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Hadron annihilation into two photons and backward virtual Compton scattering in the scaling regime of QCD

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We study the scaling regime of hadron-(anti)hadron annihilation into a deeply virtual photon and a real photon, $H\bar{H} \to \gamma^* \gamma$, and backward virtual Compton scattering, $\gamma^* H \to H\gamma$. We advocate that there is a kinematical region where the scattering amplitude factorizes into a short-distance matrix element and a long-distance dominated object: a transition distribution amplitude which describes the hadron to photon transition.

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I. INTRODUCTION

There now exists a successful description of deep exclusive reactions in terms of distribution amplitudes [1] and/or generalized parton distributions [2] on the one side and perturbatively calculable coefficient functions describing hard scattering at the partonic level on the other side. The pioneering papers [1,3] have opened the way to a real understanding of many processes that share with hadronic form factors and deeply virtual Compton scattering (dVCS) in the nearly forward region some basic properties and, in particular, the factorization [4] of the long-distance dominated matrix element of the nonlocal correlator of two quark or gluon fields between two hadronic states. Many processes have been studied in this framework [5]. Cases have been studied where the generalized parton distribution describes the transition of a nucleon to a nucleon, to a resonance [6], a nucleon-meson continuum state [7], or the exotic pentaquark [8]. We consider here a somewhat different class of reaction, where the t-channel exchange carries most of the quantum numbers of the hadron. The simplest examples are the annihilation:

$$\bar{p}p \to \gamma^* \gamma \to e^+ e^- \gamma \tag{1}$$

in the near forward region and large virtual photon invariant mass Q, which will be studied in detail at Gesellschaft für Schwerionenforschung (GSI) [9], and the VCS reaction in the backward region [i.e. when the real photon is emitted in the direction of the proton momentum in the center of mass (CM) system]

$$\gamma^* p \to p\gamma, \tag{2}$$

which is currently under experimental investigation at JLab [10].

The crucial observation that we want to make is that the amplitude for the process (1) is quite analogous to the one for process

$$\bar{p}p \to \gamma^* \to e^+ e^-, \qquad \gamma^* p \to p, \tag{3}$$

which are controlled by the timelike and spacelike proton form factors, respectively, the asymptotic form of which may be calculated in QCD [1], through the factorization of proton and antiproton distribution amplitudes on the one hand and the hard processes $\bar{q}\,\bar{q}\,\bar{q}\,qqq \to \gamma^*$ or $\gamma^* qqq \to qqq$ on the other hand. The only change needed is to replace the proton distribution amplitude which is defined from the correlator $\langle 0|u(z_1)u(z_2)d(z_3)|p\rangle$ by a quantity derived from a slightly different matrix element of the same operator, i.e. the correlator $\langle\gamma|u(z_1)u(z_2)d(z_3)|p\rangle$. This symbolic notation omits the gauge links between quarks and their spin and color indices.

This is what we define (see Fig. 1) as the hadron to photon transition distribution amplitude (TDA), which enters in the expression of the amplitude for process (1) in the kinematical region where hard and soft processes decouple, namely, when the real photon momentum is almost collinear to the incoming (π^+ or p in Fig. 1) hadron momentum as in the case of QCD factorization in dVCS. Another useful reaction of reference is the reaction $\pi^- p \to \gamma^* n$ [11], which has been analyzed in the framework of QCD factorization. If we restrict this reaction to a meson target, the obtained reaction $\pi^-\pi^+ \to \gamma^* \pi^0$ looks much the same as the one we consider here. One can apply to this last process the proof of factorization given in Ref. [4] for the vector meson production [12].

Such an extension of the factorization proof has already been briefly advocated in Ref. [13] for the electroproduction of leading baryons or antibaryons. As was advocated there, factorization in hard exclusive processes such as $\gamma_L^* p \to \rho p'$ is basically due to the fact that the meson originates from a small size $q\bar{q}$ object which is generated by a hard scattering (the hardness of the scattering controlling the initial size of the color singlet). In the case under study, the only difference is that the small size qqq object carries a baryon number.

