PHYSICAL REVIEW D **71,** 111301(R) (2005)

Connection between the neutrino seesaw mechanism and properties of the Majorana neutrino mass matrix

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(Received 10 January 2005; published 24 June 2005)

If it can be ascertained experimentally that the 3×3 Majorana neutrino mass matrix $\mathcal{M}_\nu$ has vanishing determinants for one or more of its 2×2 submatrices, it may be interpreted as supporting evidence for the theoretically well-known canonical seesaw mechanism. I show how these two things are connected and offer a realistic $\mathcal{M}_\nu$ with two zero subdeterminants as an example.

DOI: 10.1103/PhysRevD.71.111301 PACS numbers: 14.60.Pq, 11.30.Hv, 14.60.St

It is a common theoretical belief in neutrino physics that the observed smallness of neutrino masses is due to the celebrated canonical seesaw mechanism [1], i.e.

$$\mathcal{M}_\nu = \mathcal{M}_D \mathcal{M}_N^{-1} \mathcal{M}_D^T. \tag{1}$$

Here $\mathcal{M}_D$ is the 3×3 Dirac mass matrix linking $(\nu_e, \nu_\mu, \nu_\tau)$ to their right-handed singlet counterparts (N_e, N_μ, N_τ), and $\mathcal{M}_N$ is the 3×3 Majorana mass matrix of the latter. Assuming $\mathcal{M}_D$ to be of order the electroweak breaking scale, a very large $\mathcal{M}_N$ would then result in a very small $\mathcal{M}_\nu$.

However, it is impossible to verify this hypothesis without reaching energies of the scale of $\mathcal{M}_N$, or extreme sensitivities in rare decay processes. Both are hopeless in the near future unless $\mathcal{M}_N$ is of order a few TeV [2]. If $\mathcal{M}_N$ is much greater than that, one may never know if Eq. (1) is really how neutrinos become massive.

The form and texture of $\mathcal{M}_\nu$ have been under theoretical study for many years. Is it possible at all to discover from its structure that it actually comes from $\mathcal{M}_N$ as given by Eq. (1)? The answer is ‘‘yes’’, provided that $\mathcal{M}_N$ has one or more texture zeros. In that case, $\mathcal{M}_N^{-1}$ has one or more 2×2 submatrices with zero determinants. If $\mathcal{M}_D$ is also diagonal, this property is preserved in $\mathcal{M}_\nu$. Finding such a structure in the latter experimentally would be provocative supporting evidence that Eq. (1) is correct!

In the basis where the charged-lepton mass matrix $\mathcal{M}_l$ is diagonal, the possible existence of texture zeros in $\mathcal{M}_\nu$ have been considered previously [3]. These zeros are derivable from Abelian discrete symmetries [4], and in the case of $(\mathcal{M}_\nu)_{\mu\mu} = (\mathcal{M}_\nu)_{\tau\tau} = 0$ also from the non-Abelian discrete groups Q_8 [5] and D_5 [6]. However, if $\mathcal{M}_N$ is the progenitor of $\mathcal{M}_\nu$, one should perhaps consider instead the texture zeros of the former [7], which may be similarly obtained from the symmetries already mentioned. For example, if $(\mathcal{M}_N)_{\mu\mu} = (\mathcal{M}_N)_{\tau\tau} = 0$, i.e.

$$\mathcal{M}_N = \begin{pmatrix} A & B & C \\ B & 0 & D \\ C & D & 0 \end{pmatrix}, \tag{2}$$

which is the analog of Scenario (1) of Ref. [5] and also that of the model of Ref. [6], then

$$\mathcal{M}_N^{-1} = \begin{pmatrix} a & b & c \\ b & e & d \\ c & d & f \end{pmatrix} \tag{3}$$

has two zero 2×2 determinants, i.e.

$$ae - b^2 = af - c^2 = 0. \tag{4}$$

[To prove this, one simply considers the identity $\mathcal{M}_N^{-1}\mathcal{M}_N = 1$.]

