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Black holes in the ghost condensate

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We investigate how the ghost condensate reacts to black holes immersed in it. A ghost condensate defines a hypersurface-orthogonal congruence of timelike curves, each of which has the tangent vector $u^\mu = -g^{\mu\nu}\partial_\nu\phi$. It is argued that the ghost condensate in this picture approximately corresponds to a congruence of geodesics. In other words, the ghost condensate accretes into a black hole just like a pressureless dust. Correspondingly, if the energy density of the ghost condensate at large distance is set to an extremely small value by cosmic expansion then the late-time accretion rate of the ghost condensate should be negligible. The accretion rate remains very small even if effects of higher derivative terms are taken into account, provided that the black hole is sufficiently large. It is also discussed how to reconcile the black-hole accretion with the possibility that the ghost condensate might behave like dark matter.

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I. INTRODUCTION AND SUMMARY

Gravity at long distances shows us many interesting and mysterious phenomena: flattening galaxy rotation curves, dimming supernovae, and so on. These phenomena have been a strong motivation for the paradigm of dark matter and dark energy, i.e. unknown components of the Universe which show up only gravitationally. As we essentially do not know what the dark matter and the dark energy are, however, it seems a healthy attitude to consider the possibility that gravity at long distances might be different from what we think we know.

This kind of consideration has been a motivation for attempts for IR modification of gravity, e.g. massive gravity [1] and the Dvali, Gabadadze, and Porrati-brane model [2]. However, they are known to have a macroscopic UV scale at around 1000 km, where effective field theories break down [3,4]. This does not necessarily mean that these theories cannot describe the real world, but implies that we need nontrivial assumptions about the unknown UV completion. The recent proposal of ghost condensation [5] at least evades this problem and can be thought to be a step towards a consistent theory of IR modification of general relativity.

In general, if we have scalar fields then there are many things we can play with them. In cosmology, inflation can be driven by the potential part of a scalar field. It is also possible to drive inflation by the kinetic part of a scalar field [6]. On the other hand, scalar fields play important roles also in particle physics. A scalar field is used for spontaneous symmetry breaking and to change force laws in the Higgs mechanism. This is usually achieved by using a potential whose global minimum is charged under the gauge symmetry. The basic idea of ghost condensation is to break a symmetry and change a force law by the kinetic part of a scalar field. In this sense the ghost condensation can be considered as an analog of the Higgs mechanism. Note that modifying gravity force law via spontaneous symmetry breaking, i.e. ghost condensation, is different from just adding a new matter in the sense that the linearized gravity is modified even in Minkowski or de Sitter background.

Since the ghost condensation modifies gravity, it is natural to ask the question "what happens to the ghost condensate when gravity is very strong?" Two such situations are in the early universe and near a black hole. Effects of gravity in the early universe were already investigated in Refs. [5,7]. Hence, the next question would be "what happens in the other regime of strong gravity, namely, near a black hole?" This is the subject of this paper. Other interesting topics related to the ghost condensate include moving sources [8,9], nonlinear dynamics [10–12], cosmology [13–15], galaxy rotation curve [16], spin-dependent force [17], and so on.

Before discussing the ghost condensate near a black hole, we begin with briefly reviewing a well-known fact about observer-dependence of gravitational force. A black-hole horizon forms when gravity is very strong in the sense that even the degenerate pressure due to neutrons cannot support the implosive gravitational force. However, as is well known, different observers feel different gravitational forces since a force is defined by acceleration of an observer's trajectory. This is particularly notable near a black-hole horizon. For a static observer, the closer to the horizon the observer's position is, the stronger the gravitational force is. Indeed, acceleration of a static observer diverges at the horizon. On the other hand, for a freely falling observer, a black-hole horizon is not a special point and actually there is nothing divergent at the horizon. Indeed, the acceleration of a freely-falling pointlike observer vanishes by definition. An extended object passing through a black-hole horizon does feel a tidal force due to the nonzero Riemann curvature, but the tidal force is negligible for a sufficiently large black hole.

Now a ghost condensate in general defines a hypersurface-orthogonal timelike vector field $u^\mu =$

$-\partial^\mu \phi$. Thus, it is possible to regard the ghost condensate as a hypersurface-orthogonal congruence of timelike curves, each of which has the tangent vector u^μ. In this paper we shall argue that, when the ghost condensate in this picture approximately corresponds to a congruence of geodesics, the accretion rate of a ghost condensate into a black hole should be negligible for a sufficiently large black hole. The essential reason for the smallness of the accretion rate is the same as that for the smallness of the tidal force acted on an extended object freely falling into a large black hole.

