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AdS_5 black holes with fermionic hair

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The study of new Bogomol'nyi-Prasad-Sommerfield (BPS) objects in AdS_5 has led to a deeper understanding of AdS/CFT. To help complete this picture, and to fully explore the consequences of the supersymmetry algebra, it is also important to obtain new solutions with bulk fermions turned on. In this paper we construct superpartners of the $1/2$ BPS black hole in AdS_5 using a natural set of fermion zero modes. We demonstrate that these superpartners, carrying fermionic hair, have conserved charges differing from the original bosonic counterpart. To do so, we find the R-charge and dipole moment of the new system, as well as the mass and angular momentum, defined through the boundary stress tensor. The complete set of superpartners fits nicely into a chiral representation of AdS_5 supersymmetry, and the spinning solutions have the expected gyromagnetic ratio, $g = 1$.

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I. INTRODUCTION

Since the recognition of their importance in connecting weakly coupled to strongly coupled physics, Bogomol'nyi-Prasad-Sommerfield (BPS) states have continued to play a major role in fulfilling the promises of strong/weak coupling duality. This is certainly evident today in the exploration of AdS/CFT, where a weakly coupled gravity system in five dimensions is dual to four-dimensional super-Yang Mills theory at strong 't Hooft coupling. In general, very few direct comparisons may be made between states at weak coupling and strong coupling. After all, following a state from weak to strong coupling involves the observation of more and more corrections, until finally the perturbative description, valid at weak coupling, breaks down altogether. In many cases, even the effective degrees of freedom are expected to change, so that keeping track of individual states would not make sense.

On the other hand, the reason BPS states are useful is that, as shortened representations of the supersymmetry algebra, they are protected against corrections by supersymmetry. Thus, in contrast with arbitrary states, they may be traced between strong and weak coupling. As such, they provide a primary means for extracting information out of systems which involve strong/weak coupling duality. For example, there is currently much interest in the $1/2$ BPS excitations of $AdS_5 \times S^5$ configurations. From the bulk point of view, these states have interpretation as either gravitational ripples or giant gravitons. These may be investigated either as classical solutions of the supergravity equations or through the world-volume dynamics of wrapped branes. Furthermore, through duality, such states are also associated with chiral primaries in the dual field theory. It is precisely the BPS nature of such excitations that allow such a rich connection to be made between seemingly different objects such as branes, classical gravity backgrounds and chiral primary operators.

In a supergravity context, extremal black holes are an obvious choice as BPS objects to explore. Such black holes have zero temperature and have a natural correspondence with pure states in a quantum theory. In this case, like all states in a supersymmetric model, the extremal black holes ought to form representations of the supersymmetry algebra. In particular, the bosonic black hole solution itself must also be related to superpartner black holes carrying fermionic hair. In fact, such superpartners may be constructed by action of a finite supersymmetry transformation δ on the original solution, represented schematically as

$$\Phi \longrightarrow e^{\delta}\Phi = \Phi + \delta\Phi + \frac{1}{2}\delta\delta\Phi + \cdots. \qquad (1)$$

Here, Φ woul -be the metric, graviphoton or any other field in the supergravity theory. For Poincaré supergravity, a typical example would be the extremal Reissner-Nordstrom solution with mass = charge. Clearly this coincides with the corresponding BPS condition $M = |Z|$ where Z is a central charge in the supersymmetry algebra. In this case, exactly half of the supersymmetries $\delta\Phi$ would vanish, namely, those related to the Killing spinors of the background. On the other hand, the remaining fermion zero-mode spinors would generate nontrivial transformations, demonstrating that the black hole lies in a shortened multiplet of supersymmetry.

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This construction of exact black hole superpartners was first carried out in [1] in the context of ungauged $\mathcal{N} = 2$ supergravity in four dimensions. The method was also used in [2] to examine the dipole moments and gyromagnetic ratios of $1/2$ and $1/4$ BPS black holes in ungauged $D = 4$, $\mathcal{N} = 8$ supergravity, in [3] to construct the fermionic partners of the supergravity description of D0-branes in ten dimensions, and in [4] to construct the M2-brane multiplet in 11 dimensional supergravity. In general, for such extremal objects in Poincaré supergravity, it may be explicitly demonstrated that all superpartners have identical masses and charges as the original black hole itself. At some level, this is simply a kinematical consequence of satisfying the supersymmetry algebra. Nevertheless, it is reassuring to see that semiclassical methods may be successfully applied to the study of new backgrounds with fermionic hair.

In this paper, we show that the same techniques for generating black hole superpartners in ungauged supergravities may also be applied to the case of gauged (or anti-de Sitter) supergravities. However, it is important to note that the AdS superalgebra is different from the Poincaré one. In particular, masses (actually energies) and charges of the superpartners are no longer identical, but are related according to the AdS superalgebra. Below, we construct explicit superpartners for black holes in gauged $D = 5$, $\mathcal{N} = 2$ supergravity and go on to calculate the masses and charges of the superpartners. We verify that the mass and charge shifts indeed follow the pattern required by supersymmetry in AdS spaces.

The main purpose behind this construction of black hole superpartners is to demonstrate that the fermion zero modes carry additional information about BPS background in supergravity. Although we work explicitly with black holes, the techniques we use are applicable to any background with partially broken supersymmetry, even those without horizons. By fully studying the BPS states and their partners, we may also hope to obtain new methods for exploring the lowest non-BPS excitations as well. While the zero-mode construction fails for non-BPS black holes (since the would-be zero modes are non-normalizable at the horizon), this obstacle may potentially be overcome in geometries without horizons.

It is worth noting that obtaining a meaningful definition of mass and angular momentum in AdS spaces involves some care. While various definitions have been provided, we use the holographic renormalization method [5–7], which is natural in an AdS/CFT context. Properties of the five-dimensional black holes which we are interested in have recently been examined in [8,9]. There it was demonstrated that a proper set of boundary counterterms was necessary to ensure the validity of the BPS algebra on the boundary.

We begin in section II with a brief overview of $\mathcal{N} = 2$ supergravity and very special geometry as well as the familiar $1/2$ BPS black hole solutions themselves. In section III we identify convenient zero-mode spinors and use these to modify the bosonic background of section II. In section IV we calculate the conserved charges of the new system, defining the mass and angular momentum through the boundary stress tensor. We conclude in section V by showing that these black holes fall into shortened chiral representations of AdS_5 supersymmetry and find the gyromagnetic ratio of the spinning superpartners to be $g = 1$.

II. BPS BLACK HOLES IN FIVE DIMENSIONS

We are interested in $1/2$ BPS black hole solutions of gauged $\mathcal{N} = 2$ supergravity in five dimensions coupled to n vector multiplets. The bosonic fields in this model consist of the metric $g_{\mu\nu}$, $n + 1$ vectors A^I_μ, and n scalars ϕ^x, while the fermionic fields are the gravitino ψ_μ and n gauginos λ_x. The gauging of a $U(1)$ subgroup of the $SU(2)$ R-symmetry group is achieved by introducing a linear combination of the $n + 1$ vectors, $\mathcal{A}_\mu = V_I A^I_\mu$ where the V_I are a set of constants. Since $\mathcal{A}_\mu$ is what couples to R-charge, it will play a prominent role in the supersymmetry analysis of section V.

