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$\bar{c}c$ purity of $\psi(3770)$ and ψ' challenged

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It is suggested that the resonance $\psi(3770)$ may contain a sizeable ($O(10\%)$ in terms of the probability weight factor) four-quark component with the up- and down-quarks and antiquarks in addition to the $c\bar{c}$ pair, which component in itself has a substantial part with isospin $I = 1$. Furthermore such a four-quark part of the wave function should also affect the properties of the ψ' charmonium resonance through the $\psi(3770) - \psi'$ mixing previously considered in the literature. It is argued that an admixture of extra light quark pairs can explain a possible discrepancy between the theoretical expectations and the recent data on the non-$D\bar{D}$ decay width of the $\psi(3770)$ and the ratio of the yield of charged and neutral D meson pairs in its decays, as well as on the extra rate of the ψ' direct decay into light hadrons and the rate of the decay $\psi' \to \pi^0 J/\psi$. It is further argued that the suggested four-quark component of the wave function of the $\psi(3770)$ should give rise to a measurable rate for the decays $\psi(3770) \to \eta J/\psi$ and $\psi(3770) \to \pi^0 J/\psi$.

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I. INTRODUCTION

The study of charmonium resonances below and above the $D\bar{D}$ threshold currently is experiencing a remarkable revival due to the efforts of the CLEO-c experiment [1]. While the general picture of the properties of these resonances is certainly in agreement with the theoretical expectations for states of $c\bar{c}$ charmonium system, the fine details emerging with the improvement in the accuracy of the data on these resonances possibly bring into light some yet unsolved issues of charmonium physics, as well as of a general understanding of QCD dynamics. The purpose of this paper is to point out four specific pieces of the recent experimental data on the ψ' and $\psi(3770)$ resonances, which are potentially at variance with the standard theoretical expectations, and which can possibly be explained by assuming a certain admixture of a four-quark component with extra $u\bar{u}$ and $d\bar{d}$ quark pairs. Namely, the data to be discussed are: the total cross section for $D\bar{D}$ pair production in e^+e^- annihilation at the $\psi(3770)$ peak [2]

$$\sigma(e^+e^- \to D\bar{D}) = (6.39 \pm 0.10^{+0.17}_{-0.08})\ \text{nb}; \qquad (1)$$

the charged-to-neutral yield ratio at the same energy [2]

$$\sigma(e^+e^- \to D^+D^-)/\sigma(e^+e^- \to D^0\bar{D}^0) = 0.776 \pm 0.024^{+0.014}_{-0.006}; \qquad (2)$$

the branching fraction for the direct decays of the ψ' resonance into light hadrons [3]

$$\mathcal{B}(\psi' \to \text{light hadrons}) = (16.9 \pm 2.9)\%; \qquad (3)$$

and the newly measured [3] ratio of the branching fractions

$$\mathcal{B}(\psi' \to \pi^0 J/\psi)/\mathcal{B}(\psi' \to \eta J/\psi) = (4.1 \pm 0.4 \pm 0.1)\%. \qquad (4)$$

In what follows the discrepancy of each of these experimental numbers with previous theoretical expectations in the picture where both the ψ' and $\psi(3770)$ resonances are pure $c\bar{c}$ states is discussed in Sec. II, and, where possible, alternative explanations of the discrepancy within the same picture are mentioned. It will be then argued in Sec. III that the listed experimental data can be explained, at least semiquantitatively, by assuming the presence of light quark pairs in a part of the wave function of the $\psi(3770)$, and also in a much lesser part of the wave function of the ψ', where the latter may arise from a $\psi' - \psi(3770)$ mixing of the type considered by Rosner [4], as discussed in Sec. IV. The presence of extra pairs of light quarks in the resonances arguably affects the data (1) and (3) in the list above, while for an explanation of the data (2) and (4) one has to assume that in the suggested four-quark admixture there is a substantial component with isospin $I = 1$, i.e. to assume an unequal weight of the states with extra $u\bar{u}$ and $d\bar{d}$ pairs. As will be further argued in Sec. V, both these assumptions can be tested experimentally by measuring the decays $\psi(3770) \to \eta J/\psi$ and $\psi(3770) \to \pi^0 J/\psi$, of which the former should be enhanced due to the four-quark component in $\psi(3770)$ with $I = 0$, while the latter is expected to be enhanced due to the component with $I = 1$. Finally, Sec. VI contains a summary and discussion of the arguments presented here and a discussion of the current and possible future experimental data.

