaluated at the b quark mass scale in next-to-leading logarithmic (NLL) order with values [8]

$$C_7^{\rm eff} = -0.308, \qquad C_9 = 4.154, \qquad C_{10} = -4.261. \quad (5)$$

The coefficient $C_9^{\rm eff}$ has a perturbative part and a resonance part which comes from the long distance effects due to the conversion of the real $c\bar{c}$ into the lepton pair l^+l^-. Hence, $C_9^{\rm eff}$ can be written as

$$C_9^{\rm eff} = C_9 + Y(s) + C_9^{\rm res}, \quad (6)$$

where the function $Y(s)$ denotes the perturbative part coming from one-loop matrix elements of the four quark operators and is given in Ref. [9]. The long distance resonance effect is given as [10]

$$C_9^{\rm res} = \frac{3\pi}{\alpha^2}(3C_1 + C_2 + 3C_3 + C_4 + 3C_5 + C_6) \times \sum_{J/\psi,\psi'} \kappa \frac{m_{V_i}\Gamma(V_i \to l^+l^-)}{m_{V_i}^2 - s - i m_{V_i}\Gamma_{V_i}}, \quad (7)$$

where the phenomenological parameter κ is taken to be 2.3, so as to reproduce the correct branching ratio $\mathcal{B}(B \to J/\psi K^* \to K^* l^+l^-) = \mathcal{B}(B \to J/\psi K^*)\mathcal{B}(J/\psi \to l^+l^-)$. In this analysis, we will consider only the contributions arising from two dominant resonances i.e., J/ψ and ψ'. The values of the coefficients C_i's in NLL order are given in [8] as $C_1 = -0.151$, $C_2 = 1.059$, $C_3 = 0.012$, $C_4 = -0.034$, $C_5 = 0.010$, and $C_6 = -0.040$.

To evaluate the transition amplitude one can generally adopt the vacuum insertion method, where the form factors of the various currents are defined as follows

$$\langle 0|\bar{s}\gamma^\mu\gamma_5 b|B_s^0\rangle = i f_{B_s} p_B^\mu, \qquad \langle 0|\bar{s}\gamma_5 b|B_s^0\rangle = i f_{B_s} m_{B_s}, \qquad \langle 0|\bar{s}\sigma^{\mu\nu}P_R b|B_s^0\rangle = 0. \quad (8)$$

Since $p_B^\mu = p_+^\mu + p_-^\mu$, the contribution from C_9 term in Eq. (4) will vanish upon contraction with the lepton bilinear, C_7 will also give zero by (8), and the remaining C_{10} term will get a factor of $2m_l$.

Thus the transition amplitude for the process is given as

$$\mathcal{M}(B_s \to l^+l^-) = i\frac{G_F\alpha}{\sqrt{2}\pi} V_{tb}V_{ts}^* f_{B_s} C_{10} m_l (\bar{l}\gamma_5 l), \quad (9)$$

and the corresponding branching ratio is given as

$$\mathcal{B}(B_s \to l^+l^-) = \frac{G_F^2 \tau_{B_s}}{16\pi^3}\alpha^2 f_{B_s}^2 m_{B_s} m_l^2 |V_{tb}V_{ts}^*|^2 C_{10}^2 \times \sqrt{1 - \frac{4m_l^2}{m_{B_s}^2}}. \quad (10)$$

Helicity suppression is reflected by the presence of m_l^2 in (10) which gives almost vanishingly small value for e^+e^- and a very small branching ratio of $(3.4 \pm 0.5) \times 10^{-9}$ for $\mu^+\mu^-$ [11]. The published Tevatron/CDF physics results with luminosity 171 pb^{-1} provides the bound on $B_s \to \mu^+\mu^-$ [12]

$$\mathcal{B}(B_s \to \mu^+\mu^-) < 5.8 \times 10^{-7} \qquad (90\% \text{ C.L.}). \quad (11)$$

Recently, this branching ratio has been further constrained by the D0 collaboration [13] with an upper bound

$$\mathcal{B}(B_s \to \mu^+\mu^-) < 5.0 \times 10^{-7} \qquad (95\% \text{ C.L.}). \quad (12)$$

It should be noted that the τ channel is free from this helicity suppression however, its experimental detection is quite hard due to the low detection efficiency and that is why we do not have any experimental upper limit for this process as yet.

