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One dimensional M5-brane intersections

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We study one-dimensional intersections of M5 branes with M5 and M2 branes. On the worldvolume of the M5-brane, such an intersection appears as a string soliton. We study this worldvolume theory in two different regimes: (1) Where the world-volume theory is formulated in flat space and (2) where the world-volume theory is studied in the supergravity background produced by a stack of M5 (or M2) branes. In both cases, we study the corresponding string solitons, and find the most general BPS configuration consistent with the fraction of supersymmetries preserved. We argue that M5 and M2 brane intersections leave different imprints on the world-volume theory of the intersecting probe brane, although geometrically they appear to be similar.

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I. INTRODUCTION

The one-dimensional intersection of M5-branes is a much neglected subject. One of the reasons this system is not better understood is simply that it does not obey the $(p-2)$-rule [1] which generates all other self-intersecting M-brane configuration [2–14].

This rule [1] states that one p-brane can intersect another along $(p-2)$ spatial directions, if the resulting system is required to be Bogomol'nyi-Prasad-Sommerfield (BPS). By extension, if each pair of branes in a particular configuration intersects along $(p-2)$ spatial directions, we are guaranteed that the resulting configurations preserves some supersymmetry. Because of its simplicity and vast jurisdiction, this rule has been used extensively to write down complicated BPS configurations.

The defiance of the $(p-2)$ rule by the M5 $\perp$ M5(1) configuration, was thus, both a mystery and an obstacle to a clear understanding of the system. However, it was claimed in [15] that this particular system is exempt from the rule! The reasoning is as follows: It turns out that the presence of any world-volume fields (other than scalars and their duals) results in a contradiction of the assumptions under which the $(p-2)$ rule was "derived." Since the M5 $\perp$ M5(1) system is the only self-intersecting brane configuration for which the world-volume twoform on the fivebrane is turned on, that rule does not apply to this configuration.

Encouraged by the fact that some inroads are finally being made into understanding this system, we turn our attention here to another unsolved issue regarding M5 $\perp$ M5(1). In [16] it is claimed that whenever M5-branes intersect over one dimension an M2-brane is always secretly present. In this paper, we try to find whether or not that claim indeed holds true. To this end, we look at one-dimensional solitons on M5-branes in several ways; our analysis will thus be confined to the world volume of a fivebrane.

We will start by considering an M5 world-volume theory in flat space and looking for string solitons in that theory [17]. We will, however, display the most general solution for the soliton preserving the appropriate amount of supersymmetries, and proceed to study an M5-brane theory in the supergravity background produced by, alternately, infinite M5 and M2 branes.

We will show that the induced metric on the M5-brane is different in the two cases. Although, in the near-horizon Maldacena limit of the background branes the geometry is $AdS_3 \times S^3$ for both backgrounds, the radii of curvature distinguish the two cases. In fact, these radii of curvature match only for very special values of the number of branes. We take this as evidence that the one-dimensional M5/M5-brane intersection is a genuine intersection and not just a secret M2 brane stretching between the two M5 branes.

It is perhaps worth mentioning at this point, that even in string theory, the lower dimensional descendants of the enigmatic M5 $\perp$ M5(1) system continue to be somewhat special and are definitely not completely understood. The one-dimensional intersection of two M5-branes can be dimensionally reduced to two D4-branes intersecting over a point. This system is known to have two supersymmetric branches in the presence of aB-field; one of which branches is continuously connected to $B=0$, whereas the other is not [18,19] T-dualizing further relates it to an intersecting D0-D8 system, which again, is interesting in its own right.

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II. STRING SOLITONS ON M5 WORLD VOLUME

A. Cause for confusion

Consider two M5-branes, one of them extended in the 12345 directions and the other along 16789. Using $\times$ to denote directions tangent to the world-volume of a brane, we can express this configuration in tabular form as follows:

	0	1	2	3	4	5	6	7	8	9	10
M5	×	×	×	×	×	×					
M5	×	×					×	×	×	×	

(1)

The two M5-branes can be separated along X^{10}. If, however, they are located at the same point in X^{10}, they must intersect along X^1. We can study this system from the point of view of the world volume of one of the M5-branes, which we denote as **M5**. The other M5-brane[1], denoted by $M5$, will then appear as a soliton in the world-volume theory of **M5**.

Similarly, one can introduce an $M2$-brane stretched along X^{10} and ending on the **M5**-brane along X^1.