This theory was constructed in [10,11], where particular attention was paid to the notion of very special geometry. We follow the conventions of [12,13] and write the bosonic action as

$$\begin{aligned} e^{-1}\mathcal{L} = \frac{1}{2}R - \frac{1}{4}G_{IJ}F^I_{\mu\nu}F^{\mu\nu J} - \frac{1}{2}g_{xy}(\phi)\partial_\mu\phi^x\partial^\mu\phi^y \\ - V(\phi) + \frac{e^{-1}}{48}\epsilon^{\mu\nu\rho\sigma\lambda}C_{IJK}F^I_{\mu\nu}F^J_{\rho\sigma}A^K_\lambda, \end{aligned} \tag{2}$$

where we use a signature $(-, +, +, +, +)$. The gauging of the $U(1)$ subgroup introduces a potential $V(\phi)$ which may be obtained from a superpotential $W(\phi)$ through the relation

$$V(\phi) = \frac{1}{2}g^{xy}\frac{\partial W}{\partial\phi^x}\frac{\partial W}{\partial\phi^y} - \frac{2}{3}W^2, \tag{3}$$

where

$$W(\phi) = 3gV_I X^I. \tag{4}$$

For very special geometry, the n-dimensional scalar manifold is obtained by introducing $n + 1$ scalar coordinates $X^I(\phi)$ along with the restriction $\mathcal{V}(X^I) = 1$ where $\mathcal{V}$ is a homogeneous cubic polynomial

$$\mathcal{V} = \frac{1}{6}C_{IJK}X^I X^J X^K. \tag{5}$$

In this case, we have

$$\begin{aligned} G_{IJ} &= -\frac{1}{2}\left(\frac{\partial}{\partial X^I}\frac{\partial}{\partial X^J}\ln\mathcal{V}\right)\bigg|_{\mathcal{V}=1}, \\ g_{xy} &= G_{IJ}\partial_x X^I\partial_y X^J, \end{aligned} \tag{6}$$

where $\partial_x \equiv \partial/\partial\phi^x$. We also find it convenient to introduce $X_I = \frac{1}{6}C_{IJK}X^JX^K$, so that $X^IX_I = 1$ as well as $X_IdX^I = X^IdX_I = 0$, so long as we restrict ourselves to $\mathcal{V} = 1$.

In addition to the bosonic sector, given by (2), we will also need the supersymmetry variations

$$\delta\psi_\mu = \left(\mathcal{D}_\mu + \frac{i}{8}X_I(\Gamma_\mu^{\nu\rho} - 4\delta_\mu^\nu\Gamma^\rho)F_{\nu\rho}^I + \frac{1}{6}W\Gamma_\mu\right)\epsilon,$$
$$\delta\lambda_x = \left(\frac{3}{8}\partial_x X_I\Gamma^{\mu\nu}F_{\mu\nu}^I - \frac{i}{2}g_{xy}\Gamma^\mu\partial_\mu\phi^y + \frac{i}{2}\partial_x W\right)\epsilon, \tag{7}$$

for the fermions, and

$$\delta g_{\mu\nu} = \mathrm{Re}(\bar\epsilon\Gamma_{(\mu}\psi_{\nu)}),$$
$$\delta A_\mu^I = \mathrm{Re}\left(\frac{1}{2}\bar\epsilon(\partial_x X^I\Gamma_\mu\lambda^x - iX^I\psi_\mu)\right), \tag{8}$$
$$\delta X_I = \mathrm{Re}\left(\frac{i}{2}\partial_x X_I\bar\epsilon\lambda^x\right),$$

for the bosons. Here, $\mathcal{D}_\mu = \nabla_\mu - \frac{3}{2}igV_IA_\mu^I$ is the $U(1)$ covariant derivative. Our notation here is that all spinors are five-dimensional Dirac spinors, and the Dirac matrices satisfy the Clifford algebra $\{\Gamma^\mu, \Gamma^\nu\} = 2g^{\mu\nu}$.

A. BPS black holes

The 1/2 BPS black hole solutions to gauged $\mathcal{N} = 2$ supergravity were obtained in [12], and have the form

$$ds^2 = -e^{-4U}f^2dt^2 + e^{2U}(f^{-2}dr^2 + r^2d\Omega_3^2),$$
$$f^2 \equiv 1 + g^2r^2e^{6U}, \qquad A^I = e^{-2U}X^Idt, \tag{9}$$
$$X_I = \frac{1}{3}e^{-2U}H_I,$$

where

$$H_I = 3V_I + \frac{q_I}{r^2} \tag{10}$$

are a set of "harmonic" functions with constant electric charges q_I. Note that the function $U(r)$ is determined implicitly through the very special geometry constraint $\mathcal{V} = 1$ where $\mathcal{V}$ is given in (5).

The solution in (9) is a 1/2 BPS solution, and was constructed by solving the Killing spinor equations $\delta\psi_\mu = 0$ and $\delta\lambda_x = 0$ arising from (7). For the above background, these equations take on the form

$$\delta\psi_t = [\partial_t - ig + (-2ie^{-3U}fU'\Gamma_1 + gf(1 + rU')\Gamma_0)P_{++}]\epsilon,$$
$$\delta\psi_r = \left[\partial_r + U' - 2U'P_{++} + \frac{1}{2}ge^{3U}f^{-1}(1 + 3rU')\Gamma_1\right]\epsilon,$$
$$\delta\psi_\alpha = \left[\left(\hat\nabla_\alpha + \frac{i}{2}\Gamma_{01}\hat\Gamma_\alpha\right) - f(1 + rU')\Gamma_1\hat\Gamma_\alpha P_{++}\right]\epsilon,$$
$$\delta\lambda^x = -i\partial_r\phi^x e^{-U}f\Gamma_1P_{++}\epsilon, \tag{11}$$

where primes denote derivatives with respect to r, and numerical indices 0, 1 denote frame indices with the obvious vielbeins

$$e^0 = e^{-2U}fdt, \qquad e^1 = e^Uf^{-1}dr. \tag{12}$$

Angular coordinates on S^3 are given by $\alpha, \beta, \ldots$, and the carets appearing in $\delta\psi_\alpha$ denote objects defined on the *unit* sphere. Here we have also introduced a family of projection operators

$$P_{\eta\tilde\eta} = \frac{1}{2}[1 + f^{-1}(\eta i\Gamma_0 + \tilde\eta gre^{3U}\Gamma_1)], \tag{13}$$

where η and $\tilde\eta$ are independently ±1 (which we denote $\pm$ in shorthand notation). Although only P_{++} shows up explicitly in (11), we will make use of the other signs of $P_{\eta\tilde\eta}$ in future sections.