Now let us analyze the decay modes $B_s \to l^+l^-$ in the model with the Z mediated FCNC occurring at the tree level [4]. The corresponding diagram is shown in Figure 1, where the blob represents the tree-level Zbs coupling. The effective Hamiltonian for $B_s \to l^+l^-$ is given as

$$\mathcal{H}_{eff} = \frac{G_F}{\sqrt{2}} U_{sb}[\bar{s}\gamma^\mu(1-\gamma_5)b][\bar{l}(C_V^l\gamma_\mu - C_A^l\gamma_\mu\gamma_5)l], \quad (13)$$

where C_V^l and C_A^l are the vector and axial vector Zl^+l^- couplings, which are given as

$$C_V^l = -\frac{1}{2} + 2\sin^2\theta_W, \qquad C_A^l = -\frac{1}{2}. \quad (14)$$

Since, the structure of the effective Hamiltonian (13) in this model is the same form as that of the SM, like $\sim(V-A)\times(V-A)$ form, therefore its effect on the various decay observables can be encoded by replacing the SM Wilson coefficients $(C_9^{\rm eff})^{SM}$ and $(C_{10})^{SM}$ by

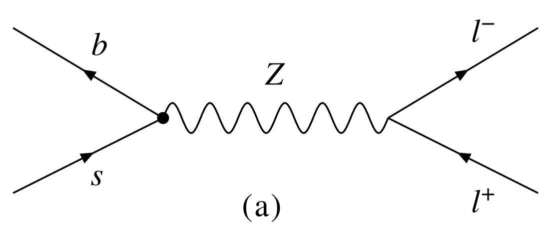

FIG. 1. Feynman diagram for $B_s \to l^+l^-$ in the model with tree-level FCNC transitions, where the blob represents the tree-level flavor changing vertex.

$$C_9^{\rm eff} = (C_9^{\rm eff})^{SM} + \frac{2\pi}{\alpha}\frac{U_{sb}C_V^l}{V_{tb}V_{ts}^*},$$
$$C_{10}^{\rm eff} = (C_{10})^{SM} - \frac{2\pi}{\alpha}\frac{U_{sb}C_A^l}{V_{tb}V_{ts}^*}. \tag{15}$$

It should be noted that U_{sb} is in general complex and hence it induces the weak phase difference (θ) between the SM and new physics contributions. Since the value of the Wilson coefficients C_9 and C_{10} are opposite to each other as seen from Eq. (5), and the new physics contributions to C_9 and C_{10} are opposite to each other, one will get constructive or destructive interference of SM and NP amplitudes for $\theta = \pi$ or zero (where θ denotes the relative weak phase between SM and NP contribution in the above equation).

Thus, one can obtain the branching ratio including the NP contributions by substituting $C_{10}^{\rm eff}$ from (15) in (10). Now using the value of $|U_{bs}| \simeq 10^{-3}$ [14], which has been extracted from the recent data on $\mathcal{B}(B \to X_S l^+ l^-)$, $\sin^2\theta_W = 0.23$, the particle masses from [15], $\alpha = 1/127$, the decay constant $f_{B_s} = 0.24$ GeV and $V_{tb}V_{ts}^* = 0.04$, we obtain the branching ratios as

$$\begin{aligned}\mathcal{B}(B_s \to \mu^+\mu^-) &= 4.2 \times 10^{-8}, \quad (\text{for} \quad \theta = \pi)\\ &= 6.8 \times 10^{-9}, \quad (\text{for} \quad \theta = 0),\\ \mathcal{B}(B_s \to \tau^+\tau^-) &= 8.9 \times 10^{-6}, \quad (\text{for} \quad \theta = \pi)\\ &= 1.4 \times 10^{-6}, \quad (\text{for} \quad \theta = 0).\end{aligned} \tag{16}$$

Thus, as seen from Eq. (16) the branching ratio for $B_s^0 \to \mu^+\mu^-$ has been enhanced by one order from its corresponding standard model value for $\theta = \pi$, and is below the present experimental upper limit. This decay mode may be observable at the Tevatron Run II [16] to the level of 2×10^{-8}. However, the predicted branching ratio for $B_s^0 \to \tau^+\tau^-$, which is $\mathcal{O}(10^{-6})$, could be observable in the currently running B factories, if we have a good efficiency for the detection of τ lepton.

Now let us consider the radiative dileptonic decay modes $B_s \to l^+l^-\gamma$, which are also very sensitive to the existence of new physics beyond the SM. Because of the presence of the photon in the final state, these decay modes are free from helicity supression, but they are further suppressed by a factor of α. However, in spite of this α suppression, the radiative leptonic decays $B_s \to l^+l^-\gamma$, $l = (\mu, \tau)$ have comparable decay rates to that of purely leptonic ones. The SM predictions for these branching ratios are $\mathcal{B}(B_s \to \mu^+\mu^-\gamma, \tau^+\tau^-\gamma) = 1.9 \times 10^{-9}$, 9.54×10^{-9}, respectively [17,18]. These decays are also studied in some beyond the standard model scenarios [19].