II. THE DATA VS THE THEORETICAL EXPECTATIONS

We proceed to the discussion of the data listed in Eqs. (1)–(4) starting with the last one. The ratio of the decay rates in Eq. (4) was considered in Ref. [5] using an extension of the chiral algebra technique from a previous work [6] and also in the context of the QCD multipole expansion for hadronic transitions in heavy quarkonium [7,8]. Within this approach both the ψ' and J/ψ are considered as compact $c\bar{c}$ states, so that in their interaction

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with the soft gluon field they are equivalent to a pointlike source. The ratio of the decay amplitudes is then fully determined by the ratio of the amplitudes for creation of the corresponding light pseudoscalar meson by the gluonic operator $G\tilde{G} = (1/2)\epsilon_{\mu\nu\lambda\sigma}G^{\mu\nu}G^{\lambda\sigma}$. The ratio of the latter amplitudes is determined by the anomaly in the axial current and the isospin and the flavor SU(3) breaking by the light quark masses [9]. In terms of the decay rates the result for the ratio reads [5]

$$\frac{\Gamma(\psi' \to \pi^0 J/\psi)}{\Gamma(\psi' \to \eta J/\psi)} = 3\left(\frac{m_d - m_u}{m_d + m_u}\right)^2 \frac{f_\pi^2}{f_\eta^2}\frac{m_\pi^4}{m_\eta^4}\frac{p_\pi^3}{p_\eta^3}, \tag{5}$$

where p_π, p_η is the momentum of the corresponding light meson in the decay, m_u and m_d are the light quark masses, and f_π and f_η are the annihilation constants of the mesons, normalized in such a way that in the limit of the flavor SU(3) symmetry $f_\pi = f_\eta$. In reality it is known from comparison of f_π and f_K that the presence of heavier strange quarks increases the constant f, so that f_η is expected to be larger than f_π. Therefore the limit $f_\pi = f_\eta$ can be used as an upper bound on the ratio of the rates in Eq. (5). The ratio of the masses of the u and d quarks, describing the explicit breaking of the chiral symmetry and the isospin violation in this breaking was studied years ago in great detail by Gasser and Leutwyler [10]. The largest value for the ratio $(m_d - m_u)/(m_d + m_u)$ allowed by that study is approximately 0.35. Thus the theoretical upper bound for the ratio of the decay rates in Eq. (5) is approximately 2.3%, which is still more than 4σ below the experimental result (4). It can be also mentioned in connection with the light quark mass ratio that the well-known Weinberg formula [11] gives

$$\frac{m_d - m_u}{m_d + m_u} = \frac{m_{K^0}^2 - m_{K^+}^2 + m_{\pi^+}^2 - m_{\pi^0}^2}{m_{\pi^0}^2} = 0.285, \tag{6}$$

and results in a still smaller ratio of the decay rates if used in Eq. (5). It is certainly understood [5] that the formula (5) may receive unexpectedly large corrections from the effects of SU(3) violation; however, such corrections can also significantly affect the analysis of the chiral phenomenology in Ref. [10], and the whole subject then would have to be revisited anew. It should be mentioned that the largest theoretical estimate of the ratio (4) found in the literature [12] corresponds to 3.4%, which is also significantly lower than the experimental number. However, the latter estimate does not fully take into account the proper QCD structure of the relevant amplitude for the meson production by the gluonic operator. A detailed consideration [13] of the terms omitted in the earlier papers [5,14] reinstates the formula in Eq. (5) for the ratio of the decay rates.

