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Inhomogenized sudden future singularities

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We find that sudden future singularities of pressure may also appear in spatially inhomogeneous Stephani models of the universe. They are temporal pressure singularities and may appear independently of the spatial finite density singularities already known to exist in these models. It is shown that the main advantage of the homogeneous sudden future singularities which is the fulfillment of the strong and weak energy conditions may not be the case in the inhomogeneous case.

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I. INTRODUCTION

It has been shown by Barrow [1] that for Friedmann cosmological models which *do not admit* an equation of state which links the energy density ϱ and the pressure p, a sudden future (SF) singularity of pressure may appear, even for the matter fulfilling the strong energy condition $\varrho > 0$, $\varrho + 3p > 0$, though violating the dominant energy condition $\varrho > 0$, $-\varrho < p < \varrho$ [2]. This is in contrast to the most of the current observational discussion in cosmology, mainly devoted to determining of the barotropic index w in a barotropic equation of state $p = w\varrho$, which tightly constraints the energy density and the pressure [3]. On the other hand, the observational data interpreted by such an equation of state cannot exclude a possibility of barotropic phantom cosmological models [4]. These models violate null energy condition $\varrho + p > 0$, and consequently all the remaining energy conditions [5]. Besides, phantom models allow for a Big-Rip (BR) curvature singularity, which appears as a result of having the infinite values of the scale factor $a(t)$ at finite future. This is in opposition to a curvature Big-Bang (BB) singularity which takes place in the limit $a \to 0$.

The common feature of BB and BR singularities is that both ϱ and p blow up equally. This is not the case with a SF singularity for which a blow up occurs only for the pressure p, but not for the energy density ϱ. It is interesting that SF singularities are similar to those appearing in spatially inhomogeneous Stephani models of the universe [6], in which they were termed finite density (FD) singularities [7,8]. However, FD singularities occur as singularities in spatial coordinates rather than in time (as SF singularities do), which means that even at the present moment of the evolution they may exist somewhere in the Universe [9–11]. In this paper we show that sudden future (SF) singularities (as temporal singularities) can be inhomogenized in the sense, that they may appear in spatially inhomogeneous models of the universe, independently of the spatial finite density (FD) singularities allowed in these models. We also show that the inhomogeneous Stephani models lead to energy conditions violation, which mainly refers to the fact that they admit FD singularities.

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The SF singularities appear in the simple framework of Friedmann cosmology with the assumption that the energy-momentum is conserved, so that one can write the energy density and pressure as follows (following [1] we assume $8\pi G = c = 1$, $k = 0, \pm1$)

$$\varrho = 3\left(\frac{\dot{a}^2}{a^2} + \frac{k}{a^2}\right), \tag{1}$$

$$p = -\left(2\frac{\ddot{a}}{a} + \frac{\dot{a}^2}{a^2} + \frac{k}{a^2}\right). \tag{2}$$

From (2) one is able to notice that the singularity of pressure $p \to \infty$ occurs, when the acceleration $\ddot{a} \to -\infty$. This can be achieved for the scale factor

$$a(t) = A + (a_s - A)\left(\frac{t}{t_s}\right)^q - A\left(1 - \frac{t}{t_s}\right)^n, \tag{3}$$

where $a_s \equiv a(t_s)$ with t_s being the SF singularity time and $A, q, n = \text{const}$. It is obvious from (3) that $a(0) = 0$ and so at zero of time a BB singularity develops. For the sake of further considerations it is useful to write down the derivatives of the scale factor (3), i.e.,

$$\dot{a} = qt_s(a_s - A)\left(\frac{t}{t_s}\right)^{q-1} + A\frac{n}{t_s}\left(1 - \frac{t}{t_s}\right)^{n-1}, \tag{4}$$

$$\ddot{a} = q(q-1)t_s^2(a_s - A)\left(\frac{t}{t_s}\right)^{q-2} - A\frac{n(n-1)}{t_s^2}\left(1 - \frac{t}{t_s}\right)^{n-2}. \tag{5}$$