In the rest of this paper, for simplicity we consider a scalar field ϕ described by the action

$$I = \int d^4x\sqrt{-g}\left[P(X) - \frac{\alpha(\Box\phi)^2}{2M^2}\right], \qquad (1)$$

where $X = -\partial^\mu\phi\partial_\mu\phi$ and the sign convention for the metric is $(- + \cdots +)$. Hereafter, we assume that $P'(M^4) = 0$, $P(M^4) = 0$, and $P''(M^4) > 0$, where the first equation just defines the scale M, the second condition corresponds to zero cosmological constant in the Higgs phase, and the third condition is required by the absence of ghost in the Higgs phase. As shown in [5], P' is set to an extremely small value, or $X \to M^4$ by the expansion of the universe.

In the following we investigate a ghost condensate interacting with a black hole and present an approximate $P' = 0$ solution, for which gravitational backreaction such as accretion rate is very small. A. Frolov [18] considered different solutions with $P' \neq 0$, which correspond to congruences of nongeodesic (namely accelerated) observers, and obtained a large accretion rate due to the stress-energy tensor of order M^4P'. However, as the accretion proceeds, the energy density around the black hole, which is also of order M^4P', should decrease and P' near the black hole should approach to zero or a small value. Therefore, the accretion should slow down because of shortage of energy and the steady-state accretion claimed in Ref. [18] cannot be established. In the following we shall obtain a much smaller accretion rate for an approximate $P' = 0$ solution. Note that setting $P' \simeq 0$ is completely natural since the expansion of the universe makes P' extremely small.

We expect that the effects of the α term (corresponding somehow to the tidal force for a freely falling extended object) are small for a sufficiently large black hole. To be more precise, for a large enough black hole, there should be an approximate $X = M^4$ solution for which the α term can be treated perturbatively. Thus, we first construct an appropriate $X = M^4$ solution and then introduce deviation from it due to the α term as a perturbation.

In a Gaussian normal coordinate system called Lemaitre reference frame [19], the Schwarzschild geometry with mass parameter m_0 is written as

$$ds^2 = -d\tau^2 + \frac{d\rho^2}{a(\tau,\rho)} + \rho^2 a^2(\tau,\rho)d\Omega^2, \qquad (2)$$

where

$$a(\tau,\rho) = \left[1 - \frac{3\tau}{4m_0}\left(\frac{2m_0}{\rho}\right)^{3/2}\right]^{2/3}. \qquad (3)$$

This coordinate system was originally found by Lemaitre [20] and independently by Rylov [21] and Novikov [22]. For completeness, the coordinate transformation from the standard coordinate system to this one is given in the appendix. In particular the usual areal radius r is given in this coordinate by $r = \rho a$ so that the event horizon is located at $\rho a = 2m_0$. Manifestly, there is nothing bad on the future (black-hole) horizon and the coordinate system covers everywhere in the shaded region in Figs. 1 and 2. The metric becomes ill only on the curvature (physical) singularity at $\rho a = 0$. Each world line with $\rho =$ const. in Fig. 2 corresponds to an observer freely falling into the black hole. With $\alpha = 0$, $\phi = M^2\tau$ satisfies the equation of motion and the Einstein equation since $X = M^4$ implies that the stress-energy tensor of ϕ vanishes. In this sense, this coordinate choice provides an analog of the unitary gauge in flat spacetime. For this solution with $\alpha = 0$, $\Box\phi$ is regular outside the horizon $\rho a > r_g$:

$$0 < \frac{\Box\phi}{M^3} = \frac{3}{2\rho a M}\sqrt{\frac{r_g}{\rho a}} < \frac{3}{2Mr_g}, \qquad (4)$$

where $r_g = 2m_0$ is the Schwarzschild radius. Hence, the effect of the term $-\alpha(\Box\phi)^2/2M^2$ in the action is suppressed by the small factor

$$\epsilon = \frac{\alpha}{M^2 r_g^2}. \qquad (5)$$

In Sec. II we shall perform perturbation with respect to ϵ, assuming that $\alpha = O(1)$ and that the black-hole radius r_g