The rate of the direct decays of the ψ' into light hadrons has been a source of concern previously, and the latest experimental result (3) emphasizes the problem at a somewhat greater statistical significance. In the standard picture of the three-gluon annihilation of the $^3S_1$ charmonium states both the hadronic and the $\ell^+\ell^-$ annihilation amplitudes are proportional to the wave function at the origin, so that the ratio of the rates of these processes should be the same for J/ψ and ψ'. However, the combination of the PDG values [15] gives[1]

$$\mathcal{B}(\psi' \to \ell^+\ell^-)\frac{\mathcal{B}(J/\psi \to \text{light hadrons})}{\mathcal{B}(J/\psi \to \ell^+\ell^-)} = (10.9 \pm 0.6)\%, \tag{7}$$

which is 2.2σ below the experimental number in Eq. (3). Clearly, the uncertainty in the latter number is still large enough to accommodate compatibility with the standard picture. Nevertheless, even given the present uncertainty, it makes sense to look into corrections to the straightforward prediction of the equality between the combination of the branching fractions in Eq. (7) and that in Eq. (3). Moreover, it is well known that the similarity of the annihilation decay rates is strongly broken in exclusive channels, most notably in the decays of J/ψ and ψ' into $\rho\pi$. (For a recent discussion see, e.g., Ref. [4].) One source of modification of the short-distance hadronic annihilation of a heavy quarkonium state is provided by the nonperturbative corrections [16] due to the gluon condensate. These corrections depend on details of the quarkonium wave function, which are not reduced to the wave function at the origin. For this reason the estimate of the nonperturbative behavior can be done in a model independent way only for very heavy quarkonium, for which the dynamics is dominated by a Coulomb-like short-distance gluon force. An extrapolation down to the realistic bottomonium and charmonium states suggests that this effect should somewhat suppress the hadronic annihilation of the 2S state relative to the 1S. The recent experimental data [17] on the $\ell^+\ell^-$ branching fractions for the Υ resonances, $\mathcal{B}_{\mu\mu}(\Upsilon) = (2.49 \pm 0.02 \pm 0.07)\%$, $\mathcal{B}_{\mu\mu}(\Upsilon') = (2.03 \pm 0.03 \pm 0.08)\%$ tend to support such behavior, since they correspond to the ratio

$$\frac{\mathcal{B}_{\mu\mu}(\Upsilon')\mathcal{B}(\Upsilon \to \text{light hadrons})}{\mathcal{B}_{\mu\mu}(\Upsilon)\mathcal{B}(\Upsilon' \to \text{light hadrons})} \approx 1.5 \pm 0.1. \tag{8}$$

If this behavior is extrapolated further down to the charmonium mass, the nonperturbative corrections in fact enhance the disagreement between the numbers in Eq. (3) and (5).

Proceeding to discussion of the data on the $\psi(3770)$ resonance we first notice that the measured charged-to-neutral yield ratio (2) significantly exceeds the ratio of the P wave kinematical factors in the corresponding channels, which is equal to $p_+^3/p_0^3 = 0.685$. If the $\psi(3770)$ and the D

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[1] Here the value $\mathcal{B}(J/\psi \to \text{light hadrons}) = (86.8 \pm 0.4)\%$ is used, as quoted in Ref. [3], in order to ensure that it is defined in the same way as the experimental number (3) for ψ'.

mesons were point particles, the yield of the D^+D^- pairs would be enhanced by the well-known Coulomb factor [18] $R_c = 1 + \pi\alpha/(2v_+) \approx 1.085$, making the expected ratio equal to 0.743 in a reasonable agreement with the data (2). It is well understood however, that in a more realistic picture taking into account the form factors of the D mesons [19] and of the $\psi(3770) \to D\bar{D}$ vertex [20] the Coulomb enhancement is significantly weaker. Moreover, if the $\psi(3770)$ resonance is considered as ‘‘strong‘‘ in the sense that the P wave dynamics of the D mesons at energies close to the resonance peak is totally dominated by the Breit-Wigner form of the wave function, the interference of the Coulomb and the resonance phase shifts results in a nontrivial energy behavior of the charged-to-neutral yield ratio, and generally almost completely eliminates the Coulomb enhancement at the energy close to that of the peak [21]. This behavior generally agrees with the measurements by CLEO [22,23], BABAR [24,25], and Belle [26] at the peak energy of the $\Upsilon(4S)$ resonance, where they find a very weak or nonexistent Coulomb enhancement of the yield in the B^+B^- channel, far below the pointlike estimate 1.19 for the Coulomb factor.