The main point is that the evolution of the Universe, as described by the scale factor (3), begins with the standard BB singularity at $t = 0$, and finishes at SF singularity at $t = t_s$, provided we choose

$$1 < n < 2, 0 < q \le 1. \tag{6}$$

For these values of n and q, the scale factor (3) vanishes, and its derivatives (4) and (5) diverge at $t = 0$, leading to a divergence of ϱ and p in (1) and (2) (BB singularity). On the other hand, the scale factor (3) and its first derivative (4) remain constant, while its second derivative (5) diverge, leading to a divergence of pressure in (2) *only*, with finite energy density (1). This behavior means, for example, that positive curvature ($k = +1$) Friedmann models may not recollapse to a second BB singularity—instead they terminate in a SF singularity [12].

II. INHOMOGENEIZED SUDDEN FUTURE SINGULARITIES

Now, let us consider inhomogeneous Stephani models. They appear as the only conformally flat perfect-fluid models which can be embedded in a 5-dimensional flat space [6,13]. Their metric in the spherically symmetric case reads as (notice that we have introduced a Friedmann-like time coordinate which eliminated one of the functions of time in the original Stephani metric [8])

$$ds^2 = -\frac{a^2}{\dot{a}^2}\frac{a^2}{V^2}\left[\left(\frac{V}{a}\right)^{\cdot}\right]^2 dt^2 + \frac{a^2}{V^2}[dr^2 + r^2(d\theta^2 + \sin^2\theta d\varphi^2)], \tag{7}$$

where

$$V(t, r) = 1 + \frac{1}{4}k(t)r^2, \tag{8}$$

and $(\ldots)^{\cdot} \equiv \partial/\partial t$. The function $a(t)$ plays the role of a generalized scale factor, $k(t)$ has the meaning of a time-dependent "curvature index", and r is the radial coordinate.

Their analogy to SF singularity models is that they do not admit *any* equation of state linking p to ϱ throughout the whole evolution of the universe, although at any given moment of the evolution, such an equation of state (varying from one spacelike hypersurface to the other) can be admitted. An analytic equation of state can also be admitted at any fixed subspace with constant radial coordinate r, but not globally [8]. For the sake of simplicity, first the spherically symmetric models will be considered (note that in [8,9] a time coordinate t analogous to the cosmic time in Friedmann models was marked by τ, which had nothing to do with a common conformal time coordinate). The energy density and pressure are given by [8]

$$\varrho(t) = 3C^2(t) \equiv 3\left[\frac{\dot{a}^2(t)}{a^2(t)} + \frac{k(t)}{a^2(t)}\right], \tag{9}$$

$$p(t, r) = -3C^2(t) + 2C(t)\dot{C}(t)\frac{\left[\frac{V(t,r)}{a(t)}\right]}{\left[\frac{V(t,r)}{a(t)}\right]^{\cdot}}, \tag{10}$$

and generalize the relations (1) and (2) to inhomogeneous models.

We now show that it is possible to extend SF singularities into inhomogeneous models. Following [8] we assume that the functions $k(t)$ and $a(t)$ are related by

$$k(t) = -\alpha a(t), \tag{11}$$

with $\alpha =$ const. In fact, the limit $\alpha \to 0$ gives the Friedmann models (cf. the discussion of the conditions to derive such a limit in [8]). Inserting (11) into (10) we get

$$\varrho(t) = 3\left[\frac{\dot{a}^2(t)}{a^2(t)} - \frac{\alpha}{a(t)}\right], \tag{12}$$

$$p(t, r) = -2\frac{\ddot{a}}{a} - \frac{\dot{a}^2}{a^2} + 2\frac{\alpha}{a} - \frac{1}{4}\alpha r^2\left(2\frac{\dot{a}^2}{a} - 2\ddot{a} - \alpha\right). \tag{13}$$

From (13) one can see, that $p \to \infty$, when acceleration $\ddot{a} \to -\infty$ for an *arbitrary* value of the radial coordinate r. We can then say that we *generalized* SF singularities (given, for example, by the scale factor (3)) onto an inhomogeneous model of the universe.

