Anthropic predictions for neutrino masses

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It is argued that small values of the neutrino masses may be due to anthropic selection effects. If this is the case, then the combined mass of the three neutrino species is expected to be $\sim$1 eV, neutrinos causing a non-negligible suppression of galaxy formation.

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I. INTRODUCTION

The major ingredients of the Universe are dark energy, $\Omega_\Lambda \sim 0.7$, and nonrelativistic matter, $\Omega_m \sim 0.3$. The latter consists of nonbaryonic dark matter, $\Omega_D \sim 0.25$, baryons, $\Omega_B \sim 0.05$, and massive neutrinos, $\Omega_\nu \gtrsim 0.001$. The fact that Ω_Λ is comparable to Ω_m is deeply puzzling; this is the notorious coincidence problem that has been much discussed in the recent literature. The only plausible explanation that has so far been suggested is that Ω_Λ is a stochastic variable and that the coincidence is due to anthropic selection effects. Anthropic bounds on the cosmological constant derived in [1–4] were followed by anthropic predictions [5–8] suggesting values not far from the presently observed dark energy density. Although controversial, such anthropic arguments have been bolstered by the discovery of mechanisms that may be capable of creating ensembles with different parameter values in the context of both cosmic inflation [9–11] and string theory [12–15], and have been applied to other physical parameters as well [16–33].

Perhaps equally puzzling are the "coincidences" $\Omega_D \sim \Omega_B$ and $\Omega_B \sim \Omega_\nu$. These three matter components are relics of apparently unrelated processes in the early Universe, and it is very surprising that their mass densities are comparable to one another. The mass density of neutrinos is $\Omega_\nu = (m_\nu/94\ \text{eV})h^{-2}$, where m_ν is the combined mass of all three neutrino flavors. In this paper, we will investigate the possibility that m_ν is a stochastic variable taking different values in different parts of the Universe and that the observed value is anthropically selected.

Before delving into details, let us briefly outline the argument. A small increase of m_ν can have a large effect on galaxy formation. Neutrinos stream out of the potential wells created by cold dark matter and baryons, slowing the growth of density fluctuations. As a result, there will be fewer galaxies (and therefore fewer observers) in regions with larger values of m_ν. If the suppression of galaxy formation becomes important for $m_\nu \gtrsim \tilde{m}_\nu$, say, then values $m_\nu \gg \tilde{m}_\nu$ will be rarely observed because the density of galaxies in the corresponding regions is very low. Moreover, unless the underlying particle-physics model strongly skews the neutrino mass distribution towards values near zero, values $m_\nu \ll \tilde{m}_\nu$ are also unlikely to be observed, simply because the corresponding range of m_ν-values is very small. A typical observer thus expects to find $m_\nu \sim \tilde{m}_\nu$, i.e., a mild but non-negligible suppression of galaxy formation by neutrinos.

II. PROBABILITY DISTRIBUTION FOR m_ν

To make the analysis quantitative, we define the probability distribution $\mathcal{P}(m_\nu)dm_\nu$ as being proportional to the number of observers in the Universe who will measure m_ν in the interval dm_ν. This distribution can be represented as a product [5]

$$\mathcal{P}(m_\nu) = \mathcal{P}_*(m_\nu)n_{obs}(m_\nu). \tag{1}$$

Here, $\mathcal{P}_*(m_\nu)dm_\nu$ is the prior distribution, which is proportional to the comoving volume of those parts of the Universe where m_ν takes values in the interval dm_ν, and $n_{obs}(m_\nu)$ is the number of observers that evolve per unit comoving volume with a given value of m_ν. The distribution (1) gives the probability that a randomly selected observer is located in a region where the sum of the three neutrino masses is in the interval dm_ν.

Of course we have no idea how to estimate n_{obs}, but what comes to the rescue is the fact that the value of m_ν does not directly affect the physics and chemistry of life. As a rough approximation, we therefore assume that $n_{obs}(m_\nu)$ is simply proportional to the fraction of all baryons that form stars, which we approximate by the fraction $F_M(m_\nu)$ of all matter that collapses into galaxy-scale haloes (with mass greater than $M = 10^{12}M_\odot$),

$$n_{obs}(m_\nu) \propto F_M(m_\nu). \tag{2}$$

The idea is that there is some average number of stars per unit mass in a galaxy and some average number of observers per star. The choice of the halo mass scale is based on the empirical fact that most stars are observed to be in large halos.

The prior distribution $\mathcal{P}_*(m_\nu)$ depends on the extension of the particle-physics model which allows neutrino masses to vary and perhaps on stochastic processes during inflation which randomize these variable masses. Some candidate prior distributions will be discussed in Sec. III.

The fraction of collapsed matter $F_M(m_\nu)$ can be approximated using the standard Press-Schechter formalism [34]. We assume a Gaussian density fluctuation field $\delta(\mathbf{x}, t)$ with a variance $\sigma(M, t)$ on the galactic scale ($M = 10^{12} M_\odot$),

$$P(\delta, t) \propto \exp\left[-\frac{\delta^2}{2\sigma(t)^2}\right]. \tag{3}$$

A collapsed halo is assumed to form when the linearized density contrast δ exceeds a critical value δ_c determined by the spherical collapse model. As detailed in Appendix A, this corresponds to $\delta_c \approx 1.69$ around the present epoch and $\delta_c \approx 1.63$ in the infinite future [4]. Using the Press-Schechter approximation, we obtain

$$F_M(t) \propto P[\delta > \delta_c] = \int_{\delta_c}^{\infty} P(\delta, t) d\delta = \mathrm{erfc}\left[\frac{\delta_c}{\sqrt{2}\sigma_M(t)}\right]. \tag{4}$$

The collapsed fraction F_M thus grows over time as the rms density fluctuations σ increase.

