Yang-Mills wave functional in Coulomb gauge

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We investigate the dependence of the Yang-Mills wave functional in Coulomb gauge on the Faddeev-Popov determinant. We use a Gaussian wave functional multiplied by an arbitrary power of the Faddeev-Popov determinant. We show, that within the resummation of one-loop diagrams the stationary vacuum energy is independent of the power of the Faddeev-Popov determinant and, furthermore, the wave functional becomes field independent in the infrared, describing a stochastic vacuum. Our investigations show, that the infrared limit is rather robust against details of the variational Ansatz for the Yang-Mills wave functional. The infrared limit is exclusively determined by the divergence of the Faddeev-Popov determinant at the Gribov horizon.

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I. INTRODUCTION

Recently Yang-Mills theory in Coulomb gauge has become the subject of intensive studies both on the lattice, Refs. [1,2] and in the continuum, Refs. [3–7]. The Coulomb gauge is a physical gauge and in this gauge confinement is realized by the statistical dominance of the field configurations near the Gribov horizon, which gives rise to an infrared enhanced static color charge potential.

For the calculation of static properties of continuum Yang-Mills theory, the Schrödinger equation approach [8] seems to be most convenient. In Refs. [5–7] the Yang-Mills Schrödinger equation was approximately solved in Coulomb gauge, using the variational principle. Different Ansätze for the vacuum wave functional and different renormalization conditions have been used, and different infrared behaviors of the gluon and ghost propagators were obtained. One might wonder, whether the different results are a consequence of the different Ansätze for the wave functional. To answer this question, in this paper we consider a more general class of wave functionals, which includes, in particular, the wave functionals previously used in Refs. [5–7]. We will show, that the different Ansatz used so far, have to yield the same unique infrared behavior of the vacuum wave functional (at least to the order considered). We will also show, that in the infrared the wave functional becomes field independent, describing a stochastic vacuum. Furthermore, the infrared limit of the wave functional agrees with the exact vacuum wave functional in $D = 1 + 1$.

II. THE VARIATIONAL ANSATZ

For the Yang-Mills vacuum we consider trial wave functionals of the form

$$\Psi[A^\perp] = J^{-\alpha}[A^\perp]\phi[A^\perp], \tag{1}$$

where

$$J[A^\perp] = \frac{\mathrm{Det}(-\hat{D}_i\partial_i)}{\mathrm{Det}(-\partial^2)} \tag{2}$$

is the Faddeev-Popov determinant, which for later convenience has been normalized to $J[A^\perp = 0] = 1$. Here $\hat{D} = \partial + \hat{A}^\perp$ is the covariant derivative and $\hat{A}^\perp = A^{\perp a}\hat{T}^a$ denotes the gauge field in the adjoint representation. Furthermore $\phi[A^\perp]$ is a Gaussian wave functional defined by

$$\begin{aligned} \phi[A^\perp] &= \mathcal{N}\exp(-S[A^\perp]) \\ S[A^\perp] &= \frac{1}{2}\int d^3x\int d^3x' A_i^{\perp a}(\mathbf{x})\omega_{ij}^{ab}(\mathbf{x},\mathbf{x}')A_j^{\perp b}(\mathbf{x}') \end{aligned} \tag{3}$$

with $\mathcal{N} = \mathcal{N}(\alpha, \omega)$ being a normalization constant to ensure $\langle\psi \mid \psi\rangle = 1$. The Faddeev-Popov determinant arises as Jacobian in the transformation from "Cartesian" coordinates $A_i^a(\mathbf{x})$ to the "curvilinear coordinates" $A_i^{\perp a}(\mathbf{x})$ satisfying the Coulomb gauge $\partial_i A_i^\perp = 0$, and defines the metric in the space of transversal gauge orbits $A_i^\perp(\mathbf{x})$. Accordingly the scalar product in the space of transversal gauge orbits is defined by

$$\langle\Psi \mid \Phi\rangle = \int DA^\perp J[A^\perp]\Psi^*[A^\perp]\Phi[A^\perp], \tag{4}$$

where the integration should in principal be restricted to the fundamental modular region [4,9]. The choice $\omega_{ij}^{ab}(\mathbf{x},\mathbf{x}') = \delta_{ij}\delta^{ab}\omega(\mathbf{x},\mathbf{x}')$ and $\alpha = 0$ was used in Refs. [5,6], while $\alpha = \frac{1}{2}$ was chosen in Ref. [7]. From Eq. (1) it is seen, that for the latter choice $\phi[A^\perp]$ represents just the "radial" wave functional.[1]