However, it is very interesting to notice that in such a generalization not only SF singularities appear, but also FD singularities are possible for the radial coordinate $r^2 \to \infty$. These seem to be far away from us, and so not very harmful, since $r^2 \to \infty$ defines an antipodal center of symmetry in the spherically symmetric Stephani models. The advantage of these FD singularities is that they are able to drive the current acceleration of the universe [10,14,15].

The procedure of inhomogenizing SF singularities may be extended into the general Stephani models for which there is no spacetime symmetry at all, and so they are completely inhomogeneous. The most general Stephani metric in cartesian coordinates (x, y, z) [6,13] reads as

$$ds^2 = -\frac{a^2}{\dot{a}^2}\frac{a^2}{V^2}\left[\left(\frac{V}{a}\right)^{\cdot}\right]^2 dt^2 + \frac{a^2}{V^2}[dx^2 + dy^2 + dz^2], \tag{14}$$

where

$$V(t, x, y, z) = 1 + \frac{1}{4}k(t)\{[x - x_0(t)]^2 + [y - y_0(t)]^2 + [z - z_0(t)]^2\}, \tag{15}$$

and x_0, y_0, z_0 are arbitrary functions of time. Now the general expression for the pressure is (the expression for the energy density (9) remains the same)

$$p(t, x, y, z) = -3C^2(t) + 2C(t)\dot{C}(t)\frac{\left[\frac{V(t,x,y,z)}{a(t)}\right]}{\left[\frac{V(t,x,y,z)}{a(t)}\right]^{\cdot}}. \tag{16}$$

Inserting the time derivative of (9) and the function $V(t, x, y, z)$ from (15) into (16) gives

$$p(t, x, y, z) = -3\frac{\dot{a}^2}{a^2} - 3\frac{k}{a^2} + \frac{\dot{a}}{a}\left[2\frac{\ddot{a}}{a} - 2\frac{\dot{a}^2}{a^2} + \frac{1}{a^2}\left(\dot{k}\frac{a}{\dot{a}} - 2k\right)\right]\frac{\left[\frac{V(t,x,y,z)}{a(t)}\right]}{\left[\frac{V(t,x,y,z)}{a(t)}\right]^{\cdot}}. \tag{17}$$

It is easy to notice that SF singularity $p \to \pm\infty$ appears with (3) for $\ddot{a} \to -\infty$, if $(V/a)/(V/a)^{\cdot}$ is regular and the sign of the pressure depends on the signs of both $\dot{a}/a$ and $(V/a)/(V/a)^{\cdot}$. This proves that we can *inhomogenize* SF singularities for a Stephani model with no symmetry.

In fact, SF singularities appear independently of FD singularities whenever $\ddot{a} \to -\infty$ and the blow-up of p is guaranteed by the involvement of the time derivative of the function $C(t)$ in (10).

That makes a complimentary generalization to the one which extends SF singularities into the theories with actions being arbitrary analytic functions of the Ricci scalar and into anisotropic (but homogeneous) models [16–18].

It appears that the main motivation for studying SF singularities [1] was the fact that, unlike phantom models [4], they obey the strong and weak energy conditions, though they do not obey the dominant energy condition. In this paper we raise the point that the question of possible violation of the energy conditions for the inhomogenized SF singularity models is a bit more complex than for the homogeneous ones. From (10) and (13) for the strong, weak and dominant energy conditions to be fulfilled we have:

$$\varrho + 3p = -6\frac{\ddot{a}}{a} + 3\frac{\alpha}{a} - \frac{3}{4}\alpha r^2\left(2\frac{\dot{a}^2}{a} - 2\ddot{a} - \alpha\right) > 0, \tag{18}$$

$$\varrho + p = -2\frac{\ddot{a}}{a} + 2\frac{\dot{a}^2}{a^2} - \frac{\alpha}{a} - \frac{1}{4}\alpha r^2\left(2\frac{\dot{a}^2}{a} - 2\ddot{a} - \alpha\right) > 0, \tag{19}$$

$$\varrho - p = 2\frac{\ddot{a}}{a} + 4\frac{\dot{a}^2}{a^2} - 5\frac{\alpha}{a} + \frac{1}{4}\alpha r^2\left(2\frac{\dot{a}^2}{a} - 2\ddot{a} - \alpha\right) > 0. \tag{20}$$