The Killing spinors corresponding to (11) were constructed in [12], and have the form

$$\epsilon_{++} = e^{igt}e^{-U}e^{-1/2\Gamma_{012}\theta}e^{1/2\Gamma_{23}\phi}e^{-i/2\Gamma_{014}\psi}(\sqrt{f+1} - \sqrt{f-1}\Gamma_1)(1 - i\Gamma_0)\epsilon_0, \tag{14}$$

where ϵ_0 is an arbitrary constant spinor. By construction, ϵ_{++} satisfies the projection $P_{++}\epsilon_{++} = 0$. Here we have used an explicit parameterization of the unit S^3 given by

$$d\Omega_3^2 = d\theta^2 + \sin^2\theta d\phi^2 + \cos^2\theta d\psi^2, \tag{15}$$

and have used 2, 3, 4 to denote frame indices on S^3, with

$$\hat e^2 = d\theta, \qquad \hat e^3 = \sin\theta d\phi, \qquad \hat e^4 = \cos\theta d\psi. \tag{16}$$

It is apparent from (14) that the Killing spinors split into two parts: one related to the t-r directions and the other corresponding to Killing spinors on S^3. This feature may be made explicit by choosing a Dirac decomposition

$$\Gamma_0 = i\sigma_2 \times 1, \qquad \Gamma_1 = \sigma_1 \times 1, \qquad \Gamma_\alpha = \sigma_3 \times \sigma_\alpha \tag{17}$$

along with the split $\epsilon = \varepsilon \times \eta$. Here it is important to realize that ε may be taken to be Majorana (real) in the 1 + 1 dimensional space spanned by t and r.

We now note that Killing spinors on S^3 corresponding to solutions of

$$\left(\hat\nabla_\alpha \pm \frac{i}{2}\hat\sigma_\alpha\right)\eta_\pm = 0 \tag{18}$$

may be written explicitly as

$$\eta_\pm = e^{\mp i/2\sigma_1\theta}e^{i/2\sigma_3(\phi\mp\psi)}. \tag{19}$$

In this case, the projection (13) becomes

$$P_{\eta\tilde\eta} = \frac{1}{2}[1 + f^{-1}(-\eta\sigma_2 + \tilde\eta gre^{3U}\sigma_1)], \tag{20}$$

while the Killing spinors (14) have the form

$$\epsilon_{++} = e^{igt}e^{-U}(\sqrt{f+1} - \sqrt{f-1}\sigma_1)(1 + \sigma_2)\varepsilon_0 \times \eta_+. \tag{21}$$

When working with supersymmetry, we will make use of both representations (14) and (21) interchangeably, whenever convenient.

III. FERMION ZERO MODES AND BLACK HOLE SUPERPARTNERS

Before proceeding to analyze the fermion zero modes, we find it useful to familiarize ourselves with the form of Killing spinors in AdS_5. Thus we first analyze the complete set of Killing spinors on the maximally supersymmetric AdS space, and then demonstrate that half of the original AdS Killing spinors naturally map into Killing spinors in the presence of the black holes, while the other half become fermion zero modes.

In general, of course, any spinor that does not solve the Killing spinor equations $\delta\psi_\mu = 0$, $\delta\lambda_x = 0$ may be considered to be zero modes. However, one has to be careful in identifying physically distinct configurations, as opposed to pure supergauge degrees of freedom. The importance here is the recognition of the global part of the supersymmetry algebra as the representation generating part. In this sense, we demand that the fermion zero modes are explicitly constructed to solve an alternate projection P_{-+}, distinct from P_{++} of the Killing spinors. Note, however, that P_{-+} is not the complement of P_{++}; that is reserved for P_{--} satisfying $P_{++} + P_{--} = 1$. It instead defines orthogonality with respect to the Dirac inner product, $\overline{P_{(-\eta)(\tilde\eta)}\epsilon_1} P_{\eta\tilde\eta}\epsilon_2 = 0$. This is because $\bar\epsilon = \epsilon^\dagger\Gamma_0$ and so the $\tilde\eta$ term changes sign when permuted past Γ_0. Because of the background AdS curvature, this situation is somewhat different from that used in [1], where in addition to satisfying a supergauge choice $\gamma^\mu\delta\psi_\mu = 0$ the fermion zero modes also satisfied the complementary projection P_- instead of P_+.

A. Supersymmetry in AdS_5

To highlight the above issues, we now consider supersymmetry in the AdS_5 vacuum. This is readily obtained by taking $U = 0$ and $q_I = 0$ (so that $X_I = V_I$ are constants) in the black hole ansatz of (9). In this case, the gaugino variation trivially vanishes, and (11) reduces to the set

$$\begin{aligned} \delta\psi_t &= \left[\partial_t - \left(\frac{3}{2} \mp \frac{1}{2}\right) ig + gf\Gamma_0 P_{\pm+}\right]\epsilon, \\ \delta\psi_r &= \left[\partial_r + \frac{1}{2} g f^{-1}\Gamma_1\right]\epsilon, \\ \delta\psi_\alpha &= \left[\hat\nabla_\alpha \pm \frac{i}{2}\Gamma_{01}\hat\Gamma_\alpha - f\Gamma_1\hat\Gamma_\alpha P_{\pm+}\right]\epsilon. \end{aligned} \tag{22}$$

Unlike (11), here we have made use of the identity $P_{++} = P_{-+} + if^{-1}\Gamma_0$ to write the Killing spinor equations using both types of projections. One may solve these equations by starting with the solution ϵ_{++} of the last section with $U = 0$. To generate the solution to the $-+$ equation, simply note that to change P_{++} into P_{-+} one needs to permute through a Γ_1. This also leaves the ψ_r equation unchanged. This implies that a spinor of the form $\exp(igt)\Gamma_1\epsilon_{++}$ solves the $-+$ equations. Pushing the Γ_1 through until it is next to ϵ_0, and then replacing $\Gamma_1\epsilon_0 \to \epsilon_0$ (since $\Gamma_1\epsilon_0$ is just as arbitrary as ϵ_0) gives the other solution. The AdS_5 Killing spinors may then be written as

$$\begin{aligned} \epsilon_{\pm+} = {} & e^{i(3/2\mp1/2)gt} e^{\mp i/2\Gamma_{012}\theta} e^{1/2\Gamma_{23}\phi} \\ & \times e^{\mp i/2\Gamma_{014}\psi}(\sqrt{f+1} - \sqrt{f-1}\Gamma_1)(1 \mp i\Gamma_0)\epsilon_0, \end{aligned} \tag{23}$$

where ϵ_0 is again an arbitrary constant spinor. The sign of $i\Gamma_0$ is directly connected to the projection appearing next to ϵ_0 giving that we can replace the $\mp$ sign in the exponential with an $+i\Gamma_0$. This results in the usual form for the AdS_5 Killing spinors except for the extra factor of $\exp(\frac{3}{2}igt)$. This extra factor is a direct result of the gauge choice for A^I, but is otherwise physically insignificant.