The matrix element for the decay $B_s \to l^+l^-\gamma$ can be obtained from that of the $B_s \to l^+l^-$ one by attaching the photon line to any of the charged external fermion lines. In order to calculate the amplitude, when the photon is radiated from the initial fermions (structure dependent (SD) part), we need to evaluate the matrix elements of the quark currents present in (4) between the emitted photon and the initial B_s meson. These matrix elements can be obtained by considering the transition of a B_s meson to a virtual photon with momentum k. In this case the form factors depend on two variables, i.e., k^2 (the photon virtuality) and the square of momentum transfer $q^2 = (p_B - k)^2$. By imposing gauge invariance, one can obtain several relations among the form factors at $k^2 = 0$. These relations can be used to reduce the number of independent form factors for the transition of the B_s meson to a real photon. Thus, the matrix elements for $B_s \to \gamma$ transition, induced by vector, axial-vector, tensor, and pseudotensor currents can be parametrized as [20]

$$\langle\gamma(k,\varepsilon)|\bar{s}\gamma_\mu\gamma_5 b|B_s(p_B)\rangle = ie[\varepsilon_\mu^*(p_B\cdot k) - (\varepsilon^*\cdot p_B)k_\mu]\frac{F_A}{m_{B_s}}, \qquad \langle\gamma(k,\varepsilon)|\bar{s}\gamma_\mu b|B_s(p_B)\rangle = e\epsilon_{\mu\nu\alpha\beta}\varepsilon^{*\nu}p_B^\alpha k^\beta\frac{F_V}{m_{B_s}},$$
$$\langle\gamma(k,\varepsilon)|\bar{s}\sigma_{\mu\nu}q^\nu\gamma_5 b|B_s(p_B)\rangle = e[\varepsilon_\mu^*(p_B\cdot k) - (\varepsilon^*\cdot p_B)k_\mu]F_{TA}, \qquad \langle\gamma(k,\varepsilon)|\bar{s}\sigma_{\mu\nu}q^\nu b|B_s(p_B)\rangle = ie\epsilon_{\mu\nu\alpha\beta}\varepsilon^{*\nu}p_B^\alpha k^\beta F_{TV}, \tag{17}$$

where ε and k are the polarization vector and the four momentum of photon, p_B is the momentum of initial B_s meson and F_i's are the various form factors.

Thus, the matrix element describing the SD part takes the form

$$\begin{aligned}\mathcal{M}_{SD} = \frac{\alpha^{3/2}G_F}{\sqrt{2\pi}}V_{tb}V_{ts}^*\{&\epsilon_{\mu\nu\alpha\beta}\varepsilon^{*\nu}p_B^\alpha k^\beta(A_1\bar{l}\gamma^\mu l\\ &+ A_2\bar{l}\gamma^\mu\gamma_5 l) + i(\varepsilon_\mu^*(k\cdot p_B) - (\varepsilon^*\cdot p_B)k_\mu)\\ &\times(B_1\bar{l}\gamma^\mu l + B_2\bar{l}\gamma^\mu\gamma_5 l)\},\end{aligned} \tag{18}$$

where

$$A_1 = 2C_7\frac{m_b}{q^2}F_{TV} + C_9\frac{F_V}{m_{B_s}}, \qquad A_2 = C_{10}\frac{F_V}{m_{B_s}},$$
$$B_1 = -2C_7\frac{m_b}{q^2}F_{TA} - C_9\frac{F_A}{m_{B_s}}, \qquad B_2 = -C_{10}\frac{F_A}{m_B}. \tag{19}$$

The q^2 dependence of the form factors are given as [20]

$$F(E_\gamma) = \beta\frac{f_{B_s}m_{B_s}}{\Delta + E_\gamma}, \tag{20}$$

where E_γ is the photon energy, which is related to the momentum transfer q^2 as

TABLE I. The parameters for $B \to \gamma$ form factors.

Parameter	F_V	F_{TV}	F_A	F_{TA}
$\beta(\mathrm{GeV}^{-1})$	0.28	0.30	0.26	0.33
$\Delta(\mathrm{GeV})$	0.04	0.04	0.30	0.30

$$E_\gamma = \frac{m_{B_s}}{2}\left(1 - \frac{q^2}{m_{B_s}^2}\right). \qquad (21)$$

The values of the parameters are given in Table I.

When the photon is radiated from the outgoing lepton pairs, the internal bremsstrahlung (IB) part, the matrix element is given as [21]

$$\mathcal{M}_{IB} = \frac{\alpha^{3/2}G_F}{\sqrt{2\pi}} V_{tb}V_{ts}^* f_{B_s} m_l C_{10}\left[\bar{l}\left(\frac{\varepsilon^* \not{p}_B}{p_+ \cdot k} - \frac{\not{p}_B \varepsilon^*}{p_- \cdot k}\right)\gamma_5 l\right]. \qquad (22)$$

Thus, the total matrix element for the $B_s \to l^+ l^- \gamma$ process is given as

$$\mathcal{M} = \mathcal{M}_{SD} + \mathcal{M}_{IB}. \qquad (23)$$

The differential decay width of the $B \to l^+ l^- \gamma$ process, in the rest frame of B_s meson is given as [21]

$$\frac{d\Gamma}{ds} = \frac{G_F^2 \alpha^3}{2^{10}\pi^4} |V_{tb}V_{ts}^*|^2 m_{B_s}^3 \Delta, \qquad (24)$$

where

$$\Delta = \frac{4}{3} m_{B_s}^2 (1-\hat{s})^2 v_l ((\hat{s}+2r_l)(|A_1|^2+|B_1|^2) + (\hat{s}-4r_l) \times (|A_2|^2+|B_2|^2)) - 64\frac{f_{B_s}^2}{m_{B_s}^2}\frac{r_l}{1-\hat{s}} C_{10}^2\left((4r_l - \hat{s}^2 - 1) \times \ln\frac{1+v_l}{1-v_l} + 2\hat{s}v_l\right) - 32 r_l (1-\hat{s})^2 f_{B_s} \mathrm{Re}(C_{10}A_1^*), \qquad (25)$$

with $s = q^2$, $\hat{s} = s/m_{B_s}^2$, $r_l = m_l^2/m_{B_s}^2$, $v_l = \sqrt{1 - 4m_l^2/q^2}$. The physical region of s is $4m_l^2 \le s \le m_{B_s}^2$.