If $\mathcal{M}_D$ is also diagonal, which may be maintained again by the symmetries already mentioned [5,6], i.e.

$$\mathcal{M}_D = \begin{pmatrix} x & 0 & 0 \\ 0 & y & 0 \\ 0 & 0 & z \end{pmatrix}, \tag{5}$$

then from Eq. (1),

$$\mathcal{M}_\nu = \begin{pmatrix} x^2a & xyb & xzc \\ xyb & y^2e & yzd \\ xzc & yzd & z^2f \end{pmatrix}. \tag{6}$$

Since

$$(x^2a)(y^2e) - (xyb)^2 = x^2y^2(ae - b^2), \tag{7}$$

$$(x^2a)(z^2f) - (xzc)^2 = x^2z^2(af - c^2), \tag{8}$$

the corresponding two subdeterminants of $\mathcal{M}_\nu$ are zero as well. Using Eqs. (4), (7), and (8), let us rewrite $\mathcal{M}_\nu$ of Eq. (6) as

$$\mathcal{M}_\nu = \begin{pmatrix} \alpha & \beta & \gamma \\ \beta & \alpha^{-1}\beta^2 & \delta \\ \gamma & \delta & \alpha^{-1}\gamma^2 \end{pmatrix}. \tag{9}$$

This is Model (D) of Ref. [7]. It is in fact a realistic neutrino mass matrix, capable of describing all present data [8]. If confirmed by future precision data, this would be provocative supporting evidence that the long-held theoretical belief in the canonical seesaw mechanism is indeed valid!

The $\mathcal{M}_\nu$ of Eq. (9) has four parameters, but it will fit all data even if it is reduced to three parameters by setting $\beta = \gamma$, i.e.

$$\mathcal{M}_\nu = \begin{pmatrix} \alpha & \beta & \beta \\ \beta & \alpha^{-1}\beta^2 & \delta \\ \beta & \delta & \alpha^{-1}\beta^2 \end{pmatrix}. \qquad (10)$$

This is a special case of the general form [9] which exhibits the symmetry $\nu_\mu \leftrightarrow \nu_\tau$, implying $\theta_{23} = \pi/4$ and $\theta_{13} = 0$ in the mixing matrix linking ν_e, ν_μ, ν_τ to their mass eigenstates. Using the general analysis of Ref. [9], where $\mathcal{M}_\nu$ is given by

$$\mathcal{M}_\nu = \begin{pmatrix} a+2b+2c & d & d \\ d & b & a+b \\ d & a+b & b \end{pmatrix}, \qquad (11)$$

we then have

$$d = \beta, b = \alpha^{-1}\beta^2, a = \delta - \alpha^{-1}\beta^2, \quad c = (\alpha - \delta - \alpha^{-1}\beta^2)/2. \qquad (12)$$

As shown in Ref. [9], $\mathcal{M}_\nu$ is exactly diagonalized by

$$\begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix} = \begin{pmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta/\sqrt{2} & \cos\theta/\sqrt{2} & -1/\sqrt{2} \\ \sin\theta/\sqrt{2} & \cos\theta/\sqrt{2} & 1/\sqrt{2} \end{pmatrix} \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{pmatrix}, \qquad (13)$$

with

$$m_{1,2} = a + 2b + c \mp \sqrt{c^2 + 2d^2}, m_3 = -a, \quad \tan^2 2\theta = 2d^2/c^2. \qquad (14)$$

Using Eqs. (12) and (14), the three parameters α, β, δ of Eq. (10) can now be fixed by the three experimental measurements of $\theta (= \theta_{12})$,

$$\Delta m^2_{\rm sol} = 4(a+2b+c)\sqrt{c^2+2d^2} = \frac{4(a+2b+c)|c|}{\cos 2\theta_{12}}, \qquad (15)$$

and

$$\Delta m^2_{\rm atm} = a^2 - (a+2b+c)^2 - c^2 - 2d^2 = a^2 - \left[\frac{(\Delta m^2_{\rm sol})\cos 2\theta_{12}}{4c}\right]^2 - \frac{c^2}{\cos^2 2\theta_{12}}. \qquad (16)$$

For example, let $\alpha = 6 \times 10^{-4}$ eV, $\beta = 4 \times 10^{-3}$ eV, and $\delta = -2.1 \times 10^{-2}$ eV, then Eq. (14) yields a normal ordering of neutrino masses ($|m_1| < |m_2| < |m_3|$), with

$$\tan^2\theta_{12} = 0.42, \Delta m^2_{\rm sol} = 7.8 \times 10^{-5}\ {\rm eV}^2, \quad \Delta m^2_{\rm atm} = 2.2 \times 10^{-3}\ {\rm eV}^2, \qquad (17)$$

in good agreement with present data.