It ought to be apparent that, taken together, the complete set of Killing spinors, ϵ_{++} and ϵ_{-+}, guarantee the maximal supersymmetry of the AdS_5 background. As seen from the $\delta\psi_\alpha$ equation of (22), the AdS_5 Killing spinors have a natural realization in terms of both types of Killing spinors on S^3, namely η_+ and η_- of (19). using the standard Dirac decomposition (17), the above Killing spinors take on the form

$$\epsilon_{\pm+} = e^{i(3/2\mp1/2)gt}(\sqrt{f+1} - \sqrt{f-1}\sigma_1)(1 \pm \sigma_2)\varepsilon_0 \times \eta_\pm. \tag{24}$$

By construction, the above spinors $\epsilon_{\pm+}$ satisfy the projections

$$P_{\pm+}\epsilon_{\pm+} = 0. \tag{25}$$

The P_{++} case gives pure AdS_5 spinors which, when multiplied by e^{-U} and with appropriate modification to $f(r)$, correspond to the preserved black hole supersymmetries identified in the last section. We should also note that the pure AdS5 $++$ spinors match the black hole $++$ spinors when $r \to \infty$ (so that $U \to 0$) because the space becomes asymptotically AdS_5.

The $-+$ solutions for pure AdS_5, on the other hand, are broken supersymmetries when generalized to the black hole solution; they correspond to fermion zero modes in this background. Although any spinor not satisfying the P_{++} projection would be sufficient to realize the fermion zero-mode algebra, the ϵ_{-+} are particularly convenient because they reduce to standard Killing spinors in an asymptotically AdS_5 spacetime and hence represent genuine fermion zero modes related to the black hole geometry (as opposed to supergauge transformations of pure AdS_5). For convenience, we will drop the label $-+$ from zero mode spinors in future sections. Thus

$$\begin{aligned}\epsilon \equiv \epsilon_{-+} &= e^{2igt} e^{\alpha U} e^{i/2\Gamma_{012}\theta} e^{1/2\Gamma_{23}\phi} \\ &\quad \times e^{i/2\Gamma_{014}\psi}(\sqrt{f+1} - \sqrt{f-1}\Gamma_1)(1+i\Gamma_0)\epsilon_0 \\ &= e^{2igt} e^{\alpha U}(\sqrt{f+1} - \sqrt{f-1}\sigma_1)(1-\sigma_2)\varepsilon_0 \times \eta_-. \end{aligned} \tag{26}$$

Here α is an arbitrary constant related to choice of supergauge condition; it will drop out in all physical quantities below. These properly identified fermion zero-mode spinors will be the starting point for the generation of the superpartners of the black hole solution of [12].

B. Zero mode identities

The black hole superpartners will be obtained via (1) up to second order in the supersymmetry transformation δ. As a result, we are often faced with the task of simplifying bilinear expressions in fermion zero-mode spinors of the form $(\bar\epsilon\Gamma_{\dots}\epsilon)$, where ϵ is given by (26). Using the projection properties (25) for the zero-mode spinors, as well as Dirac conjugation, $\bar\epsilon = \epsilon^\dagger\Gamma^0$, one may obtain several useful identities:

$$\begin{aligned} &(\bar\epsilon\Gamma_1\epsilon) = 0, \quad (\bar\epsilon\Gamma_0\epsilon) = -if(\bar\epsilon\epsilon), \\ &(\bar\epsilon\Gamma_{01}\epsilon) = igre^{3U}(\bar\epsilon\epsilon), \quad (\bar\epsilon\Gamma_0\hat\Gamma_\alpha\epsilon) = 0, \\ &(\bar\epsilon\Gamma_1\hat\Gamma_\alpha\epsilon) = \frac{f}{gre^{3U}}(\bar\epsilon\hat\Gamma_\alpha\epsilon), \quad (\bar\epsilon\Gamma_{01}\hat\Gamma_\alpha\epsilon) = -\frac{i}{gre^{3U}}(\bar\epsilon\hat\Gamma_\alpha\epsilon). \end{aligned} \tag{27}$$

We will make use of these identities below.

C. Black hole superpartners

To generate black hole superpartners, we will consider fermion zero-mode transformations up to second order in ϵ starting from the bosonic background (9) constructed in [12]. The first order variations using the zero modes will generate a fermionic (gravitino and gaugino) background. Rewriting (11) with the substitution $P_{++} = P_{-+} + if^{-1}\Gamma_0$, and noting from (25) that $P_{-+}\epsilon = 0$ for a fermion zero mode ϵ, we obtain

$$\begin{aligned} \delta\psi_t &= -U'[2e^{-3U}\Gamma_{01} + igr]\epsilon, \\ \delta\psi_r &= U'[\alpha + 1 - 2if^{-1}\Gamma_0]\epsilon, \\ \delta\psi_\alpha &= -irU'\Gamma_{01}\hat\Gamma_\alpha\epsilon, \\ \delta\lambda^x &= -e^{-U}\partial_r\phi^x\Gamma_{01}\epsilon. \end{aligned} \tag{28}$$

Note that $\hat\Gamma_\alpha$ are Dirac matrices on the unit sphere, and are related to the full five-dimensional matrices by $\Gamma_\alpha = re^U\hat\Gamma_\alpha$.

We now turn to the terms second order in the supersymmetry variation, where the bosonic fields receive corrections. To obtain the second order variations $\delta\delta$(boson), we may simply take their first variations in (8), and replace the fermions with their first variations given above in (28). All other contributions would be set to zero when evaluated for a bosonic background. Using the identities (27) we find that the nonzero variations of the metric are

$$\begin{aligned} \delta\delta g_{tt} &= -grU'f^2e^{-2U}\,\mathrm{Re}(\bar\epsilon\epsilon) \\ &= 8grU'e^{2(\alpha+1)U}g_{tt}(\bar\varepsilon_0\varepsilon_0)N, \\ \delta\delta g_{t\alpha} &= \frac{3}{2}(gr)^{-1}U'e^{-6U}\,\mathrm{Re}(i\bar\epsilon\Gamma_\alpha\epsilon) \\ &= -12rU'e^{2(\alpha-1)U}(\bar\varepsilon_0\varepsilon_0)\hat K_\alpha, \\ \delta\delta g_{rr} &= -2grU'f^{-2}e^{4U}\,\mathrm{Re}(\bar\epsilon\epsilon) \\ &= -16grU'e^{2(\alpha+1)U}g_{rr}(\bar\varepsilon_0\varepsilon_0)N, \\ \delta\delta g_{\alpha\beta} &= grU'e^{2U}g_{\alpha\beta}\,\mathrm{Re}(\bar\epsilon\epsilon) \\ &= 8grU'e^{2(\alpha+1)U}g_{\alpha\beta}(\bar\varepsilon_0\varepsilon_0)N. \end{aligned} \tag{29}$$

To obtain the final expressions on each line, we have decomposed the fermion zero-mode spinors according to (26) and taken $\sigma_2\varepsilon_0 = -\varepsilon_0$ to satisfy the projection $(1-\sigma_2)$ in (26) for the zero-mode spinors. We have also defined

$$N = (\eta_-^\dagger\eta_-), \qquad \hat K_\alpha = (\eta_-^\dagger\hat\sigma_\alpha\eta_-). \tag{30}$$

Here, $\hat K_\alpha$ is a Killing vector on the unit S^3, and the decomposition (26) yields the relation $(i\bar\epsilon\Gamma_\alpha\epsilon) = -8gr^2e^{2(\alpha+2)U}(\bar\varepsilon_0\varepsilon_0)\hat K_\alpha$. In addition, the nontrivial double supersymmetry variations of the matter fields are

$$\begin{aligned} \delta\delta A_t^I &= -\frac{3}{2}grU'X^I\,\mathrm{Re}(\bar\epsilon\epsilon) \\ &= -12grU'e^{2\alpha U}X^I(\bar\varepsilon_0\varepsilon_0)N, \\ \delta\delta A_\alpha^I &= \frac{1}{2}(gr)^{-1}(U'X^I + \partial_rX^I)e^{-4U}\,\mathrm{Re}(i\bar\epsilon\Gamma_\alpha\epsilon) \\ &= -4r(U'X^I + \partial_rX^I)e^{2\alpha U}(\bar\varepsilon_0\varepsilon_0)\hat K_\alpha, \end{aligned} \tag{31}$$

for the gauge fields, and

$$\begin{aligned} \delta\delta X_I &= \frac{1}{2}gr\partial_rX_Ie^{2U}\,\mathrm{Re}(\bar\epsilon\epsilon) \\ &= -4gr\partial_rX_Ie^{2(\alpha+1)U}(\bar\varepsilon_0\varepsilon_0)N, \end{aligned} \tag{32}$$

for the scalars.