The forward backward asymmetry is given as

$$A_{FB} = \frac{1}{\Delta}\left[2m_{B_s}^2 \hat{s}(1-\hat{s})^3 v_l^2 \mathrm{Re}(A_1^* B_2 + B_1^* A_2) + 32 f_{B_s} r_l (1-\hat{s})^2 \ln\left(\frac{4r_l}{\hat{s}}\right)\mathrm{Re}(C_{10}B_2^*)\right]. \qquad (26)$$

Now using the form factors from (19), we plot the dilepton mass spectrum (24), and the forward backward asymmetries (26) for $B_s \to l^+ l^- \gamma$ decays which are shown in Figs. 2 and 3. In these plots we have used the weak phase difference between the SM and NP amplitudes θ to be π to get the maximum possible contributions. From Figs. 2 and 3, we see that the branching ratio for $B_s \to l^+ l^- \gamma$ enhanced significantly from their corresponding SM values. However, the forward backward asymmetries are reduced slightly from the corresponding SM values and for the $B_s \to \mu^+\mu^-\gamma$ process, there is a backward shifting of the zero position.

To obtain the branching ratios it is necessary to eliminate the backgrounds, coming from the resonances $J/\psi(\psi')$ with $J/\psi(\psi') \to l^+ l^-$. We use the following veto windows to eliminate these backgrounds

$$B_s \to \mu^+\mu^-\gamma : \quad m_{J/\psi} - 0.02 < m_{\mu^+\mu^-} < m_{J/\psi} + 0.02;$$
$$: \quad m_{\psi'} - 0.02 < m_{\mu^+\mu^-} < m_{\psi'} + 0.02,$$
$$B_s \to \tau^+\tau^-\gamma : \quad m_{\psi'} - 0.02 < m_{\tau^+\tau^-} < m_{\psi'} + 0.02.$$

Furthermore, it should be noted that the $|\mathcal{M}_{IB}|^2$ has infrared singularity due to the emission of soft photon. Therefore, to obtain the branching ratio, we impose a cut on the photon energy, which will correspond to the experimental cut imposed on the minimum energy for the detectable photon. Requiring the photon energy to be larger than 25 MeV, i.e., $E_\gamma \ge \delta m_{B_s}/2$, which corresponds to $s \le m_{B_s}^2(1-\delta)$ and therefore, we set the cut $\delta \ge 0.01$.

Thus, with the above defined veto windows and the infrared cutoff parameter, we obtain the branching ratios as

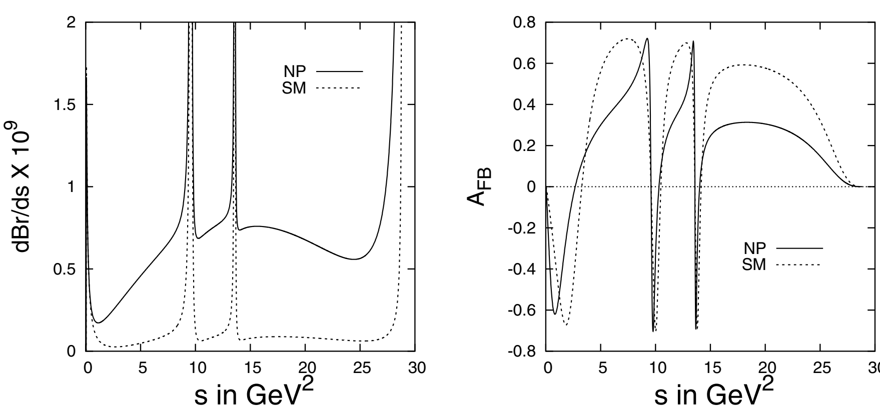

FIG. 2. The differential branching ratio and the forward backward asymmetry (A_{FB}) for the process $B_s \to \mu^+\mu^-\gamma$, in the standard model and in the NP model with the nonuniversal Z boson effect.