If $\theta_{13} \neq 0$ is required by future data, the unrestricted Eq. (9) itself should be considered. Instead of $(\mathcal{M}_N)_{\mu\mu} = (\mathcal{M}_N)_{\tau\tau} = 0$ in Eq. (2), another interesting possibility is to have $(\mathcal{M}_N)_{\mu\tau} = (\mathcal{M}_N)_{\tau\mu} = 0$. In that case, $\mathcal{M}_N^{-1}$ of Eq. (3) has $ad - bc = 0$. This has in fact been implemented in a model [10] based on $D_4 \times Z_2$.

Experimentally, it will be a daunting task to measure each of the six elements of $\mathcal{M}_\nu$. Only the absolute value of $(\mathcal{M}_\nu)_{ee}$ is subject to direct experimental measurement from neutrinoless double beta decay, which is being pursued vigorously by several international collaborations. The absolute values of $(\mathcal{M}_\nu)_{e\mu}$ and $(\mathcal{M}_\nu)_{\mu\mu}$ may be obtained from future experiments searching for μ^- to e^+ and μ^- to μ^+ conversion in nuclei, but the sensitivity required is many orders of magnitude beyond present capability.

However, a partial test of the idea of zero subdeterminants is possible because such a requirement reduces the number of independent parameters in $\mathcal{M}_\nu$. If both $|m_{\nu_e}|$ and $|(\mathcal{M}_\nu)_{ee}|$ are measured in the future, as well as the CP-nonconserving Dirac phase of the neutrino mixing matrix and its three angles, together with more precise values of $\Delta m^2_{\rm atm}$ and $\Delta m^2_{\rm sol}$, these eight quantities can be used to check if Eq. (9) [or any of the other possible forms of $\mathcal{M}_\nu$ with one or more zero subdeterminants] is still valid. If so, then it is at least indirect confirmation of this hypothesis. [Note that two zero subdeterminants imply four real parameters and one phase, and one zero subdeterminant implies five real parameters and two phases.]

Naturally small Majorana neutrino masses are obtainable in the Standard Model in three and only three tree-level mechanisms [11]. The canonical seesaw mechanism using heavy Majorana right-handed neutrino singlets N_i has dominated the literature, but the use of a heavy Higgs scalar triplet ξ without N_i is just as natural [12]. In the latter case, $\mathcal{M}_\nu$ is obtained directly through the naturally small vacuum expectation value of ξ, and it makes sense to consider the possible texture zeros of $\mathcal{M}_\nu$ which may be derived from some discrete family symmetry [5,6]. On the other hand, if $\mathcal{M}_N$ is truly the progenitor of $\mathcal{M}_\nu$ as dictated by Eq. (1), then it makes more sense to consider the structure of $\mathcal{M}_N$ for its imprint on $\mathcal{M}_\nu$. This is indeed possible if $\mathcal{M}_N$ has one or more texture zeros [7] and $\mathcal{M}_D$ is diagonal. It is pointed out in this paper that a simple way to know is to look for zero subdeterminants in $\mathcal{M}_\nu$. Finding them would go a long way in bolstering the neutrino community's faith in the correctness of Eq. (1). Present data are in fact consistent with such a prediction, as exemplified by Eqs. (9) and (10). [If the neutrino mass matrix comes from a different mechanism, a zero subdeterminant may also occur accidentally, so there can never be a decisive proof.] Just as the original observation which led to the canonical seesaw mechanism is essentially trivial, the present observation that a zero entry in a matrix is reflected by a zero determinant in its inverse is also essentially trivial, but both may in fact be important clues to the origin of the observed neutrino mass matrix.

This work was supported in part by the U. S. Department of Energy under Grant No. DE-FG03-94ER40837.

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