	0	1	2	3	4	5	6	7	8	9	10
M5	×	×	×	×	×	×					
M2	×	×									×

(2)

The one-dimensional end of this M2-brane will again appear as a world-volume soliton in the M5-brane.

From the bulk vantage point, these two configurations are very similar - perhaps even confusingly so. In both cases the surrounding space-time is static and exhibits translation symmetry along X^1. Both configurations have an $SO(4) \times SO(4)$ isometry corresponding to rotational symmetry in the $X^2 \cdots X^5$ and $X^6 \cdots X^9$ and X^{10}.

In addition, these two intersecting brane systems require identical projection conditions to be satisfied by Killing spinors, and thus preserve the same amount of supersymmetry. Killing spinors for the $M5 \perp M5$ configuration described in (1), obey

$$\gamma_{012345}\epsilon = \epsilon, \qquad \gamma_{016789}\epsilon = \epsilon, \tag{3}$$

whereas the Killing spinors for the $M5 \perp M2$ system of (2) are such that

$$\gamma_{012345}\epsilon = \epsilon, \tag{4}$$

$$\gamma_{01(10)}\epsilon = \epsilon. \tag{5}$$

Using the identity $\gamma_{(10)} = \gamma_0\gamma_1 \cdots \gamma_9$, it is trivial to see that the two sets of constraints given above are in fact identical; any one set implies the other.

Since we are currently limiting ourselves to looking at the system in its probe approximation, the bending of space-time due to any of the above M-branes need not be taken into account; in this section, the background is thus considered to be flat.

Given the fact that the one-dimensional intersection of an M5 brane with M5-branes is similar in so many ways to the one-dimensional ending of a membrane on it, it is important to ask how these two configurations can be distinguished; could they perhaps have different manifestations on the world volume of **M5**? It is to answer this question that we investigate how $M2$ and $M5$ appear, when viewed from the point of view of (the probe) **M5**. As we will show, in the flat background, for every integer value of the charge Q with respect to the self-dual threeform field strength $\hat{H}$ on the **M5** brane world volume, there exists a unique BPS soliton which respects the isometries required by a 1-dimensional intersection. In this situation, the world-volume string soliton will not be able to distinguish $M2$ ending on **M5** from M5 $\perp$ M5(1). However, in the next section, we will show that there is an interesting twist to this story when we take into account curvature effects due to background $M2/M5$ branes.

B. The view from the world volume

In the next subsection we will obtain the BPS string soliton on the **M5** brane world volume in the flat background, however, in this subsection we will set up BPS conditions for the soliton without making any specific choice of background metric. The world-volume theory we will study is that of an M5-brane oriented along 12345. The bosonic content of this theory consists of five scalars (X^a, $a = 6, ..10$) corresponding to the transverse fluctuations of the M5-brane and a 2-form whose associated field strength is self-dual.

Let us now consider a one-dimensional soliton oriented along, say, the X^1 direction. As discussed previously, there is an SO(4) symmetry in the remaining world-volume directions, $X^2, \cdots, X^5$. If we define a radial coordinate $\tilde{r}^2 = \sum_{i=2}^5 (X^i)^2$, this isometry is reflected in the fact that scalar fields depend only on $\tilde{r}$.

The scalar fields X^a, $a = 6, \cdots, 10$ can be split into two groups: one consisting of $X^6, \cdots, X^9$ which are scalars representing directions along the $M5$-brane soliton (or transverse to the $M2$-brane), and X^{10} which is transverse to the $M5$-brane soliton (or along the $M2$-brane).

Since we are looking for a string soliton, we expect the twoform to have components along the 0,1 directions and to depend only on $\tilde{r}$. The self-duality of the corresponding threeform field strength implies that we have not only the

[1]Throughout this paper we use boldface notation for the probe brane, and italics for background branes.

components $\hat{H}_{01\tilde{r}}$, but also its dual components along the transverse S^3. For ease of notation we now define a "reduced twoform" $H_{ab} \equiv \hat{H}_{0ab}/\sqrt{g_{00}}$.