In this paper, we shall restrict to the case of spinless hadrons and study the reactions

$$\pi^-\pi^+ \to \gamma^*\gamma, \qquad \gamma^*\pi^+ \to \pi^+\gamma, \tag{4}$$

with a few qualitative comments on the extension of this

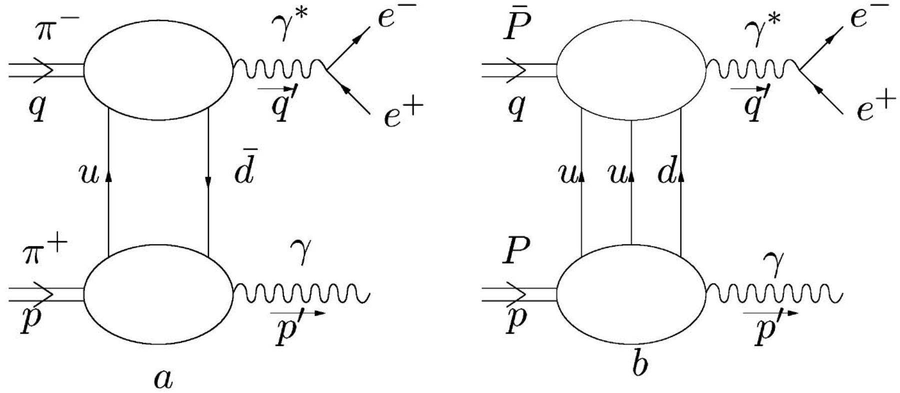

FIG. 1. The factorization of the annihilation process $\bar{H}H \to \gamma^*\gamma$ into a hard subprocess (upper blob) and a transition distribution amplitude (lower blob) for (a) the meson case and (b) the baryon case.

analysis to the phenomenologically more interesting proton case, which will be studied in detail in a future work. We also restrict to the longitudinal polarization of the virtual photon.

II. THE $H \to \gamma$ TRANSITION DISTRIBUTION AMPLITUDE

Let us take a closer look at the transition distribution amplitudes that occur in the processes in which we are interested. For their definition we introduce light-cone coordinates $v^\pm = (v^0 \pm v^3)/\sqrt{2}$ and transverse components $v_T = (v^1, v^2)$ for any four-vector v. The skewness variable $\xi = (p - p')^+/2P^+$ with $P = (p + p')/2$ describes the loss of plus momentum of the incident hadron. The momentum transfer is $\Delta = p' - p$.

We define the corresponding $H \to \gamma$ leading twist TDAs in the mesonic case as (ε is the polarization of a photon)

$$\int \frac{dz^-}{2\pi} e^{ixP^+z^-} \langle \gamma(p', \varepsilon)| O_V^\mu |\pi^+(p)\rangle|_{z^+=0, z_T=0} = \frac{1}{P^+} \frac{ie}{f_\pi} \epsilon^{\mu\nu\rho\sigma} \varepsilon_{\perp\nu} P_\rho \Delta_{\perp\sigma} V(x, \xi, t), \qquad O_V^\mu = \bar{d}(-z/2)\gamma^\mu u(z/2), \tag{5}$$

$$\int \frac{dz^-}{2\pi} e^{ixP^+z^-} \langle \gamma(p', \varepsilon)| O_A^\mu |\pi^+(p)\rangle|_{z^+=0, z_T=0} = \frac{1}{P^+} \frac{e}{f_\pi} (\vec{\varepsilon} \cdot \vec{\Delta}) P^\mu A(x, \xi, t), \qquad O_A^\mu = \bar{d}(-z/2)\gamma^\mu \gamma_5 u(z/2), \tag{6}$$