Finally, as recently pointed out by Rosner [27,28], the measured total cross section for the $D\bar{D}$ production in the e^+e^- annihilation at the $\psi(3770)$ peak (Eq. (1)) is lower than the previous measurements (by other groups) of the total resonant cross section, for which he estimates the average of (7.9 ± 0.6) nb. The approximately (1.5 ± 0.7) nb deficit of the cross section, if confirmed by the new data, would have to be covered by direct decays of the $\psi(3770)$ into light hadrons, since the expected hadronic and radiative transitions from $\psi(3770)$ to lower charmonium states can account [28] for at most about 2% of its total width 23.6 ± 0.7 MeV. If $\psi(3770)$ is a pure $c\bar{c}$ state with $J^{PC} = 1^{--}$, it would be practically impossible to explain an annihilation width in excess of few tens of KeV, and thus to understand any significant non-$D\bar{D}$ cross section at the resonance.

III. THE FOUR-QUARK COMPONENT AND THE PROPERTIES OF $\psi(3770)$

If the discussed discrepancies between the experimental data and the theoretical expectations based on the picture where both the $\psi(3770)$ and ψ' are pure $c\bar{c}$ states are taken seriously, it is quite likely that such picture has to be modified. In particular Rosner considers [28] a model of reannihilation of the $D\bar{D}$ pairs in order to explain possible non-$D\bar{D}$ decays of the $\psi(3770)$. The reannihilation model can be viewed as a particular dynamical implementation of the picture suggested here that the wave function of the $\psi(3770)$ resonance contains at the characteristic hadronic distances a sizeable admixture of four-quark states with the $u\bar{u}$ and $d\bar{d}$ light quark pairs in addition to a $c\bar{c}$ pair. In other words it is assumed that the ‘‘core‘‘ of the wave function of the $\psi(3770)$ resonance consists of the following three essential parts:

$$\Psi_{\psi''} = a\, c\bar{c} + b_0\, c\bar{c}(u\bar{u} + d\bar{d})/\sqrt{2} + b_1\, c\bar{c}(u\bar{u} - d\bar{d})/\sqrt{2}, \tag{9}$$

where a, b_0, and b_1 are coefficients, and the $I = 1$ part proportional to b_1 is written for the reasons to be discussed further. Naturally, the expression (9) is rather symbolic, since each of the terms in it contains further specifications, such as the coordinate wave functions as well as color combinations for the four-quark part, etc. Lacking the knowledge of these details we are bound to limit ourselves to a discussion of only the generic properties following from the flavor structure of $\psi(3770)$ assumed in Eq. (9). The assumed presence of the four-quark states can certainly be a result of a very strong coupling between the $D\bar{D}$ channel and the $c\bar{c}$ state. Unlike the reannihilation scheme this picture does not need to rely on the assumption that the D mesons can be considered as individual ‘‘intact‘‘ mesons at the distances, where they actually overlap. On the other hand the ‘‘generic‘‘ scheme discussed here only allows to make general semiquantitative predictions.