While the exponential factor $\exp(\alpha U)$ in (26) appears in the above expressions, this factor goes to unity asymptotically as $r \to \infty$. Since α enters nowhere else, the actual value of α is unphysical. For convenience, we take $\alpha = -1$ and furthermore define the spinor bilinear

$$\lambda \equiv 4(\bar\varepsilon_0\varepsilon_0). \tag{33}$$

Using the expression (1) for a finite supersymmetry transformation, we now observe that, up to second order in the supersymmetry variation (i.e. to lowest order in λ), the bosonic fields may be expressed as

$$
\begin{aligned}
ds^2_{(\mathrm{tot})} &= -e^{-4U} f^2 (1 + grU'\lambda N) dt^2 \\
&\quad + e^{2U}[f^{-2}(1 - 2grU'\lambda N) dr^2 \\
&\quad + r^2 (1 + grU'\lambda N) d\Omega_3^2] - 3rU' e^{-4U} \lambda dt \hat{K}, \\
A^I_{(\mathrm{tot})} &= e^{-2U} X^I \Big(1 - \frac{3}{2} grU'\lambda N\Big) dt \\
&\quad - \frac{1}{2} r (U' X^I + \partial_r X^I) e^{-2U} \lambda \hat{K}, \\
X_I^{(\mathrm{tot})} &= X_I - \frac{1}{2} gr \partial_r X_I \lambda N.
\end{aligned} \tag{34}
$$

Here, $\hat{K} = \hat{K}_\alpha d\theta^\alpha$ is the 1-form associated with the Killing vector $\hat{K}^\alpha (\partial/\partial\theta^\alpha)$. where the α index is raised and lowered using the metric on the *unit* sphere, (15). In the following section, we will examine the superpartner solutions (34), and, in particular, extract the superpartner shifts to the energy (mass), angular momentum and R-charge of the original black hole.

IV. PROPERTIES OF THE BLACK HOLE SUPERPARTNERS

Having constructed a set of black hole superpartners, (34), in the $\mathcal{N} = 2$ theory, we now set out to explore their properties. We start by observing from (34) that angular momentum (spin) is generated for the superpartners because of the off-diagonal metric component proportional to $dt\hat{K}$. This is of course expected, as from a semiclassical point of view we expect the superpartners of the spinless black hole to carry precisely spin-1/2. We also see that the effective Newtonian potential in g_{tt} is shifted by a multiplicative factor $(1 + grU'\lambda N)$. It is this shift that indicates that the superpartner energies no longer coincide with that of the original solution. This is a feature of supersymmetry in AdS spacetimes, and the energy shift clearly vanishes in the Minkowski limit $g \to 0$.

A. Energy and angular momentum

In order to make these observations on energy and angular momentum more precise, we make use of holographic renormalization in AdS and, in particular, the boundary stress tensor method for defining asymptotically conserved quantities [5–7]. Given a gravitational action $I[g_{\mu\nu}]$, the boundary stress tensor is simply [14]

$$
T^{ab} = \frac{2}{\sqrt{-h}} \frac{\delta I}{\delta h_{ab}} = -\frac{1}{8\pi G_5} (\Theta^{ab} - \Theta h^{ab}), \tag{35}
$$

where h_{ab} is the boundary metric, $h_{\mu\nu} = g_{\mu\nu} - n_\mu n_\nu$, with n_μ a unit normal to the boundary. In addition, Θ^{ab} is the extrinsic curvature tensor, which may be expressed as

$$
\Theta_{ab} = -\frac{1}{2} (\nabla_a n_b + \nabla_b n_a - n_a n^c \nabla_c n_b - n_b n^c \nabla_c n_a). \tag{36}
$$

Note that the covariant derivatives are with respect to the 5-dimensional metric. We have also used $a, b, c, \ldots$ to denote indices on the boundary. The expression (35), while divergent, may be regulated via the addition of boundary counterterms. For this particular situation, the appropriate counterterms were determined using the Hamilton-Jacobi method in [9]. The resulting renormalized stress tensor is given by

$$
\begin{aligned}
8\pi G_5 T_{ab} &= -(\Theta_{ab} - \Theta h_{ab}) \\
&\quad + \Big(W(\phi) h_{ab} - \frac{1}{4g} (2\mathcal{R}(h)_{ab} - \mathcal{R}(h) h_{ab})\Big).
\end{aligned} \tag{37}
$$

Here $W(\phi)$ is the superpotential given in (4) and $\mathcal{R}_{ab}$ is the Ricci curvature on the boundary. We note that although there are fermions present in the superpartner configuration, (34), their behavior in the stress tensor is dominated at large r by a factor of $(U')^2$, and so the fermions fall off too fast at the boundary to contribute to (37).

To explore the boundary stress tensor, we take the black hole solution of (34) and expand near the boundary at $r \to \infty$. Using r as the natural radial direction, the unit normal vector n_μ has as its only nonvanishing component $n_r = e^U f^{-1}(1 - grU'\lambda N)$, where we are only concerned with the lowest order in λ. Corresponding to this normal direction, the four-dimensional constant r surfaces are given by

$$
\begin{aligned}
ds_4^2 \equiv h_{ab} dx^a dx^b &= -e^{-4U} f^2 (1 + grU'\lambda N) dt^2 \\
&\quad + e^{2U} r^2 (1 + grU'\lambda N) \\
&\quad \times \hat{g}_{\alpha\beta} \Big(d\theta^\alpha + \frac{3}{2} r^{-1} U' e^{-6U} \lambda \hat{K}^\alpha dt\Big) \\
&\quad \times \Big(d\theta^\beta + \frac{3}{2} r^{-1} U' e^{-6U} \lambda \hat{K}^\beta dt\Big),
\end{aligned} \tag{38}
$$

where we have only worked to linear order in λ. Note that we have further chosen an Arnowitt-Deser-Misner (ADM)-like foliation of the boundary metric, with shift vectors related to angular momentum. Furthermore, given the unit normal, it is straightforward to compute the extrinsic curvature tensor from the four-dimensional metric:

$$
\begin{aligned}
\Theta_{ab} &= -\frac{1}{2} (\nabla_a n_b + \nabla_b n_a) = -\frac{1}{2} n^r \partial_r h_{ab} \\
&= -\frac{1}{2} e^{-U} f (1 + grU'\lambda N) \partial_r h_{ab}.
\end{aligned} \tag{39}
$$