Let us now quantify the effect of neutrino masses on this process. For the small scale M that we are considering, assuming a flat Universe, this fluctuation growth is well approximated by

$$\sigma_M(x) \approx \left[1 + \frac{3}{2} A_\Lambda(f_\nu) G_\Lambda(x)\right]^{p(f_\nu)} \sigma_M(0), \tag{5}$$

as shown in Appendix A. The functions A_Λ, G_Λ and p are defined below. Here we have replaced t by a new time variable

$$x \equiv \frac{\rho_\Lambda}{\rho_\mathrm{m}} = \frac{\Omega_\Lambda}{(1+z)^3 \Omega_\mathrm{m}} = \frac{1 - \Omega_\mathrm{m}}{\Omega_\mathrm{m}} (1+z)^{-3}, \tag{6}$$

i.e., the dark-energy-to-matter density ratio—we will consider several values of x below, corresponding to the infinite future ($x = \infty$), the present epoch ($x = 7/3$, our default value) and redshift unity ($x = 7/24$). The function

$$G_\Lambda(x) \approx x^{1/3}\left[1 + \left(\frac{x}{G_\infty^3}\right)^\alpha\right]^{-1/3\alpha}, \tag{7}$$

where $\alpha = 0.795$ and

$$G_\infty \equiv \frac{5\Gamma(\frac{2}{3})\Gamma(\frac{5}{6})}{3\sqrt{\pi}} \approx 1.437\,28, \tag{8}$$

describes how, in the absence of massive neutrinos, fluctuations grow as the cosmic scale factor a as long as dark energy is negligible [$G_\Lambda(x) \approx x^{1/3} \propto a \propto (1+z)^{-1}$ for $x \ll 1$] and then asymptote to a constant value as $t \to \infty$ and dark energy dominates [$G_\Lambda(x) \to G_\infty$ as $x \to \infty$].

We are considering the case where m_ν varies from place to place whereas the physics that determined the amount of baryons and cold dark matter per photon is the same everywhere, so the neutrino fraction f_ν is given by

$$f_\nu \equiv \frac{\Omega_\nu}{\Omega_\mathrm{m}} = \frac{\rho_\nu}{\rho_\mathrm{bc} + \rho_\nu} = \left[1 + \frac{\rho_\mathrm{bc}}{\rho_\nu}\right]^{-1} = \left[1 + \frac{m_\mathrm{bc}}{m_\nu/3}\right]^{-1}, \tag{9}$$

where ρ_bc denotes the non-neutrino density, i.e., that of baryons and cold dark matter, and $m_\mathrm{bc} \approx (4.75 \pm 0.30)$ eV gives the measured amount of such matter per neutrino. In other words, increasing the neutrino mass from zero will increase the total matter density per photon by a factor $\rho_\mathrm{m}/\rho_\mathrm{bc} = (1 - f_\nu)^{-1}$:

$$A_\Lambda(f_\nu) \equiv x_\mathrm{eq}^{-1/3} = \left(\frac{\Omega_\mathrm{m}}{\Omega_\Lambda}\right)^{1/3} (1 + z_\mathrm{eq}) \tag{10}$$

is the factor by which the Universe has expanded between matter-radiation equality at $x = x_\mathrm{eq}$ (when fluctuations effectively start to grow) and dark energy domination at $x = 1$ (when fluctuations gradually stop growing). Since massive neutrinos boost the matter density by a factor $(1 - f_\nu)^{-1}$, they delay vacuum domination until the scale factor is larger by a factor $(1 + f_\nu)^{1/3}$ and also, in the approximation that neutrinos are nonrelativistic at the matter-radiation equality epoch (valid for $m_\nu \gtrsim 1$ eV), cause matter-radiation equality to occur earlier, when the scale factor is smaller by a factor $(1 + f_\nu)$. We thus have

$$A_\Lambda(f_\nu) = (1 - f_\nu)^{-4/3} A_\Lambda(0). \tag{11}$$

Finally, neutrinos with nonzero mass suppress the galaxy density through the exponent $p(f_\nu)$ in Eq. (5), which is given by [35]

$$p(f_\nu) = \frac{\sqrt{25 - 24 f_\nu} - 1}{4} \approx (1 - f_\nu)^{3/5}, \tag{12}$$

and drops from unity for $f_\nu = 0$ to smaller values as f_ν increases.

In summary, Eq. (5) shows that the galaxy fluctuation evolution $\sigma_M(x)$ depends on the three cosmological parameters $A_\Lambda(0)$, $\sigma_M(0)$ and f_ν. To study the galaxy density as a function of neutrino fraction f_ν using Eq. (4), we thus need to measure $A_\Lambda(0)$ and $\sigma_M(0)$ from observational data without making any assumptions about f_ν. We cannot do this using the values of Ω_m and z_eq reported by, say, the WMAP team [36], since these assume that $f_\nu = 0$ in our part of the Universe; if $f_\nu > 0$ here, then matter-radiation equality occurred earlier. We therefore repeat the Monte Carlo Markov Chain (MCMC) analysis reported in column 5 of Table 3 in [37], measuring $A_\Lambda(0)$ and $\sigma_M(0)$ from the WMAP cosmic microwave background (CMB) power spectrum [38] combined with the Sloan Digital Sky Survey galaxy power spectrum [39]. These measurements are independent of f_ν since this is a free parameter in the analysis and therefore effectively marginalized over. This gives $A_\Lambda(0) = 3057 \pm 502$, $\sigma_M(0) \approx 0.000\,579 \pm 0.000\,064$.[1] The above-mentioned m_bc-value was measured using this same MCMC analysis. We will use the central

[1]As our galactic scale, we take $M = 10^{12} M_\odot$, specifically a top-hat smoothing scale of $R = 1.3 h^{-1}$ Mpc. This corresponds to length scales about 100 times smaller than the matter-radiation equality scale where the matter power spectrum turns over.

values for our main analysis and quantify the effect of the uncertainties in the discussion section. Equation (5) thus shows that, for $f_\nu = 0$, fluctuations grow by a factor $1 + 1.5A_\Lambda(0)G_\Lambda(x) \approx 4700$ by the present epoch, which we take to be $x = \Omega_\Lambda/\Omega_{\rm m} \approx 0.7/0.3 \approx 2.3$, giving $\sigma_M \approx 2.7$. In the infinite future $x \to \infty$, fluctuations will have grown by a factor 6600, giving $\sigma_M \approx 3.8$. The basic reason that neutrinos have such a dramatic effect is that these growth factors are so large, implying that even a modest change in the exponent $p(f_\nu)$ makes a large difference. Taylor expanding Eq. (5) in f_ν gives

$$\sigma_M(x, f_\nu) \approx \sigma_M(x, 0)e^{-\gamma(x)f_\nu} \tag{13}$$

for $f_\nu \ll 1$, where $\gamma(x) \equiv 0.6\ln[1 + 1.5A_\Lambda(0)G_\Lambda(x)] - 4/3 \approx 3.7$ for the present epoch and $\gamma \approx 3.9$ for the infinite future. Although Eq. (13) is quite a crude approximation, underestimating the suppression, it shows that small changes in A_Λ or x are unimportant since they affect this exponential fluctuation suppression only logarithmically.