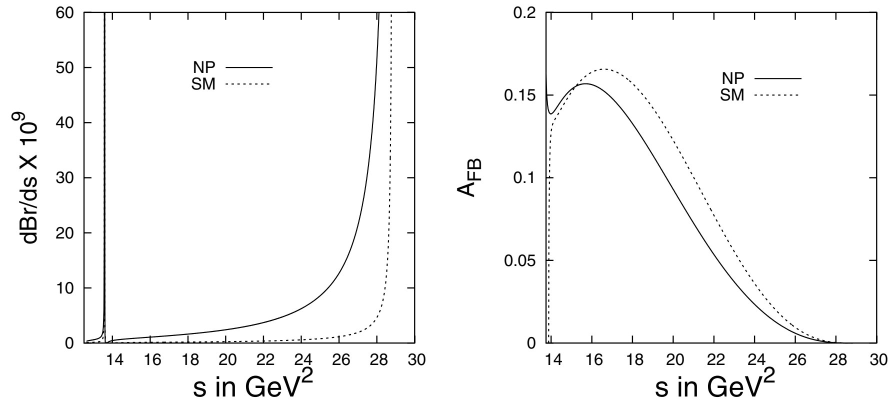

FIG. 3. Same as Fig. 2, for the $B_s \to \tau^+\tau^-\gamma$ process.

$$\mathcal{B}(B_s^0 \to \mu^+\mu^-\gamma) = 1.94 \times 10^{-8},$$
$$\mathcal{B}(B_s^0 \to \tau^+\tau^-\gamma) = 1.37 \times 10^{-7}, \tag{27}$$

which are enhanced by an order from their SM values. It should be mentioned that the $B_s^0 \to \tau^+\tau^-\gamma$ could be observable in the Run II of Tevatron. The contribution to the branching ratio due to bremsstrahlung photon is small for $B_s \to \mu^+\mu^-\gamma$ and is found to be $\mathcal{B}(B_s^0 \to \mu^+\mu^-\gamma)|_{IB} \sim 0.5 \times 10^{-8}$ whereas it has dominant contribution to the $B_s \to \tau^+\tau^-\gamma$ process, i.e., $\mathcal{B}(B_s^0 \to \tau^+\tau^-\gamma)|_{IB} \sim 1.3 \times 10^{-7}$.

Now let us consider the flavor changing rare Z decays $Z \to b\bar{s}$. Rare Z decays have been studied extensively in order to yield the signature of new physics. In the standard model this mode originates from one loop diagram with branching ratio $\sim 3 \times 10^{-8}$ [22]. While the sensitivity of the measurement for the branching ratios for rare Z decays reached at LEP2 is about 10^{-5} [15], future linear colliders (NLC, TESLA) will bring this sensitivity up to 10^{-8} level [23]. Various beyond the standard model scenarios has been employed in [24] where the branching ratio can be found to reach the sensitivity of the order of 10^{-6}. We would now be interested to analyze this decay mode in the model with an additional vectorlike down quark, where it can originate in the leading order. For completeness, we would also include the one-loop corrections to the branching ratio, although their effect is negligibly small compared to the tree-level contribution.

The tree-level amplitude is given as

$$\mathcal{M}(Z \to b\bar{s}) = \epsilon^\mu U_{sb} \frac{g}{2\cos\theta_W} \bar{u}_b \gamma_\mu P_L v_s, \tag{28}$$

where ϵ^μ denotes the polarization vector of Z boson. The decay width is given as

$$\Gamma(Z \to b\bar{s}) = \frac{m_Z}{32\pi} \frac{g^2}{\cos^2\theta_W} |U_{sb}|^2, \tag{29}$$

and the branching ratio to be

$$\mathcal{B}(Z \to b\bar{s} + \bar{b}s) = \frac{1}{\Gamma_Z} \frac{m_Z}{16\pi} \frac{g^2}{\cos^2\theta_W} |U_{sb}|^2, \tag{30}$$

where Γ_Z is the total Z boson decay width.

Now we consider the one-loop corrections arising from Figs. 4(b) and 4(c). The one-loop amplitude arising from Fig. 4(b) is given as

$$\begin{aligned}\mathcal{M}(Z \to b\bar{s})|_{\text{1-loop}} = &\frac{1}{16\pi^2} \epsilon^\mu U_{sb} \frac{g}{2\cos\theta_W} \bar{u}_b a_L^2 \gamma_\mu P_L [-2\tilde{C}_0(m_Z^2, m_s^2, m_s^2, m_b^2, m_Z^2, m_s^2) + B_1(m_s^2, m_s^2, m_Z^2) + B_0(m_s^2, m_s^2, m_Z^2) \\ &- \tilde{C}_{11}(m_Z^2, m_s^2, m_s^2, m_b^2, m_Z^2, m_s^2) + 2C_{24}(m_Z^2, m_s^2, m_s^2, m_b^2, m_Z^2, m_s^2) + m_Z^2(2C_{11}(m_Z^2, m_s^2, m_s^2, m_b^2, m_Z^2, m_s^2) \\ &+ 3C_0(m_Z^2, m_s^2, m_s^2, m_b^2, m_Z^2, m_s^2) - C_{22}(m_Z^2, m_s^2, m_s^2, m_b^2, m_Z^2, m_s^2) + C_{23}(m_Z^2, m_s^2, m_s^2, m_b^2, m_Z^2, m_s^2))] v_s,\end{aligned} \tag{31}$$