In fact, the dominant energy condition requires fulfilling both (19) and (20). Notice that in view of (9) the energy density in inhomogeneous Stephani models is always positive, i.e.,

$$\varrho > 0. \tag{21}$$

This means that the strong and weak energy conditions are still not violated if $\ddot{a} \to -\infty$, in analogy to homogeneous models, provided

$$\frac{1}{a} > \frac{\alpha}{4}r^2, \tag{22}$$

since the first term with $\ddot{a}$ in (18) and (19) must dominate the second (remember that $\ddot{a} \to -\infty$). Notice that the equality $1/a = \alpha r^2/4$ may lead to a pressure singularity avoidance in (13). Assuming that the generalized scale factor $a(t) > 0$, we conclude from (22) that the strong and weak energy conditions are always fulfilled, if $\alpha < 0$. However, an accelerated expansion for an observer at the center of symmetry at $r = 0$ can only be achieved, if $\alpha > 0$ [15] (the spatial acceleration scalar reads as $\dot{u} = -2\alpha r$—lower pressure regions are away from the center). This means that the strong and weak energy conditions are *not necessarily fulfilled* for the models with $\alpha > 0$. In particular, they cannot be fulfilled at an antipodal center of symmetry at $r^2 \to \infty$, unless $a \to 0$ (where Big-Bang singularity appears and so BB and FD singularities coincide—see Sec. III).

On the other hand, the first part of the dominant energy condition may not be violated if the contribution from the last term with $\ddot{a}$ in (20), which includes r^2, does not outweigh the first one, i.e., when

$$\frac{1}{a} < \frac{\alpha}{4}r^2. \tag{23}$$

This should be appended by the condition (19) which is equivalent to (22), i.e., the dominant energy condition is fulfilled if

$$\frac{1}{a} < \frac{\alpha}{4}r^2 < \frac{1}{a}. \tag{24}$$

This is obviously a contradiction which means that, similarly as in the isotropic Friedmann models, SF singularities violate the dominant energy condition.

Such a violation of the dominant energy condition also appears in M-theory-motivated ekpyrotic models in which $p \gg \varrho$ during recollapse [19].

Let us now discuss the problem of the possible energy conditions violation in the general Stephani model. Using (9) and (17) we get for the strong, weak, and dominant energy conditions

$$\varrho + 3p = -6\frac{\dot{a}^2}{a^2} - 6\frac{k}{a^2} + 3\frac{\dot{a}}{a}\left[2\frac{\ddot{a}}{a} - 2\frac{\dot{a}^2}{a^2} + \frac{1}{a^2}\left(\dot{k}\frac{a}{\dot{a}} - 2k\right)\right]\frac{\left[\frac{V(t,x,y,z)}{a(t)}\right]}{\left[\frac{V(t,x,y,z)}{a(t)}\right]^{\cdot}} > 0, \tag{25}$$

$$\varrho + p = 2\frac{\dot{a}}{a}\left[2\frac{\ddot{a}}{a} - 2\frac{\dot{a}^2}{a^2} + \frac{1}{a^2}\left(\dot{k}\frac{a}{\dot{a}} - 2k\right)\right]\frac{\left[\frac{V(t,x,y,z)}{a(t)}\right]}{\left[\frac{V(t,x,y,z)}{a(t)}\right]^{\cdot}} > 0, \tag{26}$$

$$\varrho - p = 6\frac{\dot{a}^2}{a^2} + 6\frac{k}{a^2} - \frac{\dot{a}}{a}\left[2\frac{\ddot{a}}{a} - 2\frac{\dot{a}^2}{a^2} + \frac{1}{a^2}\left(\dot{k}\frac{a}{\dot{a}} - 2k\right)\right] \times \frac{\left[\frac{V(t,x,y,z)}{a(t)}\right]}{\left[\frac{V(t,x,y,z)}{a(t)}\right]^{\cdot}} > 0. \tag{27}$$