In order for a configuration to be supersymmetric, it must saturate a BPS bound, the world-volume formulation of which for flat space is given in [20,21]. This can be written in a form which generalizes to curved space:

$$\sqrt{\det(g+H)} = \epsilon^\dagger \gamma^0 \bigg[\frac{1}{5!} \Gamma_{abcde} \epsilon^{abcde} - \frac{\sqrt{\det(g)}}{2} \Gamma_{ab} H^{ab} + \Gamma_a t^a \bigg] \epsilon, \quad (6)$$

Here, g_{ab} and Γ denote the world-volume pull-backs of the space-time metric and γ-matrices. In static gauge we have the expressions

$$\Gamma_a = \gamma_a + \partial_a X^i \gamma_i g_{ab} = h_{ab} + \partial_a X^i \partial_b X^j h_{ij}, \quad (7)$$

where $a = 0, \cdots 5$ is a world-volume index, $i = 6 \cdots 10$ labels directions transverse to the probe **M5** brane, and (h_{ab}, h_{ij}) comprises the full metric in space-time. In addition to solving the BPS saturation condition, the field strength H_{ab} should also satisfy the Bianchi identity constraint corresponding to the gauge invariance of the twoform potential. Using the isometries of our solution this condition can be expressed as

$$\partial_{\tilde{r}}(\sqrt{g} H^{1\tilde{r}}) = 0. \quad (8)$$

C. The string soliton solution

As mentioned in the previous subsection, here we will restrict ourselves to the case of flat metric in the 11 dimensional space-time. In a flat background, the pullback of the space-time metric onto the world volume (012345) of the **M5** brane is

$$ds_6^2 = -dt^2 + dX_1^2 + \bigg(1 + \sum_{i=6}^{9} (\partial_{\tilde{r}} X_i)^2 + (\partial_{\tilde{r}} X_{10})^2 \bigg) d\tilde{r}^2 + \tilde{r}^2 d\Omega_{s^3}^2. \quad (9)$$

Given this induced metric, the general form of H^{ab}, and the spinor projection conditions in Eq. (6), we can use the BPS condition to determine the functional dependence of H on the transverse scalars. A general solution is

$$H_{1\tilde{r}} = \sqrt{1 + \sum_{k=6}^{9} (\partial_{\tilde{r}} X_k)^2 + (\partial_{\tilde{r}} X_{10})^2} \times \bigg[-\partial_{\tilde{r}} X_{10} \pm \sqrt{\{-1 - \sum_{i=6}^{9} (\partial_{\tilde{r}} X_i)^2 - (\partial_{\tilde{r}} X_{10})^2\} \sum_{j=6}^{9} (\partial_{\tilde{r}} X_j)^2} \bigg]. \quad (10)$$

The field strength is real if and only if we set

$$\sum_{j=6}^{9} (\partial_{\tilde{r}} X_j)^2 = 0, \quad (11)$$

which in turn means

$$H_{1\tilde{r}} = -(\partial_{\tilde{r}} X_{10}) \sqrt{1 + (\partial_{\tilde{r}} X_{10})^2}. \quad (12)$$

Notice that for the calculations we carried out in this section, we never had to state explicitly whether we were considering the $M5 \perp M5(1)$ configuration of (1), or the $M5 \perp M2(1)$ configuration of (2). All that we needed in order to solve the BPS equation was a knowledge of the symmetries on the world volume (which dictate the form of H), and the projection conditions on the Killing spinor. Since the preserved supersymmetries and isometries of both (1, 2) are identical, it is not within the scope of our current calculation to distinguish between these two scenarios; the results we have obtained thus far are hence equally valid whether the background contains an M5 brane in 16789 directions or an M2 brane in the 1(10) directions.

As we have shown, the only scalar field we can turn on while preserving spherical symmetry and world-volume supersymmetry is the X^{10} field. The other scalar fields remain constant. This solution is subject to the Bianchi identity constraint [20,21] generalized to general curved space background:

$$\partial_{\tilde{r}}(\sqrt{g} H^{1\tilde{r}}) = 0. \quad (13)$$

Writing this condition in term of X_{10} implies it satisfies the equation

$$\partial_{\tilde{r}}(\tilde{r}^3 \partial_{\tilde{r}} X_{10}) = 0 \quad (14)$$

or

$$X_{10} = \text{const.} + q_0/r^2, \quad (15)$$

where q_0 is proportional to the soliton number N_1.

III. PROBING M2/M5 BACKGROUNDS

In this section we study the world-volume theories of an **M5**-brane probe in the background geometries produced, alternately, by an $M2$-brane and an $M5$-brane. We will start with the gravitational background of an $M2$ brane extended in the spatial directions x^1 and x^{10}. Our probe **M5** brane is extended, as usual, in the 12345 directions.