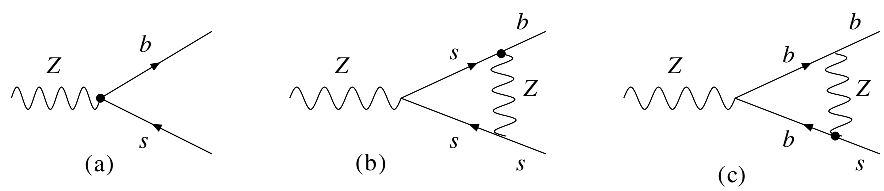

FIG. 4. Tree-level and one-loop Feynman diagrams for $Z \to b\bar{s}$ process.

where

$$a_L = \frac{g}{\cos\theta_W}\left(\frac{1}{3}\sin^2\theta_W - \frac{1}{2}\right). \quad (32)$$

The contribution from Fig. 4(c) can be obtained from 4(b) by replacing m_s by m_b and vice versa. Including the one-loop correction the branching ratio for $Z \to (b\bar{s} + \bar{b}s)$ is given as

$$\mathcal{B}(Z \to b\bar{s} + \bar{b}s) = \frac{1}{\Gamma_Z}\frac{m_Z}{16\pi}\frac{g^2}{\cos^2\theta_W}|U_{sb}|^2|1 + R_1 + R_1(m_s \leftrightarrow m_b)|^2, \quad (33)$$

where

$$R_1 = \frac{a_L^2}{16\pi^2}[-2\tilde{C}_0(m_Z^2, m_s^2, m_s^2, m_b^2, m_Z^2, m_s^2) + B_1(m_s^2, m_s^2, m_Z^2) + B_0(m_s^2, m_s^2, m_Z^2) - \tilde{C}_{11}(m_Z^2, m_s^2, m_s^2, m_b^2, m_Z^2, m_s^2) + 2C_{24}(m_Z^2, m_s^2, m_s^2, m_b^2, m_Z^2, m_s^2) + m_Z^2(2C_{11}(m_Z^2, m_s^2, m_s^2, m_b^2, m_Z^2, m_s^2) + 3C_0(m_Z^2, m_s^2, m_s^2, m_b^2, m_Z^2, m_s^2) - C_{22}(m_Z^2, m_s^2, m_s^2, m_b^2, m_Z^2, m_s^2) + C_{23}(m_Z^2, m_s^2, m_s^2, m_b^2, m_Z^2, m_s^2))]. \quad (34)$$

Using the quark masses (in GeV) as $m_s = 0.15$ and $m_b = 4.4$, $\sin^2\theta_W = 0.23$, $\alpha = 1/127$, and the mass and width of Z boson from [15], we obtain the branching ratio including one-loop corrections as

$$\mathcal{B}(Z \to b\bar{s}) = 4.08 \times 10^{-7}. \quad (35)$$

Since the expected sensitivity of giga-Z collider is of the order of 10^{-8} (which is at the level of SM expectation), we emphasize that new physics effect could be detectable in the rare decay $Z \to b\bar{s}$, if indeed it affects this mode.

Here, we have analyzed the rare decay modes $B_s \to l^+l^-$ and $B_s \to l^+l^-\gamma$ which are mediated by the $b \to s$ FCNC transitions. We have considered the model which induces tree-level FCNC coupling of Z boson, due to the addition of an extra vectorlike down quark to the matter sector. We found that the branching ratios for the radiative leptonic decay modes $B_s \to \mu^+\mu^-\gamma$, $(\tau^+\tau^-\gamma)$ are of the order of 10^{-8} (10^{-7}) which could be observable in the Tevatron Run II. Furthermore, the branching ratio of the pure leptonic mode $B_s \to \tau^+\tau^-$ found to be $\mathcal{O}(10^{-6})$. This mode can be observed in the currently running B factories with improved τ tagging efficiency.

We have also analyzed the flavor changing decay of Z boson to a pair of down quarks $Z \to b\bar{s}$. This Z decay channel may prove useful in searching for new flavor physics beyond the SM at the TESLA or any other future collider which may be designed to run at the Z pole with high luminosities, thus accumulating more than 10^9 on shell Z bosons. With improved b tagging efficiencies, the flavor changing decay $Z \to b\bar{s}$ is the most likely and the easiest one to detect among the flavor changing hadronic Z decays. It may be accessible to the giga-Z option even for branching ratio as small as $\mathcal{B}(Z \to b\bar{s}) \sim 10^{-7} - 10^{-6}$.

To conclude, the standard model results of the rare decays studied here which are induced by FCNC transitions, are very small and cannot be detected in the current or near future experiments. These rare decays provide very sensitive probe of new physics beyond the SM. Detection of these decays at visible levels by any of the future colliders would be a clear evidence of new physics.

This work is partly supported by the Department of Science and Technology, Government of India, through Grant No. SR/FTP/PS-50/2001.