To compute the counterterm contributions in (37), we also need the superpotential and the Ricci curvature of the boundary metric (38). Since r may be taken as constant with respect to the four-dimensional metric, its intrinsic curvature has a simple form, with only the S^3 being curved. In other words, we have

$$
\begin{aligned}
&\mathcal{R}_{tt} = 0, \qquad \mathcal{R}_{t\alpha} = 0, \\
&\mathcal{R}_{\alpha\beta} = 2r^{-2} e^{-2U} (1 - grU'\lambda N) h_{\alpha\beta}.
\end{aligned} \tag{40}
$$

In addition, using (4) and the form of $X_I^{(\mathrm{tot})}$ given in (34), the superpotential has the form

$$W = 3gV_I X^I = 3g\Big(1 - \frac{1}{2} g r \lambda N \partial_r\Big)(V_I X^I) = 3g\Big(1 - \frac{1}{2} g r \lambda N \partial_r\Big)[e^{2U}(1 + rU')]. \tag{41}$$

To proceed, we now need to specify the functional form of $U(r)$. Although in some cases (such as the STU model) a closed form expression may be given for U, here it is sufficient for us to assume that U has an expansion in inverse powers of r^2 of the form

$$U = \frac{\alpha_1}{r^2} + \frac{\alpha_2}{r^4} + \cdots. \tag{42}$$

In this case, Θ_{ab}, $\mathcal{R}_{ab}$ and W given in (39)–(41) may be expanded for large r and inserted into (37). We find, to lowest nontrivial order

$$\begin{aligned} 8\pi G_5 T_{tt} &= -\frac{g}{r^2}\Big(6\alpha_1\Big(1 + \frac{1}{2} g\lambda N\Big) + \frac{3}{8g^2}\Big), \\ 8\pi G_5 T_{t\alpha} &= -\frac{g}{r^2}(6\alpha_1 \lambda \hat{K}_\alpha). \end{aligned} \tag{43}$$

Note that there is also a contribution to $T_{\alpha\beta}$, which we have not computed (since it has no role to play in extracting conserved charges).

We must now extract the energy and angular momentum from the above expressions for the boundary stress tensor. To obtain the charge Q_ξ associated with a Killing vector ξ, we take

$$Q_\xi = -\lim_{r\to\infty} \int d^3\theta \sqrt{\gamma}(u^a T_{ab} \xi^b), \tag{44}$$

where u defines the unit normal time direction, and γ is the induced metric for a constant time slice. For the metric (38), this expression takes the form

$$Q_\xi = -\lim_{r\to\infty} \frac{\omega_3 r^2}{g}(T_{ta}\xi^a), \tag{45}$$

where $\omega_3 = 2\pi^2$ is the volume of the unit 3-sphere. In particular, conjugate to the Killing vector $\xi = \frac{\partial}{\partial t}$, we obtain the energy

$$E = -\lim_{r\to\infty} \frac{2\pi^2 r^2}{g} T_{tt} = \frac{\pi}{4G_5}\Big(6\alpha_1\Big(1 + \frac{1}{2} g\lambda N\Big) + \frac{3}{8g^2}\Big). \tag{46}$$

To obtain the angular momentum, we must consider some properties of Killing vectors on the unit 3-sphere. In general, the unit S^3 admits a set of SO(4) Killing vectors, which we may denote $\hat{K}_\alpha^{(ij)}$, where ij is an antisymmetric SO(4) index pair. These Killing vectors may be normalized according to

$$\int d^3\theta \sqrt{\hat{g}} \hat{K}_\alpha^{(ij)} \hat{K}^{(lm)\alpha} = \frac{1}{2}(\delta^{il}\delta^{jm} - \delta^{im}\delta^{jl})\omega_3. \tag{47}$$

On the other hand, the Killing vector $\hat{K}^\alpha$ constructed from Killing spinors in (30) is naturally given as an $SU(2)_-$ Killing vector, corresponding to the decomposition $SO(4) = SU(2)_+ \times SU(2)_-$. In particular, Killing vectors corresponding to $SU(2)_+$ and $SU(2)_-$ arise from $(\eta_+^\dagger \hat{\sigma}^\alpha \eta_+)$ and $(\eta_-^\dagger \hat{\sigma}^\alpha \eta_-)$, respectively. While $\hat{K}^\alpha \equiv (\eta_-^\dagger \hat{\sigma}^\alpha \eta_-)$ depends on the explicit Killing spinor η_-, we may always choose coordinates such that $\hat{K}^\alpha$ is aligned along the T^3 direction of SU(2). For the unit S^3 given in (15), this corresponds to taking

$$\hat{K}^\alpha = \frac{\partial}{\partial\phi} + \frac{\partial}{\partial\psi} = \hat{K}^{(12)\alpha} + \hat{K}^{(34)\alpha}, \tag{48}$$

where we have identified ϕ and ψ with rotations in the 1-2 and 3-4 planes, respectively. This also agrees with the natural embedding of $SU(2)_\pm$ in SO(4). Using these expressions in (44), we now read off the angular momentum

$$J^{ij} = -\lim_{r\to\infty} \frac{r^2}{g} \int d^3\theta \sqrt{\hat{g}} T_{t\alpha} \hat{K}^{(ij)\alpha} = \frac{3\alpha_1\lambda}{4\pi G_5} \int d^3\theta \sqrt{\hat{g}} \hat{K}_\alpha \hat{K}^{(ij)\alpha}, \tag{49}$$

so that

$$J^{12} = J^{34} = \frac{\pi}{4G_5}(3\alpha_1\lambda). \tag{50}$$

We should note that, while these definitions for energy and angular momentum were obtained for AdS black holes, they exactly match their Minkowski black hole counterparts in the case where the black hole is "small." When we say that the black hole is small, we mean that all length scales associated with it are small compared to the radius of AdS. In such a case there is a region of space such that $gr \ll \delta_<$, $\alpha_k/r^{2k} \ll \delta_>$, $\delta_< \ll \delta_> \ll 1$. The black hole's energy and angular momentum may then be read off from the metric using a standard ADM prescription; these expressions should furthermore agree with the above (up to the Casimir energy, which is absent in the ADM mass). In fact, we would have *chosen* the definitions of the conserved charges in such a way as to have this happen (by modifying them with multiplicative constants). The fact that they do agree merely confirms that we have defined them in an appropriate manner.

B. The *R*-charge and magnetic dipole moment

The conserved gauge charges are straightforward to obtain, and do not require a counterterm prescription. Based on the Maxwell equation of motion from (2), we obtain the conserved Noether (electric) charges

$$Q_I = \lim_{r\to\infty} \frac{1}{\omega_3} \int d^3\theta \sqrt{-g} G_{IJ} F^{Jrt}. \tag{51}$$

Now, we simply note that the first order corrections in λ to $\sqrt{-g}G_{IJ}$, when contracted with F^{Jrt}, fall off too fast to contribute. The only modification to the charge of the superpartners therefore comes from a direct shift in A. From (34), we obtain

$$Q_I = q_I - 9V_I\alpha_1\lambda N, \tag{52}$$

where q_I were the original black hole electric charges, given in (10).