It is well known [29] that the $c\bar{c}$ pair inside a four-quark component can annihilate in second order in the QCD coupling α_s, i.e. much faster than a colorless $J^{PC} = 1^{--}$ heavy quark pair, which is bound by the conservation laws to annihilate via three gluons. In particular a ${}^3S_1$ $c\bar{c}$ pair in a color octet state annihilates into light quarks, $c\bar{c} \to q\bar{q}$ via one gluon [30,31]. On the other hand it is quite likely that a color octet ${}^3S_1$ $c\bar{c}$ state is present in the four-quark part with a weight comparable to one. Indeed the spin-triplet states of the heavy quark pair should dominate in the Fock decomposition in Eq. (9) due to the spin selection rule [32], since the leading term proportional to a corresponds to spin-triplet ${}^3S_1$ and ${}^3D_1$ states. Furthermore, if in the four-quark component the relative color of the c and $\bar{c}$ is randomized, the states with the octet overall color of the pair carry the statistical weight of 8/9. Once the annihilation of the heavy quark pair from the four-quark component in order α_s^2 is allowed, the proper decay rate of this component can be deduced from a typical α_s^2 annihilation rate of $c\bar{c}$ in an S wave state, of which the only measured example is the decay width of the η_c. Thus one arrives at an (rather approximate) estimate

$$\mathcal{B}[\psi(3770) \to \text{light hadrons}] \sim (|b_0|^2 + |b_1|^2)\frac{\Gamma(\eta_c)}{\Gamma[\psi(3770))]}. \tag{10}$$

Experimentally, the total width of η_c is still somewhat uncertain, but is comparable to that of the $\psi(3770)$. Thus an $O(10\%)$ non-$D\bar{D}$ branching fraction in the decays of $\psi(3770)$ requires the weight factor $(|b_0|^2 + |b_1|^2)$ for the four-quark component in Eq. (9) of a similar value. Given the uncertainty in the current data on the non-$D\bar{D}$ decays of

$\psi(3770)$, it would be premature to quantitatively pursue this point any further.

If the assumed four-quark component in the $\psi(3770)$ arises from a strong mixing with the $D\bar{D}$ channel, one can expect an enhanced breaking of the isospin in this component due to a substantial difference in the excitation energy for the D^+D^- and $D^0\bar{D}^0$ mesons: approximately 30 MeV vs 40 MeV. This breaking, corresponding to a nonzero value of the coefficient b_1 in Eq. (9), can be related to the larger than expected relative yield of the charged meson pairs D^+D^- (cf. Eq. (2)). Indeed, the calculation [21] resulting in essentially no Coulomb effect at the peak of the resonance assumes a perfect isotopic symmetry in the "core" of the resonance, i.e. in the region of strong interaction. If this assumption is invalidated by the presence of an $I = 1$ four-quark component at those distances, the ratio of the wave functions for the channels with charged and neutral mesons would be biased proportionally to the coefficient b_1, resulting in a shift of the yield ratio by

$$\Delta\left[\frac{\sigma(e^+e^- \to D^+D^-)}{\sigma(e^+e^- \to D^0\bar{D}^0)}\right] \propto -4b_1\left(\frac{p_+}{p_0}\right)^3. \qquad (11)$$

Although the proportionality coefficient in this relation is not known at the present level of understanding, it looks quite reasonable to conclude that an $O(-3\%)$ value of the coefficient b_1 would be sufficient for explaining an $O(10\%)$ difference between the data (2) and the theoretical predictions based on the assumption that the $\psi(3770)$ is a pure isoscalar.

IV. THE EFFECTS OF THE FOUR-QUARK COMPONENT IN ψ'

The presence of a four-quark component in the $\psi(3770)$ can also affect the properties of the ψ' resonance through the $\psi(3770) - \psi'$ mixing. Such mixing was recently analyzed by Rosner [4] in the "pure" charmonium model as a $^3S_1 - {}^3D_1$ mixing. Based on the experimental values of the $\ell^+\ell^-$ widths of the discussed resonances and on model calculations of these widths for $^3S_1$ and $^3D_1$ charmonium, he found the mixing angle to be $(12 \pm 2)^o$. It should be understood however that this estimate of the mixing is far from being final, not only due to its theoretical model dependence, but also because the experimental data, especially for $\psi(3770)$, are currently in flux and are likely to change with further progress of the experiment. Indeed, the current data on $\Gamma_{\ell\ell}[\psi(3770)]$, $\Gamma_{\rm tot}[\psi(3770)]$, and on the e^+e^- cross section at the peak are in an apparent contradiction with the unitarity formula for the cross section at the maximum:

$$\sigma[e^+e^- \to \psi(3770)] = \frac{12\pi}{M^2}\mathcal{B}_{ee}. \qquad (12)$$