The effect of neutrino free streaming on the galactic density is illustrated in Fig. 1 (top), which shows that the suppression is non-negligible already for $m_\nu \sim 1$ eV. We use Eq. (5) in our calculations for the plots—the approximation Eq. (13) was merely to provide qualitative intuition for the effect.

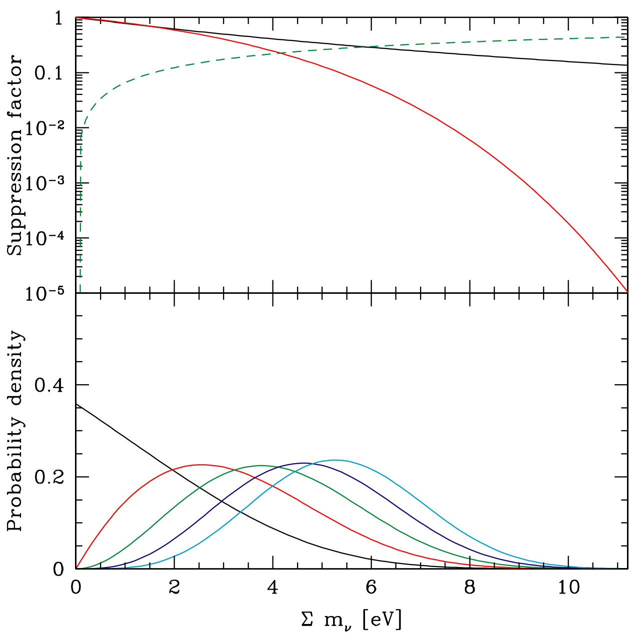

FIG. 1 (color online). The upper panel shows the factor by which the neutrino fraction f_ν (dashed curve) suppresses the current fluctuation amplitude σ_M (upper solid curve) and consequently the galaxy number density n_G (lower solid curve). The lower panel shows the resulting probability distribution for the neutrino mass sum for priors m_ν^n with $n = 0$, 1, 2, 3 and 4, peaking from left to right, respectively.

The probability distribution $\mathcal{P}(m_\nu)$ is shown in Fig. 1 (bottom) for power law priors

$$\mathcal{P}_*(m_\nu) \propto m_\nu^n, \tag{14}$$

with n ranging from 0 to 4. For $n \geq 1$, these distributions are peaked at $m_\nu \gtrsim 2$ eV, while in the case of a flat prior, $\mathcal{P}_*(m_\nu) = \text{const}$, the expected values are $m_\nu \sim 1$ eV. This is also seen in Fig. 2, where the distribution for a flat prior is shown using a logarithmic scale for m_ν.

In this discussion, we have assumed that $f_\nu \ll 1$, that is, $m_\nu \ll 10$ eV. Very heavy neutrinos (with $m_\nu \gg 1$ MeV) would annihilate well before nucleosynthesis and cause no problems with structure formation. If *all* neutrinos were heavy, neutrons would be stable, leading to an equal number of protons and neutrons. As a result, most of the matter would end up in helium instead of hydrogen. This lack of hydrogen would clearly suppress $n_{obs}(m_\nu)$ for observers like us who rely on long-lived (hydrogen burning) stars and water-based chemistry. Moreover, heavy neutrinos would not be able to blow off the envelope in supernova explosions. This means that heavy elements formed in stellar interiors would not be dispersed to form planets and observers. The possibility of the electron neutrino being light and one or two others very heavy is allowed anthropically, but it is already ruled out by the neutrino oscillation experiments, which constrain the mass differences to be within 0.05 eV.

For $m_\nu \gg 100$ eV (but $\ll 1$ MeV), keeping all other physical parameters fixed, neutrinos would have sufficiently low thermal velocities to act approximately as cold dark matter, thereby allowing galaxy-size halos to form. However, the baryon fraction in these halos would be strongly diluted, and it is therefore far from clear whether they would be able to cool and form self-

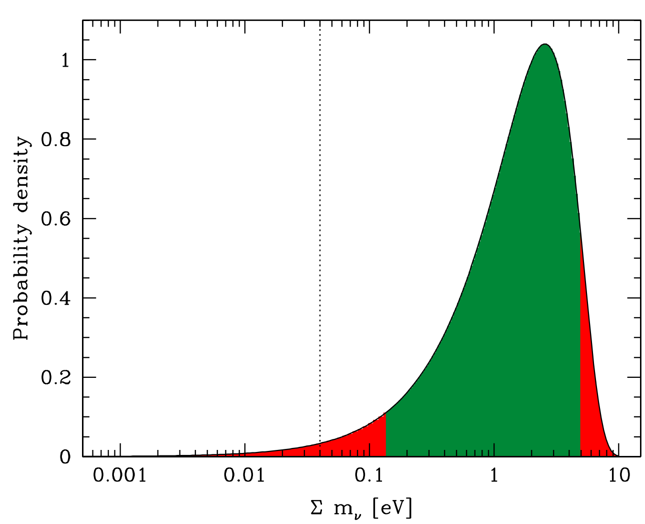

FIG. 2 (color online). The probability distribution for the neutrino mass sum for flat prior ($n = 0$). The dark/red tails contain 5% probability each. The dotted line shows the lower limit 0.05 eV from atmospheric neutrino oscillations [49–52].

gravitating baryonic disks, let alone stars or observers, with an efficiency comparable to that in our observable universe. In other words, the calculation of anthropic constraints on very large neutrino masses becomes essentially equivalent to the calculation of an anthropic upper bound on the dark matter abundance. We will not attempt to address this issue here, but simply assume that the number of observers $n_{obs}(m_\nu)$ is strongly suppressed for $m_\nu \gg 10$ eV.

We have also assumed that there are $N_\nu = 3$ stable neutrinos. Generalizing our result to $N_\nu > 3$ is straightforward: as long as the masses are low enough for neutrino infall to be negligible, the galaxy number density depends only on the total neutrino mass density, which for standard neutrino freeze-out is proportional to the sum of the neutrino masses. If the neutrinos are unstable on cosmological time scales, they suppress fluctuation growth only before decaying, with their decay radiation redshifting away to gravitationally negligible levels within a few expansion times.