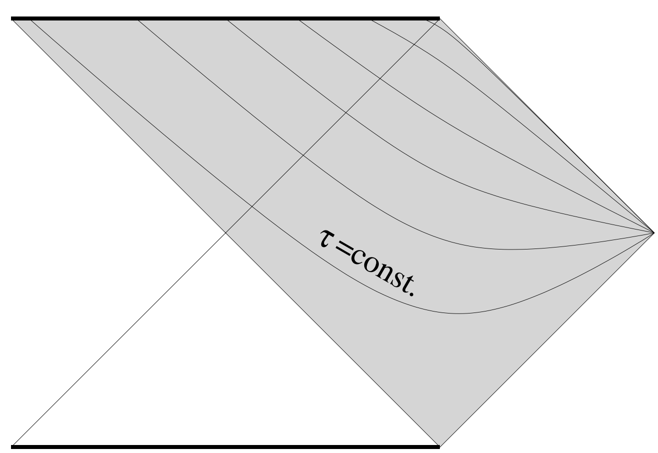

FIG. 1. Constant-τ surfaces are drawn for the Gaussian normal coordinate system (2) of Schwarzschild metric. The coordinate system covers the shaded region.

is sufficiently larger than the microscopic length scale $\sqrt{\alpha}/M$. We shall see that the accretion rate is negligible as expected.

As pointed out in Ref. [5], deviation from $P' = 0$ in homogeneous, isotropic cosmology behaves exactly like dark matter. Hence, it is also interesting to estimate the mass increase of a black hole due to the accretion of ghost condensate with nonvanishing P', whose value at large distance is set by the energy density of dark matter. Since the α term gives negligible contribution to the accretion rate, we set $\alpha = 0$ in this analysis.

Let us first estimate the maximum volume from which a black hole can in principle swallow the energy density of the ghost condensate. For this purpose, suppose that a volume with radius R at $t = 0$ falls into a black hole and passes the black-hole horizon with radius r_g at $t = T$. A crude estimate for the relation between R and T is obtained by considering a fiducial collapsing FRW universe filled with dark matter: $R \propto T^{2/3}$. The proportionality coefficient is guessed by dimensional analysis as

$$\frac{R}{r_g} \sim \left(\frac{T}{r_g}\right)^{2/3}. \tag{6}$$

Thus, the maximum mass increase $\Delta M_{\rm BH}$ during the time interval T is estimated as

$$\frac{\Delta M_{\rm BH}}{M_{\rm BH}} \sim \frac{\rho_{\pi\infty} R^3}{M_{\rm Pl}^2 r_g} \sim \frac{\rho_{\pi\infty}}{\rho_{\rm tot\infty}} (H_0 T)^2, \tag{7}$$

where $\rho_{\pi\infty}$ ($\propto a^{-3}$) is the energy density of the ghost condensate at large distance, $\rho_{\rm tot\infty} \sim M_{\rm Pl}^2 H_0^2$ is the total energy density at large distance, and H_0 is the Hubble expansion rate today. A more systematic and detailed calculation is given in Sec. III and the result is qualitatively the same, provided that the mass increase is for the Misner-Sharp energy on the black-hole horizon and that T is replaced by the advanced time v normalized at past null infinity. It is easy to see that the mass increase $\Delta M_{\rm BH}$ given by (7) is not too large. Indeed, even the integration over the age of the universe ($T \sim H_0^{-1}$) gives

$$\left.\frac{\Delta M_{\rm BH}}{M_{\rm BH}}\right|_{T \sim H_0^{-1}} \sim \frac{\rho_{\pi\infty}}{\rho_{\rm tot\infty}} < 1. \tag{8}$$

The rest of this paper is organized as follows. In Sec. II we explain the main result of this paper, the small accretion rate for an approximate $X = M^4$ solution. In Sec. III we estimate the mass increase of a black hole in the case that the ghost condensate is dark matter. Section IV is devoted to discussions.

II. ACCRETION RATE

In this section we consider corrections to a Schwarzschild geometry due to the ghost condensate. In particular, we calculate corrections to the Misner-Sharp energy to estimate the mass increase of a black hole due to accretion of the ghost condensate.