Obviously, the dominant energy condition requires fulfilling both (26) and (27). Before we go any further, using (15), we note that

$$\frac{V(t,x,y,z)}{a(t)} = \tag{28}$$

$$\frac{1}{a}+\frac{1}{4}\frac{k}{a}[(x-x_0)^2+(y-y_0)^2+(z-z_0)^2],$$

$$\left[\frac{V(t,x,y,z)}{a(t)}\right]^{\cdot} = -\frac{\dot a}{a^2}+\frac{1}{4a}\left(\dot k - k\frac{\dot a}{a}\right)[(x-x_0)^2+(y-y_0)^2 + (z-z_0)^2] - \frac{k}{2a}[(x-x_0)\dot x_0 + (y-y_0)\dot y_0 + (z-z_0)\dot z_0]. \tag{29}$$

It is important to notice that the ratio of (28) and (29) which appears in the conditions (25)–(27) allows to cancel $1/a$ from both the numerator and the denominator. Apart from that, one is able to take $\dot a/a$ out in (29) and cancel it with the same term standing in front of the last term in these conditions. This basically means that, bearing in mind the fact that $\ddot a \to -\infty$, the strong and weak energy conditions are fulfilled provided one of the expressions

$$V_1 \equiv 1 + \frac{1}{4}k[(x-x_0)^2 + (y-y_0)^2 + (z-z_0)^2], \tag{30}$$

$$V_2 \equiv \frac{1}{4}\left(\dot k\frac{a}{\dot a} - k\right)[(x-x_0)^2 + (y-y_0)^2 + (z-z_0)^2] - \frac{ka}{\dot a}[(x-x_0)\dot x_0 + (y-y_0)\dot y_0 + (z-z_0)\dot z_0] - 1, \tag{31}$$

is negative. It is clear that for $k(t) > 0$ (30) is always positive, so that (31) must necessarily be negative and it certainly does, at least in some regions of space. On the other hand, for $k(t) < 0$ (30) can be both positive and negative which requires (31) to be negative and positive, respectively. In conclusion, similarly as in the spherically symmetric case, the strong and weak energy conditions *may be violated* for inhomogenized SF singularities in Stephani models. This is different from what we have for isotropic SF singularities. Finally, one can easily notice that in order to fulfill the dominant energy condition the ratio of (30) to (31) should be simultaneously positive and negative, which is a contradiction. This means that, like in the isotropic case, a general Stephani model allows SF singularities which *always violate* the dominant energy condition. In conclusion, one can say that the problem of the energy conditions violation by SF singularities in the inhomogeneous models is more complex than in the isotropic ones.

III. INHOMOGENEOUS FINITE DENSITY SINGULARITIES

In the context of temporal SF singularities of pressure we will further briefly discuss the occurrence of spatial FD singularities of pressure in the Stephani models and possible energy conditions violation. From (10) we can see that FD singularities appear whenever the radial coordinate

$$r^2 = -4\frac{(\frac{1}{a})^{\cdot}}{(\frac{k}{a})^{\cdot}}. \tag{32}$$

Under the choice of (11), we have $(k/a)^{\cdot} = 0$, and so the singularities appear at $r^2 \to \infty$. In general, it may not be so, which was explicitly shown in [8]. For example, by choosing

$$a(t) = \alpha t^2 + \beta t + \gamma, \tag{33}$$

$$k(t) = 1 - \dot a^2 = -4\alpha a(t) + \Delta, \tag{34}$$

with

$$\Delta = 4\alpha\gamma + 1 - \beta^2, \tag{35}$$

the FD singularities appear for [8]

$$|r| = 2/\sqrt{-\Delta}. \tag{36}$$

Of course the condition (11) is obtained in the limit $\Delta \to 0$ from (34) which moves FD singularities to an antipodal center of symmetry at $r^2 \to \infty$. Having chosen $\gamma = 0$, $\Delta = 1 - \beta^2$ in (33) (Model I of Ref. [10], called Dąbrowski model in Ref. [11]) the energy density and pressure are then given by

$$\varrho = 3\frac{1}{t^2(\alpha t + \beta)^2}, \tag{37}$$

$$p = -\frac{1 + 2\alpha t(\alpha t + \beta)r^2}{t^2(\alpha t + \beta)^2}. \tag{38}$$