III. PRIOR DISTRIBUTION

Following [40], we shall now discuss possible modifications of the standard particle-physics model that could make neutrino masses variable. For early work on how masses of elementary particles can vary randomly in the context of stochastic gauge theories, see [41–43].

Dirac-type neutrino masses can be generated if the standard model neutrinos ν^α mix through the Higgs doublet vacuum expectation value Φ to some gauge-singlet fermions ν_c^β,

$$g_{\alpha\beta}\Phi\bar{\nu}^\alpha\nu_c^\beta. \tag{15}$$

The couplings $g_{\alpha\beta}$ will generally be variable in string theory inspired models involving antisymmetric form fields F_a interacting with branes. (Here, the index a labels different form fields.) F_a changes its value by $\Delta F_a = q_a$ across a brane, where q_a is the brane charge. In the low-energy effective theory, the Yukawa couplings $g_{\alpha\beta}$ become functions of the form fields,

$$g_{\alpha\beta} = g_{\alpha\beta}^{(0)} + \sum g_{\alpha\beta}^a \frac{F_a}{M_p^2} + \cdots, \tag{16}$$

where the summation is over all form fields, the coefficients $g_{\alpha\beta}^{(0)}$, $g_{\alpha\beta}^a$ are assumed to be numbers ~ 1, and M_p is the effective cutoff scale, which we assume to be the Planck mass.

In such models, closed brane bubbles nucleate and expand during inflation [44], creating exponentially large regions with different values of the neutrino masses. When F_a changes in increments of q_a, m_ν changes in increments of

$$\Delta m_\nu \sim \Phi q_a/M_p^2. \tag{17}$$

To be able to account for neutrino masses $\lesssim 1$ eV, we have to require that $\Delta m_\nu \lesssim 1$ eV, that is,

$$q_a \lesssim 10^{-11}M_p^2, \tag{18}$$

for at least some of the brane charges. Such small values of the charges can be achieved using the mechanisms discussed in [12,13,45,46].

It should be noted that the Higgs potential and the Higgs expectation value Φ will also generally depend on F_a. Moreover, each field F_a contributes a term $F_a^2/2$ to the vacuum energy density ρ_Λ, and regions with different values of F_a will generally have different values of ρ_Λ. However, in the presence of several form fields with sufficiently small charges, variations of all these parameters are not necessarily correlated, and here we shall assume that there are enough form fields to allow independent variation of the relevant parameters. We can then consider a subensemble of regions with m_ν variable and all other parameters fixed. The probability distribution for m_ν that we calculated in Sec. II corresponds to such a subensemble.

Let us now turn to the prior distribution $\mathcal{P}_*(m_\nu)$. The natural range of variation of F_a in Eq. (16) is the Planck scale, and the corresponding range of the neutrino masses is $0 \le m_\nu^{(i)} \lesssim \Phi$. (Here, the index i labels the three neutrino mass matrix eigenvalues.) Only a small fraction of this range corresponds to values of anthropic interest, $m_\nu \lesssim 10$ eV. In this narrow anthropic range, we expect that the probability distribution for F_a after inflation is nearly flat [47],[2]

$$d\mathcal{P}_* = \text{const} \cdot dF_1 dF_2 \ldots, \tag{19}$$

and that the functions $g_{\alpha\beta}(F_a)$ are well approximated by linear functions. If all three neutrino masses vary independently, this implies that

$$d\mathcal{P}_* = \text{const} \cdot dm_\nu^{(e)} dm_\nu^{(\mu)} dm_\nu^{(\tau)}. \tag{20}$$

The probability for the combined mass $m_\nu = \sum m_\nu^{(i)}$ to be between m_ν and $m_\nu + dm_\nu$ is then proportional to the volume of the triangular slab of thickness $\sim dm_\nu$ in the 3-dimensional mass space,

$$d\mathcal{P}_* \propto m_\nu^2 dm_\nu. \tag{21}$$

Alternatively, the neutrino masses can be related to one another, for example, by a spontaneously broken family symmetry. If all three masses are proportional to a single

[2]A very different model for the prior distribution was considered by Rubakov and Shaposhnikov [48]. They assumed that $\mathcal{P}_{prior}(X)$ is a sharply peaked function with a peak outside the anthropic range $\mathcal{A}$ and argued that the observed value of X should then be very close to the boundary of $\mathcal{A}$. We note that this is unlikely to be the case for the neutrino mass, since it is observed to be comfortably inside the anthropically allowed range. If the model of [48] applied, the peak of the full distribution would most likely be in a life-hostile environment, where both $\mathcal{P}_{prior}(X)$ and $n_{obs}(X)$ are very small. In the case of the neutrino mass, this would mean that the number density of galaxies is very low. This is not the case in our observable region, indicating that the model of [48] does not apply.

variable mass parameter, then we expect

$$d\mathcal{P}_* \propto dm_\nu. \tag{22}$$

Let us now assess how well the predictions derived from the prior distributions (21) and (22) agree with observations. We first consider the distribution (21), corresponding to independently varying neutrino masses. The most probable value of m_ν for $n = 2$ is $m_\nu \sim 3$ eV, and we expect both the neutrino masses and mass differences to be $\sim$1 eV. This expectation, however, is in conflict both with neutrino oscillation experiments suggesting $\Delta m_\nu \lesssim$ 0.05 eV [49–52] and with astrophysical bounds [36,37] which indicate a combined mass $m_\nu \lesssim 1$ eV.

For a flat prior distribution (22), the most probable value is $m_\nu \sim 1$ eV. If m_ν is close to this value, then the three neutrino masses must be nearly degenerate, with $\Delta m_\nu \ll m_\nu$. This could be interpreted as a sign of a family symmetry. A 90% confidence level prediction for m_ν based on this distribution can be obtained as outlined in Sec. II. This gives

$$0.1 \text{ eV} < m_\nu < 5 \text{ eV}. \tag{23}$$

The lower bound in (23) is somewhat stronger than the bound from the neutrino oscillation data [49–52] ($m_\nu \gtrsim$ 0.05 eV), while the upper bound is somewhat weaker than the current astrophysical bounds (e.g., [36,37,53–56]). Note that the strength of current astrophysical bounds is limited not by statistical errors but by systematic uncertainties in non-CMB data. For instance, the recently claimed evidence for $m_\nu > 0$ [57] may result from underestimated galaxy cluster modeling uncertainties.