$$\int \frac{dz^-}{2\pi} e^{ixP^+z^-} \langle \gamma(p', \varepsilon)| O_T^{\mu\nu} |\pi^+(p)\rangle|_{z^+=0, z_T=0} = \frac{e}{P^+} \epsilon^{\mu\nu\rho\sigma} P_\sigma \left[\varepsilon_{\perp\rho} T_1(x, \xi, t) - \frac{1}{f_\pi} (\vec{\varepsilon} \cdot \vec{\Delta}) \Delta_{\perp\rho} T_2(x, \xi, t) \right], \quad O_T^{\mu\nu} = \bar{d}(-z/2)\sigma^{\mu\nu} u(z/2), \tag{7}$$

where the first two TDAs $V(x, \xi, t)$ and $A(x, \xi, t)$ are chiral even and the latter ones $T_i(x, \xi, t)$, $i = 1, 2$ are chiral odd. In definitions (5)–(7) it is assumed that, like in the usual GPDs, a Wilson link along the light cone is included to ensure the QCD-gauge invariance for nonlocal operators. We omit this link to simplify the notation. f_π is the pion decay constant. The four leading twist TDAs are linear combinations of the four independent helicity amplitudes for the process $q\pi \to q\gamma$.

As in the case of GPDs, the signs of $x \pm \xi$ determine the partonic interpretation of the TDAs. One thus distinguishes in complete analogy with usual GPDs the Dokshitzer-Gribov-Lipatov-Altarelli-Parisi region where $x + \xi$ and $x - \xi$ have the same sign, where the physics resemble the usual parton distribution case, from the Efremov-Radyushkin-Brodsky-Lepage region for which $-\xi < x < \xi$ and where a quark-antiquark pair is extracted from the meson state. The interpretation of TDAs is more explicit when they are expressed as the overlap of light-cone wave functions for the hadron and the photon. Let us emphasize that all possible spectator configurations have to be summed over in the wave function overlap, including Fock states with additional partons in the hadron and in the photon.

Interesting sum rules may be derived for the meson to photon TDAs. Since the local matrix element describes—up to a charge factor—the photon to neutral meson transition form factor, measurable in $\gamma\gamma^*$ collisions, we obtain

$$\int_0^1 dx (Q_u V^u(x, \xi, t) - Q_d V^d(x, \xi, t)) = \sqrt{2} f_\pi F_{\pi\gamma}(t), \tag{8}$$

where $Q_u = 2/3$, $Q_d = -1/3$, and $V^q(x, \xi, t)$ is the TDA related to the operator built from the quark q; see Eq. (5).

Current algebra fixes the value of the right-hand side at $t = 0$ since $F_{\pi\gamma}(t=0) = \sqrt{2}/4\pi^2 f_\pi$, $f_\pi = 131$ MeV [14].

In an analogous way one can derive the sum rule for the axial TDAs, related to the axial-vector form factors measured in the weak decays of mesons $\pi^+ \to l^+ \nu_l \gamma$ [15]. We get

$$\int_0^1 dx A(x, \xi, t) = f_\pi F_A(t). \qquad (9)$$

These two sum rules allow one to constrain possible parametrizations of the TDAs. Note, in particular, the ξ independence of both relations. On the other hand, we do not know any sum rule constraining chirally odd TDAs.

QCD radiative corrections lead as usual to logarithmic scaling violations. The scale dependence of the meson to photon TDAs is governed by evolution equations which are the evolution equations for usual nonsinglet GPDs [3],

$$\mu^2 \frac{d}{d\mu^2} A(x, \xi, t) = \int_{-1}^1 dx' \frac{1}{\xi} V_{NS}\left(\frac{x}{\xi}, \frac{x'}{\xi}\right) A(x', \xi, t), \qquad (10)$$

where to the leading order accuracy

$$V_{NS}(x, x') = \frac{\alpha_s}{4\pi} C_F \left[\rho(x, x') \left[\frac{1+x}{1+x'} \left(1 + \frac{2}{x'-x} \right) \right] + [x \to -x, x' \to -x'] \right]_+, \qquad (11)$$

$$\rho(x, x') = \theta(x' \geq x \geq -1) - \theta(x' \leq x \leq -1),$$

with $C_F = (N_c^2 - 1)/(2N_c)$, and the same equations are satisfied by the other leading order TDAs $V(x, \xi, t)$ and $T_i(x, \xi, t)$, $i = 1, 2$.