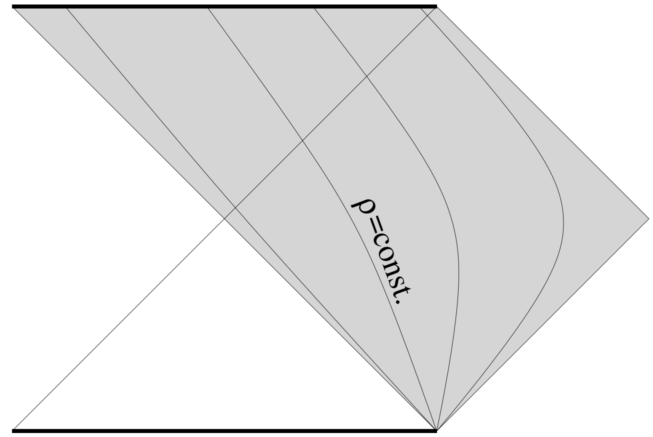

FIG. 2. Constant-ρ surfaces are drawn for the Gaussian normal coordinate system (2) of Schwarzschild metric. The coordinate system covers the shaded region.

The variation of the action (1) is

$$\delta I = \int d^4x\sqrt{-g}\left(\frac{1}{2}T^{\mu\nu}\delta g_{\mu\nu} + E_\phi \delta\phi\right), \tag{9}$$

where

$$\begin{aligned} T^{\mu\nu} &= 2P'\partial^\mu\phi\partial^\nu\phi - \frac{\alpha\partial^\mu(\Box\phi)\partial^\nu\phi}{M^2} - \frac{\alpha\partial^\mu\phi\partial^\nu(\Box\phi)}{M^2} \\ &\quad + \left[P + \frac{\alpha(\Box\phi)^2}{2M^2} + \frac{\alpha\partial^\rho(\Box\phi)\partial_\rho\phi}{M^2}\right]g^{\mu\nu}, \\ E_\phi &= 2\nabla^\mu(P'\nabla_\mu\phi) - \frac{\alpha\Box^2\phi}{M^2}, \end{aligned} \tag{10}$$

so that the relevant equations of motion is

$$M_{\rm Pl}^2 G_{\mu\nu} = T_{\mu\nu}, \qquad E_\phi = 0. \tag{11}$$

We expect that the effects of the α term are small for a sufficiently large black hole and is of order $O(\epsilon)$, where ϵ is defined by (5). Hence, we first seek a solution for $\alpha = 0$ and introduce a nonzero α as a perturbation later. In order to find a solution with $\alpha = 0$, we consider Schwarzschild spacetime,

$$\begin{aligned} g_{\mu\nu}dx^\mu dx^\nu &= -f_0(r)dt^2 + \frac{dr^2}{f_0(r)} + r^2 d\Omega^2, \\ f_0(r) &= 1 - \frac{2m_0}{r}, \end{aligned} \tag{12}$$

and seek a solution to the equation $X = M^4$ in this background. This is because, with $\alpha = 0$, any configurations with $X = M^4$ satisfy not only the ϕ-equation of motion but also the Einstein equation. After finding the solution with $\alpha = 0$, we shall calculate the order $O(\epsilon^1)$ corrections and see that the corrections due to the nonzero α are small enough and that the expansion with respect to ϵ makes sense.

To give an explicit solution to $X = M^4$, let us consider the ansatz

$$\phi = M^2[t + g(r)], \tag{13}$$

for which

$$\frac{X}{M^4} = \frac{1}{f_0} - (\partial_r g)^2 f_0. \tag{14}$$

Hence, $X = M^4$ can easily be solved to give

$$\phi = \phi_\pm \equiv M^2\left\{t \pm 2m_0\left[2\sqrt{\frac{r}{2m_0}} + \ln\left(\frac{\sqrt{r}-\sqrt{2m_0}}{\sqrt{r}+\sqrt{2m_0}}\right)\right]\right\}. \tag{15}$$

For this solution, $\Box\phi$ is finite except at $r = 0$:

$$\frac{\Box\phi_\pm}{M^2} = \pm\frac{3}{2r}\sqrt{\frac{2m_0}{r}}. \tag{16}$$

Note that the $+$ sign in (15) is appropriate for a black hole formed by gravitational collapse since, for the $+$ sign, $\phi/M^2 \sim v \equiv t + r^*$ near the horizon, where v is the advanced time and $r^* = r + 2m_0\ln(r/2m_0 - 1)$. On the other hand, the $-$ sign is appropriate for a white hole. Hereafter, we choose the $+$ sign since we are interested in a black hole.