We wish to study the dependence of the vacuum Yang-Mills wave functional (1) on the power of the Faddeev-Popov determinant α, which can take, in principle, any real value as long as $\Psi[A^\perp]$ is normalizable. The integral kernel ω as well as the parameter α have to be determined

[1]For a point particle in a s state the wave function is of the form $\Psi(r) = \phi(r)/r$, where $\phi(r)$ is the radial wave function and the Jacobian is given by $J = r^2$

by minimizing the expectation value of the energy

$$\langle H\rangle = \int DA^{\perp} J[A^{\perp}]\Psi^{*}[A^{\perp}]H\Psi[A^{\perp}]. \qquad (5)$$

III. MINIMIZATION OF THE ENERGY

Inserting the explicit form of the wave functional Eq. (1) into Eq. (5) variation of the energy with respect to the kernel ω yields the "gap equation":

$$\frac{\delta\langle H\rangle}{\delta\omega} = 2\left\langle\frac{\delta S}{\delta\omega}\right\rangle\langle H\rangle - \left\langle\left\{\frac{\delta S}{\delta\omega}, H\right\}\right\rangle = 0. \qquad (6)$$

Here the first term arises from the variation of the normalization constant ($\delta\mathcal{N}/\delta\omega = \mathcal{N}\langle\delta S/\delta\omega\rangle$) and $\{,\}$ denotes the anticommutator. Minimization of the energy $\langle H\rangle$ (5) with respect to the power α yields the condition

$$\frac{d\langle H\rangle}{d\alpha} = 2\langle \ln J[A^{\perp}]\rangle\langle H\rangle - \langle\{\ln J[A^{\perp}], H\}\rangle = 0, \qquad (7)$$

where we have used $d\mathcal{N}/d\alpha = \mathcal{N}\langle \ln J[A^{\perp}]\rangle$.

Consider now the structure of the Faddeev-Popov determinant (2), which obviously satisfies $\ln J[A^{\perp} = 0] = 0$. Furthermore, by definition (2) we have

$$\frac{\delta \ln J[A^{\perp}]}{\delta A_i^{\perp a}(\mathbf{x})} = -\mathrm{Tr}(G\Gamma_k^{o,a}(\mathbf{x})), \qquad (8)$$

where

$$G = (-\hat{D}_i\partial_i)^{-1} \qquad (9)$$

is the inverse Faddeev-Popov operator and $\Gamma_k^{0,a}(\mathbf{x}) = \delta G^{-1}/\delta A_k^{\perp a}(\mathbf{x})$ is the bare ghost-gluon vertex [7]. Since $\Gamma^{0,a} \sim \hat{T}^a$ (group generator in the adjoint representation) and $\hat{T}^a$ occurs in G only in the combination $\hat{A}^{\perp} = A^{\perp a}\hat{T}^a$ it is clear, that the quantity (8) has to be proportional to $A^{\perp}$, since $tr\hat{T}^a = 0$. Therefore we find the representation

$$\ln J[A^{\perp}] = \int d^3x d^3x' C_{ij}^{ab}[A^{\perp}](\mathbf{x},\mathbf{x}')A_i^{\perp a}(\mathbf{x})A_j^{\perp b}(\mathbf{x}') \quad (10)$$

with some, not explicitly known functional $C_{ij}^{ab}[A^{\perp}]$. Let us stress, this representation is exact and does not rely on an expansion in powers of the gluon fields.