We finally mention the possibility that the right-handed neutrinos ν_c^α have a large Majorana mass $M_R \gg \Phi$. In this case, small neutrino masses can be generated through the seesaw mechanism,

$$m_\nu \sim g^2\Phi^2/M_R. \tag{24}$$

If M_R is variable, say, within a range $M_R \lesssim M_p$, then its most probable values are likely to be $\sim M_p$, and the prior distribution will be peaked at very small values of $m_\nu \sim 10^{-6}$ eV.

This discussion suggests that the most promising scenario with variable neutrino masses is the one with Dirac-type masses determined by a single variable mass parameter. It yields a flat prior distribution for m_ν, Eq. (22), and the prediction (23) at 90% confidence level.

After we submitted the original version of this paper, Jaume Garriga pointed out to us that seesaw-type models can yield cosmologically interesting prior distributions for m_ν if the Majorana mass is restricted to the range $M_R < M_R^{(\max)} \ll M_p$. Assuming first that M_R is a fixed constant, while g is variable, Eq. (24) yields the distribution

$$d\mathcal{P}_{prior} \propto dg \propto m^{-1/2}dm. \tag{25}$$

This would give a somewhat smaller predicted neutrino mass than the distribution (22) that we used in most of our calculations.

The distribution (25) applies up to $m_{\max} \sim \Phi^2/M_R$. In order to have $m_{\max} \gtrsim 0.1$ eV, we need $M_R \lesssim 10^{13}$ GeV.

If both M_R and g are variable, then, assuming a flat prior for M_R, Eq. (25) still applies, but now $m_{\max} \sim \Phi^2/M_R^{(\max)}$, so we need $M_R^{(\max)} \lesssim 10^{13}$ GeV. An attractive feature of this scenario is that the increment of m_ν in Eq. (17) gets suppressed by an additional factor Φ/M_R, and Eq. (18) gets replaced by a much weaker constraint $q_a/M_p^2 \lesssim (M_R/10^{13}\text{ GeV})^{-1}$.

IV. DISCUSSION

In conclusion, we have found that the small values of the neutrino masses may be due to anthropic selection. If so, then the most promising model appears to be the one with a flat prior distribution, $\mathcal{P}_*(m_\nu) = \text{const}$. The range of m_ν predicted in this model, Eq. (23), has interesting implications for both particle physics and cosmology. On the particle-physics side, neutrino masses in this range are nearly degenerate, suggesting extensions of the standard model involving a spontaneously broken family symmetry. On the cosmological side, a combined neutrino mass of $\gtrsim$ 1 eV has a non-negligible effect on galaxy formation. This means that it must be taken into account in precision tests of inflation that measure the shape of the primordial power spectrum by combining microwave background and large-scale structure data.

Let us close by discussing the importance of the assumptions we have made and outlining some open problems for future work. The purpose of this brief paper and the prediction of Eq. (23) is merely to demonstrate that anthropic selection effects *may* be able to explain the neutrino masses, and much work needs to be done to place this hypothesis on a firmer footing.

A. Robustness to approximations and measurement errors

To quantify the robustness of our results, Fig. 3 shows how the probability distribution for m_ν changes when various assumptions are altered.

First of all, the parameters A_Λ and $\sigma_M(0)$ that we used have non-negligible measurement uncertainties. We see that lowering $\sigma_M(0)$ by 25% (by about twice its measurement uncertainty) lowers the m_ν-prediction slightly. Changing A_Λ within its observational uncertainty has an even weaker effect since, as we saw, it enters only logarithmically. Altering the galactic scale M affects σ_M and hence the results only weakly, because of the flatness of the dimensionless power spectrum $k^3P(k)$ on galactic scales.

Second, our calculations involved various approximations. We used the Press-Schechter approximation with density threshold $\delta_c \approx 1.69$ as per Appendix A; lowering this to account for postvirialization infall as discussed in

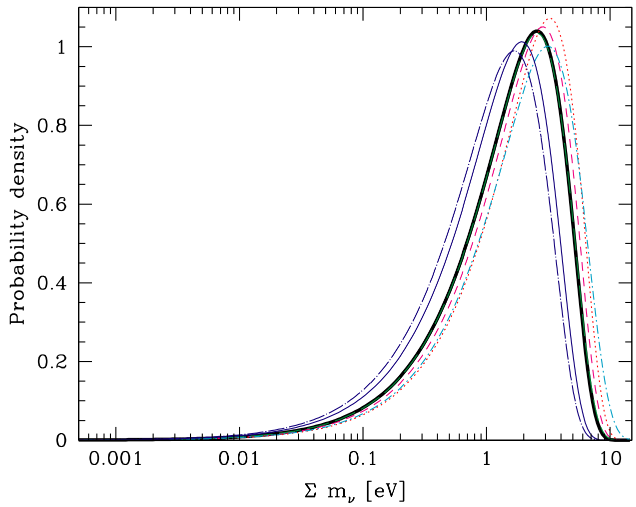

FIG. 3 (color online). Same as Fig. 2, but showing the robustness of the results to changing various assumptions. We have changed the baseline calculation from Fig. 2 (heavy black curve) by evaluating the galaxy density in the infinite future $x \to \infty$ (dotted red/ gray curve) and at redshift unity, $x = 7/3(1 + z)^3 = 7/24$ (dot-long-dashed blue/ gray curve), decreasing the density threshold to $\delta_c = 1.5$ (short-dashed magenta/ gray curve), lowering the primordial fluctuation amplitude on the galactic scale by 25% (solid blue/ gray curve), including the baryon correction as per Eisenstein and Hu (long-dashed green curve) and additionally including the neutrino infall correction as per Eisenstein and Hu (dot-dashed cyan/ gray curve).