Now let us treat the α term as a perturbation. For this purpose we consider the spherically symmetric, time-dependent ansatz

$$g_{\mu\nu}dx^\mu dx^\nu = -f(t,r)e^{-2\delta(t,r)}dt^2 + \frac{dr^2}{f(t,r)} + r^2 d\Omega^2, \tag{17}$$

$$\phi = \phi_+(t,r) + \pi(t,r), \tag{18}$$

where

$$f(t,r) = 1 - \frac{2m(t,r)}{r}, \qquad m(t,r) = m_0 + m_1(t,r), \qquad \delta(t,r) = 0 + \delta_1(t,r), \tag{19}$$

and consider π, m_1, and δ_1 as quantities of order $O(\epsilon)$, where ϵ is defined by (5).

In the order $O(\epsilon)$, the Einstein equation becomes

$$\begin{aligned}
&\partial_r m_1 - 2m_0F'(r)\partial_t m_1 = -\frac{9M^2}{8M_{\rm Pl}^2}\left(1 - \frac{m_0}{r}\right), \qquad \partial_r\delta_1 = -\frac{1+2m_0/r}{r(1-2m_0/r)}\partial_r m_1 - \frac{9M^2m_0(3-2m_0/r)}{8M_{\rm Pl}^2r^2(1-2m_0/r)},\\
&\sqrt{\frac{2m_0}{r}}[\partial_r\pi - 2m_0F'(r)\partial_t\pi] = \frac{M^2\delta_1}{1-2m_0/r} - \frac{(1-2m_0/r)M_{\rm Pl}^2}{2P_0''M^6r^2}\partial_r m_1 + \frac{M^2(1+2m_0/r)}{r(1-2m_0/r)^2}m_1 + \frac{9m_0(1-2m_0/r)}{16M^4P_0''r^3},
\end{aligned} \tag{20}$$

where $P_0'' = P''(M^4)$ and

$$\begin{aligned}
F(r) &= \sqrt{\frac{r}{2m_0}}\frac{r+6m_0}{3m_0} + \ln\left(\frac{\sqrt{r}-\sqrt{2m_0}}{\sqrt{r}+\sqrt{2m_0}}\right),\\
F'(r) &= \frac{1}{2m_0}\sqrt{\frac{r}{2m_0}}\frac{1}{1-2m_0/r}.
\end{aligned} \tag{21}$$

The solution to the first equation is

$$\frac{m_1}{m_0} = \frac{9M^2}{4M_{\rm Pl}^2}\left[-\frac{r}{2m_0} + \frac{1}{2}\ln\left(\frac{r}{2m_0}\right) + C(x_+)\right], \tag{22}$$

where

$$x_+ = F(r) + \frac{t}{2m_0}, \tag{23}$$

and $C(x_+)$ is an arbitrary function of x_+. Note that the $x_+ =$ const. hypersurface is timelike and that $x_+ \sim v/2m_0 + (5/3 - 2\ln 2)$ near the horizon, where $v = t + r_*$ is the advanced time normalized at past null infinity, and $r_* = r + 2m_0\ln(r/2m_0 - 1)$. The finiteness of m_1 in the limit $r \to \infty$ with initial, finite t requires that the leading asymptotic behavior of $C(x_+)$ for large positive x_+ should be

$$C(x_+) \sim \left(\frac{3}{2}x_+\right)^{2/3}. \tag{24}$$

This implies that the leading asymptotic behavior of m_1 on the black-hole horizon for large positive v is

$$\frac{m_1}{m_0} \sim \frac{9M^2}{4M_{\rm Pl}^2}\left(\frac{3v}{4m_0}\right)^{2/3}. \tag{25}$$

This formula shows that the accretion rate $\partial_v m_1$ is very small at late time. Indeed, the accretion rate is suppressed by the factor $M^2/M_{\rm Pl}^2$, reflecting the fact that there is no gravitational backreaction in the decoupling limit $M_{\rm Pl}^2 \to \infty$.

For $M \sim 10$ MeV, the Hubble expansion rate today H_0 is rewritten as $M^3/M_{\rm Pl}^2$. Thus, the formula (25) at $v = H_0^{-1}$ is

$$\left.\frac{m_1}{m_0}\right|_{v=H_0^{-1}} \sim \left(\frac{M_{\rm Pl}}{M_{\rm BH}}\right)^{2/3} \ll 1, \tag{26}$$

where the black-hole mass $M_{\rm BH}$ is assumed to be much larger than the Planck mass $M_{\rm Pl}$ and we have set $\alpha = O(1)$. This result says that the accretion of ghost condensate is negligible even if it is integrated over the age of the universe.