Consider now the expectation value

$$\langle \ln J[A^{\perp}]\rangle = \langle \mathrm{Tr}(\ln G^{-1} - \ln(-\partial^2))\rangle. \qquad (11)$$

In a diagrammatic expansion this quantity is given by closed ghost loops from which an even number of gluon lines are emitted or absorbed, which are pairwise contracted to gluon propagators $\langle AA\rangle$. We use here the rainbow-ladder approximation (used also in [5–7]) which consists in replacing the full ghost-gluon vertex in the one-particle irreducible ghost self-energy by the bare one, see Fig. 1. Within this approximation, in leading order of the loop expansion (in terms of the full ghost propagator) the quantity $\langle \ln J[A^{\perp}]\rangle$ (11) is given by the 2-loop diagram shown in Fig. 2(a). Note that this diagram contains only a *single* gluon propagator. Therefore, to leading order in the loop expansion (in full ghost propagators) the expectation value of Eq. (10) is given by

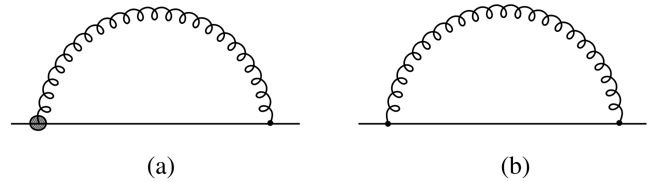

FIG. 1. Diagrammatic representation of the ghost self-energy. (a) full, (b) in rainbow-ladder approximation. Throughout the paper full and curly lines stand, respectively, for the full ghost and gluon propagators. Furthermore, dots and fat dots represent, respectively, bare and full ghost-gluon vertices.

$$\langle \ln J[A^{\perp}]\rangle \simeq \int d^3x d^3x' \langle C_{ij}^{ab}[A^{\perp}](\mathbf{x},\mathbf{x}')\rangle\langle A_i^{\perp a}(\mathbf{x})A_j^{\perp b}(\mathbf{x}')\rangle. \qquad (12)$$

Furthermore, to this order we can neglect terms of the form $\langle(\delta C/\delta A^{\perp})A^{\perp}\rangle$ and $\langle\delta^2 C/\delta A^{\perp}\delta A^{\perp}\rangle$ and find from (10) for the curvature in orbit space [7]

$$\chi_{ik}^{ab}(\mathbf{x},\mathbf{x}') = -\frac{1}{2}\left\langle\frac{\delta^2 \ln J}{\delta A_i^{\perp a}(\mathbf{x})\delta A_j^{\perp b}(\mathbf{x}')}\right\rangle = -\langle C_{ij}^{ab}[A^{\perp}](\mathbf{x},\mathbf{x}')\rangle, \qquad (13)$$

which is diagrammatically illustrated in Fig. 2(b). Note that diagram (b) arises from (a) by removing the gluon propagator $\langle A^{\perp}A^{\perp}\rangle$ [cf. Eqs. (12) and (13)]. To the considered order we can also replace $C[A]$ in (10) by its expectation value ($-\chi$) yielding

$$\ln J[A^{\perp}] = -\int d^3x d^3x' \chi_{ij}^{ab}(\mathbf{x},\mathbf{x}')A_i^{\perp a}(\mathbf{x})A_j^{\perp b}(\mathbf{x}'). \quad (14)$$

Inserting Eq. (14) into Eq. (7) we find

$$\frac{d\langle H\rangle}{d\alpha} = -\int d^3x d^3x' \chi_{ij}^{ab}(\mathbf{x},\mathbf{x}')[2\langle A_i^{\perp a}(\mathbf{x})A_j^{\perp b}(\mathbf{x}')\rangle\langle H\rangle - \langle\{A_i^{\perp a}(\mathbf{x})A_j^{\perp b}(\mathbf{x}'), H\}\rangle]. \qquad (15)$$

On the other hand for the Gaussian wave functional (1) and (3) we have

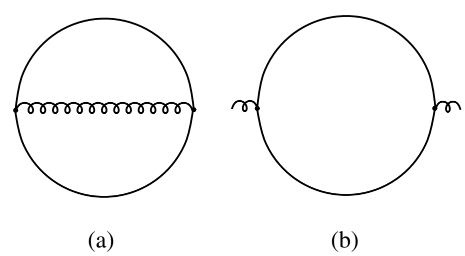

FIG. 2. Leading order contribution to (a) $\langle \ln J[A^{\perp}]\rangle$ (11) and (b) to the curvature χ (13).