[7] is seen to raise the m_ν-prediction slightly. Our fluctuation growth treatment of Eq. (5) is highly accurate in the limit of small mass scales M and low baryon fraction $\Omega_b/\Omega_m \ll 1$, agreeing with a CMBFAST [58] numerical calculation to within a few percent. Figure 3 shows that for the observed baryon fraction $\Omega_b/\Omega_m \approx 0.15$, switching to an exact treatment of baryon effects makes virtually no difference. The cosmic expansion eventually slows neutrinos enough for them to start clustering on galaxy scales, and if this happens before dark energy domination, then it reduces γ [Eq. (13)]. Since a small fraction of the neutrinos will be in the low tail of their velocity distribution, there is a slight infall correction even for the low m_ν-range we have considered, and Fig. 3 shows that this increases our m_ν-prediction slightly. Finally, we have used the cutoff value of $x \approx 7/3$, which amounts to using the reference class of observers in galaxies that have formed by now. Figure 3 shows that if we ask instead what would be observed from a random galaxy among all galaxies that ever form (setting $x = \infty$), then the m_ν-prediction increases slightly. Conversely, it also shows that considering only observers in galaxies that formed by redshift unity decreases the m_ν-prediction. In conclusion, Fig. 3 shows that although many of these assumptions make marginal differences, none of them affect the qualitative conclusions, since they all shift the predicted probability distribution by much less than 1 standard deviation.

B. Effects of other parameters

The standard models of cosmology and particle physics involve of order 10 and 28 free parameters, respectively. In order to apply anthropic constraints to them, it is crucial to know both which of them can vary, and what the interdependencies or correlations between them are. It is likely that at least some of the cosmological parameters (the baryon-to-photon ratio, say, via baryogenesis) are determined by particle-physics parameters in a way that we have yet to understand, and many particle-physics parameters may in turn be determined by a smaller number of parameters or vacuum expectation values of some deeper underlying theory. A proper analysis of anthropic predictions should therefore be done in the multidimensional space spanned by all fundamental variable parameters.

Such correlations between parameters must ultimately be taken into account not only for computing the theoretical prior $\mathcal{P}_*$ of Eq. (1), but also when computing the factor n_{obs} in this equation, which incorporates the observational selection effect. The reason is that strong degeneracies are present which can in many cases offset a detrimental change in one parameter by changes in others. For instance, suppressed galaxy formation caused by increased m_ν can to some extent be compensated by decreasing ρ_Λ, by increasing the dark-matter-to-photon ratio or by increasing the CMB fluctuation amplitude Q above the value $\sim 10^{-5}$ that we observe [24,32]—if any of these three parameters can vary, that is. In the present paper, we have merely considered the simple case where all relevant parameters except m_ν (i.e., the comoving densities of baryons and dark matter, the physical density of dark energy, and the fluctuation amplitude Q) are kept fixed at their observed values, with no account for scatter due to variation across an ensemble or from measurement uncertainties. A more detailed study of this issue is given in [59] and shows that our present results for f_ν are rather robust to assumptions about ρ_Λ.

This is closely related to the issue of how much information one wishes to include in the factor n_{obs} in Eq. (1) [32,60]. One extreme is including only the existence of observers, the other extreme is including all available knowledge (even, say, experimental constraints on m_ν). As one includes more such information, the anthropic factor becomes progressively less important, and the calculation acquires the flavor of a prediction rather than an explanation. In the context of a multiparameter analysis, the question is whether to use the measured values of other parameters (in our case non-neutrino parameters) or marginalize over them. Our fixing non-neutrino parameters at their observed values is therefore equivalent to factoring in the information from the measurements of these parameters.

Arguably the most interesting outstanding question is whether the fundamental equations that govern our Universe do or do not allow physical quantities such as the neutrino masses to vary from place to place. Calculations of anthropic selection effects may prove useful for shedding light on this. In any case, for quantities that do vary, the inclusion of anthropic selection effects such as the one we have evaluated in this paper is clearly not optional when calculating what the theory predicts that we should observe.

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APPENDIX A: GROWTH OF LINEAR PERTURBATIONS

In this appendix, we derive and test the approximation given by Eq. (5) for how small-scale matter fluctuations grow in the presence of radiation, cold dark matter and neutrinos. There are two reasons why this simple approximation complements an exact "black box" calculation with CMBFAST [58] or a nearly exact approximation with the Eisenstein and Hu fitting software [61]. First, for a qualitative argument like the one we make in this paper, it is desirable to have a simple intuitive understanding of the underlying physics that includes only those complications that really matter for the argument. Second, neither CMBFAST nor the Eisenstein and Hu package were designed to be valid for extreme cosmological parameters such as those corresponding to the infinite future, and indeed break down in this limit.

1. The ΛCDM case

For a flat Universe with only pressureless matter (dark and baryonic) and a cosmological constant, the growth of density fluctuations is given by $\delta \propto G_\Lambda(x)$, where [7,62]

$$G_\Lambda(x) = \frac{5}{6}\sqrt{1+\frac{1}{x}}\int_0^x \frac{dy}{y^{1/6}(1+y)^{3/2}}. \tag{A1}$$

We find that our $G_\Lambda(x)$ fit defined by Eq. (7) is accurate to better than 1.5% for all x and becomes exact both in the limits $x \to 0$ (when $G_\Lambda \to x^{1/3}$) and $x \to \infty$ (when $G_\Lambda \to G_\infty$). Figure 4 shows that this approximation greatly improves on the standard Carroll, Press and Turner [63] and power law fits for our present purposes, since these were designed to be accurate only in the past and present and have the wrong limiting behavior in the future as $x \to \infty$, $\Omega_m \to 0$ and $\Omega_\Lambda \to 1$. For flat models, we have $\Omega_m = 1/(1+x)$, $\Omega_\Lambda = x/(1+x)$ and $x = (\Omega_m^{-1} - 1)^{-1}$, so in terms of the standard linear growth factor $D \equiv G_\Lambda/x^{1/3}$, the three approximations shown in Fig. 4 are