III. ACCRETION WITH $P' \neq 0$

We have obtained a negligible accretion rate, assuming that $P' = 0$ in the lowest order in the ϵ-expansion. This assumption is natural since the expansion of the universe makes P' extremely small, $P' \propto a^{-3} \to 0$ $(a \to \infty)$.

On the other hand, it is also interesting to consider nonzero P' by its own since the energy density associated with homogeneous, nonvanishing P' behaves exactly like dark matter. The linear perturbation on top of the homogeneous background also behaves like dark matter, but its nonlinear behavior remains to be seen [12]. In this section we analyze how nonzero P' changes the accretion of the ghost condensate to a black hole.

Technically speaking, what we shall do in this section is the analysis of spherically symmetric, time-dependent perturbation of the Schwarzschild solution (12) with (15). Throughout this section we set $\alpha = 0$ since we have already seen in the previous section that the accretion due to nonzero α is negligibly small. We consider the spherically symmetric, time-dependent ansatz (17) with (18) and consider π, m_1, and δ_1 as first-order quantities.

The perturbed Einstein equation is

$$\begin{aligned} &\partial_r m_1 - 2m_0 F'(r)\partial_t m_1 = 0, \qquad \partial_r \delta_1 = -\frac{1 + 2m_0/r}{r(1 - 2m_0/r)}\partial_r m_1, \\ &\sqrt{\frac{2m_0}{r}}[\partial_r \pi - 2m_0 F'(r)\partial_t \pi] = \frac{M^2\delta_1}{1 - 2m_0/r} + \frac{M^2(1 + 2m_0/r)}{r(1 - 2m_0/r)^2}m_1 - \frac{(1 - 2m_0/r)M_{Pl}^2}{2P_0''M^6r^2}\partial_r m_1, \end{aligned} \tag{27}$$

where $F(r)$ is defined by (21). The solution to the first equation is

$$m_1 = \tilde{m}_1(x_+), \tag{28}$$

where $\tilde{m}_1$ is an arbitrary function and x_+ is defined by (23).

What we would like to know is the asymptotic behavior of $m_1 = \tilde{m}_1(x_+)$ for large v on the black-hole horizon, where v is the advanced time. For this purpose we just have to specify a boundary condition at large r with initial (finite) t since $x_+ \sim v/2m_0$ on the horizon and $x_+ \sim (2/3)\cdot(r/2m_0)^{3/2}$ for large r with finite t. For this purpose we give a formula relating the perturbation X_1 of X around M^4 to $\partial_t m_1$:

$$\begin{aligned} X_1 &= 2M^2\left\{-\sqrt{\frac{2m_0}{r}}[\partial_r\pi - 2m_0F'(r)\partial_t\pi] + \frac{M^2\delta_1}{1 - 2m_0/r} + \frac{M^2(1 + 2m_0/r)}{1 - 2m_0/r}m_1\right\}, \\ &= \frac{(1 - 2m_0/r)M_{Pl}^2}{P_0''M^4r^2}\partial_r m_1, \\ &= \frac{M_{Pl}^2}{\sqrt{2m_0}P_0''M^4}\frac{\partial_t m_1}{r^{3/2}}. \end{aligned} \tag{29}$$

We have used the last equation in (27) to obtain the second line and used the first equation in (27) to obtain the last line. This formula can be rewritten as a relation between $\tilde{m}_1'(x_+)$ and the energy density of π excitation ρ_π:

$$\rho_\pi = 2M^4P_0''X_1 = \frac{2M_{\rm Pl}^2}{\sqrt{2m_0}r^{3/2}}\partial_t m_1 = \frac{2M_{\rm Pl}^2}{\sqrt{2m_0}r^{3/2}}\frac{\tilde{m}_1'(x_+)}{2m_0}, \tag{30}$$

or

$$\frac{\tilde{m}_1'(x_+)}{2m_0} = \frac{\sqrt{2m_0}r^{3/2}}{2M_{\rm Pl}^2}\rho_\pi. \tag{31}$$