$$\frac{\delta S}{\delta \omega_{ij}^{ab}(\mathbf{x}, \mathbf{x}')} = \frac{1}{2} A_i^{\perp a}(\mathbf{x}) A_j^{\perp b}(\mathbf{x}'), \tag{16}$$

so that Eq. (6) becomes

$$2 \frac{\delta \langle H \rangle}{\delta \omega_{ij}^{ab}(\mathbf{x}, \mathbf{x}')} = 2 \langle A_i^{\perp a}(\mathbf{x}) A_j^{\perp b}(\mathbf{x}') \rangle \langle H \rangle - \langle \{ A_i^{\perp a}(\mathbf{x}) A_j^{\perp b}(\mathbf{x}'), H \} \rangle. \tag{17}$$

Comparison of Eqs. (15) and (17) yields

$$\frac{d\langle H \rangle}{d\alpha} = -2 \int d^3x d^3x' \chi_{ij}^{ab}(\mathbf{x}, \mathbf{x}') \frac{\delta \langle H \rangle}{\delta \omega_{ij}^{ab}(\mathbf{x}, \mathbf{x}')}. \tag{18}$$

Thus stationarity of the energy with respect to $\omega_{ij}^{ab}(\mathbf{x})$, $\delta \langle H \rangle / \delta \omega = 0$ implies also stationarity with respect to α, $d\langle H \rangle / d\alpha = 0$. Let us emphasize, that Eq. (18) is exact to leading order in the number of loops (i.e. to one-loop order in the equation of motion, to two-loop order in $\langle H \rangle$).

IV. THE ENERGY FUNCTIONAL

The above obtained result [Eq. (18)] can be also immediately inferred from the explicit expression of the expectation value of the Yang-Mills Hamiltonian in the state (1), which to leading order in the loop expansion (see above and Ref. [7]] is given by

$$\langle H \rangle = E_k + E_B + E_C \tag{19}$$

$$E_k = \delta^3(0) \frac{N_C^2 - 1}{2} \int d^3k \frac{[\Omega(\mathbf{k}) - \chi(\mathbf{k})]^2}{\Omega(\mathbf{k})}$$

$$E_B = \delta^3(0) \frac{N_C^2 - 1}{2} \int d^3k \left(\frac{\mathbf{k}^2}{\Omega(\mathbf{k})} + \frac{N_C g^2}{8} \int \frac{d^3k'}{(2\pi)^3} [3 - (\hat{\mathbf{k}} \hat{\mathbf{k}}')^2] \frac{1}{\Omega(\mathbf{k}) \Omega(\mathbf{k}')} \right)$$

$$E_C = \delta^3(0) \frac{N_C(N_C^2 - 1)}{16} \int \frac{d^3k d^3k'}{(2\pi)^3} [1 + (\hat{\mathbf{k}} \hat{\mathbf{k}}')^2] \cdot \frac{d^2(\mathbf{k} + \mathbf{k}') f(\mathbf{k} + \mathbf{k}')}{(\mathbf{k} + \mathbf{k}')^2} \frac{[(\Omega(\mathbf{k}) - \chi(\mathbf{k})) - (\Omega(\mathbf{k}') - \chi(\mathbf{k}'))]^2}{\Omega(\mathbf{k}) \Omega(\mathbf{k}')},$$

where $d(\mathbf{k})$ and $f(\mathbf{k})$ are the ghost and Coulomb form factors defined in Ref. [7] and χ is the scalar curvature defined in terms of the curvature tensor (13) by

$$t_{kn}(\mathbf{x}) \chi_{nl}^{ab}(\mathbf{x}, \mathbf{y}) = \delta^{ab} t_{kl}(\mathbf{x}) \chi(\mathbf{x}, \mathbf{y}) \tag{20}$$

$$\chi(\mathbf{k}) = \frac{N_C}{4} \int \frac{d^3q}{(2\pi)^3} [1 - (\hat{\mathbf{k}} \hat{\mathbf{q}})^2] \frac{d(\mathbf{k} - \mathbf{q}) d(\mathbf{q})}{(\mathbf{k} - \mathbf{q})^2} \tag{21}$$

with $t_{kl}(\mathbf{x}) = \delta_{kl} - \partial_k \partial_l / \partial^2$ being the transversal projector.[2] Furthermore

$$\Omega(\mathbf{k}) = \omega(\mathbf{k}) - (2\alpha - 1)\chi(\mathbf{k}) \tag{22}$$

is the inverse of the gluon propagator

$$\langle A_i^{\perp a}(\mathbf{k}) A_j^{\perp b}(-\mathbf{k}) \rangle = \frac{1}{2} \delta^{ab} t_{ij}(\mathbf{k}) \Omega^{-1}(\mathbf{k}). \tag{23}$$