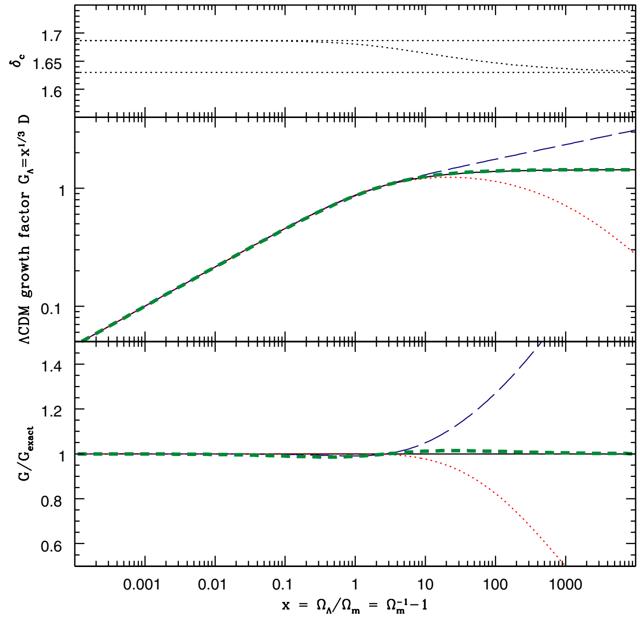

FIG. 4 (color online). The ΛCDM growth function G_Λ is shown as a function of cosmic time, first growing as the scale factor $a \propto x^{1/3}$, then asymptoting to 1.44 as dark energy halts the fluctuation growth at $x \gtrsim 1$. The middle panel shows the exact result of Eq. (A1) (solid curve), our approximation given by Eq. (7) (green thick dashed curve), the Carroll, Press and Turner approximation [63] (red dotted curve) and the power law approximation $G_\Lambda = \Omega_m^{0.21}$ [66] (blue long-dashed curve). In the bottom panel, the various approximations have been divided by the exact result, showing that Eq. (5) is accurate to better than 1.5% for all x. The top panel shows the collapse density threshold $\delta_c(x)$ dropping from 1.6865 early on to 1.629 78 in the infinite future.

$$D \approx \left[1 + \left(\frac{1-\Omega_m}{\Omega_m G_\infty^3}\right)^\gamma\right]^{-1/3\gamma}, \tag{A2}$$

$$D \approx \frac{5}{2}\Omega_m\left[\Omega_m^{4/7} - \Omega_\Lambda + \left(1+\frac{\Omega_m}{2}\right)\left(1+\frac{\Omega_\Lambda}{70}\right)\right]^{-1} = \frac{350\Omega_m}{140\Omega_m^{4/7} + (209-\Omega_m)\Omega_m + 2} \tag{A3}$$

and

$$D \approx \Omega_m^{-0.21}, \tag{A4}$$

respectively.

2. Including radiation

Early on, dark energy was negligible but radiation was gravitationally important, causing density fluctuations to grow as $\delta \propto G_\gamma(x)$, where [64]

$$G_\gamma(x) = 1 + \frac{3}{2}\left(\frac{x}{x_{eq}}\right)^{1/3}. \tag{A5}$$

Since both $G_\gamma(x)$ and $G_\Lambda(x)$ accurately describe the growth during the matter-dominated epoch $x_{\rm eq} \ll x \ll 1$, with $G_\gamma \propto G_\Lambda \propto x^{1/3}$ during this period, we can combine them to obtain the approximation

$$G(x) \approx 1 + \frac{3}{2} A_\Lambda G_\Lambda(x), \tag{A6}$$

which is accurate for all x. Here the constant A_Λ is defined by Eq. (10). In essence, fluctuations grow as $\delta \propto a \propto x^{1/3}$ between matter domination ($x = x_{\rm eq}$) and dark energy domination ($x = 1$), giving a net growth of $A_\Lambda = x_{\rm eq}^{-1/3}$. Equation (A6) shows that they grow by an extra factor of 1.5 by starting slightly before matter domination and by an extra factor $G_\Lambda(\infty) \approx 1.44$ by continuing to grow slightly after dark energy domination.

3. Including neutrinos

As shown by [35], the result $\delta \propto a$ is generalized to $\delta \propto a^p$ when a fraction of the matter is clustering inert and remains spatially uniform. The new exponent $p < 1$ is given by Eq. (12) where, in the case that we are focusing on here, the inert fraction is the neutrino fraction f_ν.[3] This motivates our approximation in Eq. (5), which simply generalizes Eq. (A5) by introducing the neutrino-dependent exponent $p(f_\nu)$.

We have tested this approximation by comparing Eq. (5) with exact results using the CMBFAST software [58] and the semianalytic approximation of Eisenstein and Hu [61], finding excellent agreement (to within a few percent) with both in the small-scale limit for $x \sim 1$ and negligible baryon fraction. In the distant future limit $x \to \infty$, both CMBFAST and the Eisenstein and Hu software break down, since they were not designed to be accurate for such unusual parameter values ($\Omega_\Lambda = 1$, etc.). For the parameter ranges of interest to us, there are small corrections for the effects of both baryons and neutrino infall, which we quantified in Fig. 3 in the discussion section.

[3] The result is more general [61,65], and applies also when the inert density components correspond to dark energy or spatial curvature. If we let $\Omega_{\rm m}$ denote the density fraction that is not inert (that clusters), then the approximation to Eq. (12) given by $p \approx \Omega_{\rm m}^{3/5}$ is quite accurate, being exact both for $\Omega_{\rm m} = 0$ and to first order in $(1 - \Omega_{\rm m})$ for all $1 - \Omega_{\rm m} \ll 1$. This is the familiar result that $d\ln\delta/d\ln a \approx \Omega_{\rm m}^{0.6}$.

4. The collapse density threshold δ_c

In the top panel of Fig. 4, we have numerically computed the collapse density threshold δ_c as a function of cosmic time x, defined as the linear perturbation theory overdensity that a top-hat fluctuation would have had at the time when it collapses. We see that it varies only very weakly with time (note the expanded vertical scale in the figure), dropping from the familiar cold dark matter value $\delta_c(0) = (3/20)(12\pi)^{2/3} \approx 1.686\,47$ early on to the limit $\delta_c(\infty) = (9/5)2^{-2/3}G_\infty \approx 1.629\,78$ [4] in the infinite future. This calculation neglects the effect of neutrinos. Since their effect is to contribute a gravitationally inert component just as dark energy, we will ignore their effect on $\delta_c(x)$, assuming that they merely cause a slight horizontal stretching of the curve (which is seen to be almost constant anyway).