For BPS states, we are more specifically interested in the R-charge, given as the electric charge of $\mathcal{A}_\mu = V_I A^I_\mu$. In this case, we may simply read off the R charge from $V_I A^I_t$:

$$V_I A^I_t = 1 - \frac{2\alpha_1}{r^2}\left(1 - \frac{3}{2}g\lambda N\right) + \cdots. \tag{53}$$

Identifying this expression with $Q/(2r^2)$ (up to the constant, which is pure gauge), we obtain

$$Q = -4\alpha_1\left(1 - \frac{3}{2}g\lambda N\right). \tag{54}$$

Finally, we may also read off the graviphoton magnetic dipole moment from

$$V_I A^I_\alpha = \frac{\alpha_1\lambda}{r^2}\tilde{K}_\alpha = \frac{\alpha_1\lambda}{r^2}(\hat{K}^{(12)}_\alpha + \hat{K}^{(34)}_\alpha). \tag{55}$$

Identifying this with $-\frac{1}{2}\mu_{ij}\hat{K}^{(ij)}_\alpha r^{-2}$ yields

$$\mu_{12} = \mu_{34} = -\alpha_1\lambda, \tag{56}$$

for the magnetic dipole moment μ_{ij}. We will discuss the relation between these charges in the next section.

V. DISCUSSION

Given the above construction of BPS black hole superpartners in AdS_5, and the further determination of their conserved charges, we now demonstrate that the structure of the superpartners is consistent with representation theory. In particular, we have worked in the context of gauged $D = 5$, $\mathcal{N} = 2$ supergravity, with superalgebra $SU(2, 2|1)$. Recall that highest weight representations [15,16] (see also Appendix B of [17]) may be labeled by $\mathcal{D}(E_0, j_1, j_2; r)$ where the lowest energy E_0, spins j_1 and j_2, and R-charge r label the compact bosonic subalgebra

$$\begin{aligned} SU(2, 2|1) &\supset SO(2, 4) \times U(1) \\ &\supset SO(2) \times SU(2) \times SU(2) \times U(1). \end{aligned} \tag{57}$$

This superalgebra allows for two types of short multiplets (chiral and nonchiral) in addition to ordinary long multiplets. The long multiplets generically contain $2^4 = 16$ states, while the short ones contain $2^2 = 4$ states. The nonchiral multiplets are of the form

$$\begin{aligned} \mathcal{D}(E_0 = 2j+1, j, j; 0) &= D(E_0, j, j)_0 \\ &+ D\left(E_0 + \frac{1}{2}, j + \frac{1}{2}, j\right)_{-1} + D\left(E_0 + \frac{1}{2}, j, j + \frac{1}{2}\right)_1 \\ &+ D\left(E_0 + 1, j + \frac{1}{2}, j + \frac{1}{2}\right)_0, \end{aligned} \tag{58}$$

while the chiral ones are

$$\begin{aligned} \mathcal{D}\left(E_0 = \frac{3}{2}r, j, 0, r\right) &= D(E_0, j, 0)_r + D\left(E_0 + \frac{1}{2}, j + \frac{1}{2}, 0\right)_{r-1} + D\left(E_0 + \frac{1}{2}, j - \frac{1}{2}, 0\right)_{r-1} + D(E_0 + 1, j, 0)_{r-2}, \\ \mathcal{D}\left(E_0 = -\frac{3}{2}r, 0, j, r\right) &= D(E_0, 0, j)_r + D\left(E_0 + \frac{1}{2}, 0, j + \frac{1}{2}\right)_{r+1} + D\left(E_0 + \frac{1}{2}, 0, j - \frac{1}{2}\right)_{r+1} + D(E_0 + 1, 0, j)_{r+2}. \end{aligned} \tag{59}$$

Since the BPS black holes of [12] carry nonzero R-charge, they ought to correspond to the chiral short multiplet given above. To see this, we identify the energy, angular momentum and R-charge obtained in the previous section as

$$\begin{aligned} E &= \frac{\pi}{4G_5}\left(6\alpha_1 + 3g\alpha_1\lambda N + \frac{3}{8}g^{-2}\right), \\ J^{12} = J^{34} &= \frac{\pi}{4G_5}3\alpha_1\lambda, \qquad Q = -4\alpha_1 + 6g\alpha_1\lambda N, \end{aligned} \tag{60}$$

Removing the Casimir energy from E, and dropping the prefactor $\pi/4G_5 = \omega_3/8\pi G_5$ from gravitational quantities, we see that the appropriate identification of $SU(2, 2|1)$ quantum numbers is as follows:

$$\begin{aligned} E_0 &= 6\alpha_1 + 3g\alpha_1\lambda N, \qquad j_1 = 0, \\ j_2 &= 3\alpha_1\lambda, \qquad r = -4\alpha_1 + 6g\alpha_1\lambda N. \end{aligned} \tag{61}$$

Setting $\lambda = 0$ for the original bosonic solution then yields

$$D(E_0, 0, 0)_r, \qquad E_0 = -\frac{3}{2}r = 6\alpha_1, \tag{62}$$

corresponding to the lowest weight component of $\mathcal{D}(E_0 = -\frac{3}{2}r, 0, j = 0, r)$ given in (59).

Turning now to the superpartners, we first note from (14) that ε_0 satisfies a projection $\sigma_2\varepsilon_0 = -\varepsilon_0$. Hence this two-component Majorana spinor in fact has only one independent real component, which may be taken as an unimportant real multiplicative constant in the product $\epsilon = \varepsilon_0 \times \eta_-$. In other words, the interesting fermion zero-mode algebra arises from the Killing spinors η_- on the sphere, and not from ε_0 itself. Based on standard representation theory techniques, we see that this algebra is essentially that of fermionic creation and annihilation operators. Thus we view the two-component Dirac spinor η_- and its conjugate $\eta_-^\dagger$ as a pair of creation and annihilation operators

$$\eta_-^\dagger = \begin{pmatrix} a_\uparrow^\dagger & a_\downarrow^\dagger \end{pmatrix}, \qquad \eta_- = \begin{pmatrix} a_\uparrow \\ a_\downarrow \end{pmatrix}, \tag{63}$$

with corresponding number operator

$$N = (\eta_-^\dagger \eta_-) = a_\uparrow^\dagger a_\uparrow + a_\downarrow^\dagger a_\downarrow = n_\uparrow + n_\downarrow. \tag{64}$$

Of course, given the semiclassical analysis of the previous sections, the normalization of these operators is not so obvious. Fortunately, we have just seen that the parameter λ, defined in (33) as $\lambda = 4(\bar{\varepsilon}_0 \varepsilon_0)$, is an ordinary c-number. Thus we simply assume that the black hole superpartners with nonvanishing spin will be normalized so that $j_2 = \frac{1}{2}$ (actually the third component of j_2). This corresponds to setting $6\alpha_1\lambda = 1$ in (61). To be somewhat more precise, there are actually two independent sets of creation and annihilation operators, as indicated in (63). In this case, the third component of angular momentum j_2 in the second SU(2) may be either "spin up" or "spin down", depending on the choice of Killing spinors η_- used to construct the Killing vector $\hat{K}^\alpha$. In fact, from (30), we may write down

$$T^3 \equiv \frac{1}{2}(\eta_-^\dagger \sigma^3 \eta_-) = \frac{1}{2}(a_\uparrow^\dagger a_\uparrow - a_\downarrow^\dagger a_\downarrow) = \frac{1}{2}(n_\uparrow - n_\downarrow). \tag{65}$$

We now see that the choice of Killing vector in (48) is overly restrictive. As a consequence, instead of having $j_2 = 3\alpha_1\lambda$ in (61), we ought to write $m_2 = 6\alpha_1\lambda T^3$ with T^3 given in (65), where m_2 is the third component of j_2.