A. M2-brane background

The metric due to the $M2$ brane background is

$$ds^2 = h_2^{1/3}(r) \bigg[h_2^{-1}(r)(-dt^2 + dx_1^2 + dx_{10}^2) + \sum_{i=2}^{9} dx_i^2 \bigg]. \quad (16)$$

The pullback of this background on the **M5** brane is

$$ds_6^2 = h_2^{-2/3}(r)(-dt^2 + dx^{12}) + h_2^{1/3}(r)\tilde{r}^2(d\Omega_{S^3}^2) + h_2^{1/3}(r)\Big(1 + \sum_{i=6}^{9}(\partial_{\tilde{r}}X^i)^2 + h_2^{-1}(r)(\partial_{\tilde{r}}X^{10})^2\Big)d\tilde{r}^2, \qquad (17)$$

where $h_2(r) = 1 + \frac{k}{r^6}$, $r^2 = \sum_{I=2}^{9}(x^I)^2$, and $\tilde{r}^2 = \sum_{i=2}^{5}(x^i)^2$. The parameter $k = 2^5\pi^2 N_2 \ell_p^6$ [22] depends on N_2, the number of $M2$ branes, and, ℓ_p, the 11 dimensional Planck length.

It is simple to see (and the general results below contain it as a subcase) that this is a BPS solution with $H_{ab} = 0$ and the $X^i = 0$, $X_{10} = 0$ in the world-volume theory. This is, of course, not surprising since the M5 brane is probing a stack of N_2 M2 branes which are infinite and therefore pass right through the probe with all the charge canceling locally. All the scalars are constants since there is no bending due to the tension of the M2-brane since the force due to it is locally cancelled.

It is generally difficult to compare different geometries (metrics) since they are expressed in different coordinates. It is, therefore, instructive to take the Maldacena (near-horizon) limit where we express the metric in the rescaled variable $u = r^2/\ell_p^3$ [22] in the low-energy limit $u\ell_p \ll 1$. In this limit, with the scalars set to constants and $H_{ab} = 0$, the geometry becomes $AdS_3 \times S^3$. With the AdS_3 radius of curvature $R_{AdS_3}^2 = \ell_p^2(\frac{\pi^2 N_2}{2})^{1/3}$ and $R_{S^3}^2 = \ell_p^2(2^5\pi^2 N_2)^{1/3}$[2]. This can be compared to the M5-brane case below.

B. M5-brane background

It is instructive to contrast this situation with the curved geometry generated by an $M5$ brane. To be able to directly compare the two situations we will assume the background $M5$ brane is extended in 16789 direction and the probe **M5** brane is still extended in 12345 directions. Like in the previous case, we will start with the background metric generated by the $M5$ brane extended in 16789 direction

$$ds^2 = h_5^{2/3}(r)\Big[h_5^{-1}(r)\Big(-dt^2 + dx_1^2 + \sum_{i=6}^{9} dx_i^2\Big) + \sum_{a=2}^{5} dx_a^2 + dx_{10}^2\Big]. \qquad (18)$$

The pullback of this metric on the probe gives the induced world-volume metric on M5 brane extended in 12345 direction

$$ds_6^2 = h_5^{-1/3}(r)(-dt^2 + dx^{12}) + h_5^{2/3}(r)\tilde{r}^2(d\Omega_{S^3}^2) + h_5^{2/3}(r)\Big(1 + h_5^{-1}(r)\sum_{i=6}^{9}(\partial_{\tilde{r}}X^i)^2 + (\partial_{\tilde{r}}X^{10})^2\Big)d\tilde{r}^2, \qquad (19)$$

where, $h_5(r) = 1 + q/r^3$, $\tilde{r}$ is as defined earlier, $r^2 = \tilde{r}^2 + x_{10}^2$, and $q = \pi N_5 \ell_p^3$ depends on N_5, the number of $M5$ branes.

Because of the different functional dependence of the harmonic function $h(r)$ in $M2$ and $M5$ case, it may appear that these are two totally different metrics. It is, therefore, useful to take the near-horizon limit to see that in both cases the near-horizon geometry is $AdS_3 \times S^3$. The formal similarity of these two backgrounds, however, ends here. To get the metric in the desired form we need to take a different limit [22], with $u^2 = r/\ell_p^3$. The radii of curvature of S^3 and that of AdS_3 are given by $R_{S^3}^2 = R_{AdS_3}^2/4 = (\pi N_5)^{1/3}\ell_p^2$. It is easy to see that these two geometries will not be the same for arbitrary integer values for N_2 and N_5.