[1] P. C. W. Davies and S. Unwin, Proc. R. Soc. **377**, 147 (1981).
[2] J. D. Barrow and F. J. Tipler, *The Anthropic Cosmological Principle* (Oxford, Clarendon Press, 1986).
[3] A. D. Linde, in *300 Years of Gravitation*, edited by S. W. Hawking and W. Israel (Cambridge University Press, Cambridge, 1987).
[4] S. Weinberg, Phys. Rev. Lett. **59**, 2607 (1987).
[5] A. Vilenkin, Phys. Rev. Lett. **74**, 846 (1995).
[6] G. Efstathiou, Mon. Not. R. Astron. Soc. **274**, L73 (1995).
[7] H. Martel, P. R. Shapiro, and S. Weinberg, Astrophys. J. **492**, 29 (1998).
[8] For a recent discussion and references, see J. Garriga and A. Vilenkin, Phys. Rev. D **67**, 043503 (2003).
[9] A. Vilenkin, Phys. Rev. D **27**, 2848 (1983).
[10] A. D. Linde, Phys. Lett. B **175**, 395 (1986).
[11] For a review, see A. D. Linde, *Particle Physics and Inflationary Cosmology* (Harwood, Chur, Switzerland, 1990).
[12] R. Bousso and J. Polchinski, J. High Energy Phys. 06 (2000) 006.
[13] T. Banks, M. Dine, and L. Motl, J. High Energy Phys. 01 (2001) 031.
[14] J. F. Donoghue, Int. J. Mod. Phys. A **16S1C**, 902 (2001).
[15] L. Susskind, hep-th/0302219.
[16] B. Carr and M. J. Rees, Nature (London) **278**, 605 (1979).
[17] M. J. Rees, Phys. Scr. **21**, 614 (1979).
[18] A. D. Linde, Phys. Lett. B **201**, 437 (1988).
[19] A. D. Linde and M. I. Zelnikov, Phys. Lett. B **215**, 59 (1988).
[20] A. D. Linde, Phys. Lett. B **351**, 99 (1995).
[21] J. García-Bellido and A. D. Linde, Phys. Rev. D **51**, 429 (1995).
[22] A. Vilenkin, in *Cosmological Constant and the Evolution of the Universe*, edited by K. Sato, T. Suginohara, and N. Sugiyama (Universal Academy Press, Tokyo, 1996); gr-qc/9512031.
[23] A. Vilenkin and S. Winitzki, Phys. Rev. D **55**, 548 (1997).

[24] M. Tegmark and M. J. Rees, Astrophys. J. **499**, 526 (1998).
[25] M. Tegmark, Classical Quantum Gravity **14**, L69 (1997).
[26] M. Tegmark, Ann. Phys. (N.Y.) **270**, 1 (1998).
[27] B. Agrawal, S. M. Barr, J. F. Donoghue, and D. Seckel, Phys. Rev. Lett. **80**, 1822 (1998); Phys. Rev. D **57**, 5480 (1998).
[28] J. F. Donoghue, Phys. Rev. D **57**, 5499 (1998).
[29] J. Garriga, T. Tanaka, and A. Vilenkin, Phys. Rev. D **60**, 023501 (1999).
[30] J. Garriga, M. Livio, and A. Vilenkin, Phys. Rev. D **61**, 023503 (2000).
[31] C. J. Hogan, Rev. Mod. Phys. **72**, 1149 (2000).
[32] A. Aguirre, Phys. Rev. D **64**, 083508 (2001).
[33] M. Tegmark, astro-ph/0302131.
[34] W. H. Press and P. Schechter, Astrophys. J. **187**, 425 (1974).
[35] J. R. Bond, G. Efstathiou, and J. Silk, Phys. Rev. Lett. **45**, 1980 (1980).
[36] D. N. Spergel *et al.*, Astrophys. J. Suppl. Ser. **148**, 175 (2003).
[37] M. Tegmark *et al.*, Phys. Rev. D **69**, 103501 (2004).
[38] G. Hinshaw *et al.*, Astrophys. J. Suppl. Ser. **148**, 135 (2003).
[39] M. Tegmark *et al.*, Astrophys. J. **606**, 702 (2004).
[40] G. Dvali and A. Vilenkin (unpublished).
[41] H. B. Nielsen and C. D. Froggatt, Nucl. Phys. **B147**, 277 (1979).
[42] H. B. Nielsen and C D. Froggatt, Nucl. Phys. **B164**, 114 (1980).
[43] C. D. Froggatt and H. B. Nielsen, *Origin of Symmetries* (World Scientific, Singapore, 1991).
[44] J. D. Brown and C. Teitelboim, Nucl. Phys. **297**, 787 (1988).
[45] G. Dvali and A. Vilenkin, Phys. Rev. D **64**, 063509 (2001).
[46] J. L. Feng, J. March-Russell, S. Sethi, and F. Wilczek, Nucl. Phys. **B602**, 307 (2001).
[47] J. Garriga and A. Vilenkin, Phys. Rev. D **64**, 023517 (2001).
[48] V. A. Rubakov and M. E. Shaposhnikov, Mod. Phys. Lett. A **4**, 107 (1989).
[49] Y. Fukuda *et al.*, Phys. Rev. Lett. **82**, 1810 (1999).
[50] E. T. Kearns, Frascati Phys. Ser. **28** (2002) 413.
[51] J. N. Bahcall, J. High Energy Phys. 11 (2003) 004.
[52] S. King, Rep. Prog. Phys. **67**, 107 (2004).
[53] S. Hannestad, J. Cosmol. Astropart. Phys. 05 (2003) 004.
[54] O. Elgaroy and O. Lahav, J. Cosmol. Astropart. Phys. 04 (2003) 004.
[55] S. Bashinsky and U. Seljak, Phys. Rev. D **69**, 083002 (2004).
[56] S. Hannestad, astro-ph/0310133.
[57] S. W. Allen, R. W. Schmidt, and S. L. Bridle, Mon. Not. R. Astron. Soc. **346**, 593 (2003).
[58] U. Seljak and M. Zaldarriaga, Astrophys. J. **469**, 437 (1996).
[59] L. Pogosian, A. Vilenkin, and M. Tegmark, J. Cosmol. Astropart. Phys. 07 (2004) 5.
[60] N. Boström, *Anthropic Bias: Observation Selection Effects in Science and Philosophy* (Routledge, New York, 2002).
[61] D. J. Eisenstein and W. Hu, Astrophys. J. **511**, 5 (1999).
[62] D. J. Heath, Mon. Not. R. Astron. Soc. **179**, 351 (1977).
[63] S. M. Carroll, W. H. Press, and E. L. Turner, Annu. Rev. Astron. Astrophys. **30**, 499 (1992).
[64] P. J. E. Peebles, *Principles of Physical Cosmology* (Princeton University Press, Princeton, New Jersey, 1993).
[65] C. P. Ma, Astrophys. J. **471**, 13 (1996).
[66] M. Tegmark and J. Silk, Astrophys. J. **441**, 458 (1995).