Note, that the curvature $\chi(\mathbf{k})$ (21) is entirely determined by the ghost form factor $d(\mathbf{k})$ and does not depend on $\omega(\mathbf{k})$. The energy (19) depends on α and $\omega(\mathbf{k})$ only through the combination $\Omega(\mathbf{k}) = \omega(\mathbf{k}) - (2\alpha - 1)\chi(\mathbf{k})$. From this fact immediately follows, that Eq. (6) implies Eq. (7), so we find again, that the wave functional (1) which minimizes the energy is independent of α. In fact, since $\langle H \rangle$ (19) depends on ω and α only through the combination $\Omega = \omega - (2\alpha - 1)\chi$ it suffices to minimize the energy with respect to Ω. The resulting gap equation[3] also depends only on Ω and its solution is independent of α. This shows, that the infrared behavior of the gluon propagator $\langle A^\perp A^\perp \rangle$ (23) is independent of the power α of the Faddeev-Popov determinant assumed in the wave functional (1). Therefore we are free to choose α for our convenience, for example, $\alpha = \frac{1}{2}$. This choice has the technical advantage, that $\Omega(\mathbf{k}) = \omega(\mathbf{k})$, which allows a straightforward application of Wick's theorem in the calculation of expectation values.

In this context let us also mention that the choice $\alpha = \frac{1}{2}$ in Eq. (1) yields the wave function used by the present authors in Ref. [7], while the wave function used in Refs. [5,6] corresponds to the choice $\alpha = 0$. In spite of the different wave functions chosen in Refs. [5–7], the same infrared behavior of the gluon propagator should be obtained in one-loop order, as shown above, provided the same renormalization condition is used. However, while Refs. [5,6] find an infrared finite gluon propagator, we find an infrared vanishing gluon propagator [7]. Two sources of the different behaviors obtained in Ref. [7], and Refs. [5,6] come to mind: (i) different choices of the renormalization

[2] The ghost and Coulomb form factor, $d(k)$ and $f(k)$, satisfy the Schwinger-Dyson equations derived in Ref. [7] with ω replaced by Ω.

[3] Although the curvature (21) does not explicitly depend on ω it depends implicitly on ω via the ghost form factor $d(k)$. However, also the ghost form factor $d(k)$ depends on ω only through Ω. This dependence is, however, a higher order effect and to one-loop order (in the equation of motion) $\delta\chi/\delta\omega$ or $\delta\chi/\delta\Omega$ can be neglected. Then the gap equation $\delta\langle H \rangle / \delta\Omega = 0$ is exactly the one obtained in Ref. [7] with ω replaced by Ω.

condition and (ii) different treatments of the curvature of orbit space. In Ref. [7] we choose the so-called horizon condition [9]

$$d^{-1}(k \to 0) = 0 \tag{24}$$

as renormalization condition while Refs. [5,6] require the kernel ω in the Gaussian wave functional to be infrared finite $\omega(k \to 0) = \text{const.}$[4] Furthermore, while the curvature of orbit space χ (13) was fully included in Ref. [7], it was completely neglected in [5] and ignored[5] in the Coulomb energy in the numerical calculations of Ref. [6]. As will be shown in the next section ignoring the curvature in the Coulomb energy will change the infrared behavior of the wave functional. It was already observed in Ref. [7], that the full inclusion of the curvature is vital for the infrared limit of the theory. This is consistent with the observation in Landau gauge, that in the Schwinger-Dyson equations the ghost loop is by far more important than the gluon loop [10].

V. THE YANG-MILLS WAVE FUNCTIONAL IN THE INFRARED

With the relation (14) the wave functional (1) becomes

$$\Psi[A^\perp] \simeq e^{\alpha \int A^\perp \chi A^\perp - (1/2) \int A^\perp \omega A^\perp}. \tag{25}$$

Furthermore, the solution of the gap equation Eq. (6) is such, that in the infrared

$$\Omega(k \to 0) = \chi(k \to 0), \qquad \omega(k \to 0) = 2\alpha\chi(k \to 0) \tag{26}$$

holds. This is an extension of the relation $\chi(k \to 0) = \omega(k \to 0)$ found in Ref. [7] for $\alpha = \frac{1}{2}$.