Using the PDG data [15] for the branching fraction $\mathcal{B}_{ee} = (1.12 \pm 0.17) \times 10^{-5}$ and the mass M of the $\psi(3770)$ one finds from this constraint the value (11.9 ± 1.8) nb for the resonant cross section, which is well above both the CLEO data (1) on the $D\bar{D}$ cross section and the average [28] for the total resonant cross section. Thus it looks reasonable to not be bound by a particular value of the mixing angle θ derived from the current data on the $\ell^+\ell^-$ decay widths, and possibly to look for alternative hints at the value of this parameter.

In the picture discussed here a mixing with the $\psi(3770)$, which has a relatively large annihilation rate into light hadrons due to the four-quark component as described by Eq. (9), should contribute to the similar decay rate of the ψ' resonance. This extra contribution to the inclusive annihilation rate (dominated by the process $c\bar{c} \to q\bar{q}$) adds incoherently to the standard annihilation rate originating in the colorless $c\bar{c}$ pair annihilation into three gluons. The measured branching fraction in Eq. (3) corresponds in absolute terms to a partial width $\Gamma(\psi' \to \text{light hadrons}) = (47.5 \pm 8.6)$ KeV, while a scaling of the J/ψ width proportionally to the $\ell^+\ell^-$ widths as shown in Eq. (7) would yield about 30 KeV. Given the uncertainty in the experimental data, and the possible contribution of the previously discussed nonperturbative effects in the three-gluon annihilation, a choice of 20 KeV as a "representative" value for the excess in the direct decay width of the ψ' resonance does not look unreasonable. The extra decay width in the mixing scheme is related to the (small) mixing angle θ as

$$\Delta\Gamma(\psi' \to \text{light hadrons}) = \theta^2\Gamma[\psi(3770) \to \text{light hadrons}], \qquad (13)$$

so that for an $O(10\%)$ branching fraction of the $\psi(3770)$ non-$D\bar{D}$ decays one approximately estimates $\theta \approx 0.1$. It can be noticed that the estimate [4] $\theta = 12^o \approx 0.2$ would result in a substantially larger extra decay rate of ψ' beyond that allowed by experiment, provided that the non-$D\bar{D}$ branching fraction for the $\psi(3770)$ is indeed around 10%.

Because of the mixing a presence of an $I = 1$ isotopic component in $\psi(3770)$ should result in a small isovector contribution in the wave function of the ψ' resonance. For reasons of G parity this part does not affect the isotopic relation between the decays $\psi' \to \pi^+\pi^-J/\psi$ and $\psi' \to \pi^0\pi^0J/\psi$, which has recently been successfully verified by CLEO [3]. However the isovector part gives an extra contribution to the amplitude of the decay $\psi' \to \pi^0J/\psi$. This extra contribution is necessarily coherent with the one derived in the standard approach [5]. Indeed, there is only one partial wave (P wave) allowed in this process, and any contributions are bound to be relatively real by the absence of essential on-shell intermediate states. Unlike in the standard approach for a pure $c\bar{c}$ charmonium, for the $I = 1$ four-quark state, assumed in this discussion, the transition amounts to a rearrangement of quarks with the $c\bar{c}$ materializing as the J/ψ and the light quarks as the neutral pion. Therefore one can reasonably assume that even a small admixture of such a four-quark component can produce a sizeable effect in the amplitude. The discrepancy between the data (4) and the theoretical formula can be explained by an extra contribution to the decay amplitude amounting to about one half of the standard one and having the same sign. In other words, the additional contribution should amount to about one third of the amplitude corresponding to the observed decay rate in Eq. (4).