For the strong, weak and dominant energy conditions to be fulfilled, respectively, we get the requirements

$$\varrho + 3p = -6\alpha\frac{r^2}{t(\alpha t + \beta)} > 0, \tag{39}$$

$$\varrho + p = 2\frac{1 - \alpha t(\alpha t + \beta)r^2}{t^2(\alpha t + \beta)^2} > 0, \tag{40}$$

$$\varrho - p = 2\frac{2 + \alpha t(\alpha t + \beta)r^2}{t^2(\alpha t + \beta)^2} > 0. \tag{41}$$

If $\alpha > 0$ (acceleration [15]), then the strong energy condition is violated if

$$t(\alpha t + \beta) > 0, \tag{42}$$

and this may happen independently of the radial coordinate r. The weak energy condition is violated for the domain of space in which

$$\frac{1}{r^2} > \alpha t(\alpha t + \beta). \tag{43}$$

With the strong energy condition violated, this gives a weak energy condition violation only in some spatial domain since the right-hand side of (43) has a positive value, but including the center of symmetry at $r = 0$. On the other hand, for the decelerated expansion, $\alpha < 0$, and the right-hand side of (43) has a negative value, so the weak energy condition is violated everywhere in the universe (i.e., for all values of r). If the strong energy condition is not violated and $\alpha > 0$ (acceleration), then again there is a weak energy condition violation for all values of r. If the strong energy condition is violated, and $\alpha < 0$, then the weak energy condition is violated only in some spatial domain which, however, includes the center of symmetry at $r = 0$. The dominant energy condition is violated for

$$\frac{1}{r^2} < -\frac{1}{2}\alpha t(\alpha t + \beta), \qquad (44)$$

which, combined with (43), gives

$$\alpha t(\alpha t + \beta) < \frac{1}{r^2} < -\frac{1}{2}\alpha t(\alpha t + \beta). \qquad (45)$$

This last condition can only be fulfilled either if $\alpha < 0$ and $t(\alpha t + \beta) > 0$, or if $\alpha > 0$ and $t(\alpha t + \beta) < 0$. Then, at least for these particular class of Stephani models, FD singularities may lead to a violation of the energy conditions in a similar way as BR singularities in phantom cosmology do.

An interesting problem is a possible avoidance of SF and FD singularities in the universe. SF singularities can easily be avoided by imposing an analytic form of the equation of state $p = p(\varrho)$. Even without this assumption, some other ways of their avoidance by introducing quadratic in Ricci curvature scalar terms [18], or by quantum effects [20], are possible. On the other hand, a necessary condition to avoid FD singularities in Stephani models comes from (10) and reads as

$$\frac{(\frac{1}{a})^{\cdot}}{(\frac{k}{a})^{\cdot}} > 0. \qquad (46)$$

In our special model (33) and (34) they can be simply avoided, if

$$\Delta > 0. \qquad (47)$$

IV. CONCLUSIONS

In conclusion, we have shown that one is able to spatially inhomogenize sudden future (SF) singularities in the sense that these singularities *do appear* in inhomogeneous models of the universe. However, despite homogeneous SF singularities, they *may* violate the strong and weak energy conditions in some regions of space, although they *share* the dominant energy condition violation with homogeneous models. It shows that the problem of the energy conditions violation for inhomogeneous models is more complex than for homogeneous ones. A possible violation of all the energy conditions by inhomogenized SF singularities is similar to what happens to Big-Rip singularities in phantom cosmologies. On the other hand, the dominant energy condition is one of the assumptions of the positive mass theorems and the cosmic censorship conjecture. In the cosmological context, the violation of the dominant energy condition leads to isotropization of a recollapsing universe and also such a violation may change the status of no-hair theorems.

Besides, we have noticed that, apart from sudden future singularities, the inhomogenized models also admit finite density (FD) singularities which are spatial rather than temporal. In relation to this we have discussed an example of an inhomogeneous model with spatial finite density singularities of pressure and studied the domains of its energy conditions violation.

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