The relation (26) holds independent of the employed renormalization condition as long as the curvature $\chi(k)$ is infrared divergent. Since the Faddeev-Popov determinant vanishes on the Gribov horizon, which contains the infrared dominant field configurations, from Eq. (14) follows that the curvature has, indeed, to be infrared divergent. The condition (26) is, however, lost when the curvature is neglected in the Coulomb energy as done in Refs. [5,6]. Given the infrared singular behavior of $\chi(k \to 0)$ the condition (26) implies that for $\alpha \neq 0$ (in particular for $\alpha = \frac{1}{2}$ [7]) the variational kernal $\omega(k)$ in the Gaussian Ansatz (3) has to be infrared singular, while the choice $\alpha = 0$ [5,6] can tolerate an infrared finite $\omega(k)$. Nonetheless, independent of the specific choice of α, by the first equation in (26), the infrared behavior of the gluon propagator (23) is exclusively determined by the curvature χ (13) in orbit space, and since $\chi(k)$ is infrared divergent the gluon propagator (23) has to vanish in the infrared. Furthermore, with the relation (26) the vacuum Yang-Mills wave functional becomes in the infrared

$$\Psi[A^\perp] = 1. \tag{27}$$

In Ref. [11] this wave functional was assumed in the infrared regime, for the sake of simplicity. We have thus shown that, to one-loop order, Eq. (27) is the correct wave function in the infrared. The infrared wave functional $\Psi[A^\perp] = 1$ suggests that gauge fields at distant positions $\mathbf{x}$, $\mathbf{x}'$, $|\mathbf{x} - \mathbf{x}'| \to \infty$ are completely uncorrelated. However, one should keep in mind that the space of gauge orbits is curved due to the presence of the Faddeev-Popov determinant in the integration measure. Indeed, from Eq. (14) and (27) it follows that in the infrared limit the vacuum expectation values of the gauge fields are given by the "Gaussian ensemble"

$$\langle \cdots \rangle = \int \mathcal{D}A^\perp J[A^\perp] \cdots = \int DA^\perp \cdots e^{-\int A^\perp \chi A^\perp}, \tag{28}$$

so that we obtain for the gluon propagator

$$\langle A^\perp(\mathbf{x}) A^\perp(\mathbf{x}') \rangle|_{|\mathbf{x}-\mathbf{x}'| \to \infty} \sim \chi^{-1}(|\mathbf{x} - \mathbf{x}'|), \tag{29}$$

which agrees with our previous result, see Eq. (23) and (26), i.e. the infrared behavior of the gluon propagator is exclusively determined by the curvature χ and not by the kernel ω in the Gaussian Ansatz, Eq. (3). In Ref. [7] it was found, in the so-called angular approximation, that $\chi(k \to 0) \sim 1/k$, so that the gluon propagator, Eq. (29) vanishes in the infrared. From the physical point of view the static gluon propagator represents (in momentum space, up to a factor of 2) the inverse of the (single) gluon energy. Thus the infrared vanishing gluon propagator (29) implies an infrared diverging gluon energy and hence the absence of free gluons in the infrared, which is a signal of confinement. Let us also emphasize that within the present approach the potential between static color charges is *not* given by the static gluon propagator but instead by the so-called Coulomb kernel

$$F(\mathbf{x}, \mathbf{x}') = g^2 \langle G(-\partial^2) G \rangle (\mathbf{x}, \mathbf{x}'), \tag{30}$$

where G denotes the ghost propagator (9) (see Ref. [7] for more details). Resorting to the angular approximation it was found in Ref. [7] that this kernel behaves in the infrared ($k \to 0$), indeed, like $F(k) \sim 1/k^4$ and thus produces a linear rising confinement potential. Thus the confinement property of static color charges is determined by the infrared behavior of the ghost rather than gluon propagator. In this context it is worth mentioning that also in

[4] In $D = 2 + 1$ we find a self-consistent solution to the coupled Schwinger-Dyson equations only when we impose the horizon condition (24).

[5] In the formal part of [6] the curvature was fully included.

Landau gauge the confining properties are encoded in the ghost propagator [12].