C. String soliton in curved background

We will now turn our attention to the BPS string soliton, which is extended along the x_1 direction, on the world volume of **M5** brane in the $M2$ brane background. As in the previous section we again take a radial ansatz for the soliton on the world volume and define the radial direction by $\tilde{r} = \sqrt{x_2^2 + \cdots + x_5^2}$. The BPS condition implies the induced metric and the threeform field strength field H should satisfy (6). We substitute the Eq. (17) into the BPS condition (6) to determine

$$\tilde{H}_{1\tilde{r}} = -\frac{h_2^{-1/3}(r)A^2(r)\partial_{\tilde{r}}X^{10} \pm B(r)}{h_2^{1/3}(r)A(r)(1 + \sum_{i=6}^{9}(\partial_{\tilde{r}}X^i)^2)}, \qquad (20)$$

where

$$A^2(r) = \Big(1 + \sum_{i=6}^{9}(\partial_{\tilde{r}}X^i)^2 + h_2^{-1}(r)(\partial_{\tilde{r}}X^{10})^2\Big), \qquad (21)$$

$$B(r) = \sqrt{-h_2^{1/3}(r)\sum_{i=6}^{9}(\partial_{\tilde{r}}X^i)^2}. \qquad (22)$$

We have put a tilde on H to distinguish it from that obtained in the flat background. We will continue to use the same notation in the $M5$ brane background as well. The BPS condition gives rise to a quadratic equation for $\tilde{H}_{1\tilde{r}}$. We, however, end up with only one solution because the term inside the square root of the solution is negative semidefinite and a real solution for $\tilde{H}_{1\tilde{r}}$ is obtained by setting that term to zero. This is achieved by setting $\partial_{\tilde{r}}X^i = 0$ for $i = 6, 7, 8, 9$. Therefore,

[2]This result is complementary to the relation obtained earlier [23] in a related context.

$$\tilde{H}_{1\tilde{r}} = -h_2^{-2/3}(r)\partial_{\tilde{r}}X^{10}\sqrt{1 + h_2^{-1}(r)(\partial_{\tilde{r}}X^{10})^2}. \tag{23}$$

This solution should also satisfy the Bianchi identity constraint (13), which in this background(17) becomes

$$\partial_{\tilde{r}}(\tilde{r}^3 h_2^{1/3}(r)\partial_{\tilde{r}}X^{10}) = 0. \tag{24}$$

It is interesting to see that for large values of $\tilde{r}$, the Bianchi identity leads to same equation for X_{10} even in the case of a curved background. Hence the solution to this equation is same as that given in Eq. (15) in this limit. The global solution, however, differs from the flat case. This can be seen, e.g., in the near-horizon limit where we can ignore constant element in the harmonic function $h_2(r)$ and taking $r = \tilde{r}$ limit,

$$X_{10}(\tilde{r}) = \frac{c_2}{N_2^{1/3}\ell_p^2}\ln\tilde{r}. \tag{25}$$

We will come back to interpretation of this solution in the next section. The fact that in the large $\tilde{r}$ limit, the Bianchi identity reduces to that in the flat space is not surprising and can be used to determine the charge carried by the string soliton. The charge N_{2s} carried by the string soliton is determined by integrating $\tilde{H}_{1\tilde{r}}$ over the asymptotic S^3 which encloses the soliton. Using this condition we can determine asymptotic behavior of X_{10} for large values of $\tilde{r}$ and we get

$$X_{10}(\tilde{r}) \sim \frac{N_{2s}}{\tilde{r}^2}. \tag{26}$$

Now we will look at the world-volume string soliton in the $M5$ brane background (19). We determine the three-form field configuration which solves the BPS condition (6) for the string soliton by using the induced metric (19). The field strength $\tilde{H}_{1\tilde{r}}$ is

$$\tilde{H}_{1\tilde{r}} = -h_5^{1/6}(r)\sqrt{1 + (\partial_{\tilde{r}}X^{10})^2}(\partial_{\tilde{r}}X^{10}). \tag{27}$$

Like in the $M2$ brane background, scalar fields X^i for $i = 6, 7, 8, 9$ are set to zero due reality condition on the field strength $\tilde{H}_{1\tilde{r}}$. The Bianchi identity gives us

$$\partial_{\tilde{r}}(\tilde{r}^3 h_5^{7/6}(r)\partial_{\tilde{r}}X^{10}) = 0. \tag{28}$$