III. $\pi^-\pi^+ \to l^+ l^- \gamma$ IN THE SCALING REGIME

Let us now study the annihilation channel $\pi^-\pi^+ \to l^+ l^- \gamma$ in the kinematical region where $Q^2, s \to \infty$ with $x_B = Q^2/s$ fixed. Q^2 is the invariant mass of the lepton pair and s is the center of mass energy squared.

The amplitude is the sum of two contributions. The bremsstrahlung process where the real photon is radiated from a final lepton yields an amplitude proportional to the timelike hadron form factors $F_\pi(s)$. Such an amplitude is strongly peaked when the photon is collinear to one of the final leptons. It should not interfere much with the process we are mostly interested in, which favors the kinematics where the photon is mostly parallel to the π^+ [16].

The $\pi^-\pi^+ \to \gamma_L^* \gamma$ contribution yields an amplitude which is proportional to the TDAs we have defined with ξ connected with x_B by $\xi \approx x_B/(2 - x_B)$ in the Bjorken limit. It reads

$$\mathcal{M}(Q^2, \xi) = \int dx dz \phi(z) M_h(z, x, \xi) A(x, \xi, t), \qquad (12)$$

where the hard amplitude is

$$M_h(z, x, \xi) = \frac{4\pi^2 \alpha_{em} \alpha_s C_F}{N_C Q} \frac{1}{z\bar{z}} \times \left(\frac{Q_u}{x - \xi - i\epsilon} + \frac{Q_d}{x + \xi + i\epsilon} \right) \vec{\varepsilon} \cdot \vec{\Delta}, \qquad (13)$$

and $\phi(z)$ is the meson distribution amplitude, $\bar{z} = 1 - z$.

The scaling law for the amplitude is

$$\mathcal{M}(Q^2, \xi) \sim \frac{\alpha_s(Q^2)}{Q}, \qquad (14)$$

up to the logarithmic corrections due to DA and TDA anomalous dimensions. This behavior is related to the one of the electromagnetic hadron form factor.

IV. BACKWARD VCS

The backward VCS reaction $eH \to e'H\gamma$ when the final photon flies approximatively in the direction of the initial hadron receives contributions from two competing subprocesses, namely, the Bethe-Heitler process, where the final photon is emitted from the lepton line, and the hadronic dVCS process, where γ is emitted from the hadron. Contrarily to the case of near forward dVCS, the Bethe-Heitler process is strongly suppressed in the backward kinematics. One may thus quite safely ignore its contribution in this study [16].

The hadronic backward VCS process is the spacelike analog of the $\bar{H}H \to \gamma_L^* \gamma$ process discussed in the previous section. The correspondence between these two amplitudes may be inferred, via crossing, from the analysis of Ref. [11]: At Born level and to leading twist, one obtains the amplitudes for backward dVCS from those of the reaction $\bar{H}H \to \gamma_L^* \gamma$ by changing the sign of the imaginary part and reversing the beam and virtual photon polarizations. To this accuracy, both processes thus carry exactly the same information on the TDAs. One should, however, remember that already at the form factor level there is an experimental difference between timelike and spacelike exclusive quantities which may be the signature of higher twist or higher order corrections [17].

The scaling law for the amplitude is the same as the one written above in Eq. (14).