Now let us estimate the right-hand side (r.h.s.) of (31) at large r with initial (finite) t. This gives the asymptotic behavior of $\tilde{m}_1'(x_+)$ for large x_+ as

$$\frac{\tilde{m}_1'(x_+)}{2m_0} = \frac{3m_0^2x_+}{M_{\rm Pl}^2}\rho_{\pi\infty} \quad \text{for } x_+ \gg 1, \tag{32}$$

where $\rho_{\pi\infty}$ is the energy density of π excitation at large r with initial (finite) t. The left-hand side of this equation is actually equal to $\partial_v m_1$ on the black-hole horizon because of (28). Thus,

$$\left.\frac{\partial_v m_1}{m_0}\right|_{r=2m_0} = \frac{3\rho_{\pi\infty}v}{2M_{\rm Pl}^2} \quad \text{for } v \gg 2m_0, \tag{33}$$

and integration with respect to v gives

$$\left.\frac{m_1}{m_0}\right|_{r=2m_0} \simeq \frac{3\rho_{\pi\infty}v^2}{4M_{\rm Pl}^2} = \frac{9}{4}\frac{\rho_{\pi\infty}}{\rho_{\rm tot\infty}}(H_0v)^2 \quad \text{for } v \gg 2m_0, \tag{34}$$

where $\rho_{\rm tot\infty} = 3M_{\rm Pl}^2H_0^2$ is the total energy density at large distance and H_0 is the present Hubble expansion rate. This formula agrees with (7) except for the $O(1)$-factor $9/4$, provided that the advanced time v is replaced by the fiducial cosmic time T.

IV. DISCUSSIONS

A tachyon is considered to be sick in the context of particle mechanics, but in field theory just indicates instability of a background. We have considered a similar possibility called ghost condensation [5] that a ghost field might be just an indication of instability of a background and that it can condense to form a different background around which there is no ghost.

We have considered the question "what happens to the ghost condensate near a black hole?" We have argued that the ghost condensate in this picture approximately corresponds to a congruence of geodesics. In other words, the ghost condensate accretes into a black hole just like a pressureless dust. Correspondingly, if the energy density of the ghost condensate at large distance is set to an extremely small value by cosmic expansion then the late-time accretion rate of the ghost condensate should be negligible. The accretion rate remains very small even if effects of higher derivative terms are taken into account, provided that the black hole is sufficiently large. This has been explicitly confirmed by a detailed calculation based on the perturbative expansion with respect to a higher derivative term. The essential reason for the smallness of the accretion rate due to the higher derivative term is the same as that for the smallness of the tidal force acted on an extended object freely falling into a large black hole. We have also given an estimate of the mass increase of a black hole in the case that the ghost condensate is dark matter and have shown that the accretion is still slow.

In Ref. [18] A. Frolov previously argued that the accretion rate is huge, contrary to our result. One of the reasons for the difference is that, while we have consistently taken into account gravitational backreaction in the present paper, he neglected gravitational backreaction. In Ref. [18], by using solutions of the equation of motion for the scalar field in a fixed geometry, a part of the stress-energy tensor is calculated to give the accretion rate via a part of the Einstein equation. However, this treatment neglects the remaining components of the Einstein equation, which could completely change both the geometry and the behavior of the scalar field. The large accretion rate is due to large P', but P' near the black hole should decrease and approach zero since the energy density, which is of order M^4P', decreases due to the accretion. Therefore, the accretion should slow down because of shortage of energy and the steady-state accretion claimed in Ref. [18] cannot be established.

It is also interesting to notice that different scalar fields can behave very differently near a black hole. We have found that the ghost condensate near $P' = 0$, i.e. within the validity of the low energy effective field theory, accretes into a black hole just like a pressureless dust. On the other hand, Frolov and Kofman [23] showed that a rolling (usual) scalar field behaves like radiation near a black hole. It seems interesting to classify the behaviors of different kinds of scalar fields near a black hole and understand their behavior in more systematic way.