V. THE DECAYS $\psi(3770) \to \pi^0 J/\psi$ AND $\psi(3770) \to \eta J/\psi$

The explanation of the somewhat enhanced rate of the hadronic transition $\psi' \to \pi^0 J/\psi$ as being due to a small admixture of the $I = 1$ four-quark component of $\psi(3770)$ can be turned around to predict the rate of the yet unobserved decay $\psi(3770) \to \pi^0 J/\psi$, since in this decay the $I = 1$ part of the four-quark component should dominate the amplitude:

$$\Gamma[\psi(3770) \to \pi^0 J/\psi] \approx \frac{(1 \div 1.5)}{\theta^2}\left(\frac{1}{3}\right)^2 \Gamma(\psi' \to \pi^0 J/\psi) \approx (4 \div 6)\left(\frac{0.1}{\theta}\right)^2 \text{ KeV}, \tag{14}$$

where the factor $(1/3)$ corresponds to the assumed fraction of the actual amplitude of $\psi' \to \pi^0 J/\psi$ being due to the mixing, and the range $(1 \div 1.5)$ refers to whether the rescaled P wave kinematical factor p_π^3 is included in the estimate or not. In terms of the branching fraction $\mathcal{B}[\psi(3770) \to \pi^0 J/\psi]$ the estimate (14) corresponds to about 2×10^{-4} (at $\theta \approx 0.1$). While certainly small, such a branching fraction does not look totally unrealistic to be measured given the total number of events in the CLEO-c sample.

The assumed $I = 0$ four-quark component naturally enters with a larger weight than $I = 1$ and its extra contribution to the amplitude of the decay $\psi' \to \eta J/\psi$ could potentially jeopardize the agreement between the experiment and the theoretical prediction [14] for the rate of this decay (relative to $\psi' \to \pi\pi J/\psi$). In order to analyze this situation, we write the SU(3) relation between the extra contributions to the amplitudes in terms of the coefficients b_0 and b_1 in Eq. (9)

$$\frac{\Delta A(\psi' \to \eta J/\psi)}{\Delta A(\psi' \to \pi^0 J/\psi)} = \frac{b_0}{\sqrt{3} b_1}, \tag{15}$$

while experimentally the ratio of the absolute values of the amplitudes, after taking into account the kinematical factor $p_{\pi,\eta}^3$, is equal to approximately 22, and, as previously suggested, the extra contribution amounts to about one third of the actual amplitude of the decay $\psi' \to \pi^0 J/\psi$. As estimated originally [14], the uncertainty in the theoretical prediction of the amplitude of the decay $\psi' \to \eta J/\psi$ is about 20%. Thus the extra contribution would not exceed this uncertainty as long as the ratio $|b_0/b_1|$ is less than approximately 23, i.e. as long as $|b_1| \gtrsim 0.043|b_0|$. This constraint is fully compatible with the estimates presented here, so that the agreement between the standard theory and the experiment for the decay $\psi' \to \eta J/\psi$ is safe from being invalidated by the extra contribution beyond its intrinsic uncertainty.

The $I = 0$ part of the four-quark component can however give rise to a realistically measurable rate of the decay $\psi(3770) \to \eta J/\psi$. Indeed, applying the same relation as in Eq. (15) to the decay amplitudes of the $\psi(3770)$ one estimates

$$\frac{\Gamma[\psi(3770) \to \eta J/\psi]}{\Gamma[\psi(3770) \to \pi^0 J/\psi]} = \frac{1}{3}\left|\frac{b_0}{b_1}\right|^2 \frac{p_\eta^3}{p_\pi^3}. \tag{16}$$

Assuming, as previously, $|b_0|^2 \sim 0.1$ and $|b_1|^2 \sim (0.03)^2 \approx 10^{-3}$, one finds $\mathcal{B}[\psi(3770) \to \eta J/\psi] \sim 7\mathcal{B}[\psi(3770) \to \pi^0 J/\psi] \sim 0.14\%(0.1/\theta)^2$. On the other hand, if the ratio $|b_0/b_1|$ is assumed to be at the discussed upper limit allowed by the uncertainty in the amplitude of the decay $\psi' \to \eta J/\psi$, the estimate for the rate of the $\psi(3770)$ decay substantially increases: $\mathcal{B}[\psi(3770) \to \eta J/\psi] \approx 36\mathcal{B}[\psi(3770) \to \pi^0 J/\psi] \approx 0.7\%(0.1/\theta)^2$.