APPENDIX: ONE-LOOP FORM FACTORS

The two-point and three-point one-loop form factors which are defined as

$$C_0; C_\mu; C_{\mu\nu}(m_1^2, m_2^2, m_3^2, p_1^2, p_2^2, p_3^2) \equiv \int \frac{d^4q}{i\pi^2}\frac{1; q_\mu; q_\mu q_\nu}{[q^2 - m_1^2][(q+p_1)^2 - m_2^2][(q-p_3)^2 - m_3^2]}, \quad (A1)$$

$$\tilde{C}_0; \tilde{C}_{\mu\nu}(m_1^2, m_2^2, m_3^2, p_1^2, p_2^2, p_3^2) \equiv \int \frac{d^4q}{i\pi^2}\frac{q^2; q^2 q_\mu q_\nu}{[q^2 - m_1^2][(q+p_1)^2 - m_2^2][(q-p_3)^2 - m_3^2]}, \quad (A2)$$

where $\sum_i p_i = 0$ is to be understood above

$$B_0; B_\mu(m_1^2, m_2^2, p^2) \equiv \int \frac{d^4q}{i\pi^2}\frac{1; q_\mu}{[q^2 - m_1^2][(q+p)^2 - m_2^2]}. \quad (A3)$$

The coefficients B_j with $j \in 0,1$, C_j with $j \in 0, 11, 12, 21, 22, 23, 24$, are defined through the following relation

$$B_\mu = p_\mu B_1, \quad C_\mu = p_{1\mu}C_{11} + p_{2\mu} + p_{2\mu}C_{12},$$
$$C_{\mu\nu} = p_{1\mu}p_{1\nu}C_{21} + p_{2\mu}p_{2\nu}C_{22} + \{p_1p_2\}_{\mu\nu}C_{23} + g_{\mu\nu}C_{24},$$
$$\tilde{C}_{\mu\nu} = p_{1\mu}p_{1\nu}\tilde{C}_{21} + p_{2\mu}p_{2\nu}\tilde{C}_{22} + \{p_1p_2\}_{\mu\nu}\tilde{C}_{23} + g_{\mu\nu}\tilde{C}_{24}, \quad (A4)$$

where $\{ab\} = a_\mu b_\nu + a_\nu b_\mu$.