V. CONCLUSIONS

We have defined the new transition distribution amplitudes $H \to \gamma$, i.e. which parametrize the matrix elements of light-cone operators between very different initial and final states; this generalizes the concept of GPDs for non-diagonal transitions. In our analysis we assumed that the momentum transfer square t is small, Q^2 is large and leads to the scaling. In a real experiment, at finite (and not so large) Q^2 and when $-t$ increases up to the order of Q^2, the real photon has to be treated as a part of the hard physics and the process becomes a fixed angle process. Such a case may be discussed along the lines of Ref. [1] or analyzed with the use of the handbag mechanism of Ref. [18].

The TDAs we introduced are, in particular, relevant for the reactions involving baryon number exchanges. We have discussed two new cases of factorization in hard exclusive reactions, the soft parts of which are described by the same hadron to photon TDA. Although, for simplicity, our study was concentrated on the mesonic case, we stress that most of our conclusions apply also for the phenomenologically more interesting, but also technically more difficult, baryonic case.

Let us emphasize the main differences between the mesonic and the baryonic cases:

(i) The relevant matrix element involves three quark fields $\langle\gamma|u^\alpha(z_1)[z_1;z_0]u^\beta(z_2)\times[z_2;z_0]d^\gamma(z_3[z_3;z_0])|p\rangle|_{z_i^+=0,z_i^T=0}$, and, consequently, it will involve more independent spinor structures than in the meson case.

(ii) The number of independent helicity amplitudes for the process $p\to qqq\gamma$ is 16 (there are in this case 2^5 helicity amplitudes but parity conservation relates amplitudes differing by all the signs of helicities). This allows one to predict that there will be 16 independent, leading twist TDAs for the $p\to\gamma$ transition. As in the meson case, where only one out of four TDAs actually enters Eq. (12), less than 16 TDAs are expected to contribute to the amplitude of the processes under study.

(iii) A slight generalization of the scaling law written in Eq. (14) dictates the high Q^2 behavior of the baryonic amplitudes, up to logarithmic corrections

$$\mathcal{M}(Q^2,\xi)\sim Q^{-2n_v+3}, \qquad (15)$$

where n_v is the number of valence partons in the hadron ($n_v=3$ for the proton).

(iv) The QCD evolution equations for the TDAs in the baryonic case are more involved, since the radiated gluon may be coupled to each pair of exchanged quarks and the evolution kernel depends on the direction of these quark momenta. New evolution equations may be written in a way which takes care of the different sectors in longitudinal momentum fractions of the three exchanged quarks.

The framework which we present in the present paper is very different from the handbag dominance model [18], which has been proposed to describe the annihilation reaction $p\bar{p}\to\gamma\gamma$. Such a model, based on an analogy with the reaction $\gamma^*\gamma\to\pi^+\pi^-$ at small energies [19], predicts a different scaling behavior at large Q^2 and a different helicity structure. Although these two approaches deal with different kinematical domains (the present one studies the forward and backward cases, that of Ref. [18] concentrates on fixed large angle kinematics) which are well distinct at asymptotic energies, actual experimental data at moderate values of Q^2 and s may not well separate these two regions. It should be important to investigate in detail the phenomenological differences of these two approaches. Experimental tests of our picture should be feasible in the near future. Already now experimental data exist on backward VCS at JLab up to $Q^2=1\ \mathrm{GeV}^2$ [10]. Further experiments on backward VCS are planned with higher energy and rates at JLab, HERMES, and COMPASS experiments. The PANDA and PAX experiments with the proposed 1.5–15 GeV high-luminosity antiproton storage ring at GSI [9] intend to study the proton antiproton annihilation reaction. Let us finally note that $p\to\pi$ TDAs may also be defined in an analogous way [20], allowing a similar description of reactions such as

$$p\bar{p}\to\gamma^*\pi \quad \text{or} \quad \gamma^* p\to p\pi.$$

Again these processes are likely to be measurable in forthcoming experiments at GSI, JLab, and HERMES.

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