We have included the homogeneous component of the energy density (in other words, cosmological energy density) of π in the formula (33). For a black hole in a galaxy, one might think that $\rho_{\pi\infty}$ should be replaced by some fraction of the energy density of dark halo if the ghost condensate behaves as dark matter in galaxies. In this case the mass increase in the unit of the initial mass would become order unity within the galaxy dynamical time. However, because of the following reason, we expect that accretion should be slower. What makes the local density of π in a galaxy higher than the cosmological value should be nonlinear dynamics. In Ref. [12] it is argued that caustics should form within the Kepler time[1] and that the ghost condensate should be described by a patchwork of regular solutions. The size of each patchwork domain can be much smaller than the size of the galaxy, depending on the dynamics. This means that the ghost condensate averaged over galactic scales should have effective rotation, i.e. angular momentum, around a black hole located in the galaxy unless the initial condition is extremely fine-tuned. (Without the patchwork, ϕ should be regular everywhere and there would be no rotation: $\partial_{[\mu}\partial_{\nu]}\phi = 0$.) The fine-tuning required to make the effective rotation vanish is expected to be very severe in the case of a tiny ratio of the black hole size to the dark halo size. With the effective rotation, it is not easy for the ghost condensate to fall into a black hole straightforwardly. Thus, even if the ghost condensate contributes to the dark halo significantly, we expect that the dark halo component cannot accrete to a black hole efficiently. On the other hand, the cosmological energy density of the excitation of the ghost condensate can smoothly fall into a black hole because of the absence of rotation. Therefore, the accretion rate for a black hole in a galaxy should be between the one given in (33) and the one which would be obtained by setting $\rho_{\pi\infty}$ to the energy density of dark matter in the galaxy. In other words, for a black hole in a galaxy, the mass increase in the unit of the initial mass should become order unity in a time scale between the galaxy dynamical time and the age of the universe. It is worthwhile to analyze this issue in more detail.

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APPENDIX: GAUSSIAN NORMAL COORDINATE

We can find a Gaussian normal coordinate system motivated by the analysis in Sec. II. First, let us consider ϕ_+ and x_+ as time and space coordinates. We can calculate metric components as

[1] See Refs. [24,25] for discussions about caustics for a different system, a rolling tachyon.

$$\partial^\mu \phi_+ \partial_\mu \phi_+ = -M^4, \qquad \partial^\mu x_+ \partial_\mu x_+ = \frac{r}{(2m_0)^3}, \qquad \partial^\mu \phi_+ \partial_\mu x_+ = 0. \tag{A1}$$

Thus, the Schwarzschild metric is expressed as

$$ds^2 = -\frac{d\phi_+^2}{M^4} + \frac{(2m_0)^3}{r(\phi_+, x_+)} dx_+^2 + r^2(\phi_+, x_+) d\Omega^2, \tag{A2}$$

where

$$r(\phi_+, x_+) \equiv 2m_0 \left[\frac{3}{2}\left(x_+ - \frac{\phi_+}{2m_0 M^2}\right)\right]^{2/3}. \tag{A3}$$

This coordinate system is nice in the sense that it covers everywhere in the region $v > -\infty$ (namely, the relevant half of the Kruskal extension) including the inside of the future (black-hole) horizon. However, it is not manifest how to deform this metric continuously to the flat metric.

Hence, let us do one more coordinate transformation $(\phi_+, x_+) \to (\tau, \rho)$, where

$$\tau \equiv \frac{\phi_+}{M^2}, \qquad \rho \equiv 2m_0 \left(\frac{3}{2} x_+\right)^{2/3}, \qquad \tau < \tau_{\max}(\rho) \equiv \frac{4m_0}{3}\left(\frac{\rho}{2m_0}\right)^{3/2}. \tag{A4}$$

In this new coordinate system, the Schwarzschild solution is

$$ds^2 = -d\tau^2 + \frac{d\rho^2}{a(\tau, \rho)} + \rho^2 a^2(\tau, \rho) d\Omega^2, \qquad \phi = M^2 \tau, \tag{A5}$$

where

$$a(\tau, \rho) = \left[1 - \frac{3\tau}{4m_0}\left(\frac{2m_0}{\rho}\right)^{3/2}\right]^{2/3}. \tag{A6}$$

This coordinate choice is an analog of the unitary gauge in flat spacetime. Actually, the metric becomes the flat metric in the $m_0 \to 0$ limit. The unbroken shift symmetry is

$$\phi \to \phi + M^2 \tau_0, \qquad \tau \to \tau + \tau_0, \qquad \left(\frac{\rho}{2m_0}\right)^{3/2} \to \left(\frac{\rho}{2m_0}\right)^{3/2} + \frac{3\tau_0}{4m_0}. \tag{A7}$$

There is nothing bad on the future (black-hole) horizon and the coordinate system covers everywhere in the region $v > -\infty$ (the shaded region in Figs. 1 and 2). The metric becomes ill only on the curvature (physical) singularity at $\rho a = 0$. As a consistency check it is easy to calculate the Ricci tensor $R_{\mu\nu}$ and the Misner-Sharp energy $M_{\rm MS}$ for this metric as

$$R_{\mu\nu} = 0, \qquad M_{\rm MS} = \frac{\rho a}{2}[1 - \partial^\mu(\rho a)\partial_\mu(\rho a)] = m_0. \tag{A8}$$

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