VI. SUMMARY AND DISCUSSION

In summary, it is suggested here that the wave function of the resonance $\psi(3770)$ contains a substantial four-quark component of the type $c\bar{c}u\bar{u}$ and $c\bar{c}d\bar{d}$ which component is not quite symmetric between the extra u and d quarks, resulting in a presence of an isovector $I = 1$ part. Moreover, a small fraction of this four-quark state also enters the wave function of the ψ' resonance due to a small $\psi(3770) - \psi'$ mixing. As shown, this assumption allows to explain the possible discrepancies between the theoretical expectations based on the picture of $\psi(3770)$ and ψ' being pure $c\bar{c}$ states and the recent experimental data. In particular the charmed quarks inside the four-quark state are allowed to annihilate into light hadrons at a much higher rate than from a colorless 1^{--} state, which results in a possible fraction of direct decays of the $\psi(3770)$ into light hadrons. The $I = 1$ four-quark component of $\psi(3770)$ should create a bias in the coupling of this resonance to D^+D^- and $D^0\bar{D}^0$ which can explain the observed charged-to-neutral yield ratio (2) at the resonance peak. The additional contribution to the direct annihilation rate feeds down to the ψ' resonance through the $\psi(3770) - \psi'$ mixing and eliminates the discrepancy between the central value of the measured branching fraction $\mathcal{B}(\psi' \to \text{light hadrons})$ in Eq. (3) and the standard scaling from J/ψ of the direct annihilation rate proportionally to the $\ell^+\ell^-$ decay width. Finally, the $I = 1$ part of the four-quark component appearing in ψ' as a result of the mixing can explain the excess of the measured rate of the decay $\psi' \to \pi^0 J/\psi$ (Eq. (3)) over the long-standing theoretical predictions. The suggested four-quark admixture in the $\psi(3770)$ resonance should result in yet unobserved decays $\psi(3770) \to \pi^0 J/\psi$ and $\psi(3770) \to \eta J/\psi$. The estimates for the rates of these decays are still quite uncertain, but the "typical" expected values for their branching fractions: $\mathcal{B}[\psi(3770) \to \pi^0 J/\psi] \sim 2 \times 10^{-4}$ and $\mathcal{B}[\psi(3770) \to \eta J/\psi] \sim 0.15\%$, illustrate that an experimental search for these processes is quite feasible.

At this point no dynamical scheme is offered for a more detailed consideration of the four-quark component of the $\psi(3770)$. Generally it may be related to the strong coupling of the resonance to the $D\bar{D}$ channel, but may also be affected by "molecular charmonium" [29] effects. I believe that a more specific scheme than presented here would receive a strong boost from improved experimental data, especially if the decays $\psi(3770) \to \pi^0 J/\psi$ and $\psi(3770) \to \eta J/\psi$ are found at or around the level indicated above. It should be also mentioned that in some of the discussed cases the deviation of the experimental data from the theoretical expectations just barely exceeds 2σ. In particular this is the case for the direct annihilation rate of the ψ' in Eq. (3), and also for the difference between the total and the $D\bar{D}$ resonant cross section at the $\psi(3770)$ resonance, where the total cross section is not yet available from the CLEO data, and one has to resort [28] to averaging the results of the previous experiments. The confusion around the data is further enhanced by that the available experimental numbers do not appear to agree well with each other in view of the unitarity formula in Eq. (12) and thus at least some of the data will have to change. Thus for a general understanding of the properties of the charmonium resonances and for a better assessment of the status of a mixing scheme along the lines discussed here it is quite important that sufficiently accurate and reliable data become available at least for such basic characteristics of the $\psi(3770)$ resonance as its total width and the total resonant e^+e^- cross section.

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