Given the above considerations, we see that the superpartner quantum numbers read off from (61) fit the representations

$$D\left(E_0 + \frac{1}{2}gN, 0, m_2 = T^3\right)_{r+gN}, \qquad E_0 = -\frac{3}{2}r, \tag{66}$$

where $N = n_\uparrow + n_\downarrow$ and $T^3 = \frac{1}{2}(n_\uparrow - n_\downarrow)$. Note here that the spin j_2 is given implicitly in terms of the angular momentum representations $|j_2, m_2\rangle$. Since the number operators $n_\uparrow$ and $n_\downarrow$ independently take on the values 0, 1, we identify precisely the 4 states of the short multiplet. In particular, we have $N = 0, 1, 2$, with corresponding spins $j_2 = 0, \frac{1}{2}, 0$. As the dimensionful quantities E_0 and r are measured with respect to the AdS inverse radius g, the above expression is in complete agreement with the chiral short representation of (59) with superspin $j = 0$. Thus we have demonstrated that, in fact, working to second order in the supersymmetry transformations is sufficient to reproduce the appropriate zero-mode algebra of the corresponding supersymmetry algebra.

A. The gyromagnetic ratio

Following [2,3], we may also compute the gyromagnetic ratio of the black hole superpartners. Here we make use of the definition

$$\mu^{ij} = \frac{\tilde{g}Q}{2M} J^{ij}, \tag{67}$$

where $\tilde{g}$ denotes the gyromagnetic ratio (to distinguish it from the inverse AdS radius). The magnetic dipole moment was identified in (56) to be

$$\mu_{12} = \mu_{34} = -\alpha_1\lambda, \tag{68}$$

which is clearly proportional to the angular momentum $J_{12} = J_{34} = (\pi/4G_4)3\alpha_1\lambda$ given in (50). To compute the gyromagnetic ratio, we further use the mass and charge of the original bosonic solution, $M = (\pi/4G_5)6\alpha_1$ and $Q = -4\alpha_1$ to obtain

$$(-\alpha_1\lambda) = \frac{\tilde{g}(-4\alpha_1)}{2(\pi/4G_5)6\alpha_1}\left(\frac{\pi}{4G_5}3\alpha_1\lambda\right), \tag{69}$$

which yields $\tilde{g} = 1$. This agrees with the asymptotically Minkowski case previously obtained in [18]. We use the mass and charge of the original system because J_{ij} and μ_{ij} were only calculated to leading order.

This result appears somewhat surprising, as it was proven in [19,20] that unbroken supersymmetry in four dimensions is sufficient to ensure that $\tilde{g}$ is *exactly* equal to 2 for the superspin-0 multiplet. In this case, the situation is somewhat different, as we are working in five dimensions and furthermore with the anti-de Sitter superalgebra. It turns out, however, that it is not the AdS nature of the system that leads to $\tilde{g} \neq 2$, but rather just the simple fact that $\tilde{g} = 1$ is natural in five dimensions, at least for superpartners of nonrotating black holes [18]. While $\mathcal{N} = 2$ supersymmetry shares many common features between four and five dimensions, there are differences as well. Consider, for example, the minimal (ungauged) supergravity multiplet $(g_{\mu\nu}, \psi_\mu, A_\mu)$, with supersymmetry transformation

$$\begin{aligned} \delta\psi_\mu &= \left[\nabla_\mu + \frac{i}{8}(\gamma_\mu{}^{\nu\rho} - 2(D-3)\delta_\mu^\nu\gamma^\rho)F_{\nu\rho}\right]\epsilon, \\ \delta g_{\mu\nu} &= \bar{\epsilon}\gamma_{(\mu}\psi_{\nu)}, \qquad \delta A_\mu = -\frac{i}{D-3}\bar{\epsilon}\psi_\mu, \end{aligned} \tag{70}$$

normalized in either $D = 4$ or $D = 5$ according to $[\delta_1, \delta_2]\Phi = \frac{1}{2}(\bar{\epsilon}_1\gamma^\mu\epsilon_2)\partial_\mu\Phi + \cdots$, with Φ any of the fields in the multiplet. This system admits BPS (Reissner-Nordstrom) black holes of the form

$$\begin{aligned} ds^2 &= -\mathcal{H}^{-2}dt^2 + \mathcal{H}^{2/(D-3)}d\vec{y}^2, \\ A_{(1)} &= \frac{2}{D-3}\mathcal{H}^{-1}dt. \end{aligned} \tag{71}$$

For a spherically symmetrical black hole with harmonic function $\mathcal{H} = 1 + q/r^{D-3}$, application of the techniques of [1–3] to generate superpartners yields

$$\delta\delta g_{ti} = (\bar{\epsilon} i \gamma_i^j \epsilon)\hat{x}_j \frac{(D-2)q}{2r^{D-2}},$$
$$\delta\delta A_i = (\bar{\epsilon} i \gamma_i^j \epsilon)\hat{x}_j \frac{q}{(D-3)r^{D-2}}. \tag{72}$$

where ϵ is a fermion zero-mode spinor. After extracting then angular momentum and magnetic moment from these expressions, and inserting them into (67), we obtain

$$\tilde{g} = \frac{2}{D-3} = \begin{cases} 2 & \text{for } D=4, \\ 1 & \text{for } D=5. \end{cases} \tag{73}$$

So we see that $\tilde{g} = 1$ is actually expected in five dimensions, regardless of whether the background is AdS or Minkowski. Noting that $\tilde{g} = 1$ is the natural value in both IIA theory in ten dimensions and maximal supergravity in eight dimensions [3], it rather appears that $\tilde{g} = 2$ is a unique feature of four dimensions.

Finally, we note that while we have focused on the stationary BPS solutions of [12], they actually have singular horizons or naked singularities in the context of the STU model. While this is a rather undesirable feature, our present analysis is unaffected by such singularities, as we only depend on the asymptotics away from the singularity. It would, however, be worthwhile to extend the fermion zero-mode construction to the case of the recently constructed supersymmetric AdS_5 black holes supported by rotation [21,22]. In addition, while to our knowledge this method has only been applied to the generation of superpartners of particlelike representations, nothing prevents it from being extended to more general backgrounds with partially broken supersymmetry. It would be of particular interest to examine the fermion zero modes in the recently constructed 1/2 BPS backgrounds [23,24], and to explore the role they may play in the AdS/CFT context.

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