VI. YANG-MILLS THEORY IN $D = 1 + 1$

Let us test the above result in $1 + 1$ dimension, where Yang-Mills theory can be solved exactly on a torus and reduces to quantum mechanics in curved space. Implementing the Coulomb gauge $\partial_1 A_1 = 0$, there is only a constant gauge field $A_1(x_1) = \text{const}$ left, which can be diagonalized in color space by exploiting the residual global gauge freedom U, not fixed by $\partial_1 A_1 = 0$. Defining the remaining quantum mechanical degree of freedom, a, by

$$gA_1 L \equiv gLA_1^a \frac{\tau^a}{2} = U \frac{a}{2} \tau^3 U^\dagger, \tag{31}$$

where L is the spatial extension of the torus the Faddeev-Popov determinant becomes [13]

$$J(a) = \sin^2 a. \tag{32}$$

The Gribov horizon occurs at $a = n\pi$ and the fundamental modular region is obviously given by $0 \leq a \leq \pi$. Furthermore the Yang-Mills Hamiltonian in the variable a is given by

$$H_{\text{kin}} = -\frac{g^2 L}{8} \frac{1}{\sin^2 a} \frac{d}{da} \sin^2 a \frac{d}{da}. \tag{33}$$

In one spatial dimension there is no magnetic field and no dynamical gluon charge ($-\hat{A}_1^{ab} \Pi_1^a = 0$), since the gauge field has only one (nonzero) color degree of freedom. Accordingly, the Coulomb term of the Yang-Mills Hamiltonian [8] vanishes in the absence of external color charges.

With the Ansatz

$$\Psi_k(a) = \frac{1}{\sqrt{J(a)}} \phi_k(a) = \frac{1}{\sin a} \phi_k(a), \tag{34}$$

which corresponds to the choice $\alpha = \frac{1}{2}$ in Eq. (1), the Schrödinger equation $H\Psi_k = E_k \Psi_k$ reduces to

$$-\frac{g^2 L}{8} \phi_k''(a) = \left(E_k + \frac{g^2 L}{8}\right) \phi_k(a), \tag{35}$$

whose solution is given by[6]

$$\phi_k(a) = \sin(ka), E_k = \frac{g^2 L}{8}(k^2 - 1). \tag{36}$$

[6]This solution was previously found in Ref. [14].

In the continuum limit $L \to \infty$ only the vacuum state $k = 1$ survives ($E_1 = 0$), while all excited states $k > 1$ acquire an infinite energy and are thus frozen out. The vacuum wave function is given by (34) and (36)

$$\Psi_{k=1}(a) = 1, \tag{37}$$

which is precisely the infrared limit of the vacuum Yang-Mills wave functional in $D = 3 + 1$ found above [see Eq. (27)]. Note also that the radial wave function $\phi_k(a)$ (36) vanishes on the Gribov horizon $a = n\pi$ to compensate for the vanishing of the Faddeev-Popov determinant $J(a)$ (32), just as in the $D = 3 + 1$ dimensional case where (for $\alpha = \frac{1}{2}$), the radial wave functional $\phi[A^\perp]$ (3) vanishes in the infrared due to the infrared divergence of $\omega(k \to 0)$.

VII. SUMMARY AND CONCLUSIONS

We have studied the variational solution of the Yang-Mills Schödinger equation in Coulomb gauge for a class of wave functionals (1) consisting of a Gaussian and an arbitrary power ($-\alpha$) of the Faddeev-Popov determinant. We have found, that up to one loop in the gap equation (i.e. two loops in the energy) the stationary solution is independent of this power α. The same is true for the transversal gluon propagator (23) which is exclusively determined by the self-consistent solution Ω of the gap equation $\delta\langle H\rangle/\delta\omega = 0$. This solution Ω is independent of the choice of α. Different choices of α will lead to different kernels ω (with possibly different infrared behaviors) in the wave functional (3). But this will not affect the gluon propagator (23). Furthermore in the infrared the Yang-Mills vacuum wave functional becomes field independent describing a stochastic vacuum, in which color cannot propagate over large distances. The infrared limit of the wave functional becomes exact in $D = 1 + 1$.

Our investigations show that the infrared behavior of Yang-Mills theory in Coulomb gauge is rather robust with respect to changes in the variational Ansätze for the wave functional as long as the curvature in orbit space induced by the Faddeev-Popov determinant is properly included.

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