Notice, in the large $\tilde{r}$, this equation is the same as that obtained in the $M2$ brane background. However, for finite values of $\tilde{r}$, this equation differs significantly from that obtained in the $M2$ brane background. Particularly, in the near-horizon limit and with $r = \tilde{r}$, we can determine behavior of $X_{10}(\tilde{r})$ using the Bianchi identity constraint,

$$X_{10}(\tilde{r}) = \frac{c_5}{N_5^{7/6}\ell_p^{7/2}}\tilde{r}^{3/2}. \tag{29}$$

In the next section we will compare this solution with that obtained in the $M2$ brane background. Here we will just mention that this behavior is significantly different from that obtained in the $M2$ brane background. Large $\tilde{r}$ behavior of $X_{10}(\tilde{r})$ is again determined by either using the Bianchi identity or using the Gauss' law constraint. The latter determines behavior of $X_{10}(\tilde{r})$ as a function of soliton charge N_{5s} for large $\tilde{r}$,

$$X_{10}(\tilde{r}) \sim \frac{N_{5s}}{\tilde{r}^2}. \tag{30}$$

IV. CONCLUSIONS AND DISCUSSION

The motivation behind this paper was to investigate the claim [16] that two M5-branes can never intersect along one direction without a membrane being present; when it can not be seen, one is to assume that the membrane has collapsed. Building on [15], (which stated that two M5-branes *can* in fact intersect along a string if the world-volume twoform is turned on), we study one-dimensional intersections of M5-branes. In particular, we focus on $M5 \perp M5(1)$ and $M5 \perp M2(1)$ intersections, studying these from the point of view of the fivebrane world volume.

$M5 \perp M5(1)$ and $M5 \perp M2(1)$ intersections preserve the same fraction of supersymmetry in addition to having the same isometries. In the world-volume theory of an M5-brane, the string solitons corresponding to each of these one-dimensional intersections are explored.

We considered, in turn, both M-brane intersections as world-volume solitons in a probe M5-brane. We were lead to the same mathematical form of the solution for the soliton. Even though the solutions have the same form one can ask whether they are in fact identical. The question thus is what the world-volume charge produced by each one of the intersections is. We find that the M5 brane in the flat background is not a suitable setup for answering this question unambiguously. If we take curvature effects due to brane in the background in account then geometry of the defect on the probe brane is different.

In our second approach to the problem we look at a probe M5-brane in a curved space-time produced, respectively, by stacks of $M2$ and $M5$-branes. In both cases the world-volume H field vanishes due to local canceling of charges when the intersecting brane "passes right through" the probe **M5**-brane. We compare the induced world-volume metrics on the probe M5-brane by taking the Maldacena decoupling limit in which the induced metrics are both $AdS_3 \times S^3$ but with differing radii of curvature. This gives the evidence that the probe M5 brane can distinguish the geometry produced by a stack of $M2$ branes from that produced by a stack of $M5$ branes. It is, however, not a very convincing result. We, therefore, focus our attention to the string like defect on the probe **M5** brane.

We find the most general string soliton solutions in the world-volume theory of **M5** brane in the $M2$ and $M5$ brane background. This solution gives us the clearest way of differentiating these two configurations. Let us first state similarities of these two configurations. Firstly, in this case the world-volume H field is nontrivial in both cases.

Secondly, behavior of X_{10} far away from the core of the intersection is also similar in both cases, see (26) and (30). The distinguishing feature of the two intersections, i.e., $M2 \perp \mathbf{M5}(1)$ and $M5 \perp \mathbf{M5}(1)$, is the behavior of the world-volume scalar field $X_{10}(\tilde{r})$ in the near-horizon geometry of the string defect generated by the intersection on the probe $M5$ brane.

In the case of $M2$ brane background, the scalar field $X_{10}(\tilde{r})$ on the probe **M5** brane depends logarithmically on $\tilde{r}$ (25), indicating that the background $M2$ brane actually ends on the **M5** brane and the singular geometry generated by it qualifies to be a string soliton on the **M5** brane world volume.

On the other hand, in the $M5$ brane background, behavior of $X_{10}(\tilde{r})$ in the near-horizon geometry is regular and in fact X_{10} vanishes at the location of the defect (29). It means the line defect doesnot really qualify to be a bona fide world-volume string soliton. In fact, a natural interpretation of this defect is that the background $M5$ brane smoothly join the probe **M5** brane at the line defect to form a nonsingular geometry generated by a bent M5 brane.

We, therefore, conclude that the $M2$ and $M5$-brane backgrounds are indeed distinguishable in the world-volume theory of the **M5** brane probe.

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