[1] A. Ali, Report No DESY 97-192.
[2] P. Ball *et al.*, "B Decays," in Proceedings of the workshop on Standard Model Physics at the LHC, CERN, 2000-004 (unpublished).
[3] P. Langacker and M. Plümacher, Phys. Rev. D **62**, 013006 (2000).
[4] Y. Nir and D. Silverman, Phys. Rev. D **42**, 1477 (1990); D. Silverman, Phys. Rev. D **45**, 1800 (1992); Int. J. Mod. Phys. A **11**, 2253 (1996); Y. Grossman, Y. Nir, and R. Rattazzi, in *Heavy Flavours II*, edited by A. J. Buras and M. Lindner (World Scientific, Singapore, 1998), p. 755.
[5] V. Barger, C. W. Chiang, P. Langacker, and H. S. Lee, Phys. Lett. B **580**, 186 (2004); V. Barger, C. W. Chiang, J. Jiang, and P. Langacker, Phys. Lett. B **596**, 229 (2004); D. Atwood and G. Hiller, hep-ph/0307251; A. K. Giri and R. Mohanta, Phys. Lett. B **594**, 196 (2004); Phys. Rev. D **69**, 014008 (2004); Mod. Phys. Lett. A **19**, 1903 (2004); N. G. Deshpande, D. K. Ghosh, and X. G. He, Phys. Rev. D **70**, 093003 (2004); N. G. Deshpande and D. K. Ghosh, Phys. Lett. B **593**, 135 (2004); X. G. He and G. Valencia, Phys. Rev. D **70**, 053003 (2004); G. Buchalla, G. Hiller, and G. Isidori, Phys. Rev. D **63**, 014015 (2001).
[6] T. Inami and C. S. Lim, Prog. Theor. Phys. **65**, 297 (1981); **65**, 1772(E) (1981); G. Buchalla and A. J. Buras, Nucl. Phys. B **400**, 225 (1993); M. Misiak and J. Urban, Phys. Lett. B **451**, 161 (1999); A. J. Buras, Phys. Lett. B **566**, 115 (2003).
[7] S. R. Choudhury and N. Gaur, Phys. Lett. B **451**, 86 (1999); K. S Babu and C. Kolda, Phys. Rev. Lett. **84**, 228 (2000); H. E. Logan and U. Nierste, Nucl. Phys. B **586**, 39 (2000); C. S. Huang, L. Wei, Q. S. Yan, and S. H. Zhu, Phys. Rev. D **63**, 114021 (2001); **64**, 059902(E) (2001); C. Bobeth, T. Ewerth, F. Krüger, and J. Urban, Phys. Rev. D **64**, 074014 (2001); **66**, 074021 (2002); G. Isidori and A. Retico, J. High Energy Phys. 11 (2001) 001; A. J. Buras, P. H. Chankowski, J. Rosiek, and L. Slawianowska, Nucl. Phys. B **659**, 3 (2003); C. S. Huang and X. H. Wu, Nucl. Phys. B **657**, 304 (2003); P. H. Chankowski and L. Slawianowska, Phys. Rev. D **63**, 054012 (2001); G. D'Ambrosio, G. F. Giudice, G. Isidori, and A. Strumia, Nucl. Phys. B **645**, 155 (2002); A. Dedes and A. Pilaftsis, Phys. Rev. D **67**, 015012 (2003); A. Dedes, H. K. Dreiner, and U. Nierste, Phys. Rev. Lett. **87**, 251804 (2001); R. Arnowitt, B. Dutta, T. Kamon, and M. Tanaka, Phys. Lett. B **538**, 121 (2002); J. K. Mizukoshi, X. Tata, and Y. Yang, Phys. Rev. D **66**, 115003 (2002); S. Baek, P. Ko, and W. Y. Song, Phys. Rev. Lett. **89**, 271801 (2002); J. High Energy Phys. 03 (2003) 054; S. Baek, Y. G. Kim, and P. Ko, J. High Energy Phys. 02 (2005) 067; S. Baek, Phys. Lett. B **595**, 461 (2004); A. Dedes and B. T. Huffman, Phys. Lett. B **600**, 261 (2004); J. Ellis, K. A. Olive, and V. C. Spanos, hep-ph/0504196.
[8] M. Beneke, Th. Fledmann, and D. Seidel, Nucl. Phys. B **612**, 25 (2001).
[9] A. J. Buras and M. Münz, Phys. Rev. D **52**, 186 (1995).
[10] C. S. Lim, T. Morozumi, and A. I. Sanda, Phys. Lett. B **218**, 343 (1989); N. G. Deshpande, J. Trampetic, and K. Panose, Phys. Rev. D **39**, 1461 (1989); P. J. O'Donnell and H. K. K. Tung, Phys. Rev. D **43**, R2067 (1991); P. J. O'Donnell, M. Sutherland, and H. K. K. Tung, Phys. Rev. D **46**, 4091 (1992).
[11] A. J. Buras of Ref. [6].
[12] A. Acosta *et al.* (CDF Collaborations), Phys. Rev. Lett. **93**, 032001 (2004).
[13] V. M. Abazov *et al.* (D0 Collaboration), Phys. Rev. Lett. **94**, 071802 (2005).
[14] A. K. Giri and R. Mohanta, Phys. Rev. D **68**, 014020 (2003).
[15] S. Eidelman (Particle Data Group), Phys. Lett. B **592**, 1 (2004).
[16] K. Anikeev *et al.*, hep-ph/0201071.
[17] T. M. Aliev, A. Özpineci, and M. Savci, Phys. Rev. D **55**, 7059 (1997); G. Eilam, C. D. Lü, and D. X. Zhang, Phys. Lett. B **391**, 461 (1997).
[18] T. M. Aliev, N. K. Pak, and M. Savci, Phys. Lett. B **424**, 175 (1998); C. Q. Geng, C. C. Lih, and W. M. Zhang, Phys. Rev. D **62**, 074017 (2000).
[19] T. M. Aliev, A. Özpineci, and M. Savci, Eur. Phys. J. C **27**, 405 (2003); U. O. Yilmaz, B. B. Sirvanh, and G. Turan, Eur. Phys. J. C **30**, 197 (2003); G. Turan, Mod. Phys. Lett. A **20**, 533 (2005).
[20] F. Krüger and D. Melikhov, Phys. Rev. D **67**, 034002 (2003).
[21] S. R. Choudhury, A. S. Cornell, N. Gaur, and G. C. Joshi, hep-ph/0504193.
[22] M. Clements *et al.*, Phys. Rev. D**27**, 570 (1983); V. Ganapathy *et al.*, Phys. Rev. D **27**, 579 (1983); W. S. Hou, N. G. Deshpande, G. Eilam, and A. Soni, Phys. Rev. Lett. **57**, 1406 (1986); J. Barnabeu, M. B. Gavela, and A. Santamaria, Phys. Rev. Lett. **57**, 1514 (1986).
[23] R.-D. Heuer *et al.*, hep-ph/0106315.
[24] C. Bush, Nucl. Phys. B **319**, 15 (1989); W. S. Hou and R. G. Stuart, Phys. Lett. B **226**, 122 (1989); B. Grzadkowski, J. F. Gunion, and P. Krawczyk, Phys. Lett. B **268**, 106 (1991); B. Mukhopadhyaya and A. Raychoudhuri, Phys. Rev. D **39**, 280 (1989); M. Duncan, Phys. Rev. D **31**, 1139 (1985); F. Gabbiani, J. H. Kim, and A. Massiero, Phys. Lett. B **214**, 398 (1988); D. Atwood, L. Reina, and A. Soni, Phys. Rev. D **55**, 3156 (1997); M. Chemtob and G. Moreau, Phys. Rev. D **59**, 116012 (1999); D. Atwood, S. Bar-Shalom, G. Eilam, and A. Soni, Phys. Rev. D **66**, 093005 (2002); C. Yue, H. Li, and H. Zong, Nucl. Phys. B **650**, 290 (2003).